DL2 : VA - Changement de variables

* Distribution normale.

on a $P_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$

donc $F_X(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(t-\mu)^2}{2\sigma^2}\right) dt$

on pose $u = \frac{t-\mu}{\sqrt{2}\sigma}$ donc on a $du = \frac{1}{\sqrt{2}\sigma}\, dt$

donc $F_X(x) = \int_{-\infty}^{\frac{x-\mu}{\sqrt{2}\sigma}} \frac{1}{\sigma\sqrt{2\pi}} \exp(-u^2) \cdot \sqrt{2}\sigma\, du$

$= \int_{-\infty}^{\frac{x-\mu}{\sqrt{2}\sigma}} \frac{1}{\sqrt{\pi}} e^{-u^2}\, du$

on a $\mathrm{erf}(x) = \int_0^x \frac{2}{\sqrt{\pi}} e^{-t^2}\, du$

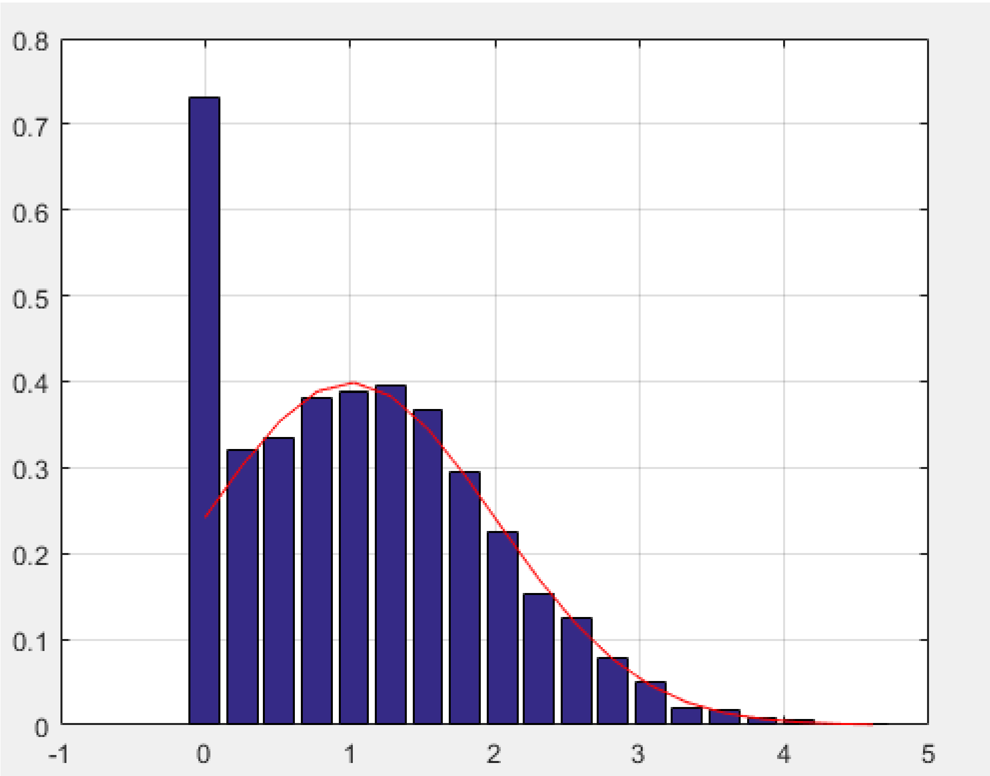
donc $F_X(x) = \int_{-\infty}^{0} \frac{1}{\sqrt{\pi}} e^{-t^2}\, du + \int_0^{\frac{x-\mu}{\sqrt{2}\sigma}} \frac{1}{\sqrt{\pi}} e^{-t^2}\, dt$

$F_X(x) = \boxed{\frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)}$

La fct inverse

on pose $y = \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right)$

donc $\boxed{x = \mu + \sqrt{2}\sigma\, \mathrm{erf}^{-1}(2y-1)}$



* Distribution log-normale.

On a $P_x(x) = \frac{1}{\sigma x\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right)$

donc $F_x(x) = \int_0^x \frac{1}{\sigma t\sqrt{2\pi}} \exp\left(-\frac{(\ln t - \mu)^2}{2\sigma^2}\right) dt$

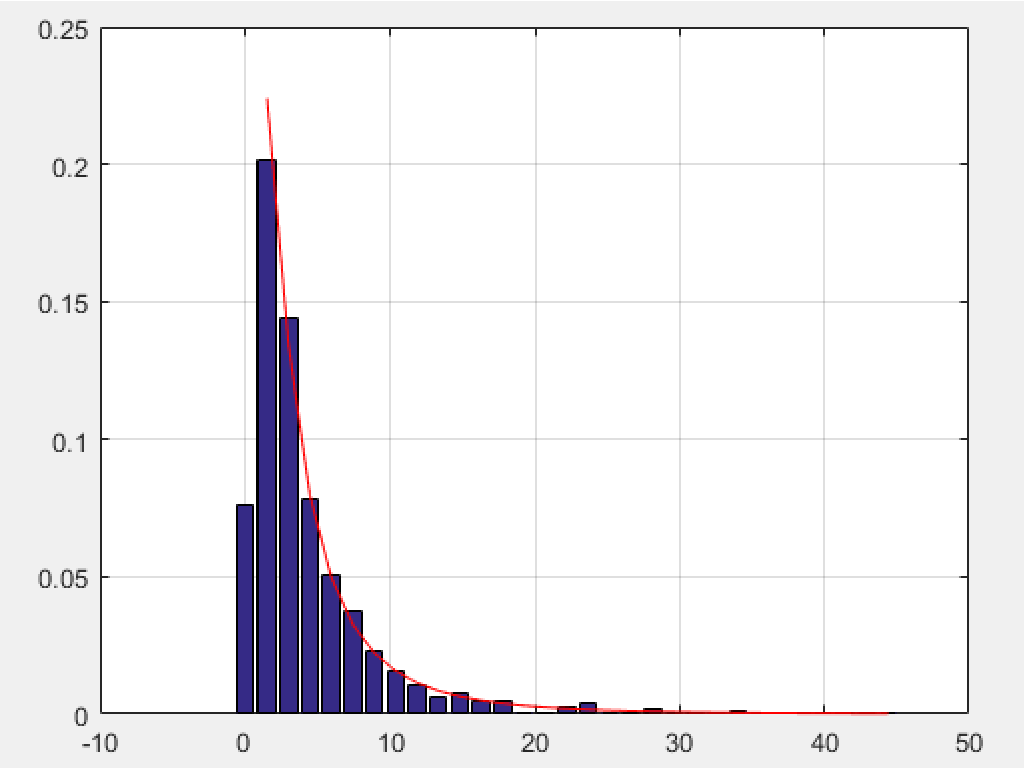
On pose $u = \frac{\ln t - \mu}{\sigma\sqrt{2}}$ et $du = \frac{dt}{t\sigma\sqrt{2}}$

donc $F_x(x) = \int_{-\infty}^{\frac{\ln x - \mu}{\sigma\sqrt{2}}} \frac{1}{\sqrt{\pi}} e^{-u^2} du$

$= \frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln x - \mu}{\sigma\sqrt{2}}\right)$

fct inverse : $y = \frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln x - \mu}{\sigma\sqrt{2}}\right)$

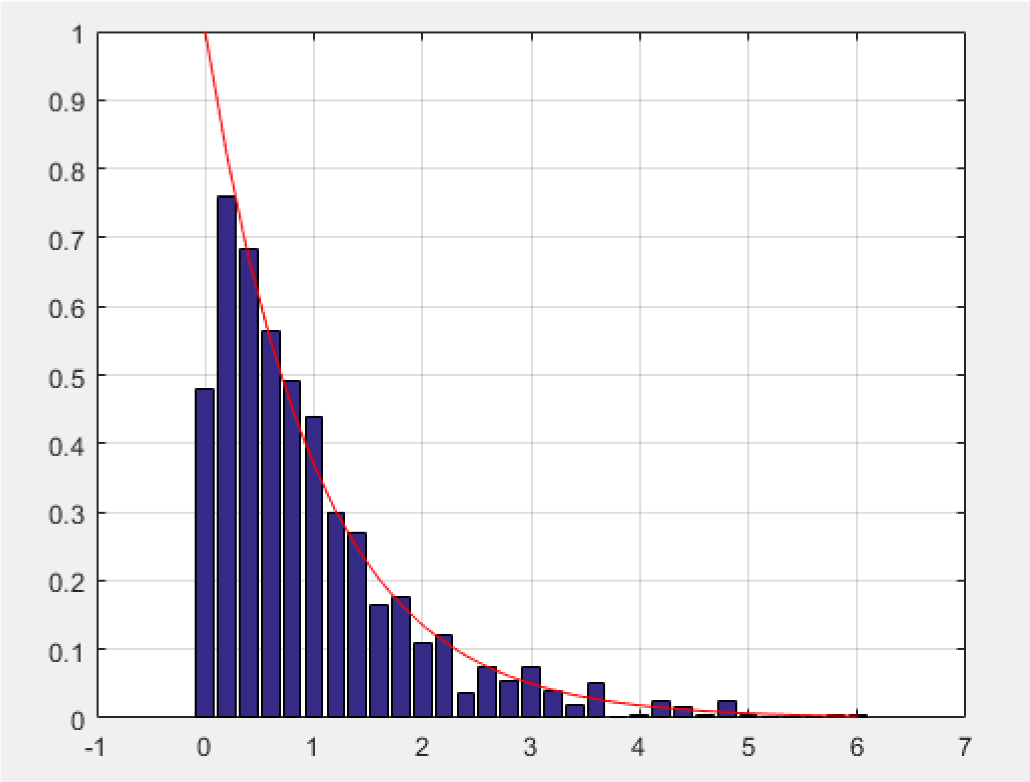
donc $x = \exp\left(\mu + \sigma\sqrt{2}\, \operatorname{erf}^{-1}(2y - 1)\right)$



* Distribution gamma.

On a $P_x(x) = \frac{\lambda^m}{\Gamma(m)} x^{m-1} e^{-\lambda x} = \lambda e^{-\lambda x}$ (pour $m=1$)

On a $x = -\frac{\operatorname{Log}(y) \times \Gamma(m)}{\lambda^m}$



* Distribution géométrique

On a $P(X=1) = p$

$P(X=k+1) = p(k).(p-1)$