DL 4 :

Exercice 2,

On a $E(X_n) = 0$ donc $X(n)$ est centrée

$$R(k) = \begin{cases} \sigma^2 \sum_{j=0}^{3-k} b_{j+k} b_j & \text{pour } 0 \leq k \leq 3 \\ \sigma^2 \sum_{j=0}^{3-|k|} b_{j+|k|} b_j & \text{pour } -3 \leq k \leq 0 \\ 0 & \text{donc } k \in \mathbb{Z} \setminus [\![-3, 3]\!] \end{cases}$$

$$R(0) = \sigma^2 \sum_{j=0}^{3} b_j^2 = 2$$

$$R(1) = R(-1) = \sigma^2 \sum_0^2 b_j b_{j+1} = \sigma^2 (b_1 + b_2 b_1 + b_3 b_2) = -0,1$$

$$R(2) = R(-2) = \sigma^2 \sum_0^1 b_j b_{j+2} = \sigma^2 (b_2 + b_3 b_1) = 0,5$$

$$R(3) = R(-3) = \sigma^2 b_3 = 0,1$$

Exercice 3 :

On a
$$\begin{bmatrix} R(0) & R(1) & R(2) \\ R(1) & R(0) & R(1) \\ R(2) & R(1) & R(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sigma^2 \\ 0 \\ 0 \end{bmatrix}$$

donc
$$\begin{cases} R(0) + a_1 R(1) + a_2 R(2) = \sigma^2 & (1) \\ R(1) + a_1 R(0) + a_2 R(1) = 0 & \Rightarrow R(1) = \dfrac{-a_1}{1+a_2} R(0) \quad (2) \\ R(2) + a_1 R(1) + a_2 R(0) = 0 & (3) \end{cases}$$

de (1) et (2) on a $R(2) = R(0) - \dfrac{\sigma^2}{1+a_2}$ (4)

de (3) et (4) on trouve $R(0) = \dfrac{\sigma^2 (1+a_2)}{(1-a_2)(1+a_1+a_2)(1-a_1+a_2)}$

et donc $R(1) = \dfrac{-\sigma^2 a_1}{(1-a_2)(1+a_1+a_2)(1-a_1+a_2)}$

$$R(2) = \frac{-\sigma^2}{1-a_2} \left[1 - \frac{1+a_2}{(1+a_1+a_2)(1-a_1+a_2)} \right]$$