Ex. motite seminar : Seminar 6-7

I $\boxed{RG \Leftrightarrow FA}$

1) Given the regular grammar $G = (\{S, A\}, \{a, b\}, P, S)$

nonterminals, productions, terminals, start symbol

P: $S \to aA$

$A \to aA \mid bA \mid a \mid b$, build the equivalent FA.

$\delta(S, a) = A$ $\qquad$ $M = (Q, \Sigma, \delta, q_0, F)$

$\delta(A, a) = \{A, K\}$ ← not DFA

$\delta(A, b) = \{A, K\}$ $\qquad$ $Q = \{S, A, K\}$?, $\Sigma = \{a, b\}$

$q_0 = S$, $F = \{K, \cancel{S}\}$

*

2) Given the reg. grammar $G = (\{S, A\}, \{a, b\}, P, S)$

P: $S \to \varepsilon \mid aA$

$A \to aA \mid bA \mid a \mid b$, build the eq. FA.

$M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{S, A, K\}$?, $\Sigma = \{a, b\}$, $q_0 = S$, $F = \{K, S\}$

$\delta(S, a) = A$

$\delta(A, a) = \{A, K\}$

$\delta(A, b) = \{A, K\}$

*

δ	a	b
S	$\{A\}$	$\emptyset$
A	$\{A, K\}$	$\{A, K\}$
K	$\emptyset$	$\emptyset$

3) Given the foll. FA $M = (Q, \Sigma, \delta, q_0, F)$, $Q = \{p, q, r\}$, $q_0 = p$,

$F = \{r\}$, $\Sigma = \{0, 1\}$, build the eq. right linear grammar.

δ	0	1
p	q	p
q	r	p
r	r	r

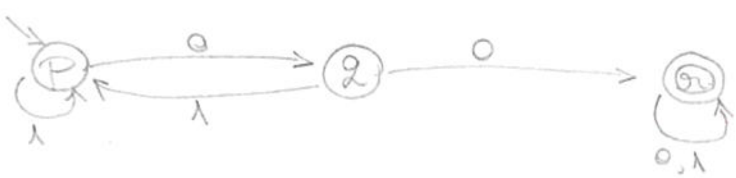
$\Rightarrow G = (N, \Sigma, P, S)$ $\qquad$ P: $p \to 0q \mid 1p$

$\Sigma = \{0, 1\}$ $\qquad$ $q \to 0r\}$ = $q \to 0r \mid 1p \mid 0$

$N = \{p, q, r\}$ $\qquad$ $r \to 0$ $\quad$ $r \to 0 \mid 1 \mid 0r \mid 1r$

bc r is final

$S = p$



construct M based on $G = (N, \Sigma, P, S)$

$Q = N \cup \{K\}$, $K \in N$

$q_0 = S$

$F = \{K\} \cup \{S \mid \text{if } S \to \varepsilon \in P\}$

δ: $A \to aB \Rightarrow \delta(A, a) = B$

$A \to a \Rightarrow \delta(A, a) = K$

4) Given the foll. FA $M = (Q, \Sigma, \delta, q_0, F)$, $Q = \{p, q, r\}$, $\Sigma = \{0, 1\}$, $q_0 = p$, $F = \{p, r\}$, build the eq. right linear gr.

δ	0	1
p	q	p
q	r	p
r	r	r

$G = \{N, \Sigma, P, S\}$ $\quad S = P$

$N = \{q, p, r\}$

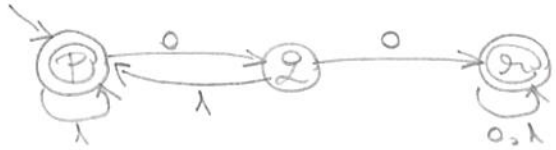
$\Sigma = \{0, 1\}$

?

$P: \ p \to 0q \mid 1p \mid 1 \mid \varepsilon$

$q \to 0r \mid 1p \mid 0 \mid 1$

$r \to 0r \mid 1r \mid 0 \mid 1$



? $F = \{p, r\}$

$\left.\begin{array}{l} p \to 1p \\ p = \text{final} \end{array}\right\} \Rightarrow p \to 1$

$\left.\begin{array}{l} p = \text{start} \\ p = \text{final} \end{array}\right\} p \to \varepsilon$

$\left.\begin{array}{l} q \to 1p \\ p = \text{final} \end{array}\right\} \to q \to 1$

$\left.\begin{array}{l} q \to 0r \\ r = \text{final} \end{array}\right\} \Rightarrow q \to 0$

II $\boxed{RG \Leftrightarrow RE}$

1) Give the RG corr. to the foll. RE $0(0 \overset{\text{sau}}{+} 1)^* 1$

$0: \ G_1 = (\{S_1\}, \{0, 1\}, \{S_1 \to 0\}, S_1)$

$1: \ G_2 = (\{S_2\}, \{0, 1\}, \{S_2 \to 1\}, S_2)$

$0+1: \ G_3 = (\{S_1, S_2, S_3\}, \{0, 1\}, \{S_1 \to 0, S_2 \to 1, S_3 \to 0 \mid 1\}, S_3)$

$G_3' = (\{S_3\}, \{0, 1\}, \{S_3 \to 0 \mid 1\}, S_3)$

$(0+1)^*: \ G_4 = (\{S_3\}, \{0, 1\}, \{S_3 \to 0 \mid 1, S_3 \to 0S_3 \mid 1S_3, S_3 \to \varepsilon\}, S_3)$

$G_4' = (\{S_3\}, \{0, 1\}; \{S_3 \to 0S_3 \mid 1S_3 \mid \varepsilon\}, S_3)$! not regular

$0(0+1)^*: \ G_5 = (\{S_1, S_3\}, \{0, 1\}, \{S_1 \to 0S_3, S_3 \to 0S_3 \mid 1S_3 \mid \varepsilon\}, S_1)$! not reg.

$0(0+1)^* 1: \ G_6 = (\{S_1, S_2, S_3\}, \{0, 1\}, \{S_1 \to 0S_3, S_3 \to 0S_3 \mid 1S_3 \mid S_2, S_2 \to 1\}, S$ not reg.

$G_6' = (\{S_1, S_3\}, \{0, 1\}, \{S_1 \to 0S_3, S_3 \to 0S_3 \mid 1S_3 \mid 1\}, S_1)$

2) Give the RE corr. to the foll. gr. $G = (\{S, A, B\}, \{a, b\}, P, S)$

$\boxed{X = aX + b \Rightarrow X = a^* b}$

$P: \left.\begin{array}{l} S \to aA \\ A \to aA \mid bB \mid b \\ B \to bB \mid b \end{array}\right\} \Leftrightarrow \begin{array}{l} S = aA \\ A = aA + \overbrace{bB + b}^{B} \\ B = bB + b = b^* b = b^+ \ ? \end{array}$

$\Rightarrow A = aA + B = a^* B = a^* b^+$

$S = aA = aa^* b^+ = a^+ b^+$?

right linear grammar $\Leftrightarrow$ all productions are of the form

1) nonterm. $\to$ term. nonterm.

2) nonterm. $\to$ term.

regular grammar $\Leftrightarrow$ 1) of right linear

2) only $S \to \varepsilon \in P \Rightarrow S$ can't appear in rhs. in other productions

$A \to \varepsilon \notin P$

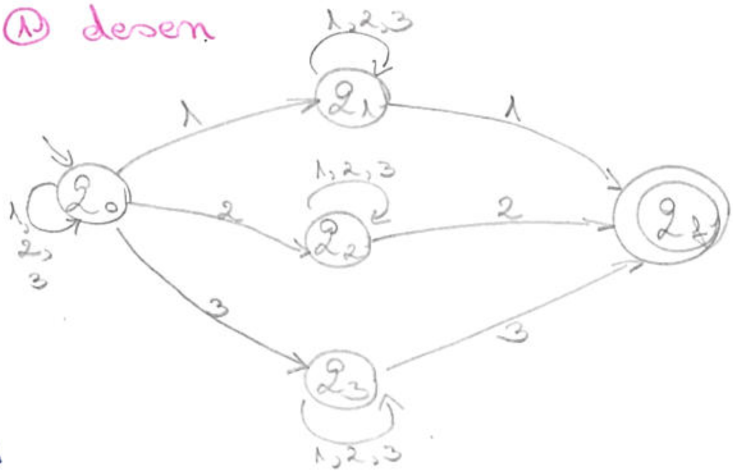
III) $\boxed{FA \Leftrightarrow RE}$

Seminar 4-5 : FA

1) Given the FA: $M=(Q,\Sigma,\delta,q_0,F)$, $Q=\{q_0,q_1,q_2,q_3,q_f\}$, $\Sigma=\{1,2,3\}$, $F=\{q_f\}$. Prove that $w=12321 \in L(M)$

δ	1	2	3
q_0	$\{q_0,q_1\}$	$\{q_0,q_2\}$	$\{q_0,q_3\}$
q_1	$\{q_1,q_f\}$	$\{q_1\}$	$\{q_1\}$
q_2	$\{q_2\}$	$\{q_2,q_f\}$	$\{q_2\}$
q_3	$\{q_3\}$	$\{q_3\}$	$\{q_3,q_f\}$
q_f	$\emptyset$	$\emptyset$	$\emptyset$

① desen



② pornește de la (q_0,w) ca să aj. la (q_f,ε)

$(\overset{q_1}{q_0},12321)\vdash(\overset{q_1}{q_1},2321)\vdash(\overset{q_1}{q_1},321)\vdash(\overset{q_1}{q_1},21)\vdash(\overset{q_f}{q_1},1)\vdash$

$\vdash(q_f,\varepsilon)\Rightarrow w\in L(M)$

2) Find the language accepted by this FA:

$\Rightarrow Q=\{p,q\}$ $\quad F=\{q\}$

$\Sigma=\{a,b\}$

↳ b, ab, aab, aa...ab...b $\Rightarrow L=\{a^n b^m \mid n\in\mathbb{N}, m\in\mathbb{N}^*\}$

? $L=L(M)$

① $L\subseteq L(M)$ (all seqs. of that shape are accepted by M)

$\forall n\in\mathbb{N}, m\in\mathbb{N}^*$ $\quad a^n b^m\in L(M)$

Let $n\in\mathbb{N}, m\in\mathbb{N}^*$

$(p,a^n b^m)\overset{n}{\underset{(a)}{\vdash}}(p,b^m)\underset{(b)}{\vdash}(q,b^{m-1})\overset{m-1}{\underset{(b)}{\vdash}}(q,\varepsilon)\Rightarrow a^n b^m\in L(M)$

a) $(p,a^n)\overset{n}{\vdash}(p,\varepsilon),\ \forall n\in\mathbb{N}$ $\quad\Rightarrow$ inducție

b) $(q,b^k)\overset{k}{\vdash}(q,\varepsilon),\ \forall k\in\mathbb{N}$

② $L(M)\subseteq L$ (M does not accept anything else but sequences of that shape)

Seminar 8 : CFG

1) Given the CFG grammars below, give a leftmost / rightmost derivation for w.

a) $G = (\{S, A, B\}, \{0, 1\}, \{S \to 0B \mid 1A,\ A \to 0 \mid 0S \mid 1AA,\ B \to 1 \mid 1S \mid 0BB\})$

$w = 0001101110$

$S \to 0B \mid 1A$ (rules 1, 2)
$A \to 0 \mid 0S \mid 1AA$ (rules 3, 4, 5)
$B \to 1 \mid 1S \mid 0BB$ (rules 6, 7, 8)

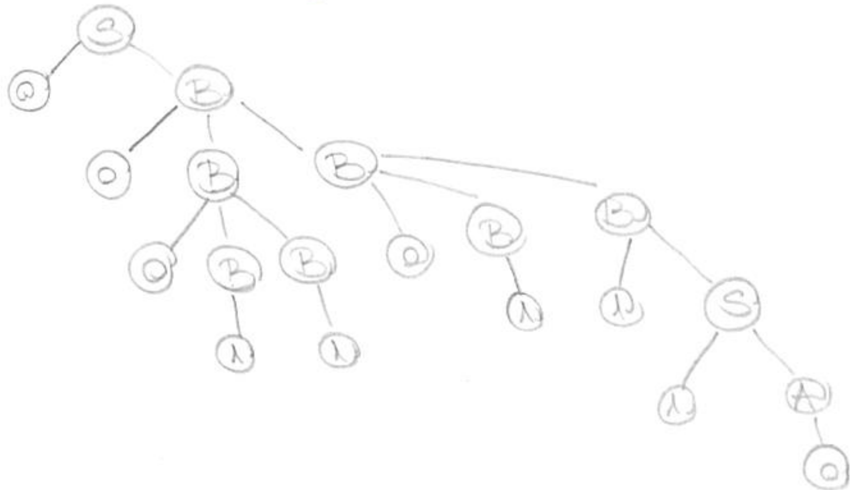
L: $S \underset{1}{\Rightarrow} 0B \underset{8}{\Rightarrow} 00BB \underset{8}{\Rightarrow} 000BBB \underset{7}{\Rightarrow}$ *

$\Rightarrow$ ~~$0001SBB \underset{2}{\Rightarrow} 00011ABB \underset{3}{\Rightarrow} 000110BB \underset{7}{\Rightarrow}$~~

$\Rightarrow$ ~~$0001101SB$~~

* $000BBB \underset{6}{\Rightarrow} 0001BB \underset{6}{\Rightarrow} 00011B \underset{8}{\Rightarrow} 000110BB \underset{6}{\Rightarrow} 0001101B \underset{7}{\Rightarrow}$

$\Rightarrow 00011011S \underset{2}{\Rightarrow} 000110111A \underset{3}{\Rightarrow} 0001101110$



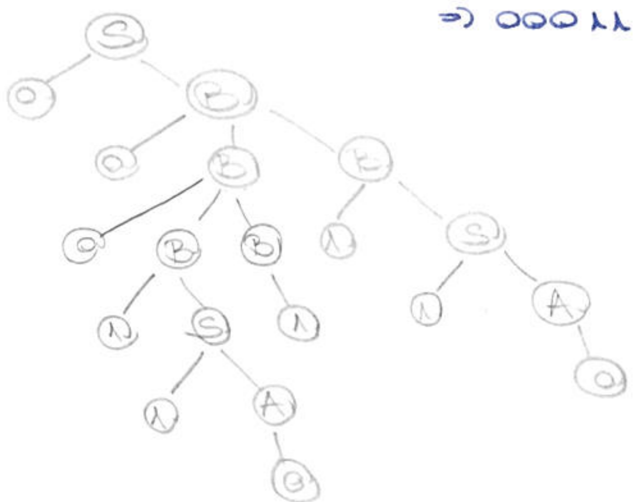
$1S \to 11A \to 110$

R: $S \underset{1}{\Rightarrow} 0B \underset{8}{\Rightarrow} 00BB \underset{8}{\Rightarrow} 00B0BB \Rightarrow$

$\underset{7}{\Rightarrow} 00B1S \underset{2}{\Rightarrow} 00B11A \underset{3}{\Rightarrow} 00B110 \underset{8}{\Rightarrow} 000BB110$ (110)

$\underset{6}{\Rightarrow} 000B1110 \underset{7}{\Rightarrow} 0001S1110 \underset{2}{\Rightarrow} 00011A1110 \underset{3}{\Rightarrow}$

$\Rightarrow 0001101110 = w$



2) Prove that the foll. gr. is ambiguous: a) $G = (\{E\}, \{a, +, *, (,)\}, P)$

$P: E \to E + E \mid E * E \mid (E) \mid a$

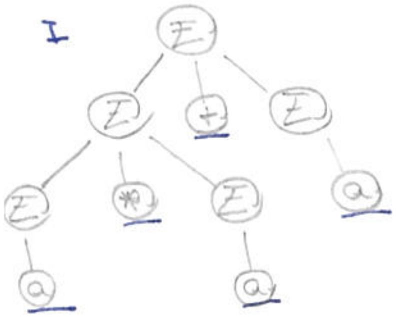
$w = a * a + a$

I (L) $E \underset{1}{\Rightarrow} \underline{E} + E \underset{2}{\Rightarrow} E * E + E \underset{4}{\overset{3}{\Rightarrow}} a * a + a$

II (R) $E \underset{2}{\Rightarrow} E * \underline{E} \underset{1}{\Rightarrow} E * E + E \underset{4}{\overset{3}{\Rightarrow}} a * a + a$

$\Rightarrow$ diff. syntax trees

I



b) $G = (\{S\}, \{if, then, else, a, b\}, P, S)$

$P: S \to$ if b then S | if b then S else S | a

$w =$ if b then (if b then a) else a

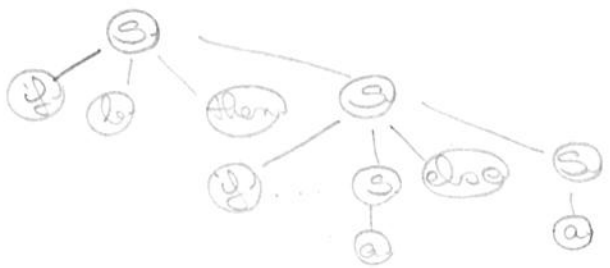
I $S \underset{1}{\Rightarrow}$ if b then $\underline{S}$ $\underset{2}{\Rightarrow}$ if b then if b then S else S $\underset{3}{\overset{2}{\Rightarrow}}$

$\Rightarrow$ if b then if b then a else a

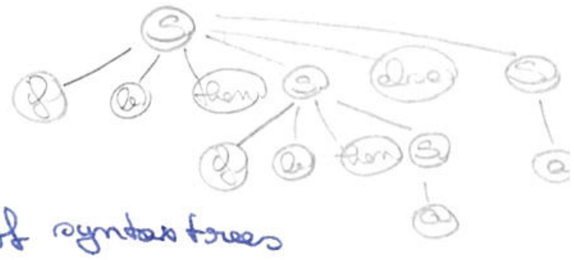
II $S \underset{2}{\Rightarrow}$ if b then $\underline{S}$ else S $\underset{1}{\Rightarrow}$ if b then if b then S else S $\underset{3}{\overset{2}{\Rightarrow}}$

$\Rightarrow$ if b then if b then a else a

I



II



$\Rightarrow$ diff syntax trees

RDP: 1) Given the CFG $G = (\{S\}, \{a, b, c\}, \{S \to aSbS \mid aS \mid c\})$,

parse the seq. $w = aacbc$ (1 2 3 4 5) using rec. desc. parser.

work stack, input stack   ① $S \to aSbS$

$(q, 1, \varepsilon, S) \vdash^{exp} (q, 1, S_1, aSbS) \vdash^{adv} (q, 2, S_1 a, SbS) \vdash^{exp}$

$\vdash (q, 2, S_1 a S_1, aSbSbS) \vdash^{adv} (q, 3, S_1 a S_1 a, \boxed{SbSbS}) \vdash^{exp}$

$\vdash (q, 3, S_1 a S_1 a S_1, aSbS\boxed{bSbS}) \vdash^{mi} (b, 3, S_1 a S_1 a S_1, \boxed{aSbSbSbS})$ (nonterm)

$\vdash^{at} (q, 3, S_1 a S_1 a S_2, aS\boxed{bSbS}) \vdash^{mi} (b, 3, S_1 a S_1 a S_2, aS\boxed{bSbS}) \vdash^{at}$

$\vdash (q, 3, S_1 a S_1 a S_3, \boxed{cbSbS}) \vdash^{adv} (q, 4, S_1 a S_1 a S_3 c, bSbS) \vdash^{adv}$

$\vdash (q, 5, S_1 a S_1 a S_3 c b, SbS) \vdash^{exp} (q, 5, S_1 a S_1 a S_3 c b S_1, aSbSbS)$

$\vdash^{mi} (b, 5, S_1 a S_1 a S_3 c b S_1, aSbS\boxed{bS}) \vdash^{at} (q, 5, S_1 a S_1 a S_3 c b S_2, aSbS)$

$\vdash^{mi} (b, 5, S_1 a S_1 a S_3 c b S_2, aS\boxed{bS}) \vdash^{at} (q, 5, S_1 a S_1 a S_3 c b S_3, cbS) \vdash^{adv}$

$\vdash (q, 6, S_1 a S_1 a S_3 c b S_3 c, bS) \vdash^{mi} (b, 6, S_1 a S_1 a S_3 c b S_3 c, bS) \vdash^{back}$ (term)

nonterm ⇒ at   term

$\vdash (b, 5, S_1 a S_1 a S_3 c b S_3, cbS) \vdash^{at} (b, 5, S_1 a S_1 a S_3 c b, SbS) \vdash^{back}$

$\vdash (b, 4, S_1 a S_1 a S_3 c, bSbS) \vdash^{back} (b, 3, S_1 a S_1 a S_3, cbSbS) \vdash^{at}$ (4-1)

$\vdash (b, 3, S_1 a S_1 a, SbSbS) \vdash^{back} (b, 2, S_1 a S_1, aSbS\boxed{bS}) \vdash^{at}$

$\vdash (q, 2, S_1 a S_2, aSbS) \vdash^{adv} (q, 3, S_1 a S_2 a, SbS) \vdash^{exp+mi+at+mi+at}_{\text{same jump to } S_3}$

$\vdash (q, 3, S_1 a S_2 a S_3, cbS) \vdash^{adv} (q, 4, S_1 a S_2 a S_3 c, bS) \vdash^{adv}$

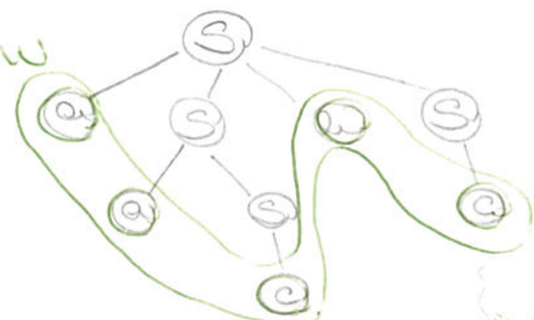
$\vdash (q, 5, S_1 a S_2 a S_3 c b, S) \vdash^{exp+mi+at+mi+at}_{\text{jump to } S_3} (q, 5, S_1 a S_2 a S_3 c b S_3, c) \vdash$ (nonterm)

$(q, 6, S_1 a S_2 a S_3 c b S_3 c, \varepsilon)$

adv. $(q, 5, S_1 a S_2 a S_3 c b S_3 c, \varepsilon) \vdash^{success} (f, 6, S_1 a S_2 a S_3 c b S_3 c, \varepsilon)$

⇒ $w = aacbc$ is syntactically correct   input stack: abcd... → head

parse tree $= S_1 S_2 S_3 S_3$



I EXPAND – when head of input is a nonterminal

ex.: $(q, 1, \varepsilon, S) \vdash^{exp} (q, 1, S_1, aS)$ (nonterm)

$(S_1): S \to aS$

II ADVANCE – when head of input stack is terminal = corr. symbol from input

input: $w = aacbc$, $i = 1\ 2\ 3\ 4\ 5$ (clock)

$(q, i, \alpha, a_i\beta) \vdash^{adv} (q, i+1, \alpha a_i, \beta)$

III MOMENTARY INSUCCESS – when head of input stack is a terminal ≠ corr. symbol from input

ex.: $(q, i, \alpha, a_i\beta) \vdash^{mi} (b, i, \alpha, a_i\beta)$

w. stack: a b c d ← head

V ANOTHER TRY – when head of working stack is a nonterminal (after m.i.)

$(b, i, \alpha A_j, \gamma_j \beta) \vdash (q, i, \alpha A_{j+1}, \gamma_{j+1}\beta)$, if ∃ $A \to \gamma_{j+1}$

$\vdash (b, i, \alpha, A\beta)$

$\vdash (e, i, \alpha, \beta)$, if $i = 1$, $A = S$ error

IV BACK – after m.i., when head of working stack is a terminal

ex.: $(b, i, \alpha a, \beta) \vdash^{back} (b, i-1, \alpha, a\beta)$

Seminar 8 : LL(1) parser

Given the cfg. $G = (\{S, A, B, C, D\}, \{+, *, a, (,)\}, P, S)$

$w = a * (a+a)$

P:
(1) $S \to BA$
(2) $A \to +BA$
(3) $A \to \varepsilon$
(4) $B \to DC$
5) $C \to *DC$
6) $C \to \varepsilon$
7) $D \to (S)$
8) $D \to a$

I) FIRST

	F_0	F_1	F_2	$= F_3$ ← stop
S	$\emptyset$	$\emptyset$	(, a	(, a
A	+, ε	+, ε	+, ε	+, ε
B	$\emptyset$	(, a	(, a	(, a
C	*, ε	*, ε	*, ε	*, ε
D	(, ~~ε~~, a	(, a	(, a	(, a

$\Rightarrow$ FIRST

only first terminal

$(, a \oplus +, \varepsilon = \{(+, (\varepsilon, a+, a\varepsilon\} = (, a$

$F_1(S) = F_0(S) \cup \{x \mid S \to BA,\ x \in F_0(B) \oplus F_0(A)\} =$
$= \emptyset \cup \{\emptyset\} = \emptyset$

$\{+, \varepsilon\} \cup \{+ \oplus \emptyset \oplus \{+, \varepsilon\}\} = \{+, \varepsilon\}$

$\{(, a\} \cup \{\emptyset \cup \{F_0(D) \oplus F_0(C)\}\} = \{(, a\}$

$(, a \oplus *, \varepsilon = \{(*, (\varepsilon, a*, a\varepsilon\}$

$\{*, \varepsilon\} \cup \{* \oplus \{(, a\} \oplus \{*, \varepsilon\}\}$
$\Downarrow$
$\{*, \varepsilon\}$

$* \oplus (, a = *(= *$
$* \oplus \varepsilon = *$

$\{(, a\} \cup \{(\oplus \emptyset \oplus) \oplus a\} = \{(, a\}$

II) FOLLOW

	L_0	L_1	L_2	L_3
S	ε	ε,)	ε,)	ε,)
A	$\emptyset$	ε	ε,)	ε,)
B	$\emptyset$	+, ε	+, ε,)	+, ε,)
C	$\emptyset$	$\emptyset$	+, ε	+, ε,)
D	$\emptyset$	*	*, +, ε	*, +, ε,)

L_4 same

L_0 always like this

$D \to (S) \Rightarrow \beta =)$
$F(\beta) =) \Rightarrow L_1(S) =) \cup L_0(S)$
$S \to BA \Rightarrow \beta = \varepsilon \Rightarrow L_1(A) = L_0(S) = \varepsilon$
$S \to BA \Rightarrow \beta = A;\ F(A) = \{+, \varepsilon\}$
$A \to +BA \Rightarrow \beta = A;\ F(A) = \{+, \varepsilon\}$
$\exists \varepsilon \Rightarrow L_0(S) = \varepsilon$
$L_0(A) = \emptyset$ } se adaugă
$B \to DC \Rightarrow \beta = \varepsilon \Rightarrow L_0(B) = \emptyset$
$C \to *DC \Rightarrow \beta = \varepsilon \Rightarrow L_0(C) = \emptyset$
$B \to DC \Rightarrow \beta = C \Rightarrow F(C) = *, \varepsilon$
$C \to *DC \Rightarrow \beta = C \Rightarrow F(C) = *, \varepsilon$
$\exists \varepsilon \Rightarrow L_0(B) = \emptyset$
$L_0(C) = \emptyset$

L_2: $D \to (S) \Rightarrow F()) =) \Rightarrow L_2(S) =)$
$A \to +BA \Rightarrow \beta = \varepsilon \Rightarrow L_1(A) = \varepsilon$
$S \to BA \Rightarrow \beta = \varepsilon \Rightarrow L_1(S) = \varepsilon,)$
$A \to +BA \Rightarrow \beta = A \Rightarrow F(A) = +, \varepsilon$
$S \to BA \Rightarrow \beta = A \Rightarrow F(A) = +, \varepsilon$
$\exists \varepsilon \Rightarrow L_1(A) = \varepsilon$
$L_1(S) = \varepsilon,)$ $\Rightarrow L_2(B) = +, \varepsilon,)$

$B \to DC \Rightarrow \beta = \varepsilon \Rightarrow L_1(B) = +, \varepsilon$
$C \to *DC \Rightarrow \beta = \varepsilon \Rightarrow L_1(C) = \emptyset$
$B \to DC \Rightarrow \beta = C \Rightarrow F(C) = *, \varepsilon$
$C \to *DC \Rightarrow \beta = C$
$\exists \varepsilon \Rightarrow L_1(B) = +, \varepsilon$
$L_1(C) = \emptyset$

$S \xrightarrow{\text{for } A} A \Rightarrow \beta = \varepsilon \Rightarrow L_{i-1}(S) = \ldots$
$S \to A* \Rightarrow \beta = * \Rightarrow F(\beta) = * \Rightarrow L_i(A) =$
$S \to AB \Rightarrow \beta = B \Rightarrow F(\beta) = F(B) = \alpha$
if $\varepsilon \in \alpha \Rightarrow L_{i-1}(S) = \ldots$

III) Parsing table

	a	+	*	(	)	\$
S	BA, 1			BA, 1		
A		+BA, 2			ε, 3	ε, 3
B	DC, 4			DC, 4		
C		ε, 6	*DC, 5		ε, 6	ε, 6
D	a, 8			(S), 7		
a	pop					
+		pop				
*			pop			
(				pop		
)					pop	
\$						acc

$\leftarrow \Sigma \cup \$$

$N \cup \Sigma \cup \$$

(term, term) = pop

$S \to BA$ (1)

$F(BA) = F(B) \oplus F(A) = (, a \oplus +, \varepsilon = \boxed{(, a}$

$\Rightarrow (S, () = (BA, 1)$

$(S, a) = (BA, 1)$

$A \to +BA$ (2)

$F(+BA) = F(+) \oplus F(B) \oplus F(A) = + \oplus \{(, a\} = +$

$\Rightarrow (A, +) = (+BA, 2)$

$A \to \varepsilon$ (3)

$F(\varepsilon) = \varepsilon \Rightarrow (\varepsilon, i)$

$(A, y) = (\varepsilon, i)$

$y \in Foll(A) = \varepsilon \to \$$

(4) $B \to DC$; $F(DC) = F(D) \oplus F(C) = (, a \oplus *, \varepsilon = \{(, a\}$

$\Rightarrow (B, () = (DC, 4)$; $(B, a) = (DC, 4)$

(5) $C \to *DC$; $F(*DC) = F(*) \oplus F(D) \oplus F(C) = * \oplus \{(, a\} = *$

$\Rightarrow (C, *) = (*DC, 5)$

(6) $C \to \varepsilon$; $F(\varepsilon) = \varepsilon \Rightarrow (\varepsilon, 6)$; $Foll(C) = \{+, \varepsilon,)\} \Rightarrow (C, +) = (\varepsilon, 6)$

...

IV) Parse the seq. $w = a * (a + a)$

$(w\$, S\$, \varepsilon) \vdash (a*(a+a)\$, S\$, \varepsilon) \vdash (a*(a+a)\$, BA\$, 1) \vdash (a*(a+a)\$, DCA\$, 14)$

$\vdash (a*(a+a)\$, aCA\$, 148) \vdash (*(a+a)\$, CA\$, 148) \vdash (*(a+a)\$, *DCA\$, 1485)$

$\vdash ((a+a)\$, DCA\$, 1485) \vdash ((a+a)\$, (S)CA\$, 14857) \vdash (a+a)\$, S)CA\$, 14857)$

$\vdash (a+a)\$, BA)CA\$, 148571) \vdash (a+a)\$, DCA)CA\$, 1485714) \vdash$

$\vdash (a+a)\$, aCA)CA\$, 14857148) \vdash (+a)\$, CA)CA\$, 14857148) \vdash$

$\vdash (+a)\$, A)CA\$, 148571486) \vdash (+a)\$, +BA)CA\$, 1485714862) \vdash$

$\vdash (a)\$, BA)CA\$, 1485714862) \vdash (a)\$, DCA)CA\$, 14857148624) \vdash$

$\vdash (a)\$, aCA)CA\$, 148571486248) \vdash ()\$, A)CA\$, 14857148624863) \vdash$

$\vdash (\$, CA\$, 148....63) \vdash (\$, A\$, 148...636) \vdash (\$, \$, 148....6363) \vdash$

$\Rightarrow 148571486248 6363$

LL(1) conflict

$A \to \alpha\beta$

$A \to \alpha\gamma$ $\Big\} \Rightarrow$ $A \to \alpha B$

$B \to \beta \mid \gamma$

Seminar 10 : LR(0)

ex.: $G = (\{S', S, A\}, \{a, b, c\}, P, S')$ $w = abbc$

P: (0) $S' \to S$ (2) $A \to bA$

(1) $S \to aA$ (3) $A \to c$

I. Compute the canonical coll. of states:

if state $s = [A \to \alpha . B\beta]$, where B = nonterm ⇒ closure + all prods. of B ($[B \to .\delta]$)

$s = (s, X)$: if the dot is in front of X ⇒ move the dot after X ⇒ apply closure ; if there is no dot in front of X, the res. is ∅

always start with this

$\boxed{s_0 = \cancel{goto}\ closure(\{[S' \to .S]\})} = \{[S' \to .S], [S \to .aA]\}$

for each new state s, we compute goto of s and $N \cup \Sigma / \{S'\}$

$s_1 = goto(s_0, S) = closure(\{[S' \to S.]\}) = \{[S' \to S.]\}$

$goto(s_0, A) = \emptyset$ $goto(s_0, b) = \emptyset$ $goto(s_0, c) = \emptyset$

$s_2 = goto(s_0, a) = goto(\{[S \to .aA]\}) = goto(\{[S \to a.A]\}) =$

$s_2 = closure(\{[S \to a.A]\}) = \{[S \to a.A], [A \to .bA], [A \to .c]\}$

$s_3 = goto(s_2, A) = goto(\{[S \to a.A]\}) = goto(\{[S \to aA.]\}) =$

$= closure(\{[S \to aA.]\}) = \{[S \xrightarrow{(1)} aA.]\}$ no dot in front of A

$s_4 = goto(s_2, b) = goto(\{[A \to .bA]\}) = goto(\{[A \to b.A]\}) =$

$= closure(\{[A \to b.A]\}) = \{[A \to b.A], [A \to .bA], [A \to .c]\}$

$s_5 = goto(s_2, c) = closure(\{[A \to c.]\}) = \{[A \xrightarrow{(3)} c.]\}$

$s_6 = goto(s_4, A) = closure(\{[A \to bA.]\}) = \{[A \xrightarrow{(2)} bA.]\}$

$goto(s_4, b) = closure(\{[A \to b.A]\}) = s_4$ $\}$ $goto(s_4, c) = s_5$

goto: $\to N \cup \Sigma / \{S'\}$

II. Parsing table

states	action	S	A	a	b	c
s_0	shift	s_1		s_2		
s_1	acc.					
s_2	shift		s_3		s_4	s_5
s_3	reduce 1*					
s_4	shift		s_6		s_4	s_5
s_5	reduce 3*					
s_6	reduce 2*					

III) <u>Parse the input sequence</u>: init. config $= (\$ s_0, w\$, \varepsilon) \to$ final $= (\$ s_{acc}, \$, \tilde{\pi})$

work stack $= \alpha \to$	$\leftarrow$ input stack $= \beta$	output band $= \pi$
$\$ s_0$ shift	$\leftarrow a b b c \$$	ε
$\$ s_0 a s_2$ always ends with a state shift	$\leftarrow b b c \$$	ε
$\$ s_0 a s_2 b s_4$ shift	$\leftarrow b c \$$	ε
$\$ s_0 a s_2 b s_4 b s_4$ shift	$c \$$	ε
search for a rhp $\$ s_0 a s_2 b s_4 b s_4 c s_5$ red 3	$\$$	ε
$\$ s_0 a s_2 b s_4 b s_4 A s_6$ red 2	$\$$	3
$\$ s_0 a s_2 b s_4 A s_6$ red 2	$\$$	2 3
$\$ s_0 a s_2 A s_3$ red 1	$\$$	2 2 3
$\$ s_0 S s_1$ acc	$\$$	1 2 2 3
	finished	

Seminar 11-12: $\boxed{SLR}$

1) $G = (\{S', E, T\}, \{+, id, const, (,)\}, P, S')$ $\quad w = id + const$

P: $S' \to E$ $\quad$ (2) $E \to E + T$ $\quad$ (4) $T \to id$

(1) $E \to T$ $\quad$ (3) $T \to (E)$ $\quad$ (5) $T \to const$

I) <u>Canonical collection:</u>

$s_0 = closure(\{[S' \to .E]\}) = \{[S' \to .E], [E \to .T], [E \to .E+T],$
$[T \to .(E)], [T \to .id], [T \to .const]\}$

$s_1 = goto(s_0, E) = closure(\{[S' \to E.], [E \to E.+T]\}) =$
$= \{[S' \to E.], [E \to E.+T]\}$

$s_2 = goto(s_0, T) = closure(\{[E \to T.]\}) = \{[E \to T.]\}$

$s_3 = goto(s_0, () = closure(\{[T \to (.E)]\}) = \{[T \to (.E)], [E \to .T]$
$, [E \to .E+T], [T \to .(E)], [T \to .id], [T \to .const]\}$

$s_4 = goto(s_0, id) = closure(\{[T \to id.]\}) = \{[T \to id.]\}$

$s_5 = goto(s_0, const) = closure(\{[T \to const.]\}) = \{[T \to const.]\}$

$s_6 = goto(s_1, +) = closure(\{[E \to E+.T]\}) = \{[E \to E+.T], [T \to .(E)]$
$, [T \to .id], [T \to .const]\}$

$s_7 = goto(s_3, E) = closure(\{[T \to (E.)], [E \to E.+T]\}) = \{[T \to (E.)],$
$[E \to E.+T]\}$

$s_8 = goto(s_6, T) = closure(\{[E \to E+T.]\}) = \{[E \to E+T.]\}$

$s_9 = goto(s_7,)) = closure(\{[T \to (E).]\}) = \{[T \to (E).]\}$

$Follow(E) = Follow(T) = \{\varepsilon, +,)\}$

II Fill the table: action = Σ, goto = N

states	+	(	)	id	const	$	E	T
s_0		shift 3		shift 4	shift 5		s_1	s_2
s_1	shift 6					acc.		
s_2	reduce 1		reduce 1			red. 1		
s_3		shift 3		shift 4	shift 5		s_7	s_2
s_4	red. 4		red. 4			red. 4		
s_5	red. 5		red 5			red 5		
s_6		shift 3		shift 4	shift 5			s_8
s_7	shift 6		shift 9					
s_8	red 2		red 2			red 2		
s_9	red 3		red 3			red 3		

1) dacă în s avem prod. cu punct care NU e la final și se află în fața unui terminal => shift i (= action(s, term)), s_i = goto (s, term)
s_3 = goto (s_0, ( ) => action(s_0, ( ) = s_3

2) dacă (1) [S' → E.] ∈ s => action(s, $) = acc.

3) dacă în s avem prod. i cu punct la final => action(s, Follow(X)) = red

ex.: (i) [X → A.] ∈ s
Follow(A) = {a, b}
=> action(s, a) = reduce i
action(s, b) = reduce i

work stack →		← input stack	output band
$ s_0	shift 4	id + const $	ε
$ s_0 id s_4	red 4) T → id	+ const $	ε
$ s_0 T s_2	red 1 E → T	+ const $	4
$ s_0 E s_1	shift 6	+ const $	1 4
$ s_0 E s_1 + s_6	shift 5	const $	1 4
$ s_0 E s_1 + s_6 const s_5	red 5 T → const	$	1 4
$ s_0 E s_1 + s_6 T s_8	red 2 E → E + T	$	5 1 4
$ s_0 E s_1		$	$\boxed{2\ 5\ 1\ 4}$
acc.			

PDA: 1) Să se constr. un PDA care acceptă limbajul $L=\{a^n b^n \mid n \geq 0\}$

$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0, q_1\}$, $\Sigma = \{a, b\}$, $\Gamma = \{Z_0, A\}$

$F = \emptyset$ (PDA acceptă limbajul după criteriul stivei vide)

$\delta : \{q_0, q_1\} \times \{a, b, \varepsilon\} \times \{Z_0, A\} \to \mathcal{P}(\{q_0, q_1\} \times \{Z_0, A\}^*)$

↳ q_0 - starea în care se citesc repetat simboluri a de pe banda de intrare, adăugându-se câte un A în stivă

↳ q_1 - starea în care se trece după citirea primului b de pe banda de intrare, apoi se vor citi repetat b-uri, ștergându-se câte un A din stivă pt. fiecare b

1. $\delta(q_0, \varepsilon, Z_0) = \{(q_0, \varepsilon)\}$ - este acc. secv. vidă ε prin vidarea stivei

2. $\delta(q_0, b, Z_0) = \emptyset$ - cuv. nu poate să înceapă cu b

3. $\delta(q_0, a, Z_0) = \{(q_0, AZ_0)\}$ - s-a citit primul a și s-a adăugat un A în stivă

4. $\delta(q_0, \varepsilon, A) = \emptyset$ - nu e o situație validă pt. că am terminat de citit cuv. și nu am citit niciun b, forma cuv. nu e cea dorită (cuv. e format numai din a)

5. $\delta(q_0, a, A) = \{(q_0, AA)\}$ - citit a ⇒ adăugat A în stivă

6. $\delta(q_0, b, A) = \{(q_1, \varepsilon)\}$ - s-a citit primul b și s-a șters un A - s-a trecut în starea 1 (stare de ștergere)

7. $\delta(q_1, \varepsilon, A) = \emptyset$ - s-a terminat de citit cuv., dar în stivă au mai rămas A ⇒ cuv. are mai multe a decât b

8. $\delta(q_1, b, A) = \{(q_1, \varepsilon)\}$ - stare de ștergere: se citește un b de pe banda de intrare ⇒ se șterge un A din stivă

9. $\delta(q_1, a, A) = \emptyset$ - * din ac. stare de ștergere nu se poate citi a de pe banda de intrare pt. că forma cuv. nu ar fi bună (ar alterna a-urile și b-urile)

10. $\delta(q_1, \varepsilon, Z_0) = \{(q_1, \varepsilon)\}$ - cuv. a fost citit, în stivă avem doar Z_0 deci îl ștergem, stiva devine vidă ⇒ cuv. este acceptat

11. $\delta(q_1, a, Z_0) = \emptyset$ - *

12. $\delta(q_1, b, Z_0) = \emptyset$ - citim b, dar în stivă nu există A pt a fi șters

$Q \times \Gamma$		$\Sigma + \{\varepsilon\}$		
		a	b	ε
Q_0	Z_0	(q_0, AZ_0)	$\emptyset$	(q_0, ε)
	A	(q_0, AA)	$\{q_1, \varepsilon\}$	$\emptyset$
Q_1	Z_0	$\emptyset$	$\emptyset$	(q_1, ε)
	A	$\emptyset$	(q_1, ε)	$\emptyset$

ex.: • $w = aabb$

$(q_0, aabb, Z_0) \vdash_3 (q_0, abb, AZ_0) \vdash_5 (q_0, bb, AAZ_0) \vdash_6$

$\vdash (q_1, b, AZ_0) \vdash_8 (q_1, \varepsilon, Z_0) \vdash_{10} (q_1, \varepsilon, \varepsilon)$

↳ s-a ajuns la o config. finală după crit. stivei vide => cuv. acc. de PD

• $w = abb$

$(q_0, abb, Z_0) \vdash_3 (q_0, bb, AZ_0) \vdash_6 (q_1, b, Z_0) \vdash_{12}$ block

↳ cuv. nu e acc. de PDA

2) Să se construiască un PDA care acc. $L = \{a^{3n} b^n \mid n \geq 0\}$

I la un grup de 3 de a, se adaugă în stivă un A
se folosesc 2 stări intermediare + crit. stivei vide

$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{a, b\}$, $\Gamma = \{A, Z_0\}$, $F = \emptyset$

$\delta : \{q_0, q_1, q_2, q_3\} \times \{a, b, \varepsilon\} \times \{Z_0, A\} \to P(\{q_0, q_1, q_2, q_3\} \times \{Z_0, A\}^*)$

$\delta(q_0, \varepsilon, Z_0) = \{(q_0, \varepsilon)\}$ ✓

$\delta(q_0, a, Z_0) = \{(q_1, AZ_0)\}$ 1

$\delta(q_1, a, A) = \{(q_2, A)\}$ 2

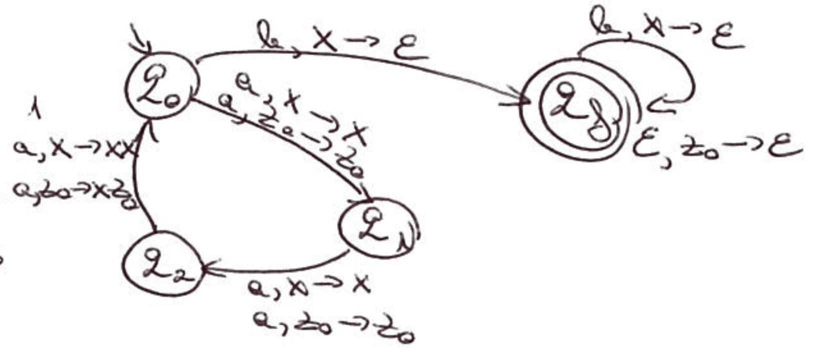
$\delta(q_2, a, A) = \{(q_0, A)\}$ 3

$\delta(q_0, a, A) = \{(q_1, AA)\}$

$\delta(q_0, b, A) = \{(q_3, \varepsilon)\}$ ✓

$\delta(q_3, b, A) = \{(q_3, \varepsilon)\}$ ✓

$\delta(q_3, \varepsilon, Z_0) = \{(q_0, \varepsilon)\}$ ✓



=> celelalte cazuri $\delta(-, -, -) = \emptyset$

II) - la fiecare a citit se adaugă în stivă A (după terminarea citirii de a, vor fi de 3 ori mai multe simboluri decât cele rămase de citit)

- la fiecare b citit se vor șterge câte 3 A din stivă; pt. că numai simbolul din vf. stivei se poate șterge la un mom. dat, ștergerea a 3 simboluri se va face folosind încă 2 stări intermediare din care au loc doar ε tranziții + crit. stivei vide

$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{a, b\}$, $\Gamma = \{Z_0, A\}$, $F = \emptyset$

$\delta : \{q_0, q_1, q_2, q_3\} \times \{a, b, \varepsilon\} \times \{Z_0, A\} \to P(\{q_0, q_1, q_2, q_3\} \times \{Z_0, A\}^*)$

$\delta(q_0, \varepsilon, Z_0) = \{(q_0, \varepsilon)\}$

$\delta(q_0, a, Z_0) = \{(q_0, AZ_0)\}$

$\delta(q_0, a, A) = \{(q_0, AAZ_0)\}$

$\delta(q_0, b, A) = \{(q_1, \varepsilon)\}$

$\delta(q_1, \varepsilon, A) = \{(q_2, \varepsilon)\}$

$\delta(q_2, \varepsilon, A) = \{(q_3, \varepsilon)\}$

$\delta(q_3, b, A) = \{(q_1, \varepsilon)\}$

$\delta(q_3, \varepsilon, Z_0) = \{(q_3, \varepsilon)\}$

rest = $\emptyset$

3) PDA for $L = \{a^n b^{3n} \mid n \geq 0\}$

- la fiecare a citit se vor adăuga 3 de A pe stivă ($\Rightarrow$ nr(A) = nr(b))
- la fiecare b citit se șterge câte un A
- criteriul stării finale $\Rightarrow F = \{q_2\}$, $Q = \{q_0, q_1, q_2\}$

$\delta(q_0, a, Z_0) = \{(q_0, AAAZ_0)\}$

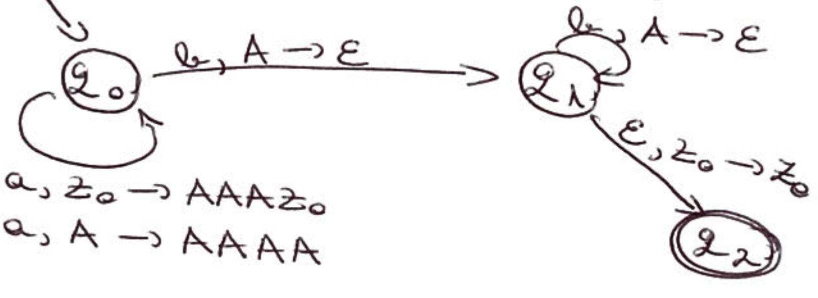
$\delta(q_0, a, A) = \{(q_0, AAAA)\}$

$\delta(q_0, b, A) = \{(q_1, \varepsilon)\}$

$\delta(q_1, b, A) = \{(q_1, \varepsilon)\}$

$\delta(q_1, \varepsilon, Z_0) = \{(q_2, Z_0)\}$

↳ stare finală ⇒ acc.



Grammars and Languages :

<digit> := 0 | 1 | 2 | . . . | 9 ;
<letter> ::= A | . . . | Z | a | . . . | z ;
<aux> := <digit> | <letter> ;
<seg> := <aux> | <seg> <aux> ;
<id> := <letter> | <letter> <seg> ;

} id

<nzdigit> := 1 | . . . | 9 ;
<digit> := <nzdigit> | 0 ;
<sign> := − | + ;
<digitSeg> := <digit> | <digit> <digitSeg> ;
<no> := <nzdigit> | <nzdigit> <digitSeg> ;
<integer> := 0 | <~~no~~> | <sign> <no> ;

} signed integer

Ex : 1) Define <u>N</u> as language

$\Sigma = \{a\} \Rightarrow \Sigma^* = \{\varepsilon, a, a^2, \dots\}$

Let $\Sigma = \{0, 1, 2, \dots, 9\}$

just 1 — multiple

$\Sigma^* \supset \{ w \mid w = a w_1,\ a \in \{1, 2, \dots, 9\},\ w_1 \in \Sigma^* \} \cup \{0\} = \underline{N}$

2) $G = (\{S, C\}, \{a, b\}, P, S)$ prove $L = ab(ab^2)^2 \in L(G)$

$\Updownarrow$

$S \overset{*}{\Rightarrow} ab(ab^2)^2$
$= ab\ abb\ abb$

P : $S \to ab \mid aCSb$ (1), (2)
$C \to S \mid bSb$ (3), (4)
$CS \to b$ (5)

$S \underset{2}{\Rightarrow} aCSb \underset{4}{\Rightarrow} abSbSb \underset{1}{\Rightarrow} ababbSb \underset{1}{\Rightarrow} ababbabb = w$

3) $G = (\{S\}, \{a, b, c\}, P, S)$, P : $S \to a^2 S$ (1)
$S \to bc$ (2)

? $L(G) = \{a^{2n} bc \mid n \geq 0)$

I $\{a^{2n} bc \mid n \geq 0) \subseteq L(G)$? $L(G) \subseteq L$

Let $w = a^{2n} bc$, prove $w \in L(G)$

$S \underset{1}{\overset{n}{\Rightarrow}} a^{2n} S \underset{2}{\Rightarrow} a^{2n} bc \Rightarrow w \in L(G)$

II $L \subseteq L(G)$

$P(n) : a^{2n} bc \in L(G), \forall n \in \mathbb{N}$

$\left.\begin{array}{l} P(0) : a^0 bc = bc \in L(G) \\ S \to bc \end{array}\right\} \Rightarrow P(0)$ is true

$P(k) \overset{?}{\Rightarrow} P(k+1)$, $\forall k \in \mathbb{N}$

we assume $P(k)$ is true $\Rightarrow a^{2k}bc \in L(G) \Rightarrow S \overset{*}{\Rightarrow} a^{2k}bc$ is true

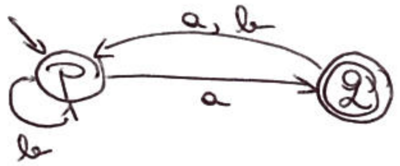
$S \underset{①}{\Rightarrow} a^2 S \overset{*}{\Rightarrow} a^2\, a^{2k}bc = a^{2(k+1)}bc = P(k+1)$

$\Rightarrow S \overset{*}{\Rightarrow} a^{2(k+1)}bc \Rightarrow P(k+1)$ is true $\Rightarrow P(n)$ true $\forall n \in \mathbb{N}$

FA:

Ex: 1) write the table for the foll. FA

a)



1, if s is final
0, otherwise

α	a	b	
p	{q}	{p}	0
q	{p}	{p}	1

b)

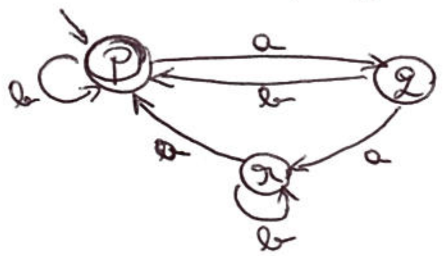
α	a	b	
p	{q}	{p}	0
q	{p}	{r}	0
r	{p}	{q}	1

c)

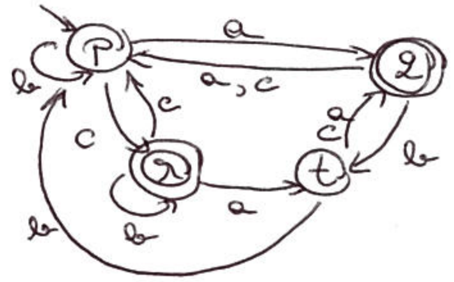
α	a	b	
p	{q}	{r}	0
q	{r}	$\emptyset$	1
r	{r}	{t}	0
t	{q}	{p}	1

2) graph for FA

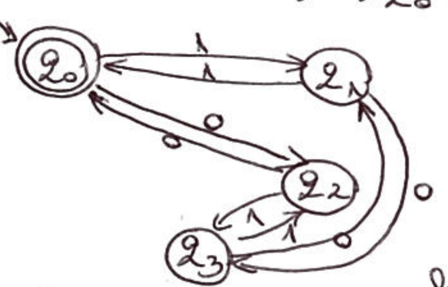
a) $M = (\{p, q, r\}, \{a, b\}, \alpha, p, \{p\})$



b) $M = (\{p, q, r, t\}, \{a, b, c\}, \alpha, p, \{q, r\})$



3) $M = (Q, \Sigma, \alpha, q_0, \{q_0\})$, $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{0, 1\}$



a) $w_1 = 1010$

$(\overset{q_1}{q_0, 1010}) \vdash (\overset{q_3}{q_1, 010}) \vdash (\overset{q_2}{q_3, 10}) \vdash$
$\vdash (\overset{q_0}{q_2, 0}) \vdash (q_0, \varepsilon) \Rightarrow w_1 \in L(M)$

b) $w_2 = 1100$

$(\overset{q_1}{q_0, 1100}) \vdash (\overset{q_0}{q_1, 100}) \vdash (\overset{q_2}{q_0, 00}) \vdash (\overset{q_0}{q_2, 0}) \vdash (q_0, \varepsilon)$
$\Rightarrow w_2 \in L(M)$

c) $w_3 = 1011 \Rightarrow (\overset{q_1}{q_0, 1011}) \vdash (\overset{q_3}{q_1, 011}) \vdash (\overset{q_2}{q_3, 11}) \vdash (\overset{q_3}{q_2, 1}) \vdash$
$\vdash (q_3, \varepsilon) \Rightarrow$ NO bc q_3 is not a final state

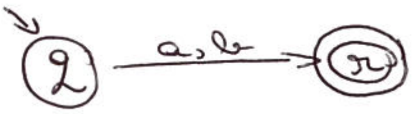
4) Find FA and RG for $(a+b)^*(ab)^* = \{\varepsilon$

$a|b$ ↳ $\varepsilon, ab, abab, \dots$

$\varepsilon, a, aa, \dots, b, bb, \dots = a^* + b^*$

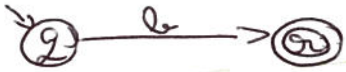
$(a+b)^* = (a|b)^* = a\,b\,a\,bb\dots$

$a+b$:



a :

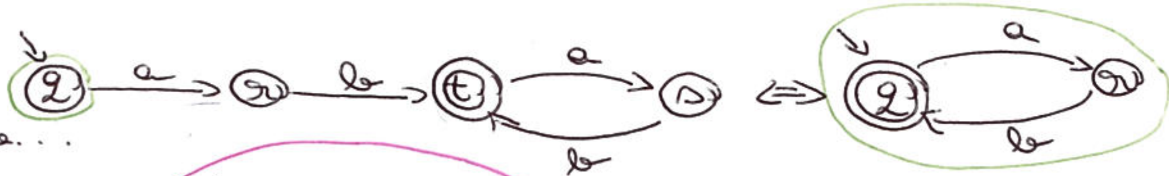
b :



$(a+b)^*$:

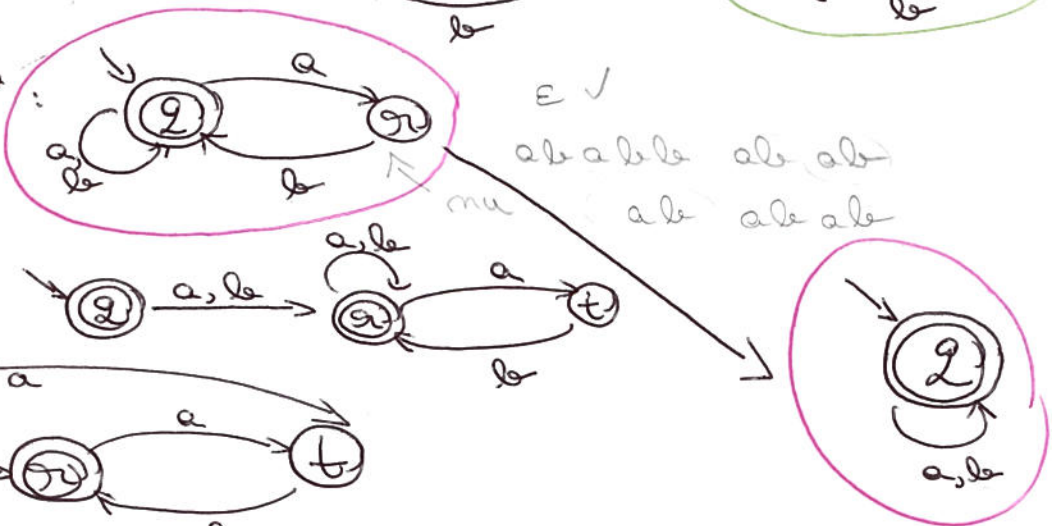
ab :

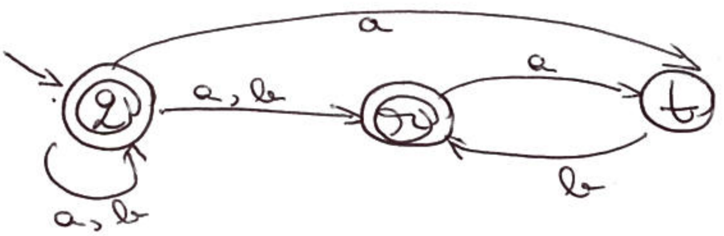
$(ab)^*$:



ε, ab, abab...

$(a+b)^*\,(ab)^*$:





$G = (N, \Sigma, P, S)$; $N = \{q\}$; $\Sigma = \{a, b\}$, $S = q$, $F = \{q$

P: $q \to aq \mid bq \mid a \mid b \mid \varepsilon$

5) RE + FA for $G = (\{S, A, B, C\}, \{a, b, c\}, P, S)$

$S \to aA \mid bB$

$A \to aC \mid bC \mid a \mid b$

$B \to aC \mid bC \mid a \mid b$

$C \to aA$

$M = (Q, \Sigma, \alpha, q_0, F)$

$Q = \{S, A, B, C, K\}$, $\Sigma = \{a, b, c\}$, $q_0 = S$, $F = \{S, K\}$

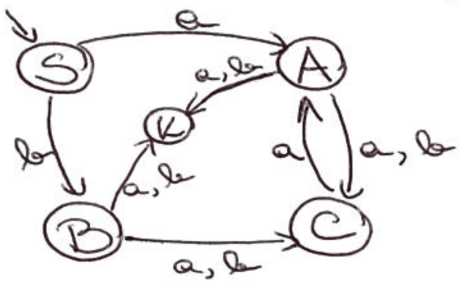
$\alpha(S, a) = A$ $\quad\alpha(A, a) = \{C, K\}$ $\quad\alpha(B, a) = \{C, K\}$

$\alpha(S, b) = B$ $\quad\alpha(A, b) = \{C, K\}$ $\quad\alpha(B, b) = \{C, K\}$

$\alpha(C, a) = A$

not DFA

α	a	b	c
S	A	B	$\emptyset$
A	C, K	C, K	$\emptyset$
B	C, K	C, K	$\emptyset$
C	A	$\emptyset$	$\emptyset$
K	$\emptyset$	$\emptyset$	$\emptyset$



$nr(a) \geq nr(b) \geq 0$

6) PDA for $L = \{a^n b^m \mid n \geq m \geq 0\}$

1.6.7 din culegere

$M = (Q, \Sigma, \Gamma, \alpha, q_0, z_0, F)$ $\quad$ crit stării finale

$Q = \{q_0, q_1, q_2\}$, $\Sigma = \{a, b\}$, $\Gamma = \{z_0, A\}$, $F = \{q_2\}$

$\alpha : \{q_0, q_1, q_2\} \times \{a, b, \varepsilon\} \times \{z_0, A\} \to \mathcal{P}(\{q_0, q_1, q_2\} \times \{z_0, A\}^*)$

$\alpha(q_0, a, z_0) = \{(q_0, Az_0)\}$ $\quad a \longrightarrow push(A)$

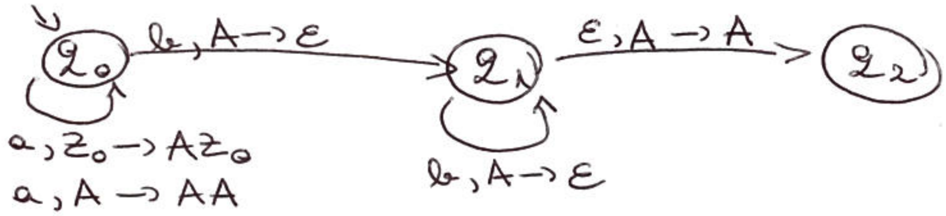
$\alpha(q_0, a, A) = \{(q_0, AA)\}$ $\quad b \longrightarrow pop(A)$

$\alpha(q_0, b, A) = \{(q_1, \varepsilon)\}$

$\alpha(q_1, b, A) = \{(q_1, \varepsilon)\}$

$\alpha(q_1, \varepsilon, A) = \{(q_2, A)\}$ → se aj. la starea q_1, A în vf. stivei și cuvântul a fost citit complet (ε) ⇒ s-au citit mai puține simboluri b decât a (pt că au rămas A pe stivă ⇒ nrcitite(a) > nrcitite(b)) ⇒ trecem într-o stare finală q_2 și cuvântul este acceptat

restul cazurilor = $\emptyset$



7) PDA for $L = \{ a^n 1^m 2^m 3^n \mid n > 0, m \geq 0 \}$?

8)
```
a := 0;
i = 1;
repeat
    if (a>3) or (i<10) then
            a := a + a * i;
            i := i + 1;
while (i>10)
```

label	op	arg1	arg2	rez
1	:=	5	—	a
2	:=	1	—	i
3	>	a	3	t_1
4	<	i	10	t_2
5	or	t_1	t_2	t_3
6	goto	t_3	—	[8] jump if
7	goto	—	—	[13] else empty jump after
8	*	a	i	t_4
9	+	a	t_4	t_5
10	:=	t_5	—	a
11	+	i	1	t_6
12	:=	t_6	—	i
13	$\leq$	i	10	t_7
14	goto	t_7	—	[16]
15	goto	—	—	[3] con
16				

2

9) if (a > 3) or (i < 10) and (a <> i) then
if i mod 2 = 0 then
a := a + 1
else a := a + a * i
i := i + 1

label	op	arg1	arg2	rez
1	>	a	3	t_1
2	<	i	10	t_2
3	<>	a	i	t_3
4	and	t_2	t_3	t_4
5	or	t_1	t_4	t_5
6	goto	t_5	—	[8] jump if
7	goto	—	—	[~~8~~ 11] else empty
8	mod	i	2	t_6
9	=	t_6	0	t_7
10	goto	t_7	—	[15]
11	*	a	i	t_8
12	+	a	t_8	t_9
13	:=	t_9	—	a
14	goto	—	—	[17]
15	+	a	1	t_{10}
16	:=	t_{10}	—	a
17	+	i	1	t_{11}
18	:=	t_{11}	—	i
19	stop			

10) First Follow? $G = (\{S, A, B\}, \{a, b, c\}, P, S)$

P : 1) $S \to AA$ 3) $A \to aA$ 5) $A \to SB$ 7) $B \to Bb$

2) $S \to AB$ 4) $A \to \varepsilon$ 6) $B \to c$

I)

	F_0	F_1	F_2	F_3
S	∅	a, ε, c	a, ε, c	a, ε, c
A	a, ε	a, ε	a, ε, c	a, ε, c
B	c	c	c	c

$F_1(S) = F_0(S) \cup \{$

$F_{(0)}(A) \oplus F_0(A) = a, \varepsilon \oplus a, \varepsilon = a,$

$a, \varepsilon \oplus b$

$F_0(A) \oplus F_0(B) = a, \varepsilon \oplus c$ ab εb

ac

$\varepsilon c = a, c$

$F_1(A) = a \oplus a, \varepsilon \oplus \varepsilon \oplus ∅ \oplus c$

$= F_0(A)$

$F_1(B) = c \oplus b = c$

$F_2(S) = a, \varepsilon \oplus a, \varepsilon = a, \varepsilon$

$a, \varepsilon \oplus c = a, c$

$F_2(A) = a, \varepsilon, c \oplus c = a, \varepsilon, c$

II)

	L_0	L_1	L_2	L_3
S	ε	ε, c	ε, c	ε, c
A	∅	a, ε, c	a, ε, c	a, ε, c
B	∅	ε, b	a, b, c, ε	a, b, c, ε

$A \to SB \Rightarrow \beta = B \Rightarrow F(B) = \{c$

$S \to AA \Rightarrow \beta = A \Rightarrow F(A) = a, \varepsilon, c$

$\varepsilon \in F(A) \Rightarrow L_0(S) = \varepsilon$

$S \to AA \Rightarrow \beta = \varepsilon \Rightarrow L_0(S) = \varepsilon$

$S \to AB \Rightarrow \beta = B \Rightarrow F(B) = \{c\}$

$A \to aA \Rightarrow \beta = \varepsilon \Rightarrow L_0(A) = ∅$

$S \to AB \Rightarrow \beta = \varepsilon \Rightarrow L_0(S) = \varepsilon$

$A \to SB \Rightarrow \beta = \varepsilon \Rightarrow L_0(A) = ∅$

$B \to Bb \Rightarrow \beta = b$

L_2: $A \to SB \Rightarrow \beta = B \Rightarrow F(B) = \{c\}$

$S \to AA \Rightarrow \beta = A \Rightarrow F(A) = a, \varepsilon, c$

$S \to AA \Rightarrow \beta = \varepsilon \Rightarrow L_1(S) = \varepsilon, c$

$S \to AB \Rightarrow \beta = B \Rightarrow F(B) = c$

$A \to aA \Rightarrow \beta = \varepsilon \Rightarrow L_1(A) = a, \varepsilon, c$

$S \to AB \Rightarrow \beta = \varepsilon \Rightarrow L_1(S) = \varepsilon, c$

$A \to SB \Rightarrow \beta = \varepsilon \Rightarrow L_1(A) = a, \varepsilon, c$

$B \to Bb \Rightarrow \beta = b$

	a	b	c	$
S	AB, 2		AB, 2 AA, 1	AA, 1
A	ε, 4 aA, 3		ε, 4	ε, 4
B			Bb, 7 c, 6	
a	pop			
b		pop		
c			pop	
$				acc

(1) $S \to \overset{\alpha}{AA}$; $F(\overset{\alpha}{AA}) = F(A) \oplus F($

$= \{a, \varepsilon, c\} \Rightarrow \varepsilon \in F(\overset{\alpha}{AA}) \Rightarrow$

$Foll(S) = \{\varepsilon, c\}$

$\Rightarrow (S, \varepsilon) = (\overset{\alpha}{AA}, 1)$

$(S, c) = (\overset{\alpha}{AA}, 1)$

(2) $S \to AB$; $F(AB) = F(A) \oplus F($

$= \{a, c\} \ (\not\ni \varepsilon) \Rightarrow (S, a) = AB,$

$(S, c) = AB, 2$

(3) $A \to aA$; $F(aA) = F(a) \oplus F($

$= \{a\} \Rightarrow (A, a) = aA, 3$

(4) $A \to \varepsilon$; $F(\varepsilon) = \varepsilon$

$Foll(A) = \{a, \varepsilon, c\}$

$\Rightarrow (A, a) = \varepsilon, 4$

$(A, \varepsilon) = \varepsilon, 4$

$(A, c) = \varepsilon, 4$

(5) $A \to SB$; $F(SB) =$

$= \{a, c\} \Rightarrow (A, a) = SB$

$(A, c) = SB, 5$

(6) $B \to c$; $F(c) = c$

$\Rightarrow (B, c) = c, 6$

(7) $B \to Bb$; $F(Bb) = c$

$(B, c) = Bb, 7$

11) Can. coll. for $G = (\{S', S, A\}, \{a, b, c\}, P, S')$ LR(0)

P: 1) $S \to aSa$ 3) $A \to cA$ 0) $S' \to S$

2) $S \to bAb$ 4) $A \to a$

$s_0 =$ closure$(\{[S' \to .S]\}) = \{[S' \to .S], [S \to .aSa], [S \to .bA$

if $[A \to \alpha.B\beta] \in$ closure(s) $\}$ ⇒ all prod. of $B \to .\alpha$ will appear in closure

↳ dot in front of nonterm.

goto(s, X) for dot in front of X $(.X)$ ⇒ move the dot after X ⇒

⇒ apply closure

$s_1 =$ goto$(s_0, S) =$ closure$(\{[S' \to S.]\}) = \{[S' \to S.]\}$

~~s_2~~ $=$ goto$(s_0, A) = \emptyset$ goto$(s_0, c) = \emptyset$

$s_2 =$ goto$(s_0, a) =$ closure$(\{[S \to a.Sa]\}) = \{[S \to a.Sa],$
$[S \to .aSa], [S \to .bAb]\}$

$s_3 =$ goto$(s_0, b) =$ closure$(\{[S \to b.Ab]\}) = \{[S \to b.Ab], [A \to .cA$
$[A \to .a]\}$

$s_4 =$ goto$(s_2, S) =$ closure$(\{[S \to aS.a]\}) = \{[S \to aS.a]\}$

$s_5 =$ goto$(s_2, a) =$ closure$(\{[S \to a.Sa]\}) = \{[S \to a.Sa],$
$[S \to .aSa], [S \to .bAb]\}$

~~s_6~~ $=$ goto$(s_2, b) =$ closure$(\{[S \to b.Ab]\}) = \{[S \to b.Ab], [A \to .cA$
$[A \to .a]\} = s_3$

$s_6 =$ goto$(s_3, A) =$ closure$(\{[S \to bA.b]\}) = \{[S \to bA.b]\}$

$s_7 =$ goto$(s_3, c) =$ closure$(\{[A \to c.A]\}) = \{[A \to c.A], [A \to .cA],$
$[A \to .a]\}$

$s_8 =$ goto$(s_3, a) =$ closure$(\{[A \to a.]\}) = \{[A \to a.]\}$

$s_9 =$ goto$(s_4, a) =$ closure$(\{[S \to aSa.]\}) = \{[S \to aSa.]\}$

$s_{10} =$ goto$(s_5, S) =$ closure$(\{[S \to aS.a]\}) = \{[S \to aS.a]\}$

~~s_{11}~~ $=$ goto$(s_5, a) =$ closure$(\{[S \to a.Sa]\}) = s_5$

$s_{11} =$ goto$(s_6, b) =$ closure$(\{[S \to bAb.]\}) = \{[S \to bAb.]\}$

$s_{12} =$ goto$(s_7, A) =$ closure$(\{[A \to cA.]\}) = \{[A \to cA.]\}$

goto$(s_7, c) =$ closure$(\{[A \to c.A]\}) = s_7$

goto$(s_7, a) =$ closure$(\{[A \to a.]\}) = \{[A \to a.]\} = s_8$

goto$(s_{10}, a) =$ closure$(\{[S \to aSa.]\}) = \{[S \to aSa.]\} = s_9$

Ex. Sub. 1:

1. $("0040"|"+40")"264"("4"|"5")("0"|"1"|\ldots|"9")^5$

2. $L_1=\{\varepsilon,0,00\}$, $L_2=\{\varepsilon,1,0\}$

$$L_1 \oplus L_2 = \begin{matrix} \varepsilon\varepsilon & 0\varepsilon & 00\varepsilon \\ \varepsilon 1 & 01 & 001 \\ \varepsilon 0 & 00 & 000 \end{matrix} = \{\varepsilon, 0, 1, 00\}$$

3. $a(ab)^*a = \{aa, a\,ab\,a, a\,abab\,a, \ldots\}$

4. $L=\{01,10\}$, $L^* = \cancel{(01+10)^*} = (01|10)^* = \{\varepsilon, 0110, 1001, \ldots$

5. $\underline{S\to aAb}$ $\quad A\to b \quad C\to c$

$A\to aA \quad C\to bA \quad S, a, A, b \Rightarrow$ inacc. C, c

6. $G=(\{S,A\},\{2,1\},P,S)$

$S\to 2S1 \mid 1A \mid 2$ (1) (2) (3)

$A\to 1A \mid 1$ (4) (5)

221? $S \underset{1}{\Rightarrow} 2S1 \underset{3}{\Rightarrow} 221$

2111? $S \underset{1}{\Rightarrow} 2S1 \underset{2}{\Rightarrow} 21A1 \underset{5}{\Rightarrow} 2111$

21? $S \underset{1}{\Rightarrow} 2S1 \Rightarrow \emptyset$

12? $S \underset{2}{\Rightarrow} 1A \Rightarrow \emptyset$

7. $L_1=\{\varepsilon,0,1\}$, $L_2=\{0,00\}$

$L_1\cup L_2=\{\varepsilon,0,1,00\}$

8. $\cancel{\{\varepsilon,0,00\}}$ $L_1=\{a,aa\}$, $L_2=\{\varepsilon,b,bb\}$

$L_1L_2=\{a, \underline{ab}, \underline{abb}, \underline{aa}, aab, aabb$

9. $G=(\{S,A,B,C\},\{;, id, +, c\},P,S)$

P: (1) $S\to A$ $\quad B\to id \mid A \mid c$ (3) (4) (5)

(2) $A\to BC$ $\quad C\to +A \mid \varepsilon$ (6) (7)

$c+id;$: $S \underset{1}{\Rightarrow} A; \underset{2}{\Rightarrow} BC; \underset{5}{\Rightarrow} cC; \underset{6}{\Rightarrow} c+A; \underset{2}{\Rightarrow} c+BC; \underset{3}{\Rightarrow} c+idC; \underset{7}{\Rightarrow} c+id;$

$id+id;$: $S \Rightarrow A; \Rightarrow BC; \underset{3}{\Rightarrow} id\,C; \underset{6}{\Rightarrow} id+A; \underset{2}{\Rightarrow} id+BC; \underset{3+7}{\Rightarrow} id+id;$

$id+$: $S\Rightarrow A \Rightarrow BC \Rightarrow idC \Rightarrow \emptyset$

$id+id$: $\emptyset$

10. $S\to 0S \mid 1A$

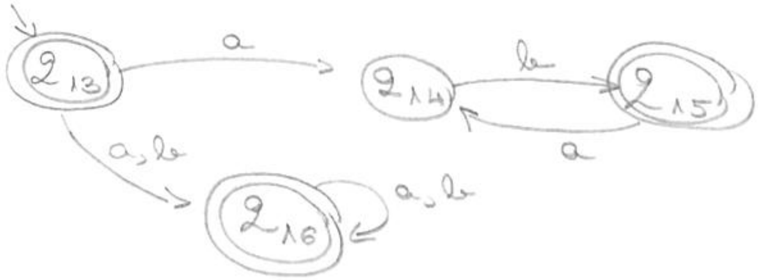
$A\to 1A \mid 1$

$\begin{cases} 0^n, n\geq 0 \\ 1^m, m>0 \end{cases}$

$\Rightarrow L=\{0^n 1^m \mid n\geq 0, m>0\}$

Subject 2 :

1. LG and FA for $(a+b)^* + (ab)^*$



$G = (N, \Sigma, P, S)$

$\Sigma = \{a, b\}$

$N = \{q_{13}, q_{14}, q_{15}, q_{16}\}$

$S = q_{13}$

$F = \{q_{13}, q_{15}, q_{16}\}$

P: $q_{13} \to a q_{14} \mid a q_{16} \mid b q_{16} \mid \varepsilon \mid a \mid b$

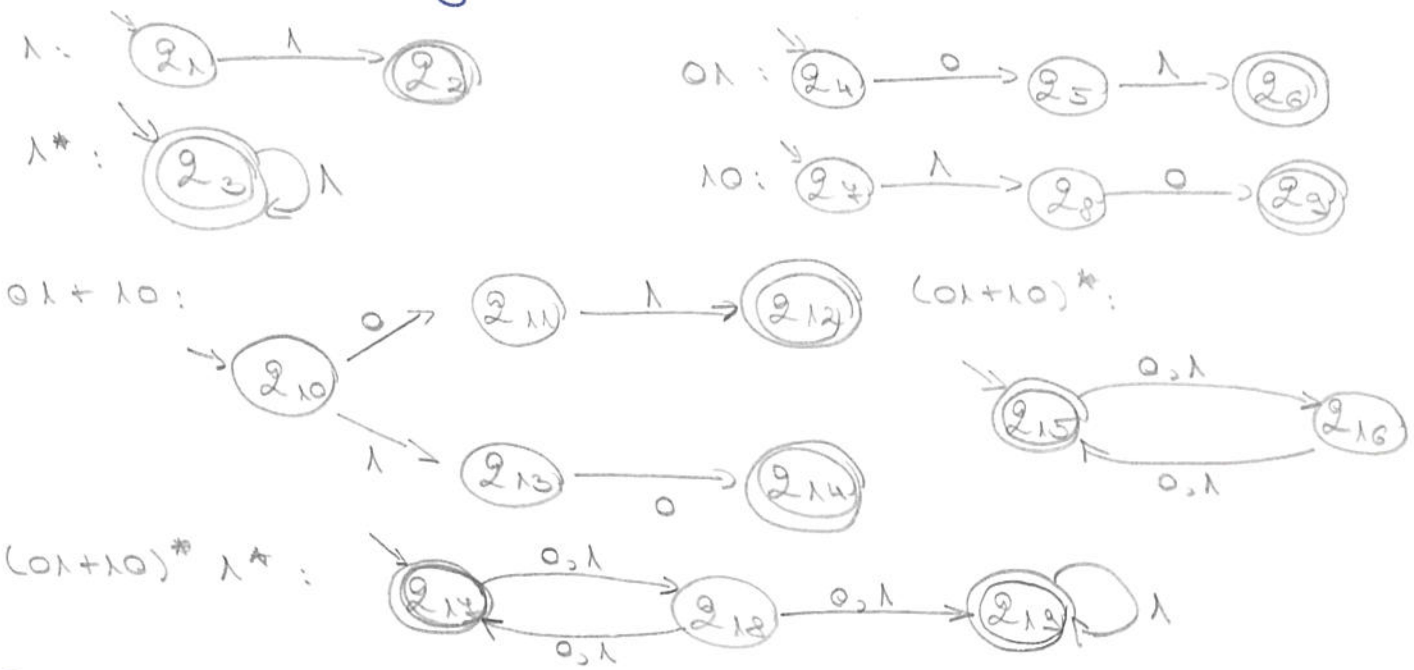
$q_{14} \to b q_{15} \mid b$

$q_{15} \to a q_{14}$

$q_{16} \to a q_{16} \mid b q_{16} \mid a \mid b$

ε, 01, 10, 01011001

2. LG and FA for $(01 + 10)^* 1^*$



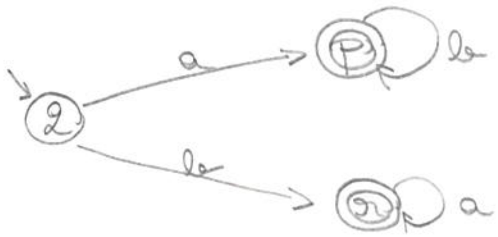
P: $q_{17} \to 0 q_{18} \mid 1 q_{18} \mid \varepsilon$

$q_{18} \to 0 q_{17} \mid 1 q_{17} \mid 0 \mid 1 \mid 0 q_{19} \mid 1 q_{19}$

$q_{19} \to 1 q_{19} \mid 1$

$G = (\{q_{17}, q_{18}, q_{19}\}, \{a, b\}, P, q_{17})$

3. RLG and RE



$G = (\{q, p, r\}, \{a, b\}, P, q)$

$P: \quad q \to ap \mid a \mid br \mid b$

$p \to \cancel{bp} \mid b$

$r \to ar \mid a$

$\boxed{RG \Leftrightarrow RE}$: $\boxed{X = aX + b \Rightarrow \text{sol}: X = a^* b}$ *

$q = ap + a + br + b$

$p = bp + b \overset{*}{\Rightarrow} p = b^* b = b^+$

$r = ar + a \overset{*}{\Rightarrow} r = a^* a = a^+$

$\Rightarrow \cancel{RE}\ q = ab^+ + a + ba^+ + b =$

4. PDA for $L = \{a^n b^m c^m \mid n > 0, m \geq 0\}$

$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$a \to$ mimic q_0 $F = \{q_f\}$

$b \to$ push q_1

$c \to$ pop q_2

$\delta(q_0, a, z_0) = \{(q_0, z_0)\}$

$\delta(q_0, a, z_0) = \{(q_1, z_0)\}$

$\delta(q_1, b, z_0) = \{(q_1, Xz_0)\}$

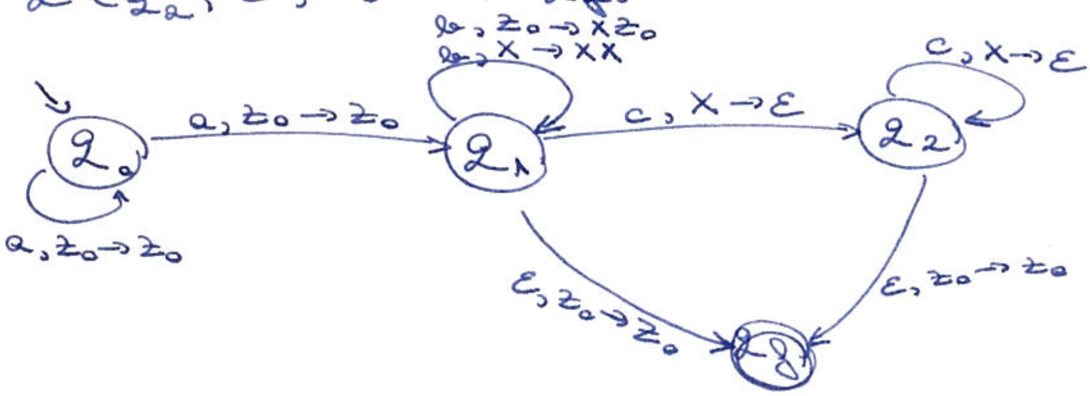
$\delta(q_1, b, X) = \{(q_1, XX)\}$

$\delta(q_1, \varepsilon, z_0) = \{(q_f, z_0)\}$

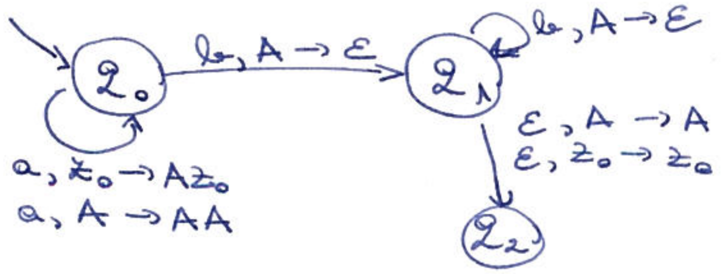
$\delta(q_1, c, X) = \{(q_2, \varepsilon)\}$

$\delta(q_2, c, X) = \{(q_2, \varepsilon)\}$

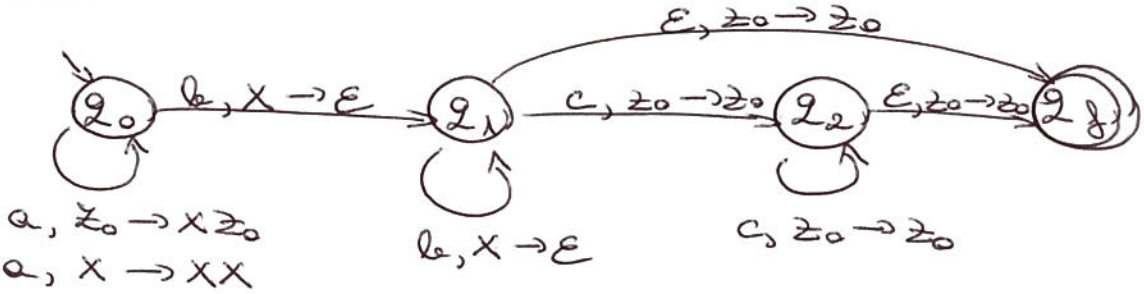
$\delta(q_2, \varepsilon, z_0) = \{(q_f, z_0)\}$



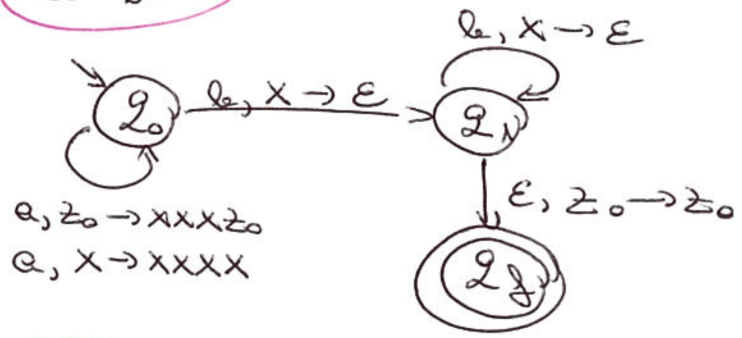
5. PDA for $L = \{a^n b^m \mid n \geq m \geq 0\}$ — a - push, b - pop



$a^n b^n c^m$, $n > 0$, $m \geq 0$ — c - do nothing, a - push, b - pop

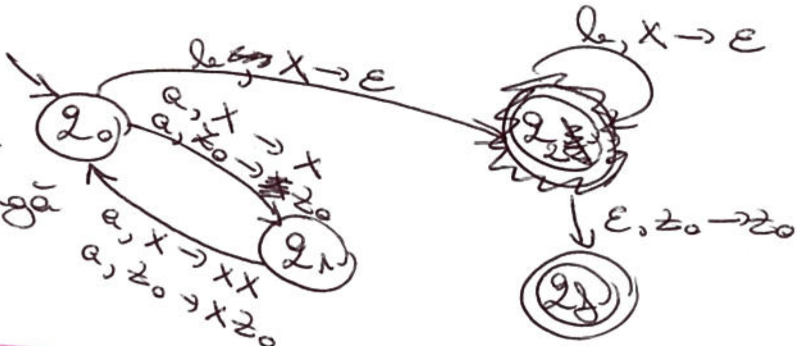


$a^n b^{3n}$



$a^{2n} b^n$

la un grup de 2 de a se adaugă doar un X



ww^R, $w \in \{a, b\}^+$ a b aa bb b aa a . . .