Seminar 1: Programming languages specification

1. $\boxed{\text{BNF}}$ (Backus - Naur Form)

Language elements

- meta - linguistic vars. (nonterminals)
  ↳ written between < >
- language primitives (terminals)
  ↳ written without any special delimiters
- meta - linguistic connectors
  a) ::= (= is defined as)
  b) | (= alternatives, OR)

$$\boxed{\langle nonterminal \rangle ::= expr_1 \mid expr_2 \mid \ldots \mid expr_n}$$

ex.1 all nonempty seq. of letters in BNF

<Letter> ::= a | . . . | z | A | . . . | Z

recursiv

<nonempty seq. of letters> ::= <Letter> | <Letter> <nonempty seq. of letters>

ex.2 $\left\{ \begin{array}{l} 0 \quad \underbrace{+1 \ -1 \ \ 1} \\ \cancel{02} \end{array} \right.$ all signed and unsigned integers with the constraints.

a) 0 is always unsigned
b) nos. with at least 2 digits should not start with 0

<non zero digit> ::= 1 | . . . | 9
<digits> ::= 0 | <non zero digit>

<sign> ::= + | -

nu

<integer> ::= ~~0 | <sign> <non zero digit> | <integer> <digit>~~
::= 0 | <no> | <sign> <no>

<no> ::= <non zero digit> | <non zero digit> <digit seq>

<digit seq.> ::= <digit> | <digit> <digit seq.>

2. $\boxed{\text{EBNF}}$ (Extended BNF) Wirth's dialect

Language elements

- nonterminals are written without < >
- terminals are written between " "
- ::= → =
- | (the same)
- { } → 0 or more
- [ ] → 0 or 1 (optional)
- ( ) grouping
- (* *) [illegible]

- rules end with .

ex 3) ex2 in EBNF

[ ]

nonzero digit = "1" | ... | "9".

digit = "0"| non-zero digits .

sign = "+" | "-".

(digit = ["0"] | "1" | ... | "9".)

integer = "0" | [sign] nonzero digit {digit} .

Seminar 2:

const int id = 4 ;

← not a lexical constant

int consts. : -12, 0, 12

string consts. : " abc def"

char consts : ',', ',t'

charconst = "'" char "'"

char = letter | digit | special word

specialword = ";" | "." | ...

stringconst = " " " {char} " " "

intconst = (seminar 1)

Scanning algorithm

- detection
- classification
- codification

testProg. txt

```
program test;
var a, b : integer;
       c : string;
begin
    a := 2;
    if (a >= 0) then
          b := -5;
     write ( " This is a message: ")
end.
```

ST (only ids and constants)

ST pos	symbol
0	test
1	a
2	b
3	c
4	2
5	0
6	-5
7	"This..."

PIF

token	ST pos
program	-1
id	0
;	-1
var	-1
id	1
,	-1
id	2
:	-1
integer	-1
;	-1
id	3
:	-1
string	-1
;	-1
begin	-1
id	1
:=	-1
const	4
;	-1
if	-1
(	-1
id	1
>=	-1
const	5
)	-1
then	-1
id	2
:=	-1
const	6
;	-1
write	-1
(	-1
const	7
)	-1
end	-1
.	-1

lexical error = unclassified tokens

ex: 1) 4a   3) 'ab'   5) +0, 012

2) "This   4) & - does not belong to the alphabet

Seminar 3: Grammars

$G = (N, \Sigma, P, S)$

$V = N \cup \Sigma$

$P \subseteq V^* N V^* \times V^*$

$\boxed{\alpha \to \beta}$, $\alpha, \beta \in V^*$

production

$\alpha \Rightarrow \beta$ direct derivation

$L(G) = \{ w \in \Sigma^* \mid S \overset{*}{\Rightarrow} w \}$

ε - empty sequence

1) $G = (N, \Sigma, P, S)$ if a sequence $\in$ language

$N = \{S, C\}$

$\Sigma = \{a, b\}$

$P: S \to aCSb \overset{1}{} \mid \overset{2}{ab}$

$C \to \overset{3}{S} \mid \overset{4}{bSb}$

$CS \to \overset{5}{b}$

$\left.\right\}$ 5 production

? $w = ab(ab^2)^2 \in L(G)$; $(ab)^2 = abab \neq aabb = a^2b^2$

$ab \; ab^2 \; ab^2$

$P:$ (1) $S \to aCSb$ (4) $C \to bSb$

(2) $S \to ab$ (5) $CS \to b$

(3) $C \to S \mid bSb$

$w = ab\,abb\,abb$

bc we derivate both S

$S \underset{(1)}{\Rightarrow} aCSb \underset{(4)}{\Rightarrow} a\,bSb\,S\,b \underset{(2)}{\overset{2}{\Rightarrow}} ab\,abb\,abb = w$

$\Rightarrow S \overset{4}{\Rightarrow} w \Rightarrow w \in L(G)$

$S \underset{(1)}{\Rightarrow} aCSb \underset{(4)}{\Rightarrow} a\,bSb\,S\,b \underset{(2)}{\overset{2}{\Rightarrow}} ab\,abb\,abb = w$

$\Rightarrow S \overset{4}{\Rightarrow} w \Rightarrow w \in L(G)$

start from S and make your way until you reach w

$\Rightarrow w \in L(G)$

2. $G = (\{S\}, \{a, b, c\}, \{S \to a^2S \mid bc\}, S)$ $\quad {}^{(1)} \xrightarrow{(1)}{}^{(2)} \Rightarrow^* a^{2m}bc$

$L(G) = ?$ $\quad$ (1) $S \to a^2S$ $\quad$ (2) $S \to bc$

$\Rightarrow S \overset{(1)}{\Rightarrow} a^2S \overset{(2)}{\Rightarrow} a^2bc$

! assume $L = \{\ldots\}$ and prove that $L = L(G)$

$\Rightarrow S \overset{(1)}{\Rightarrow} a^2S \overset{(1)}{\Rightarrow} a^2a^2S = a^4S \overset{(1)}{\Rightarrow} a^6S \Rightarrow$

$\Rightarrow \ldots \Rightarrow a^{2m}S \overset{(2)}{\Rightarrow} a^{2m}bc$

(G) $L = \{a^{2m}bc^* \mid m \in \mathbb{N}\}$

I $L \subseteq L(G)$ - G generates all the seqs. of that shape

II $L(G) \subseteq L$ - G does not generate anything else but seqs. of that shape

(I) $\forall w \in L,\ w \in L(G)$

$\forall m \in \mathbb{N},\ a^{2m}bc \in L(G)$? $\Big\}\ L \subseteq L(G)$

$\boxed{P(m):\ a^{2m}bc \in L(G),\ m \in \mathbb{N}}$

$*$? $P(m)$ is true $\forall m \in \mathbb{N}$ (by math. induction)

$P(0):\ a^0bc = bc \in L(G)\ (S \underset{(2)}{\Rightarrow} bc) \Rightarrow P(0)$ is true

$P(k) \overset{?}{\Rightarrow} P(k+1),\ \forall k \in \mathbb{N}$

we assume $P(k)$ is true $\Rightarrow a^{2k}bc \in L(G) \Rightarrow$

$\Rightarrow S \overset{*}{\Rightarrow} a^{2k}bc$ (ind. hypoth) true

$S \underset{(1)}{\overset{*}{\Rightarrow}} a^2S \underset{\text{(ind hyp)}}{\overset{*}{\Rightarrow}} a^2 a^{2k}bc = a^{2(k+1)}bc = P(k+1)$

$\Rightarrow S \overset{*}{\Rightarrow} a^{2(k+1)}bc \Rightarrow P(k+1)$ is true $\Rightarrow$ $*$ is true

(II) $\boxed{L(G) \subseteq L}$

$S \underset{(2)}{\Rightarrow} bc = a^{2\cdot 0}bc \in L$

$\overset{(1)}{\Rightarrow} a^2S \underset{(2)}{\Rightarrow} a^2bc \in L$

$\overset{(1)}{\Rightarrow} a^4S \underset{(2)}{\Rightarrow} a^4bc \in L$

$\overset{(1)}{\Rightarrow} \ldots$? $L(G)$

$\Rightarrow a^{2k}bc,\ k \in \mathbb{N}$

all generated prod. in all poss. combinations

3) find the grammar that generates the foll. language:

$L = \{0^n 1^n 2^m \mid n, m \in \mathbb{N}^*\}$ ($n, m \neq 0$) $= \{012, 00112, 0122, \dots\}$

(1) $S \to AB$ (2) $A \to 0A1$ (5) $B \to B2$

$A \to 0A1 \mid 01$ (3) $A \to 01$

$B \to 2 \mid B2$ (4) $B \to 2$

$? \ L = L(G) \Leftrightarrow \begin{cases} L(G) \subseteq L \\ L \subseteq L(G) \end{cases}$

I $? \ L \subseteq L(G)$ $\forall m, n \in \mathbb{N}^*$, $0^n 1^n 2^m \in L(G)$

$P(n,m) := 0^n 1^n 2^m \in L(G)$, $\forall n, m \in \mathbb{N}^*$

$?^{*} \ P(n,m)$ is true $\forall n, m \in \mathbb{N}^*$ (by math. induction)

$P(1,1) = 012 \in L(G)$ $(S \overset{(1)}{\Rightarrow} AB \underset{(3)}{\Rightarrow} 01B \underset{(4)}{\Rightarrow} 012) \Rightarrow$

$\Rightarrow P(1,1)$ is true

$P(k, l) \overset{?}{\Rightarrow} P(k+1, l+1)$, $\forall k, l \in \mathbb{N}^*$

we assume $P(k,l)$ is true $\Rightarrow 0^k 1^k 2^l \in L(G) \Rightarrow$

$\Rightarrow S \overset{*}{\Rightarrow} 0^k 1^k 2^l$ (ind. hypth.)

$S \underset{(1)}{\Rightarrow} AB \underset{(2)}{\Rightarrow} 0A1B \underset{(2)}{\overset{k}{\Rightarrow}} \underbrace{00\dots0}_{k} A \underbrace{11\dots1}_{k} B = 0^k A 1^k B \underset{(3)}{\Rightarrow}$

$\underset{(3)}{\Rightarrow} 0^k 01 1^k B = 0^{k+1} 1^{k+1} B \underset{(5)}{\Rightarrow} 0^{k+1} 1^{k+1} B2 \underset{(5)}{\overset{l}{\Rightarrow}} 0^{k+1} 1^{k+1} B \underbrace{22\dots2}_{l}$

$= 0^{k+1} 1^{k+1} B 2^l \underset{(4)}{\Rightarrow} 0^{k+1} 1^{k+1} 2 2^l = 0^{k+1} 1^{k+1} 2^{l+1} = P(k+1)$

$\Rightarrow S \overset{*}{\Rightarrow} 0^{k+1} 1^{k+1} 2^{l+1} \Rightarrow P(k+1)$ is true $\Rightarrow$ * is true

II $L(G) \subseteq L$

$S \Rightarrow AB \underset{(3)}{\Rightarrow} 01B \underset{(4)}{\Rightarrow} 012$

$\underset{(2)}{\Rightarrow} 0A1B \underset{(3)}{\Rightarrow} 0011B \underset{(5)}{\Rightarrow} 0^2 1^2 B2 \underset{(4)}{\Rightarrow} 0^2 1^2 2^2$

$\underset{(4)}{\Rightarrow} 0^2 1^2 2$

...

$\Rightarrow 0^k 1^k 2^l$, $\forall k, l \in \mathbb{N}^*$

all generated prod. in all poss. combinations

Seminar 4 : Finite Automata

1. $M = (Q, \Sigma, \alpha, q_0^{\in Q}, F^{\subseteq Q})$

$Q = \{q_1, q_2, q_3, q_4, q_0\}$

$\Sigma = \{1, 2, 3\}$

$F = \{q_4\}$

? $w = \boxed{12321} \in L(M)$

α	1	2	3
q_0	$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_0, q_3$
q_1	$\{q_1, q_4\}$	$\{q_1\}$	$\{q_1$
q_2	$\{q_2\}$	$\{q_2, q_4\}$	$\{q_2$
q_3	$\{q_3\}$	$\{q_3\}$	$\{q_3, q$
q_4	$\emptyset$	$\emptyset$	$\emptyset$

$\alpha : Q \times \Sigma \to P(Q)$

$(q, x) \to$ configuration

$x \in \Sigma^*$ sequence over Σ

$(q_0, w) \to$ initial config

$(q_f, \varepsilon) \to$ final config

$q_f \in F$ final state

$(p^{\in Q}, ax^{a \in \Sigma,\, x \in \Sigma^*}) \vdash (q, x)$ iff. $q \in \alpha(p, a)$

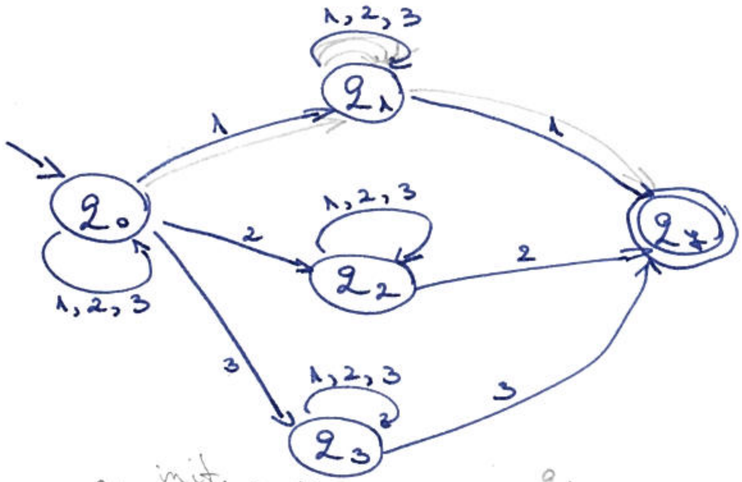
↳ direct transition

$\vdash^n \quad n \in \mathbb{N}^*$

$\vdash^*$

$\vdash^+$

$L(M) = \{ w \in \Sigma^* \mid (q_0, w) \vdash^* (q_4, \varepsilon), q_4 \in F \}$



$(q_0, 12321) \vdash (q_1, 2321) \vdash$

$\vdash (q_1, 321) \vdash (q_1, 21) \vdash (q_1, 1) \vdash$

$\vdash (q_4, \varepsilon) \Rightarrow (q_0, w) \overset{5}{\Rightarrow} (q_4, \varepsilon)$

$\Rightarrow w \in L(M)$

$(q_0 \overset{q_1}{\frown}, 12321) \vdash (q_1 \overset{q_1}{\frown}, 2321) \vdash (q_1 \overset{q_1}{\frown}, 321) \vdash (q_1 \overset{q_1}{\frown}, 21) \vdash$ init. config

$\vdash (q_1 \overset{q_4}{\frown}, 1) \vdash (q_4, \varepsilon)$ final config $\Rightarrow (q_0, w) \overset{5}{\Rightarrow} (q_4, \varepsilon) \Rightarrow w \in L(M)$

2)

? $L(M)$

① $\delta(p,a)=p$ ③ $\delta(q,b)=q$

② $\delta(p,b)=q$

Let $L=\{a^n b^m \mid n\in\mathbb{N}, m\in\mathbb{N}^*\}$ and prove $\boxed{L=L(M)}$

$a^* b^+$

1) ? $\boxed{L\subseteq L(M)}$

$\forall n\in\mathbb{N}, \forall m\in\mathbb{N}^* : a^n b^m \in L(M)$

$(\overset{p}{p}, a^n b^m) \underset{(a)}{\overset{n}{\vdash}} (\overset{q}{p}, b^m) \vdash (\overset{q}{q}, b^{m-1}) \underset{(b)}{\overset{m-1}{\vdash}} (q,\varepsilon) \Rightarrow$

$$\left[\begin{array}{l}(a)\ (p,a^n)\overset{n}{\vdash}(p,\varepsilon),\ \forall n\in\mathbb{N}\\ (b)\ (q,b^m)\overset{m}{\vdash}(q,\varepsilon),\ \forall m\in\mathbb{N}^*\end{array}\right] \Rightarrow (p,a^n b^m)\overset{n+m}{\vdash}(q,\varepsilon) \Rightarrow a^n b^m \in L(M)$$

(a) $P(n): (p,a^n)\overset{n}{\vdash}(p,\varepsilon), n\in\mathbb{N}$ + same for (b)

I $P(0): (p,\varepsilon)\overset{0}{\vdash}(p,\varepsilon)$ true

II Let $P(k): (p,a^k)\overset{k}{\vdash}(p,\varepsilon)$ be true

prove $P(k+1): (p,a^{k+1})\overset{k+1}{\vdash}(p,\varepsilon)$ true

$(p,a^{k+1}) \underset{\text{ind. hyp.}}{\overset{k}{\vdash}} (p,a) \vdash (p,\varepsilon)$ true

① când îs grupuri care se multiplică ⇒ cicluri

② când ai ε ⇒ initial = final

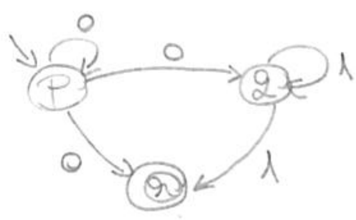
I, II ⇒ $P(n)$ is true $\forall n\in\mathbb{N}$

2) $\boxed{L(M)\subseteq L}$

3) $\boxed{?\ FA}$: a) $L_a=\{0^n 1^m 2^q \mid \underbrace{n,m\in\mathbb{N}^*}_{\geq 1}, \underbrace{q\in\mathbb{N}}_{\geq 0}\}$

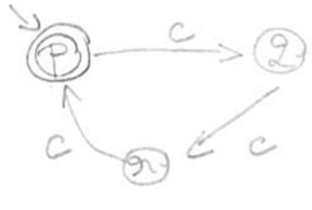
$=\{01, 012, \dots\}$

$=\{0, 0101, 010101, \dots\}$



? nu-i bun

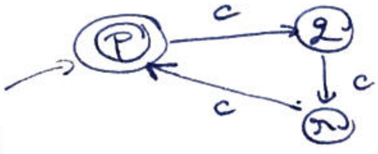
b) $L_b=\{0(01)^n \mid n\in\mathbb{N}\}$



ε, ccc, cccccc, ...

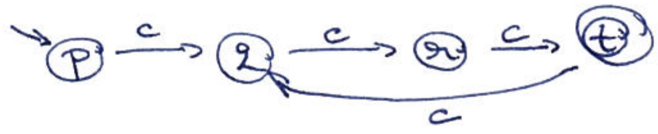
(ccc)

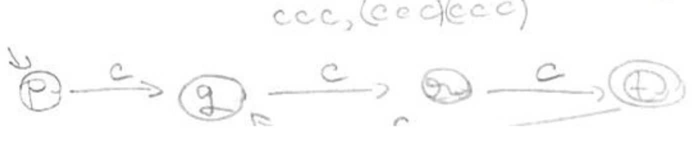
c) $L_c=\{c^{3n} \mid n\in\mathbb{N}\}$



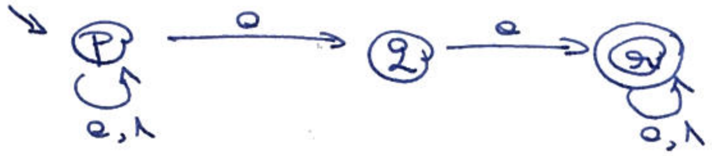
$L_{c'}=\{c^{3n} \mid n\in\mathbb{N}^*\}$

ccc, (ccc)(ccc)





d) All seq. over $\Sigma = \{0,1\}$ have at least 2 cons. 0's.



e) integer numbers

f) variable declaration (Pascal, C, Java, ...)

Seminar 5:

$\boxed{RE \Leftrightarrow RG \Leftrightarrow FA}$ (the corr. languages are the same)

I) $RG \Leftrightarrow FA$

$S \subseteq RLG \subseteq CFG$: $\boxed{\begin{array}{l} A \to aB \\ A \to b \end{array}}$

$S \to \varepsilon$, $\varepsilon \in \Sigma^*$

! $\varepsilon \notin \Sigma$

! If $S \to \varepsilon \in P$ then S does not appear in the rhs

$S \to \varepsilon \in P \Rightarrow \cancel{A \to bS}$

nonterm. N   terminals Σ

1. $G = (\{S, A\}, \{a, b\}, P, S)$

$P: S \to aA \mid \varepsilon$

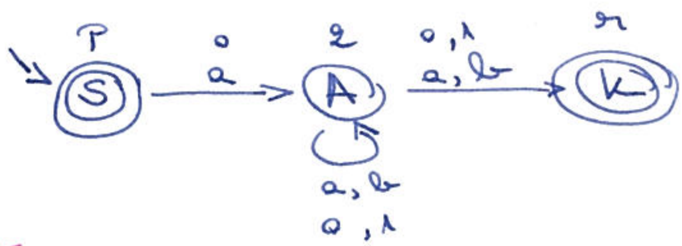
$A \to aA \mid bA \mid a \mid b \Rightarrow K$

? $\Leftrightarrow$ FA

final states

$M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{S, A, K\}$



$\Sigma = \{a, b\}$

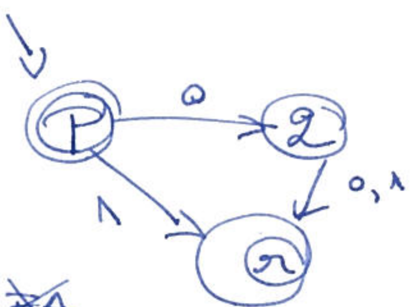
$q_0 = S$

$F = \{K, S\}$

$\delta(S, a) = \{A\}$ not DFA

$\delta(A, a) = \{A, K\}$

$\delta(A, b) = \{A, K\}$



p 0 q

p 0 r ~~DFA~~

p, 0, {q, r} ~~DFA~~

p, 0 → {q, r}

2. $M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{p, q, r\}$

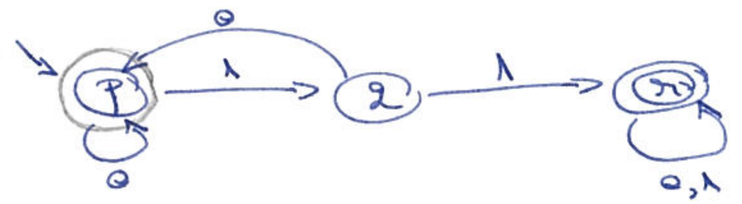
$\Sigma = \{0, 1\}$

$q_0 = p$

$F = \{r\}, p\}$

δ	0	1
p	p	q
q	p	r
r	r	r

? ⟺ RLG



$G = (\{p, q, r\}, \{0, 1\}, P, p)$

$P: p \to 0p \mid 1q \mid \varepsilon$

$q \to 0p \mid 1r \mid 1$

$r \to 0r \mid 1r \mid 0 \mid 1$

since r is final loops

II. $\boxed{RE \Leftrightarrow RG}$ $\Sigma = \{0, 1\}$

3. RE: $01(0 \overset{sau}{+} 1)^* 1$? ⟺ RG

$\{01, 001, 011, 0001, 0011, \dots\}$

[$\Sigma = \{a, b, c\}$

$a \to \{a\}$ $P = \{S \to a\}$

$\varepsilon \to \{\varepsilon\}$ $P = \{S \to \varepsilon\}$

$\emptyset \to \emptyset$ $P = \emptyset$]

01: $S \to 0A$

$A \to 1$

inaccessible

$S_1 \to 0$

$S_2 \to 1$

$S_3 \to 0 \mid 1$

0: $G_1 = (\{S_1\}, \{0, 1\}, \{S_1 \to 0\}, S_1)$

1: $G_2 = (\{S_2\}, \{0, 1\}, \{S_2 \to 1\}, S_2)$

0 + 1: $G_3 = (\{S_3, S_2, S_1\}, \{0, 1\}, \{S_1 \to 0,$ $S_3 \to S_1 \mid S_2$

$S_2 \to 1, S_3 \to 0 \mid 1\}, S_3\}$ it's not regular

$G'_3 = (\{S_3\}, \{0, 1\}, \{S_3 \to 0 \mid 1\}, S_3\}$

01: $G_4 = (\{S_1, S_2\}, \{0, 1\}, \{S_2 \to 1, S_1 \to 0S_2\}, S_1)$

$(0+1)^*$: $G_5 = G_3'^*$

$G_5 = (\{S_3\}, \{0, 1\}, \{S_3 \to 0 \mid 1, S_3 \to 0S_3 \mid 1S_3 \mid \varepsilon\}$
$S_3)$

$G'_5 = (\{S_3\}, \{0, 1\}, \{S_3 \to 0S_3 \mid 1S_3 \mid \varepsilon\}, S_3)$! not RG

1

$0(0+1)^*: G_6 = (\{S_1, S_3\}, \{0,1\}, \{S_1 \to 0S_3, S_3 \to 0S_3 | 1S_3 | \varepsilon\}, S_1)$! not RG

$0(0+1)^* 1: G_7 = (\{S_1, S_2, S_3\}, \{0,1\}, \{S_1 \to 0S_3, S_3 \to 0S_3 | 1S_3 | S_2, S_2 \to 1\}, S_1)$! not RG

eliminate S_2

$G_7' = (\{S_1, \cancel{S_2}, S_3\}, \{0,1\}, \{S_1 \to 0S_3, S_3 \to 0S_3 | 1S_3 | 1\}, S_1)$ RG

4. RG: $G = (N, \Sigma, P, S)$? $\Leftrightarrow$ RE

$N = \{S, A, B\}$

$\Sigma = \{a, b\}$

$P: S \to aA$

$A \to aA | bB | b$

$B \to bB | b$

(*) $X = aX + b$

$\Rightarrow$ sol: $X = a^* b$

! $b b^* = b^+$

$b b^+ \neq b^+$

$$\Rightarrow \begin{cases} S = aA \quad (1) \\ A = aA + \overbrace{bB + b}^{B} \\ B = bB + b \overset{(*)}{\Rightarrow} B = b^* b = b^+ \end{cases}$$

$\Rightarrow A = aA + b \cdot b^+ + b$

$A = aA + b(b^+ + \varepsilon)$

$A = aA + b b^*$

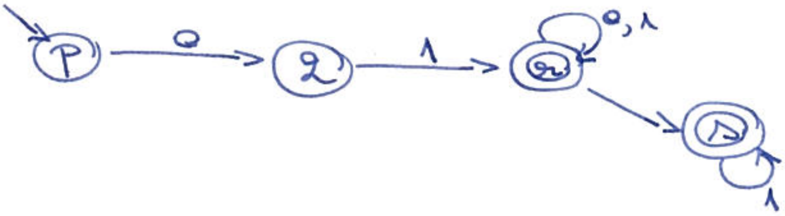
$A = aA + b^+ \overset{(*)}{\Rightarrow} A = a^* b^+$

$(1) \Rightarrow S = a\, a^* b^+$

$S = a^+ b^+$

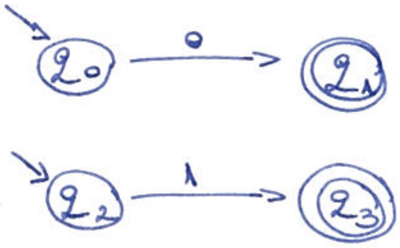
III) FA ⇔ RE

5) RE: $01(0+1)^*1^* \Leftrightarrow 01(0+1)^*$? ⇒ FA

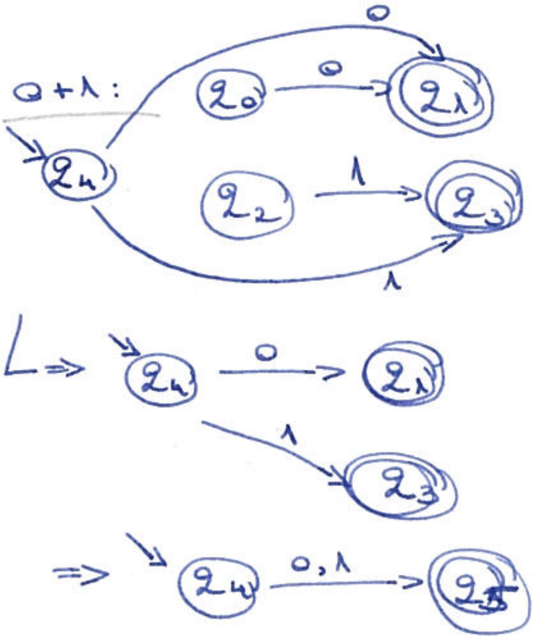


0 :

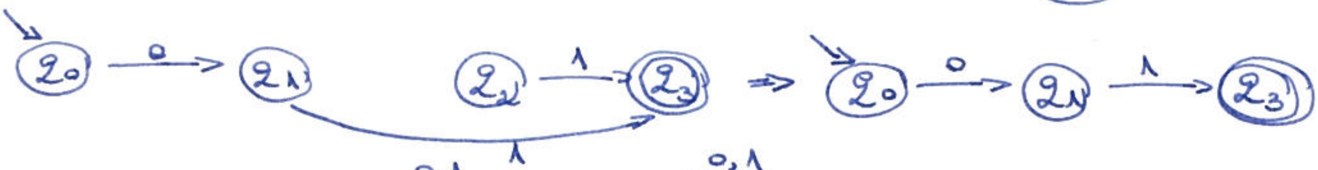
1 :



$0+1$:

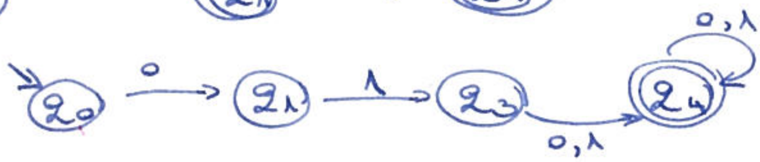


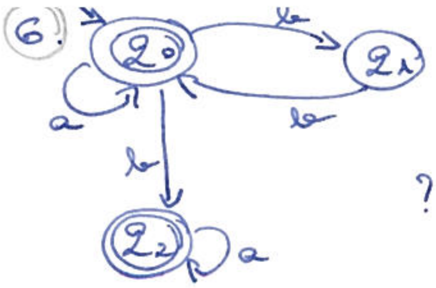
01 :



$(0+1)^*$:

$01(0+1)^*$:





$? \Leftrightarrow RE$

$a^*(bb)^* b a^*$

$$\begin{cases} q_0 = \varepsilon + q_0 a + q_1 b \\ q_1 = q_0 b \\ q_2 = q_0 b + q_2 a \end{cases}$$

$X = Xa + b$

sol : ba^*

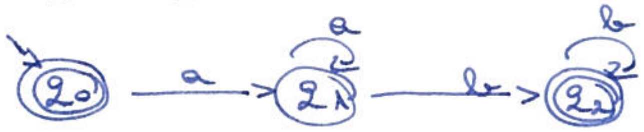
$q_0 = \varepsilon + q_0 a + q_1 b = \varepsilon + q_0 a + q_0 b^2 = \varepsilon + q_0(a+b^2) = (a+b^2)^*$

$q_2 = (a+b^2)^* \cdot b + q_2 \cdot a = (a+b^2)^* b a^*$

$RE = q_0 + q_2 = (a+b^2)^* + (a+b^2)^* \cdot b \cdot a^*$

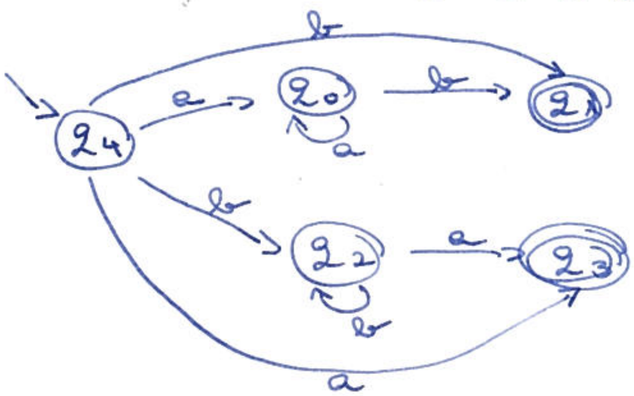
$= (a+b^2)^* (\varepsilon + ba^*)$

$a^+ b^+ + \varepsilon$



$a^* b + b^* a$; $a^* b = b, ab, aab, \ldots$

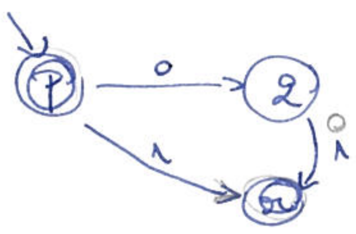
$b^* a = a, ba, bba, \ldots$



" " $\neq$

$P \; 0 = q$

$p, 0 = r$



$\delta(p, 0) = q$

$\delta(p, 1) = r$

set of states : p, q

alphabet : 0, 1

one initial state

set of final states

$p \xrightarrow{0} q$

$p \xrightarrow{0} r$

$\begin{cases} 1 \\ 00 \\ 01 \end{cases}$

0 , 1 accepted

$p \xrightarrow{0} q \quad q \xrightarrow{1} r$ final

FA id.in

a, b, c

```
isIdentifier (token) {
    FA faid = new FA("FA id.in")
    isIdentifier {
        faid.checkSequence(token);
    }
}
```

Seminar 7: CFG (= context free grammar)

1. $G = (\{S, A, B\}, \{0, 1\}, P, S)$

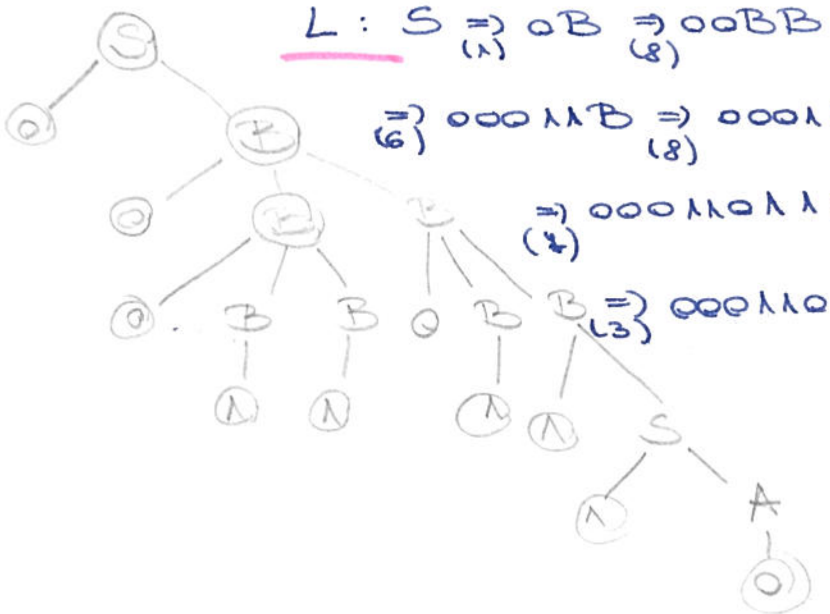
$P: S \to \overset{1}{0B} \mid \overset{2}{1A}$

$A \to \overset{3}{0} \mid \overset{4}{0S} \mid \overset{5}{1AA}$

$B \to \overset{6}{1} \mid \overset{7}{1S} \mid \overset{8}{0BB}$

$w = 0001101110$

? left (right) most derivation for w + associated parse trees

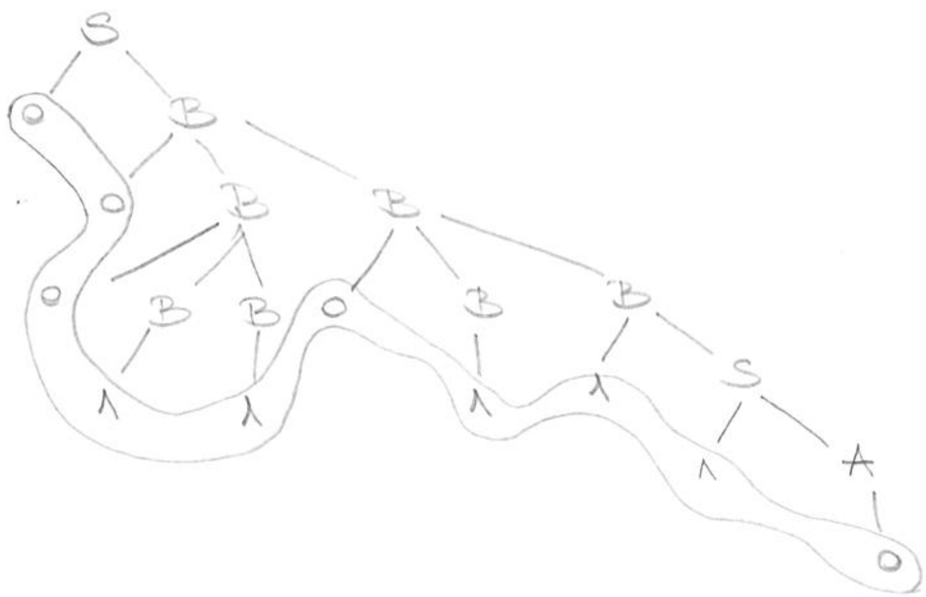


L: $S \underset{(1)}{\Rightarrow} 0B \underset{(8)}{\Rightarrow} 00BB \underset{(8)}{\Rightarrow} 000BBB \underset{(6)}{\Rightarrow} 0001BB \underset{(6)}{\Rightarrow} 00011B \underset{(8)}{\Rightarrow} 000110BB \underset{(6)}{\Rightarrow} 0001101B \underset{(7)}{\Rightarrow} 00011011S \underset{(2)}{\Rightarrow} 000110111A \underset{(3)}{\Rightarrow} 0001101110$

R: $S \underset{(1)}{\Rightarrow} 0B \underset{(8)}{\Rightarrow} 00BB \underset{(8)}{\Rightarrow} 00B0BB \underset{(7)}{\Rightarrow} 00B0B\lambda S \underset{(2)}{\Rightarrow}$

$\underset{(2)}{\Rightarrow} 00B0B\lambda\lambda A \underset{(3)}{\Rightarrow} 00B0B\lambda\lambda 0 \underset{(6)}{\Rightarrow} 00B0\lambda\lambda\lambda 0 \underset{(8)}{\Rightarrow} 000BB0\lambda\lambda\lambda 0$

$\underset{(6)}{\Rightarrow} 000\lambda B0\lambda\lambda\lambda 0 \underset{(6)}{\Rightarrow} 000\lambda\lambda 0\lambda\lambda\lambda 0$



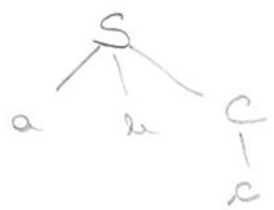
2. Prove G is ambiguous:

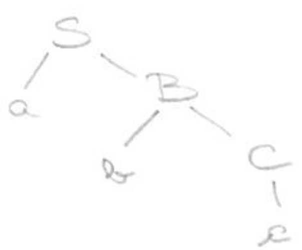
a) $G = (\{S, B, C\}, \{a, b, c\}, P, S)$

P: $S \to abC \mid aB$ $\qquad\qquad w = abc$

$B \to bC$

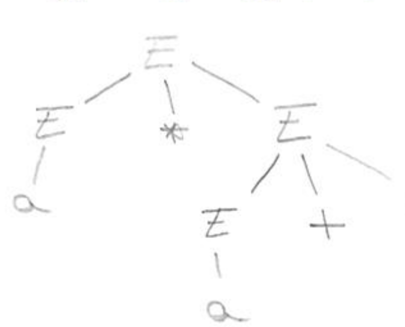
$C \to c$

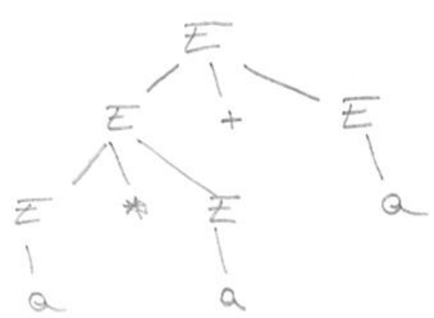




b) $G = (\{E\}, \{a, +, *, (,)\}, \{E \to E+E \mid E*E \mid (E) \mid a\}, E)$

$w = a*a+a$





RDP: $G = (\{S\}, \{a, b, c\}, \{S \to aSbS \mid aS \mid c\}, S)$

$w = aacbc$? $w \in L(G)$ using RDP

config (s, i, α, β), $i \in \{1, 2, \dots, n+1\}$, $n = \text{length}(w)$

$s \in (q, b, t, e)$

initial config: $(q, 1, \varepsilon, S)$

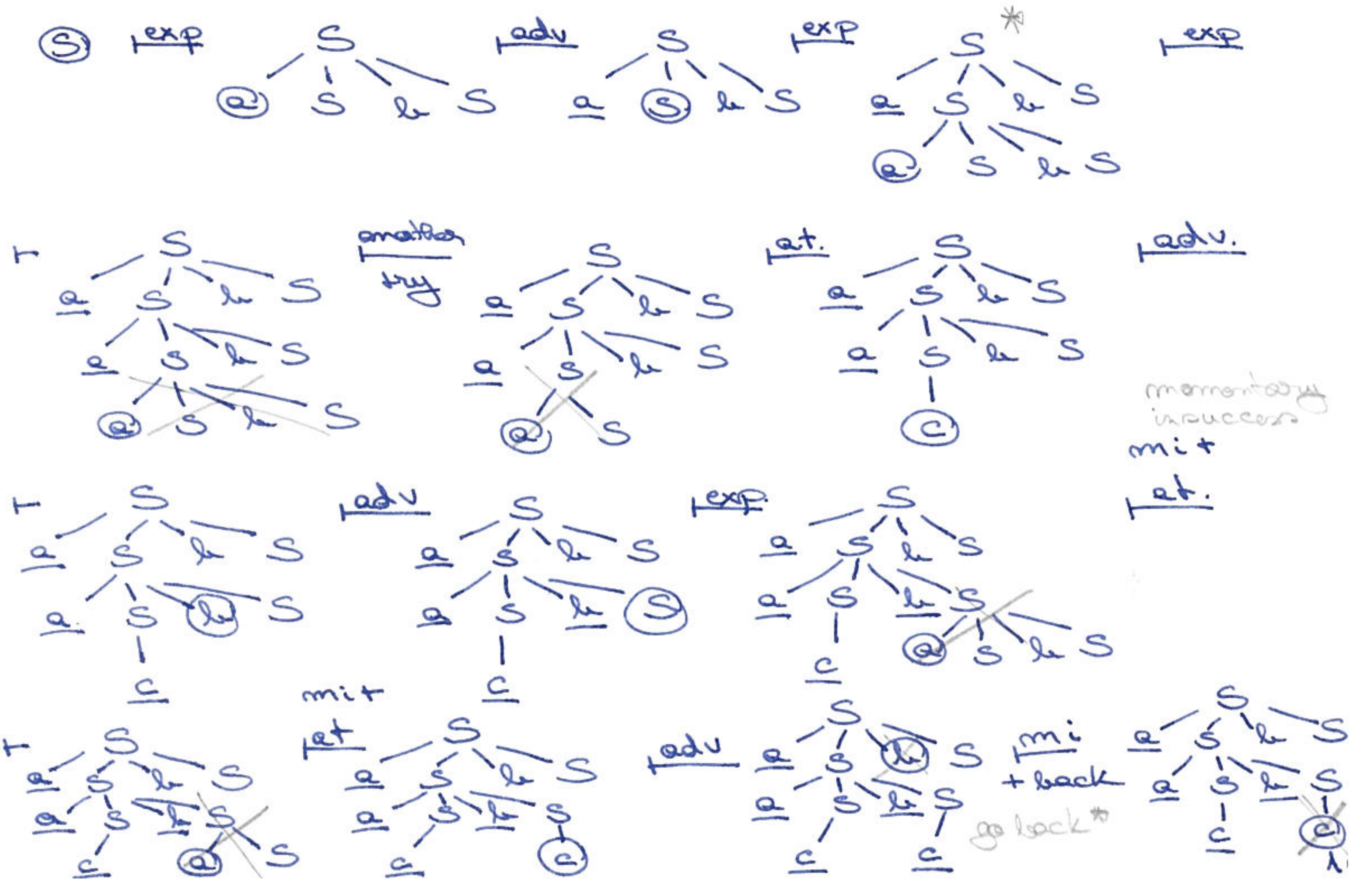
final succ. config: $(t, n+1, \alpha, \varepsilon)$

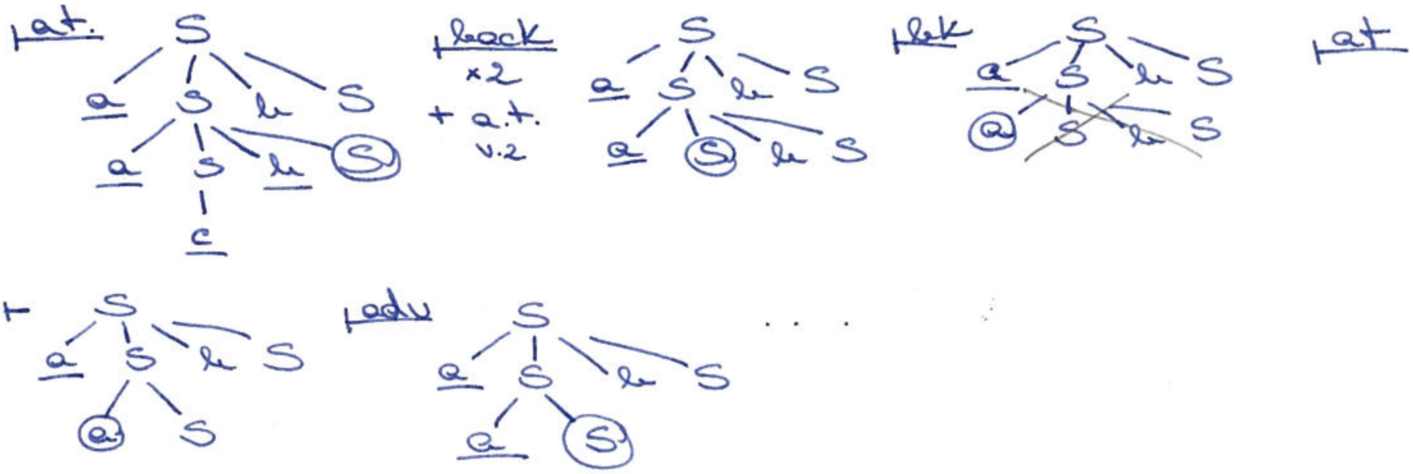
$(q, 1, \varepsilon, S) \vdash_{exp} (q, 1, S_1, aSbS) \vdash_{adv} (q, 2, S_1 a, SbS) \vdash$

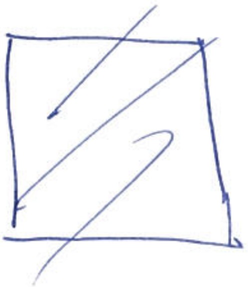
$\vdash_{exp} (q, 2, S_1 a S_1, aSbSbS) \vdash_{adv} (q, 3, S_1 a S_1 a, SbSbS)$

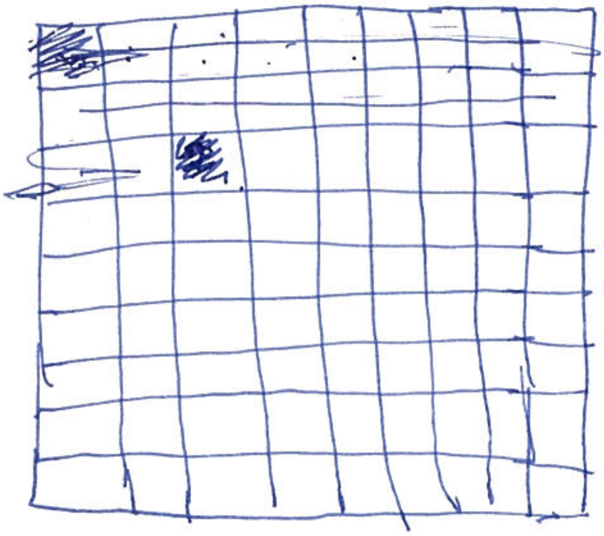
$\vdash_{exp} (q, 3, S_1 a S_1 a S_1, aSbSbSbS) \vdash$

$\vdash_{\substack{mom \\ insucc}}$




at.
back
×2
+ a.t.
v.2
bak
at
T
adv
. . .





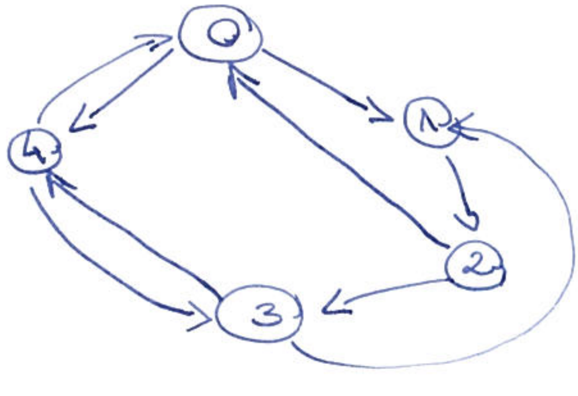
4 tasks. $9 \cdot 9 = \underline{81} : 4 = 20$

1* $20 / 9 = 2.$

$20 \% 9 = 2$

$$F_1^{(S)} = F_0(S) \cup \left(F_0(B) \oplus F_0(A) \right)$$
$$(+, \varepsilon) \oplus \varnothing = \varnothing$$
$$= F_0^{(S)} \cup \varnothing = F_0(S)$$

$$F_1(A) = F_0(A) \cup \left(F_0(B) \oplus F_0(A) \right) = F_0(A)$$



Seminar 8: LL(1)

1. $G = (\{S, A, B, C, D\}, \{a, +, *, (,)\}, P, S)$

P:
(1) $S \to BA$
(2) $A \to +BA$
(3) $A \to \varepsilon$
(4) $B \to DC$
(5) $C \to *DC$
(6) $C \to \varepsilon$
(7) $D \to (S)$
(8) $D \to a$

$w = a*(a+a)$

I) Build First and Follow

• FIRST(α), $\alpha = x_1 x_2 \ldots x_n$, $x_i \in V$, $i = \overline{1,n}$

$FIRST(x_1 x_2 \ldots x_n) = FIRST(x_1) \oplus FIRST(x_2) \oplus \ldots \oplus FIRST(x_n)$

only terminals or ε on first pos.

	F_0	F_1	F_2	$= F_3$
S	$\emptyset$	$\emptyset$	a, (	a, (
A	+, ε	+, ε	+, ε	+, ε
B	$\emptyset$	a, (	a, (	a, (
C	*, ε	*, ε	*, ε	*, ε
D	a, (	a, (	a, (	a, (

$\Rightarrow F_2 = F_3 = FIRST$

First(S) = {a, (}
(A) = {+, ε}
(B) = {a, (}
(C) = {*, ε}
(D) = {a, (}

$B \to DC \Rightarrow \{a, (\}\ \{*, \varepsilon\} \Rightarrow$ rhs a, a, (, ($\Rightarrow \{a, (\}$

$\cancel{\{+, \varepsilon\}} \oplus \{a, c\} \oplus \{+, \varepsilon\} = \boxed{a, c}$

• FOLLOW

$S \overset{0}{\Rightarrow} S$ is a sentencial form

	L_0	L_1	L_2	L_3	$= L_4$
S	ε	ε,)	ε,)	ε,)	ε,)
A	$\emptyset$	① ε	ε,)	ε,)	ε,)
B	$\emptyset$	② +, ε	+, ε,)	+, ε,)	+, ε,)
C	$\emptyset$	$\emptyset$	+, ε	+, ε,)	+, ε,)
D	$\emptyset$	*	*, +, ε	*, +, ε,)	*, +, ε,)

$\Rightarrow L_3 = L_4 = FOLLOW$

$S \to BA \Rightarrow Fol(A) = Fol(S)$

$\left.\begin{array}{l} B \to DC \\ C \to \varepsilon \end{array}\right\} \Rightarrow Fol(D) = Fol(B)$

$B \to DC \Rightarrow Fol(D) = First(C)$

$Follow(A) = \{w \in \Sigma \mid S \overset{*}{\Rightarrow} \alpha S \beta,\ w \in FIRST(\beta)\}$

① $S \to BA \Rightarrow Fol(A) = Fol(S) = \varepsilon$

② $Fol(B) = First(A) \oplus Fol(S) = \{+, \varepsilon\}$ bc. fol(A) = ε? from one step back (L_0)

IV) LL(1) parsing table

	a	+	*	(	)	$
S	BA, 1			BA, 1		
A		+BA, 2			ε, 3	ε, 3
B	DC, 4			DC, 4		
C		ε, 6	*DC, 5		ε, 6	ε, 6
D	a, 8			(S), 7		
a	pop					
+		pop				
*			pop			
(				pop		
)					pop	
$						acc.

S → BA
FIRST (BA) = {a, (}

initial config.: (w$, S$, ε)
final config.: ($, $, α)

III) parse the sequence

i.c.: (a*(a+a)$, S$, ε) ⊢ (a*(a+a)$, BA$, 1) ⊢
⊢ (a*(a+a)$, DCA$, 14) ⊢ (a*(a+a)$, aCA$, 148) ⊢
⊢ (*(a+a)$, CA$, 148) ⊢ (*(a+a)$, *DCA$, 1485) ⊢
⊢ ((a+a)$, DCA$, 1485) ⊢ ((a+a)$, (S)CA$, 14857) ⊢
⊢ (a+a)$, S)CA$, 14857) ⊢ (a+a)$, BA)CA$, 148571) ⊢
⊢ (a+a)$, DCA)CA$, 1485714) ⊢ (a+a)$, aCA)CA$, 14857148) ⊢
⊢ (+a)$, CA)CA$, 14857148) ⊢ (+a)$, A)CA$, 148571486) ⊢
⊢ (+a)$, +BA)CA$, 1485714862) ⊢ (a)$, DCA)CA$, 148571 48624) ⊢
⊢ (a)$, aCA)CA$, 14857148624 8) ⊢ ()$, A)CA$, 14857148624 6) ⊢
⊢ ()$,)CA$, 14857148624 63) ⊢ ($, A$, 1485714862463 6) ⊢
⊢ ($, $, $\boxed{14857148624636 3}$) transition to success

1

Seminar 9: LR(0) parser

$G = (\underbrace{\{S, A\}}_{N}, \underbrace{\{a, b, c\}}_{\Sigma}, P, S)$

P: (1) $S \to aA$? $w = abbc \in L(G)$

(2) $A \to bA$

(3) $A \to c$

$[\underbrace{A \to \alpha . \beta}_{\text{kernel}}, \underbrace{u}_{\text{prediction}}^{\in \Sigma^k}]$ - LR(k) item

$[A \to \alpha . \beta]$ - LR(0) item

ws →	← is
$ s₀	abbc $

if $[A \to \alpha . B\beta] \in s_i$ then $[B \to . \gamma] \in s_i$, $\forall B \to \gamma \in P$

I) Compute LR(0) canonical collection $\mathcal{C} = \{s_0, s_1, \dots\}$

$s_0 = \text{closure}(\{[S' \to . S]\}) = \{[S' \to . S], [S \to . aA]\}$ $S \to aA$. in front of terminal → STOP

$\forall X \in N \cup \Sigma = \{S, A, a, b, c\}$

$s_1 = \text{goto}(s_0, S) = \text{closure}(\{[S' \to S.]\}) = \{[S' \to S.]\}$

$\text{goto}(s_0, A) = \emptyset$

$\boxed{\text{goto}(s, X) = \text{closure}(\{[A \to \alpha X . \beta] \mid [A \to \alpha . X\beta] \in s\})}$

$\text{goto}(s_0, b) = \emptyset$ $\text{goto}(s_0, c) = \emptyset$

$s_2 = \text{goto}(s_0, a) = \text{closure}(\{[S \to a . A]\}) = \{[S \to a.A], [A \to .bA],$
$[A \to .c]\}$

$s_3 = \text{goto}(s_2, A) = \text{closure}(\{[S \to aA.]\}) = \{[S \to aA.]\}$

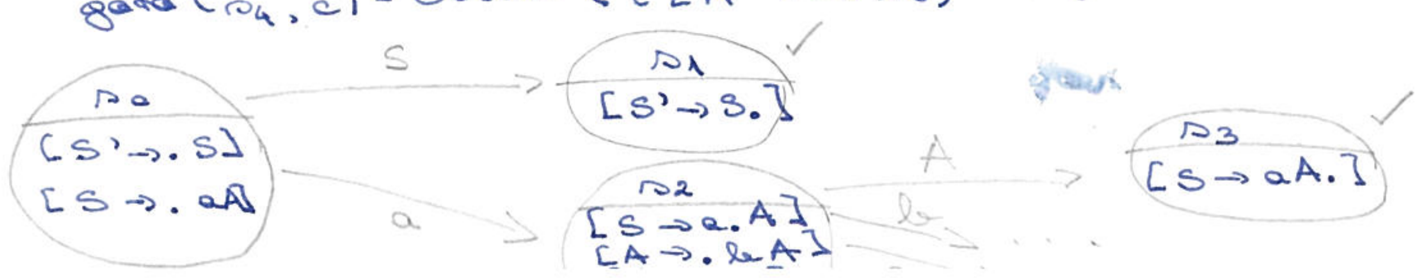
$s_4 = \text{goto}(s_2, b) = \text{closure}(\{[A \to b.A]\}) = \{[A \to b.A], [A \to .bA],$
$[A \to .c]\}$

$s_5 = \text{goto}(s_2, c) = \text{closure}(\{[A \to c.]\}) = \{[A \to c.]\}$

$s_6 = \text{goto}(s_4, A) = \text{closure}(\{[A \to bA.]\}) = \{[A \to bA.]\}$

$\text{goto}(s_4, b) = \text{closure}(\{[A \to b.A]\}) = s_4$

$\text{goto}(s_4, c) = \text{closure}(\{[A \to c.]\}) = s_5$



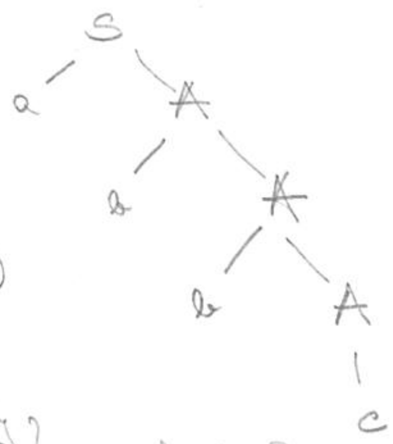
II) LR(0) parsing table *

	action	goto S	A	a	b	c
s_0	shift	s_1		$\boxed{s_2}$		
s_1	(= reduce 0) accept					
s_2	shift		s_3 *		$\boxed{s_4}$	s_5
s_3	reduce 1					
s_4	shift		$\boxed{s_6}$ *		s_4	s_5
s_5	reduce 3					
s_6	reduce 2					

III) Parse w : initial config = ($\$ s_0, w\$, \varepsilon$), final config = ($\$ s_{acc}, \$, \pi$)

work stack = α →←		input stack = β	output band = π
$\$ s_0$	shift	$abbc\$$	ε
$\$ s_0 a s_2$	shift	$bbc\$$	ε
$\$ s_0 a s_2 b s_4$	shift	$bc\$$	ε
$\$ s_0 a s_2 b s_4 b s_4$	shift	$c\$$	ε
$\$ s_0 a s_2 b s_4 b s_4 c s_5$	reduce 3	$\$$	ε
$\$ s_0 a s_2 b s_4 b s_4 A s_6$ *	reduce 2	$\$$	3
$\$ s_0 a s_2 b s_4 A s_6$	reduce 2	$\$$	2 3
$\$ s_0 a s_2 A s_3$ *	reduce 1	$\$$	2 2 3
$\$ s_0 S s_1$		$\$$	1 2 2 3

accept



* 1) if $[A \to \alpha . \beta] \in s_i$ ⇒ shift (→ dot is not at the end)

2) if $[A \to \beta .] \in s_i$ and $A \neq S'$ ⇒ reduce l,
where l = no. of prod. $A \to \beta$

3) if (only) $[S' \to . S] \in s_i$ ⇒ accept (or reduce 0)

4) if $goto(s_i, X) = s_j$ ⇒ $goto(s_i, X) = s_j$

5) otherwise error

$s_3 = \{[S \to aA.]\}$, $S \neq S'$, (1) $S \to aA$ ⇒ reduce 1

$s_5 = \{[A \to c.]\}$, $A \neq S'$, (3) $A \to c$ ⇒ reduce 3

$s_6 = \{[A \to bA.]\}$, $A \neq S'$, (2) $A \to bA$ ⇒ reduce 2

SLR Parser:

$G = (\{E, T\}, \{+, (,), id, const\}, P, E)$, $S' \to E$

$P:$ (0) $S' \to E$

(1) $E \to T$ $w = id + const$

(2) $E \to E + T$

(3) $T \to (E)$

(4) $T \to id$

(5) $T \to const$

I $s_0 = closure(\{[S' \to .E]\}) = \{[S' \to .E], [E \to .T], [E \to .E+T],$
$[T \to .(E)], [T \to .id], [T \to .const]\}$

$s_1 = goto(s_0, E) = closure(\{[S' \to E.], [E \to E.+T]\}) =$
$= \{[S' \to E.], [E \to E.+T]\}$

$s_2 = goto(s_0, T) = closure(\{[E \to T.]\}) = \{[E \to T.]\}$

$s_3 = goto(s_0, ( ) = closure(\{[T \to (.E)]\}) = \{[T \to (.E)], [E \to .T],$
$[E \to .E+T], [T \to .(E)], [T \to .id], [T \to .const]\}$

$s_4 = goto(s_0, id) = closure(\{[T \to id.]\}) = \{[T \to id.]\}$

$s_5 = goto(s_0, const) = closure(\{[T \to const.]\}) = \{[T \to const.]\}$

$s_6 = goto(s_1, +) = \{[E \to E+.T], [T \to .(E)], [T \to .id], [T \to .const]$

$s_7 = goto(s_3, E) = \{[T \to (E.)], [E \to E.+T]\}$

$goto(s_3, T) = s_2$ $goto(s_3, () = s_3$ $goto(s_3, id) = s_4$

$goto(s_3, const) = s_5$

$s_8 = goto(s_6, T) = closure(\{[E \to E+T.]\}) = \{[E \to E+T.]\}$

$s_3 = goto(s_6, () = closure(\{[T \to (.E)]\}) = \{[T \to (.E)], [E \to .T],$
$[E \to .E+T], [\cancel{E}T \to .(E)], [T \to .id], [T \to .const]\}$

$goto(s_6, id) = closure(\{[T \to id.]\}) = s_4$

$goto(s_6, const) = closure(\{[T \to const.]\}) = s_5$

$s_9 = goto(s_7,)) = closure(\{[T \to (E).]\}) = \{[T \to (E).]\}$

$goto(s_7, +) = closure(\{[E \to E+.T]\}) = s_6$

$FOLLOW(E) = \{\varepsilon, +,)\}$

$FOLLOW(T) = \{\varepsilon, +,)\}$

II SLR table

	action						goto	
	id	const	+	(	)	$	E	T
s0	shift s4	shift s5		shift s3			s1	s2
s1			shift s6			acc.		
s2			reduce 1		reduce 1	reduce 1		
s3	shift s4	shift s5		shift s3			s7	
s4			reduce 4		reduce 4	reduce 4		
s5			reduce 5		reduce 5	reduce 5		
s6	shift s4	shift s5		shift s3				s8
s7			shift s6		shift s9			
s8			reduce 2		reduce 2	reduce 2		
s9			reduce 3		reduce 3	reduce 3		

III

work stack		input stack	output band
$ s0	shift s4	id + const $	ε
$ s0 id s4	reduce 4	+ const $	ε
$ s0 T s2		+ const $	4
$ s0 E s1		+ const $	14
$ s0 E s1 + s6		const $	14
$ s0 E s1 + s6 const s5		$	14
$ s0 E s1 + s6 T s8		$	514
$ s0 E s1		$	2514

acc.

LR(1) : LR(1) item : $[A \to \alpha . \beta , a]$

$G = (\{S, A\}, \{a, b\}, P, S)$

$S' \to S$

P :

(1) $S \to AA$

(2) $A \to aA$

(3) $A \to b$

$w = abab \in L(G)$?

I) Canonical collection

II) LR(1) parsing table

	action			goto	
	a	b	$	S	A
s_0	shift s_3	shift s_4		s_1	s_2
s_1			acc		
s_2	shift s_6	shift s_7			s_5
s_3	shift s_3	shift s_4			s_8
s_4	reduce 3	reduce 3			
s_5			reduce 1		
s_6	shift s_6	shift s_7			s_9
s_7			reduce 3		
s_8	reduce 2	reduce 2			
s_9			reduce 2		

III)

work stack	input stack	outputband
$ s_0 shift s_3	a b a b $	ε
$ s_0 a s_3 shift s_4	b a b $	ε
$ s_0 a s_3 b s_4 red 3	a b $	ε
$ s_0 a s_3 A s_8 red 2	a b $	3
$ s_0 ~~a~~ A s_2 shift s_6	~~a~~ b $	2 3
$ s_0 A s_2 a s_6 shift s_7	~~b~~ $	2 3
$ s_0 A s_2 a s_6 b s_7	$	2 3
$ s_0 A s_2 a s_6 A s_9	$	3 2 3
$ s_0 A s_2 A s_5	$	2 3 2 3
$ s_0 S s_1	$	1 2 3 2 3

accept

Pair = < Row, Column >

HashSet < Pair > = String

LALR(1)

I Canonical coll. - starting from LR(1) can. coll. (same G)

$\mathcal{C} = \{ s_0, s_1, s_2, s_{36}, s_{47}, s_5, s_{89} \}$

$s_{47} = \{ [A \to b., a|b|\$] \}$

$s_{36} = \{ [A \to a.A, a|b|\$], [A \to .aA, a|b|\$], [A \to .b, a|b|\$] \}$

$s_{89} = \{ [A \to aA., a|b|\$] \}$

	a	b	\$	S	A
s_0	shift s_{36}	shift s_{47}		s_1	s_2
s_1			acc		
s_2	shift s_{36}	shift s_{47}			s_5
s_{36}	shift s_{36}	shift s_{47}			s_{89}
s_{47}	red 3	red 3	red 3		
s_5			red 1		
s_{89}	red 2	red 2	red 2		

work stack	input stack	output band
$\$ s_0$	abab\$	ε
$\$ s_0 a s_{36}$	bab \$	ε
$\$ s_0 a s_{36} b s_{47}$	ab \$	ε
$\$ s_0 a s_{36} A s_{89}$	ab \$	3
$\$ s_0 A s_2$	ab \$	23
$\$ s_0 A s_2 a s_{36}$	b \$	23
$\$ s_0 A s_2 a s_{36} b s_{47}$	\$	23
$\$ s_0 A s_2 a s_{36} A s_{89}$	\$	323
$\$ s_0 A s_2 A s_5$	\$	2323
$\$ s_0 S s_1$	\$	12323
acc		

PDA (= Push Down Automata)

$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

initial symbol from stack

stack memory alphabet

[FA: $\delta: Q \times \Sigma \to P(Q)$
config.: $(q, x \in \Sigma^*)$]

$\delta: Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \to P(Q \times \Gamma^*)$

config. $(q \in Q, x \in \Sigma^*, \gamma \in \Gamma^*)$

initial config: (q_0, w, Z_0)

final config: $(q, \varepsilon, \varepsilon) \to$ using "empty stack" criteria

$(q_f, \varepsilon, \gamma) \to$ using "final state" criteria, $q_f \in F$

- trans. $(p, ax, Z\gamma) \vdash (q, x, \alpha\gamma)$ iff. $(q, \alpha) \in \delta(p, a, Z)$

$(p, x, Z\gamma) \vdash (q, x, \alpha\gamma)$ iff. $(q, \alpha) \in \delta(p, \varepsilon, Z)$

$\vdash^k, \vdash^+, \vdash^*$

- $L_\varepsilon(M) = \{w \in \Sigma^* \mid (q_0, w, Z_0) \vdash^* (q, \varepsilon, \varepsilon), q \in Q\}$

$\Updownarrow$

$L_f(M) = \{w \in \Sigma^* \mid (q_0, w, Z_0) \vdash^* (q_f, \varepsilon, \gamma), q_f \in F, \gamma \in \Gamma^*\}$

PDA $L = \{0^n 1^n \mid n \in \mathbb{N}^*\}$

Ex: 1) $L_1 = \{a^n b^{2n} \mid n \in \mathbb{N}^*\}$? PDA

$M = (\{q_0, q_1, q_2\}, \{a, b\}, \{X, Z_0\}, \delta, q_0, Z_0, \{q_2\})$

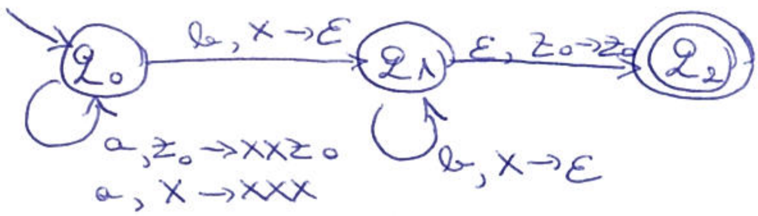
$\delta(q_0, a, Z_0) = (q_0, XXZ_0)$

$\delta(q_0, a, X) = (q_0, XXX)$

$\delta(q_0, b, X) = (q_1, \varepsilon)$ $b \to$ pop = replace $(X) = \varepsilon$

$\delta(q_1, b, X) = (q_1, \varepsilon)$

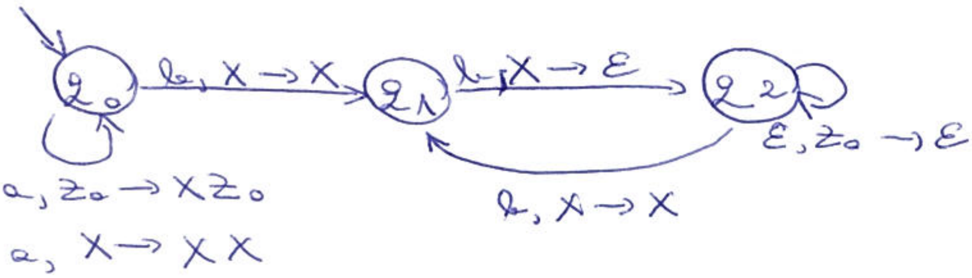
$\delta(q_1, \varepsilon, Z_0) = (q_2, \cancel{Z_0})$



$(q_0, ab, Z_0) \vdash (q_0, b, X^2Z_0) \vdash (q_1, \varepsilon, XZ_0)$ not acc.

$(q_0, ab^3, Z_0) \vdash (q_0, b^3, X^2Z_0) \vdash (q, b^2, XZ_0) \vdash (q_1, b, Z_0)$

$\vdash (q_2, b, Z_0)$ not acc.



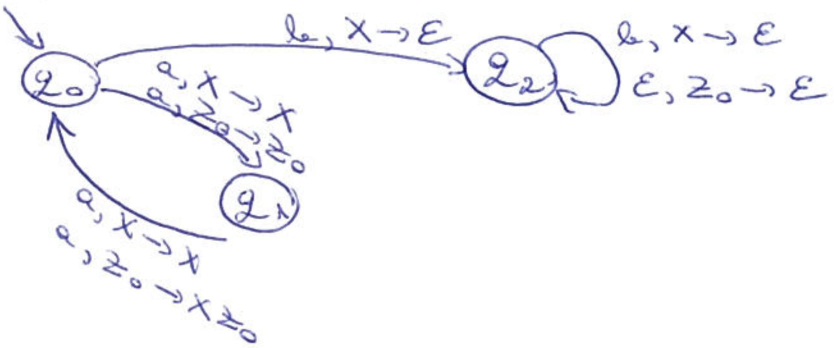
$(q_0, a^2b^4, Z_0) \vdash (q_0, ab^4, XZ_0) \vdash (q_0, b^4, XXZ_0) \vdash$

$\vdash (q_1, b^3, XXZ_0) \vdash (q_2, b^2, XZ_0) \vdash (q_1, b, XZ_0) \vdash$

$\vdash (q_2, \varepsilon, Z_0) \vdash (q_2, \varepsilon, \varepsilon)$ empty stack

$(q_0, ab, Z_0) \vdash (q_0, b, XZ_0) \vdash (q_1, \varepsilon, XZ_0)$ not acc

2) $L_2 = \{a^{2n}b^n \mid n \in \mathbb{N}^*\}$



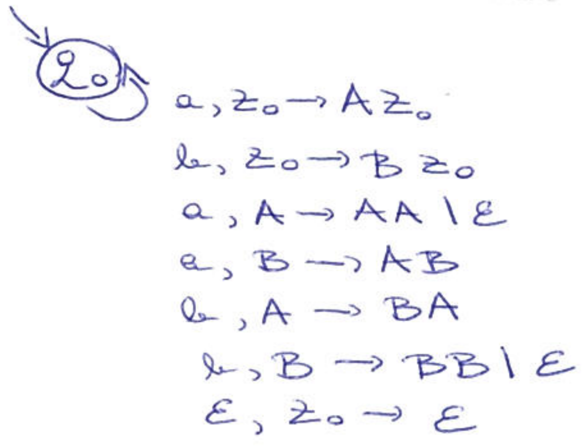
$(q_0, a^4b^2, Z_0) \vdash (q_1, a^3b^2, Z_0) \vdash (q_0, a^2b^2, XZ_0)$

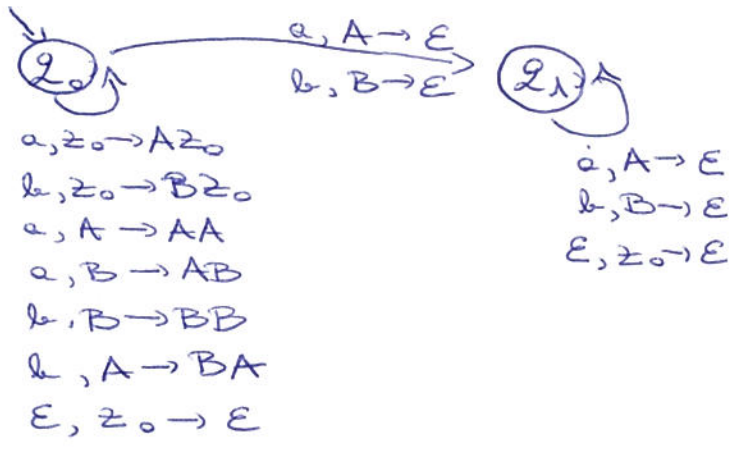
$\vdash (q_1, ab^2, XZ_0) \vdash (q_0, b^2, XXZ_0) \vdash (q_2, b, XZ_0)$

$\vdash (q_2, \varepsilon, Z_0) \vdash (q_2, \varepsilon, \varepsilon)$ acc.

$(q_0, ab, Z_0) \vdash (q_1, b, Z_0) \vdash \emptyset$

3) $L_3 = \{ w w^R \mid w \in \{a, b\}^+ \}$





Attribute grammars: $AG = (\overset{\nearrow \text{cfg}}{G}, A, R)$

Ex. 1) ? AG for counting the no. of vowels in a non empty string

$$\left.\begin{array}{l} S \to L \\ S \to L S \end{array}\right\} = \text{word} \qquad \begin{array}{l} S.no = L.no \\ S_1.no = L.no + S_2.no \end{array}$$

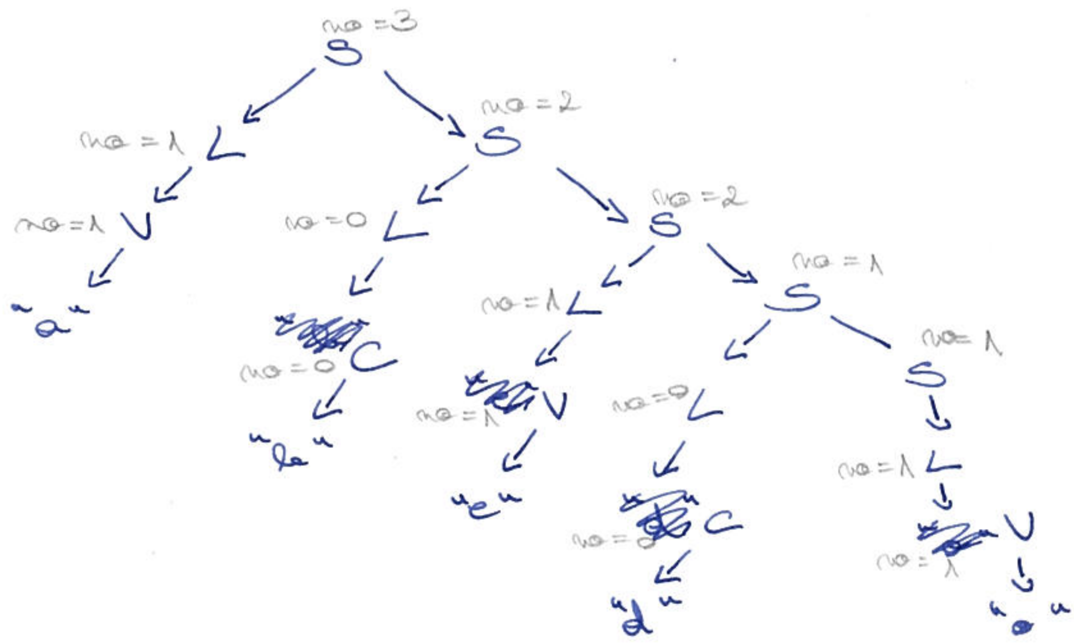
$L \to V$ $\qquad L_{(V)}.no = V.no = 1$

$L \to C$ $\qquad L_{(C)}.no = 0 = C.no$

$V \to \text{"a"} \mid \text{"e"} \mid \ldots$ $\qquad V.no = 1$

$C \to \text{"b"} \mid \text{"c"} \mid \ldots$ $\qquad C.no = 0$

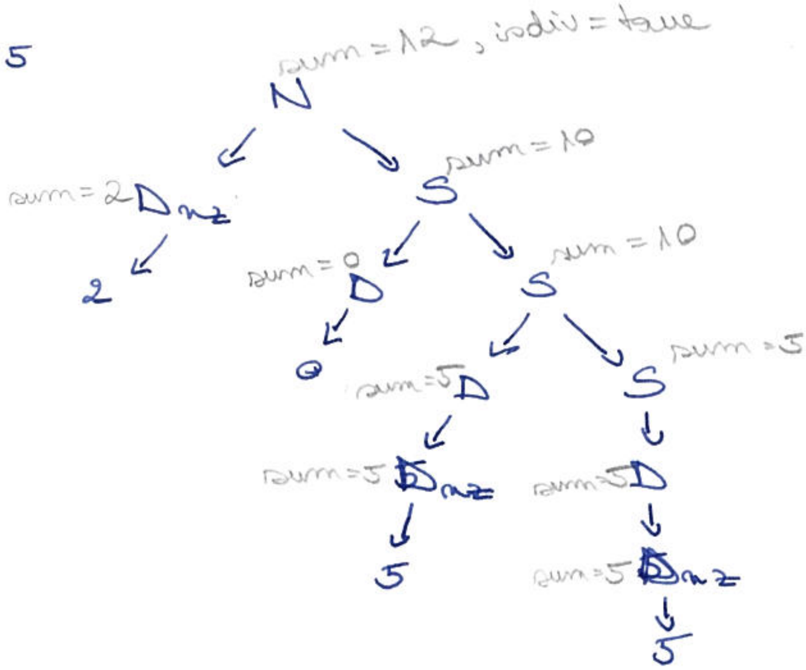
$w = abeda$



2) AG no : 3 $\quad A = \{sum, isdiv\}$

$N \to D$	$N.sum = D.sum$, $\underbrace{N.isdiv = (N.sum \% 3) == (}$
$N \to D_{nz} S$	$N.sum = D_{nz}.sum + S.sum$,
$D_{nz} \to 1$ ~~...~~	$D_{nz}.sum = 1$
⋮	⋮
$D_{nz} \to 9$	$D_{nz}.sum = 9$
$D \to 0$ ~~...~~	$D.sum = 0$
$D \to D_{nz}$	$D.sum = D_{nz}.sum$
$S \to D$	$S.sum = D.sum$
$S \to DS$	$S_1.sum = D.sum + S_2.sum$

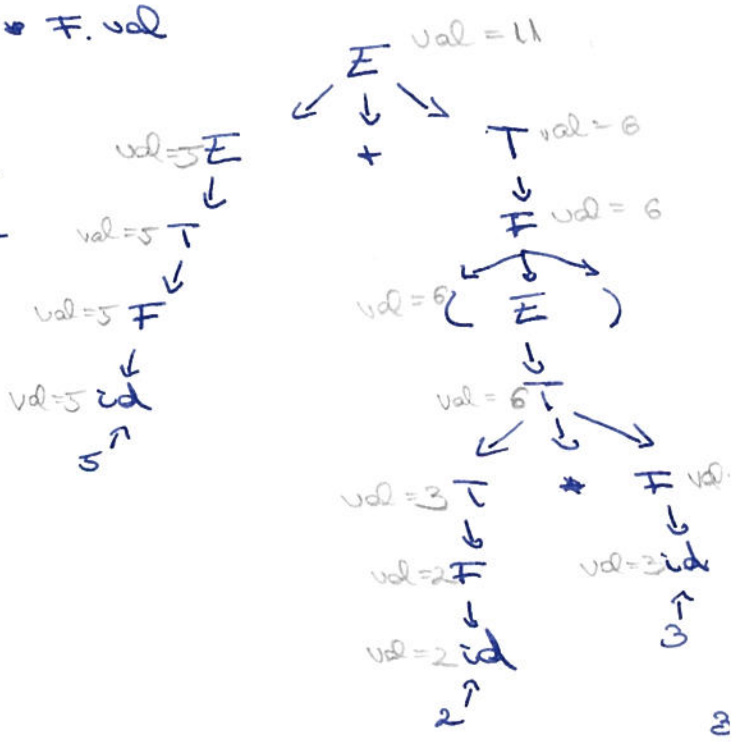
$w = 2055$



3) ? AG = value of an arithm. expr. with id, +, *, (,) $\quad A = \{val\}$

$E \to E + T$	$E_1.val = E_2.val + T.val$
$E \to T$	$E.val = T.val$
$T \to T * F$	$T_1.val = T_2.val * F.val$
$T \to F$	$T.val = F.val$
$F \to (E)$	$F.val = E.val$
$F \to id$	$F.val = id.val$

$w = id + (id + id)$



3 - Address Code

Ex. 1) if (a > 2) and (b < 7) or (a < b)
then
c := -2;
a := a + b;
else
a := a - 1;
endif
□

Label	op	arg1	arg2	rez
1	>	a	2	t_1
2	<	b	7	t_2
3	<	a	b	t_3
4	and	t_1	t_2	t_4
5 ~~or~~	or	t_3	t_4	t_5
6	goto	t_5		[10]
7	−	a	1	t_6 } else
8	:=	t_6		a } block
9	goto			13
10	:=	-2		c } if
11	+	a	b	t_7 } block
12	:=	t_7		a
13				

2) s := 0;

```
for i=1, n do
    s := s + i;
endfor
```

label	op	arg1	arg2	rez
1	:=	0		s
2	:=	1		i
3	~~=~~ >	i	n	t_1
4	goto	t_1		[10]
5	+	s	i	t_2
6	:=	t_2		s
7	+	i	1	t_3
8	:=	t_3		i
9	goto			[3]
10				

3) a := 0; flag := true;

```
for i=1 to n do
    if (a>3) or (j<0) and flag
      then
        flag := false
      else
        a := a + j * i
```

label	op	arg1	arg2	rez
1	:=	0		a
2	:=	true		flag
3	:=	1		i
~~3~~ 4	>	i	n	t_1
5	goto	t_1		[20]
6	>	a	3	t_2
7	<	j	0	t_3
8	==	flag	true	t_4
9	or	t_3	t_4	t_5
10	and	t_5	t_4	t_6
11	goto	t_6		[16]
~~12~~	:=	false		flag → else
~~13~~	goto			

label	op	arg1	arg2	rez
12	*	j	i	t_7
13	+	a	t_7	t_8
14	:=	t_8		a
15	goto			[17]
16	:=	false		flag
17	+	i	1	t_9
18	:=	t_9		i
19	goto			[4]
20				

} else block

→ if block

*) if a < b or "ceva" and "ceva"
then a:= expl
else a:= expr

label	op	arg1	arg2	rez
1	<	a	b	t_1
2	or	t_1	"ceva"	t_2
3	and	t_2	"ceva"	t_3
4	goto	t_3		[7]
5	:=	expr		a → else
6	goto			[8]
7	:=	expl		a → if
8				

5) repeat → while b == 0
cod

cod { a := 5
if a > 10 then b := a * 10 * c
else b := - c + a * b
a := a + 1 }

until b <> 0

label	op	arg1	arg2	rez
1	:=	5		a
2	>	a	10	t_1
3	goto	t_1		[8]
~~[illegible]~~	~~[illegible]~~ -	0	c	~~[illegible]~~
~~[illegible]~~	:=	t_2		
4	*	a	b	t_2
5	-	t_2	c	t_3
6	:=	~~[illegible]~~ t_3		b
7	goto			[11]
8	*	a	10	t_4
9	*	t_4	c	t_5
10	:=	t_5		b
11	==	b	0	t_6
12	goto	t_6		[13]
	da pașii 1-10			
13				

$\left.\begin{matrix} t_2 \\ t_3 \\ b \end{matrix}\right\}$ else

$\left.\begin{matrix} t_4 \\ t_5 \\ b \end{matrix}\right\}$ if