Optimization of Land Use in Afforestation Areas Using Evolutionary Self-Organization

Niels Strange, Henrik Meilby, and Bo Jellesmark Thorsen

ABSTRACT. This article presents a cellular automaton approach to the optimization of land use planning in afforestation areas. The case study deals with 110 ha of recently afforested land in Denmark. A modified two-dimensional automaton containing 2,345 active cells is used for optimizing the land use setup in this area. Four land use alternatives are considered: pasture, beech (*Fagus sylvatica* L.), Norway spruce (*Picea abies* [L.] Karst.), and oak (*Quercus robur* L.). The objective is to maximize a weighted sum of soil expectation value, scale-dependent costs, recreational value, and the value of structural variation. It is demonstrated that for this sort of problem, appropriately modified cellular automata may yield acceptable solutions within a comparatively low number of iterations. Furthermore, for a simplified hypothetical problem, it is shown that the cellular automaton approach was able to identify optimal solutions within reasonable time. For. Sci. 48(3):543–555.

Key Words: Combinatorial analysis, cellular automata, multiple use, forest management planning, policy analysis, spatial economics.

A FFORESTATION HAS BECOME an important element in Danish environmental policy. Afforestation is seen as a way to increase the amount of renewable natural resources, environmental and other social goods, e.g., recreational services, carbon sequestration, and protection of groundwater resources. In the 1980s surplus agricultural production and the impact of the EU Common Agricultural Policy made afforestation politically attractive and increased the incentive for a Danish policy within the field. The comments to the 1989 Act on Natural Resource Management included the target of doubling the Danish forest area from the present 12% to about 25% of the land area within one forest rotation (70–150 yr). To reach this target, an annual afforestation rate of about 5,000 ha will be required. Therefore, since 1989, funding for state afforestation has been granted, and since 1991, subsidies for private afforestation have been available.

In afforestation areas, the number of stands, the size and shape of each individual stand, and the spatial allocation of various tree species can be decided freely. Therefore, to maximize the economic output of the forest, the choice of tree species may be adapted to topography and soil quality. Moreover, scale-dependent costs may be minimized by maintaining a certain minimum stand size. On the other hand, landscape and biological diversity may increase with decreasing stand size, and the recreational value of a given stand is likely to depend on the distance to inhabited areas. This decision problem is complex and may be difficult to solve using traditional optimization methods.

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A series of recent studies have investigated land use optimization in general and spatial forest planning in particular. Several approaches have been considered. They include Lagrangian relaxation (Torres-Rojo 1989), column generation (Weintraub et al. 1994), map-coloring theory (Roise 1990), random search (Clements et al. 1990, Nelson et al. 1991), biased sampling search (O'Hara et al. 1989), random ordering and one-stage dynamic programming (Yoshimoto et al. 1994), simulated annealing (Lockwood and Moore 1993, Murray and Church 1995, Tarp and Helles 1997), tabu search (Thompson et al. 1994, Murray and Church 1995, Bettinger et al. 1997), and mixed integer programming (Hof et al. 1994, Hof et al. 1997).

Most of these approaches are centralized in the sense that the objective function of the entire system is evaluated in each iteration and used to determine which changes to introduce in the next step. A classic group of centralized approaches is the multi-level or hierarchal planning optimization methods (Dirickx and Jennergren 1979, Weintraub et al. 1994).

Centralistic approaches possess the advantage that they directly address the overall problem and usually are conceptually easy to understand. On the other hand, when the resolution of the system is high and the number of potential states of each element is considerable, the number of feasible configurations is tremendous, rendering (most) centralistic approaches impracticable. However, a property shared by many spatial systems is that the interaction between different locations decreases dramatically with distance. This suggests that local trade-offs may sometimes be more important than global ones. Hence, it is conceivable that a decentralized approach, where decisions are taken in parallel, may find good solutions to huge spatial problems within reasonable time. Therefore, we present here a cellular approach where evolutionary self-organization is used to find good solutions to a complicated tree species allocation problem for an afforestation area. We conclude that the algorithm is able to find high-quality solutions with relatively low computational efforts. Furthermore, in the appendix we use a simpler example to demonstrate that such an approach can actually find optimal solutions within reasonable time. It illustrates the potential advantages of the approach.

The current problem involves four management criteria, a weighted sum of which is optimized. One criterion is to maximize the total soil expectation value from timber production. Another is to reduce the costs by acknowledging the economies of scale characterizing timber production. The third is to maximize a measure of structural variation, and the fourth is to maximize the aesthetic/recreational value. Inhabited areas, roads, and lakes are considered fixed landscape elements. The first and fourth objective function components do not change as a consequence of changes in the use of neighboring locations. The economies of scale and the structural variation components, on the other hand, depend on the land use composition of a neighborhood around each cell. The complexity caused by the structural variation component in particular makes it difficult to solve the problem with an exact method such as mixed integer programming.

Cellular Automata and Land Use Problems

Spatial systems, whatever their dimension, are characterized by a decreasing flow of information with increasing distance. This implies that any specific element of the system reacts upon, and adjusts, its behavior in response to the occurrences of the nearby surroundings mainly. All elements of the system will, in time, be influenced by all changes of the system, but information from far away will be delayed as well as restrained. This applies to a wide variety of spatial systems, and most likely also to the economics of land use.

Properties of Cellular Automata

In cellular automata, the values of the state variables (here, land use) evolve in discrete steps, and all cells of an automaton are evaluated simultaneously. In general, the cells of a cellular automaton can be perceived as spatially ordered, parallel processing computers even though, in this case, they are simulated on a serial computer. For a thorough introduction to cellular automata, see Wolfram (1984).

Cellular automata are characterized by five properties: (1) they consist of a discrete lattice of cells; (2) they evolve in discrete steps; (3) each cell can take on a finite set of possible values; (4) all cell values evolve according to a common set of rules, and (5) the rules determining the development of each cell take a local neighborhood into consideration.

In most cases, the evolution of cellular automata is irreversible. Trajectories of the system's configuration (land use allocation) merge, and, after many iterations, trajectories starting from various random initial states become concentrated onto attractors. One may say that the evolution from arbitrary initial states towards an attractor constitutes self-organizing behavior. Structure evolves from less structured initial states, and the rules characterizing the automaton determine the resulting patterns.

If cellular automata shall be useful for optimization purposes, then the properties of the attractors are crucial. If a cellular automaton does not evolve towards a steady state, or at least a limited steady cycle, there is no obvious way to extract a solution from the dynamic behavior. Moreover, the globally optimal solution may not be part of an attractor.

Geographic Applications of Cellular Automata

Cellular automata appear to be a natural modeling tool in cases where the dynamic system is characterized by a certain amount of homogeneity and where the interactions between elements of the systems are comparatively well understood. In such cases, it seems evident that cellular automata may form the basis of efficient solutions to problems emphasizing the dynamic behavior of the system. A wide range of geographic applications appears in the literature. For instance, Hogeweg (1988) used cellular automata theory in modeling landscape dynamics, Green et al. (1985) applied cellular automata for simulating fire dispersal in a forest ecosystem, and Solé and Manrubia (1995) used cellular automata to simulate gap dynamics of natural forests. Cellular automata have also been used to simulate secondary succession and spatial patterns of vegetation development (Colasanti and Grime 1993). Finally, Itami (1994) presented a grid-based geographic information system within a cellular automaton framework.

Economic Applications

Spatial competition in economic systems was originally analyzed by Hotelling (1929). Prasad (1988) describes an economy as a cellular automaton where the self-organization or adaptation corresponds to the evolution of the system towards a more efficient disposition of social resources. In Prasad's economic model, each cell is an individual economic agent, and a neighbor is an individual with whom the agent interacts i.e., a potential buyer-seller relationship. Keenan and O'Brien (1993) use cellular automata to demonstrate cartel formation in an economy where myopic firms compete through prices in finite time.

The structure of the rural landscape may be determined by a number of spatially related functions. In Balmann (1993, 1997) structural changes in an agricultural landscape are analyzed within a two-dimensional cellular automaton framework. Individual farms are located at different points in a region and compete on various markets. The overall maximization problem is solved as a decentralized optimization problem, where all the farms maximize profit in parallel subject to their expectations, financial state, and existing assets. The model is mainly used to demonstrate structural change. It is concluded that the rules are reliable since developmental paths from different initial states converge towards similar equilibria. Similarly, a cellular automaton based model was developed by Deadman et al. (1993) to model historical rural settlement patterns. It is evident that such models might be useful for policy analysis, i.e., as tools which can be used for testing which policies may help achieve desired goals.

Urban development can be modeled in a similar manner, and the agricultural and urban landscape may interact. For example, urban development may increase the demand and price on available land and, in turn, this might lead to changing land use in the vicinity of urban areas. Engelen (1988), although not working with cellular automata, developed an evolutionary model of urban systems using the concepts of the theory of self-organization. The model was used to explore the many possible futures and alternative structures of complex urban systems in Belgium, e.g., for calculating the magnitude and spatial distribution of future energy consumption. Moreover, it was used for testing the impacts of small as well as larger interventions, changing policies, and technological evolution. In both Engelen (1988) and Balmann (1993, 1997) economic models are applied within a self-organizing spatial structure to analyze changes in rural or urban landscapes.

As described above, cellular automata implemented on a regular two-dimensional lattice have previously been used to describe spatial economic systems. Although cellular automata have primarily been used to simulate the consequences of presumed cell behavior, they may also be formulated as optimization procedures, either by introducing appropriate cell behavior or by modifying the updating scheme of the automaton. It is, however, an open question how objectives pursued by the individual cells are related to the system-wide objective. In other words, what cell behavior should be introduced in order to pursue a given overall objective? In the afforestation problem presented here, there is a natural relationship between the cell and the system objectives, and this is what makes cellular automata useful for optimization of the current system. However, as local optimization is not guaranteed to lead to a system-wide optimum, case-specific modifications of the automaton behavior are required to ensure that system-wide good solutions are produced. In a previous paper (Strange et al. 2001) we tested a variety of modifications and found that probabilistic evolutionary strategies were more efficient than most other strategies. Accordingly, in the present study we apply an evolutionary strategy. Preliminary results for a somewhat different model are found in Strange and Meilby (1999).

The Afforestation Planning Problem

As mentioned, considerable effort is currently being made to promote afforestation in Denmark. For this study we have chosen an area (110 ha), which has recently been afforested by The National Forest and Nature Agency. Four land use alternatives were available: pasture, beech (*Fagus sylvatica* L.), Norway spruce (*Picea abies* (L.) Karst.), and oak (*Quercus robur* L.). Pastures are assumed to be used for forage production and are intended as an open land alternative.

Lattice Structure

The two-dimensional cellular automaton is defined as a lattice of $I \times J = 56 \times 84 = 4,704$ square cells. In the lattice, the afforestation area contains 2,345 cells, where each cell is approximately $469 \, m^2$. The remaining 2,359 cells outside the afforestation area consist of inhabited areas, roads, lakes, existing forest and farmland. We denote this set of cells K. The inhabited areas are villages, farm houses, and summer cottages.

Let C be a set of cells representing the afforestation area and let $c = (c_1, c_2)$ be a specific cell. Each cell, $c \in C$, may be used in four different ways. Let D denote the four possible land use alternatives and the specific alternative chosen for each cell. In this application $D = \{$beech, Norway spruce, oak, pasture$\}$. A specific land use configuration can be denoted $d(C) = [d(c), c \in C]$. In this model, the number of such land use configurations is $\#\{d \in D\}^{\#\{c \in C\}} = 4^{2,345}$.

Cell Characteristics

Based on Madsen (1983) and Granat (1995), the water retention capacity was estimated for each cell of the automaton. The water retention capacity is related to the soil texture and the potential rooting depth.

Neighborhood

In cellular automata, all cells respond to the current state of a local neighborhood of some size. However, in this study, the edge of the cellular grid cuts off part of the afforestation area, implying that for some cells of the automaton, part of the neighborhood is undefined. To provide these cells with a neighborhood, we assume that the shares of various land uses in the undefined part of the neighborhood are similar to the shares currently found in the known part of the neighborhood. Thereby we avoid more specific assumptions regarding areas outside the edge of the lattice.

We define the neighborhood $\Psi(c)$ around c

$$\Psi(c) = \left\{ c' \in C \middle| (|\, c_1 - c_1' \,| \;\leq 1) \wedge (|\, c_2 - c_2' \,| \;\leq 1) \wedge (c' \neq c) \right\} \quad (1)$$

i.e., those cells c' in C that are located within one cell from the eight directions 0°, 45°, 90°, 135°, 180°, 225°, 270°, 315°, excluding cell c itself. This is also called the Moore neighborhood (Wolfram 1984). To further characterize the neighborhood, we introduce three neighborhood functions Φ_1, Φ_2, and Φ_3.

$\Phi_1(c, d(C))$ is the number of immediate neighbors (0–8) with the same use as the cell in question for a given configuration $d(C)$, i.e.,

$$\Phi_1(c, d(C)) = \# \{ c' \in \Psi(c \mid d(c') = d(c) \} \quad (2)$$

Assuming scale-dependent costs, the higher this number, the lower the costs associated with the current use of the cell.

We assume that the recreational value is related to the distance between a cell and the nearest cell that is part of an inhabited area. Accordingly, $\Phi_2(c)$ measures the geographical distance to the nearest inhabited area.

It is presumed that structural variation is considered a desirable property and, therefore, we specify a function, Φ_3, which evaluates the variation characterizing a suitable neighborhood surrounding c. Hence, $\Phi_3(c, d(C), d(c), r)$ yields the distance weighted share of $d(c)$, the current land use of cell c, within a circular area with radius r. Denoting the geographical distance from cell $c \in C$ to another cell $c' \in C \cup K$ as $\Lambda(c, c')$, the neighborhood $\Gamma(c, r)$ of cells within the distance r from c is given by

$$\Gamma(c, r) = \left\{ c' \in C \cup K \middle| (\Lambda(c, c')) \leq r \wedge (c' \neq c) \right\} \quad (3)$$

and for $d(c)$ we define the subset $\Gamma'(c, r, d(c)) = \{ c' \in \Gamma(c, r) \mid d(c') = d(c) \}$. The neighborhood function $\Phi_3(c, d(C), d(c), r)$ is then given by

$$\Phi_3(c, d(C), d, (c), r) = \sum_{c' \in \Gamma'(c, r, d(c))} \Lambda(c, c')^{-1} \Bigg/ \sum_{c' \in \Gamma(c, r)} \Lambda(c, c')^{-1} \quad (4)$$

Objective Function and Transition Rules

Our objective is to maximize the overall economic value of the land use configuration $d(C)$. Basically, we assume that the overall value of the configuration, $V(d(C))$, is additive in cell contributions, $v_c(d(C))$, i.e.,

$$V(d(C)) = \sum_{c \in C} v_c(d(C)) \quad (5)$$

To be more specific, we assume that the value generated in cell c is the soil expectation value G minus scale-dependent costs F, plus the recreational value R and the value of structural variation component A. For each of these value components, an appropriate weight λ can be specified. Consequently, the contribution of cell c to the total value of $d(C)$ is given by:

$$V(d(C)) = \lambda_1 G(c) - \lambda_2 F(c, d(C)) + \lambda_2 R(c, d(C)) + \lambda_4 A(c, d(C)d(c), r) \quad (6)$$

The relationship between water retention capacity and soil expectation value, which is specific to each land use, is based on Holstener-Jørgensen (1958), Møller (1933), and the Danish Forest Society et al. (1995). For a given land use in D, the soil expectation value is $G(c) = \beta_i + \alpha_i s(c)$, where $s(c)$ is the water retention capacity of cell c and $i = 1, 2, 3, 4$ for beech, Norway spruce, oak, and pasture, respectively. Model parameters are presented in Table 1. The scale-dependent cost is assumed to be independent of land use and is modeled as

$$F(c, d(C)) = \omega_1 - \omega_2 \Phi_1(c, d(C)), \quad (7)$$

i.e., the costs are assumed to decrease linearly with the number of neighbors with similar land use. Furthermore, we assume that the recreational value is given by

$$R(c, d(C)) = \omega_3 (d(C) / \Phi_2(c)), \quad (8)$$

i.e., the recreational contribution to $v_c(d(C))$ decreases asymptotically towards zero with increasing distance to inhabited areas. Specific values of $\omega_3(d(C))$ are associated with each land use (Table 1). Finally, the contribution of structural variation to $v_c(d(C))$ is expressed as

$$A(c, d(C), d(c), r) = -\kappa_1 \Phi_3^2(c, d(C), d(c), r) + \kappa_2 \Phi_3(c, d(C), d(c)r) + \kappa_3, \quad (9)$$

where $\kappa_1 \ldots \kappa_3$ are chosen such that this quadratic function attains its maximum value where $\Phi_3(c, d(C), d(c), r) = 0.25$, i.e., $(-0.5\kappa_1 + \kappa_2 = 0) \wedge (\kappa_1 > 0)$.

Local Versus Global Equilibrium

We emphasize that even though the overall system objective can be decomposed into cell contributions as above, we cannot be sure that cell-based optimization will lead to the system-wide optimum. Indeed, system-wide optimality may not even be consistent with cell-level optimization.

At the general level, the possible conflict between system and cell optimization is not surprising. In economics, it is well known that pursuing individual interests can lead to a globally inferior outcome. Moreover, a global optimum is not guaranteed to constitute an equilibrium from which no individual could gain by making unilateral deviations. Therefore, either the economic system must have properties that ensure that locally optimal changes

Table 1. Model parameters.

α_1	441	β_1	−38,582
α_2	293	β_2	29,152
α_3	382	β_3	−14,828
α_4	167	β_4	23,333
ω_1	65,000	ω_2	10,000
$\omega_3\,(1)$	70,000	$\omega_3\,(2)$	1,000
$\omega_3\,(3)$	50,000	$\omega_3\,(4)$	70,000
κ_1	1	κ_2	0.5
κ_3	0	r	200 m

are also globally advantageous, or the behavior of the automaton must be modified in such a way that global improvements become much more common than global deteriorations.

In the model presented above, the conflict between cell and system-wide optimization is related to the maximization of F, A, and sometimes R. The optimization of the G function leads to the same result whether it is done locally or globally since, in terms of G, the cells are fundamentally independent. Similarly, if $\Phi_2(c)$ measures the distance to inhabited areas, local optimization of R will not conflict with system-wide objectives. However, if $\Phi_2(c)$ measures the distance to a land use in D, then conflicts may arise because, in that case, the gain realized by changing land use in one cell may be offset by the cumulated loss of hundreds of cells for which the current cell was the nearest neighbor with the land use in question. In the current application, $\Phi_2(c)$ measures the distance to inhabited areas and, therefore, no such conflict exists for the R component.

The local optimization of the F component may sometimes conflict with system-wide optimization. Consider a simple two-cell problem with two land use alternatives where the two cells hold land uses 1 and 2, respectively, in iteration t. In a plain cellular automaton all cells optimize their behavior at each iteration step presuming no change in the behavior of the surrounding cells. Accordingly, to maximize F both cells will change land use in iteration $t + 1$. This behavior is repeated in the following iterations, implying that the land uses switch back and forth between the cells. Clearly, this does not lead to a system-wide optimum where the land use of both cells is the same.

The properties of the A component of the model are somewhat different. By maximizing A, each individual cell avoids land uses with either a high or a very low local share of the area, as such land uses will lead to low or even negative A values. Therefore, land uses that are relatively rare within the neighborhood are given priority. It is important to note that as the neighborhoods considered by each of the other cells of the actual neighborhood are different, the gain of the current cell may be offset by the cumulated loss experienced by the other cells of the neighborhood.

In order to control the behavior of the automaton, we consider an evolutionary variant of the algorithm below, which is applied in the present study. It does not assure system-wide optimization either, but it increases the probability of identifying a good solution.

Evolutionary Optimization Procedure

As described above, the behavior of crude cellular automata is not generally consistent with system-wide optimization. Therefore, we introduce a probabilistic algorithm that implies that the number of currently optimizing cells gradually decreases during the optimization process. In the beginning of the process, most cells are allowed to optimize, and at this stage many local decisions will immediately turn out to be second-best (or worse) due to simultaneous decisions of other cells within the neighborhood. At this stage, only those parts of the final solution are formed which are mainly determined by the basic geographical information. Later,

only few cells are active, meaning that the basis of local decisions does not immediately change. At this stage, those parts of the solution that have already crystallized will to some extent determine the solution in neighboring regions. Gradually, a solution will be found even for remote regions where the value of various land uses depends almost exclusively on the land use of the neighborhood.

The algorithm involves the possibility of local improvement (innovation) as well as random land use changes (mutation). Figure 1 presents a flow chart of the algorithm.

In each iteration, all cells are evaluated with some probability. If evaluation is carried out, the potential improvement of the current land use is accepted (innovation). Cells that are not evaluated remain unchanged, as long as mutation does not occur. The probability of innovation is high in the beginning of the process and gradually decreases towards zero. The probability function is specified as

$$P_I(t) = P_I^0 \cdot (1 - t/T)^{\tau_I} \qquad (10)$$

where P_I^0 is the initial probability of innovation, t is the current iteration number, T is the total number of iterations, and the exponent $\tau_I > 0$ (Table 2).

The probability of mutations is also high in the initial iterations and approaches zero in the final ones. The mutation probability function is

$$P_M(t) = P_M^o \cdot (1 - t/T)^{\tau_M} \qquad (11)$$

where P_M^o is the initial probability of mutation ($t = 0$). When the exponents are chosen such that $\tau_M > \tau_I$, the mutation probability will decline faster than the probability of innovation, implying that the degree of stochasticity characterizing the movements of the system will be highest in the initial stages of the process.

The software was developed using Borland Turbo Pascal. Several hundred optimization runs were executed with varying parameters: P_I^o, P_M^o, τ_M, τ_I, and T, to find an appropriate parameter setup. The resulting parameters are presented in Table 2. It turned out that, for the final objective value, the single most crucial parameter was T, and therefore, special care was taken to identify the minimum number of iterations required for the algorithm to reach the highest possible average objective value. On the basis of 70 optimization runs with T ranging from 10 to 2,500, it was found that no significant improvement of the average objective value could be observed for T values exceeding approximately 250. Hence, all results reported here are based on optimization runs with $T = 500$.

Results

To investigate the properties of the self-organizing algorithm, a simpler version of the above described model was formulated and solved using the self-organizing algorithm, simulated annealing, and an IP solver. Details on this test are given in the Appendix. The encouraging result of the test is that the self-organizing algorithm is actually capable of finding good solutions within reasonable time. Furthermore,
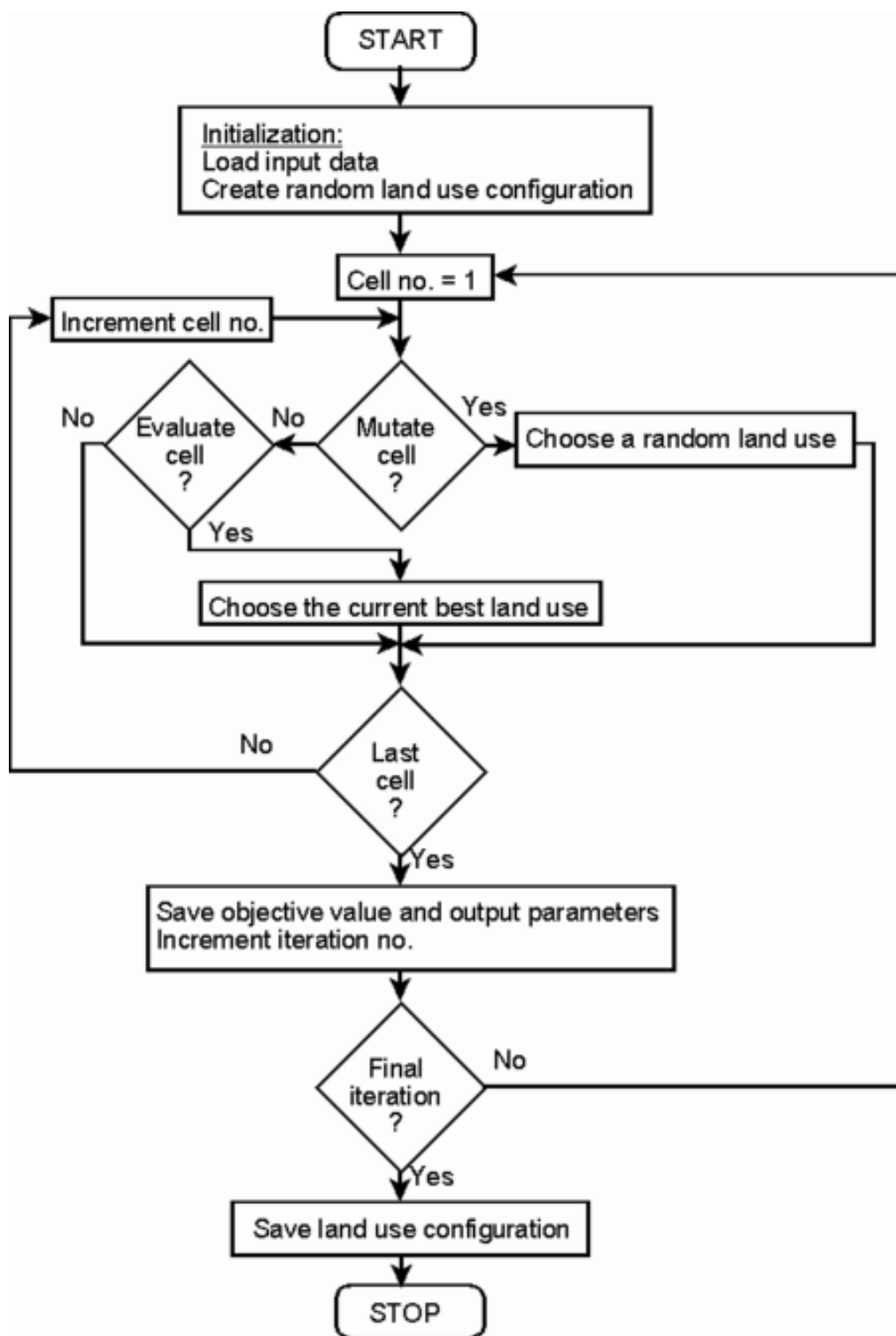
Figure 1.   A flowchart representation of the evolutionary algorithm.

Table 2. Parameters of the self-organizing algorithm.

P_I^0	0.9
P_M^0	0.01
τ_I	2
τ_M	4

for larger problems, it outperforms simulated annealing. When we include the structural variation component A, the problem becomes very difficult to formulate and solve using IP solvers. Thus, it appears reasonable to proceed with the self-organizing algorithm to solve the more complicated problem presented above.

If we maximize G and disregard scale-dependent costs, recreational value, and structural variation (i.e., $\lambda_2 = \lambda_3 = \lambda_4 = 0$), the decision problem is simple. Norway spruce should be chosen in all cells, as it has the highest G of all land uses irrespective of the water retention capacity. The problem is completely separable, as the cells are not interacting. However, realistic problems are not separable, as the decision in one cell will usually affect the value of the land use in neighboring cells. The chosen model parameters are presented in Table 1. Moreover, as a test case we chose the following weights for the four components of the objective function, $\lambda_1 = 1$, $\lambda_2 = 1$, $\lambda_3 = 225$, and $\lambda_4 = 10,000$.

Generally, we assume that structural variation is a matter of concern, as this would be advantageous to wildlife, hunting, and presumably also recreation. Hence, we maximize an objective function, which, in addition to various other objectives, emphasizes structural variation (A). Due to the scale-dependent cost function F, however, cells with similar land use are encouraged to gather, and hence, a certain large-scale patchiness is promoted. Furthermore, the recreational value of a cell, R, is modeled as inversely proportional to the distance to inhabited areas, reflecting that the intensity of recreational use depends on accessibility. Finally, it is assumed that people generally prefer pastures and broadleaved species (in particular beech) in the vicinity of inhabited areas, and we therefore prioritize pasture and beech followed by oak, and finally Norway spruce.

An initial solution was created by assigning a random land use $d(c) \in D$ to each $c \in C$ with equal probabilities. Then, the optimization process was allowed to work for a certain number of iterations (T). Figure 2 presents an example solution showing the spatial distribution of land uses resulting from an optimization run with 500 iterations. The result corresponds to the policy formulation given by the weights $\lambda_1,.., \lambda_4$. Consequently, pastures are frequently chosen in cells that are located in the vicinity of inhabited areas; further away, patches with beech, oak, and Norway spruce become increasingly common. The neighborhood function, $\Phi_3(c, d(C), d(c), r)$, generates some structural variation. However, due to the scale-dependent cost function, the various land use categories are arranged in groups that may form oblong, parallel structures or assume more organic looking shapes. It should be noted that the outer edges of the afforestation area influence the solution even at some distance.

The resulting land use composition of the afforestation area is approximately: 24% pasture, 17% beech, 8% Norway spruce, and 1% oak (percent of all 56×84 cells). The evolution of the objective value and the shares of each of the four land uses are shown in Figures 3a and 3b, respectively. Early in the process, at iteration 5, the objective value is $DKK\,3.73 \cdot 10^7$, which is low compared to the final value of $DKK\,4.56 \cdot 10^7$ at iteration 500. However, apparently the system approaches the final land use distribution after only 100–150 iterations. From the state resulting
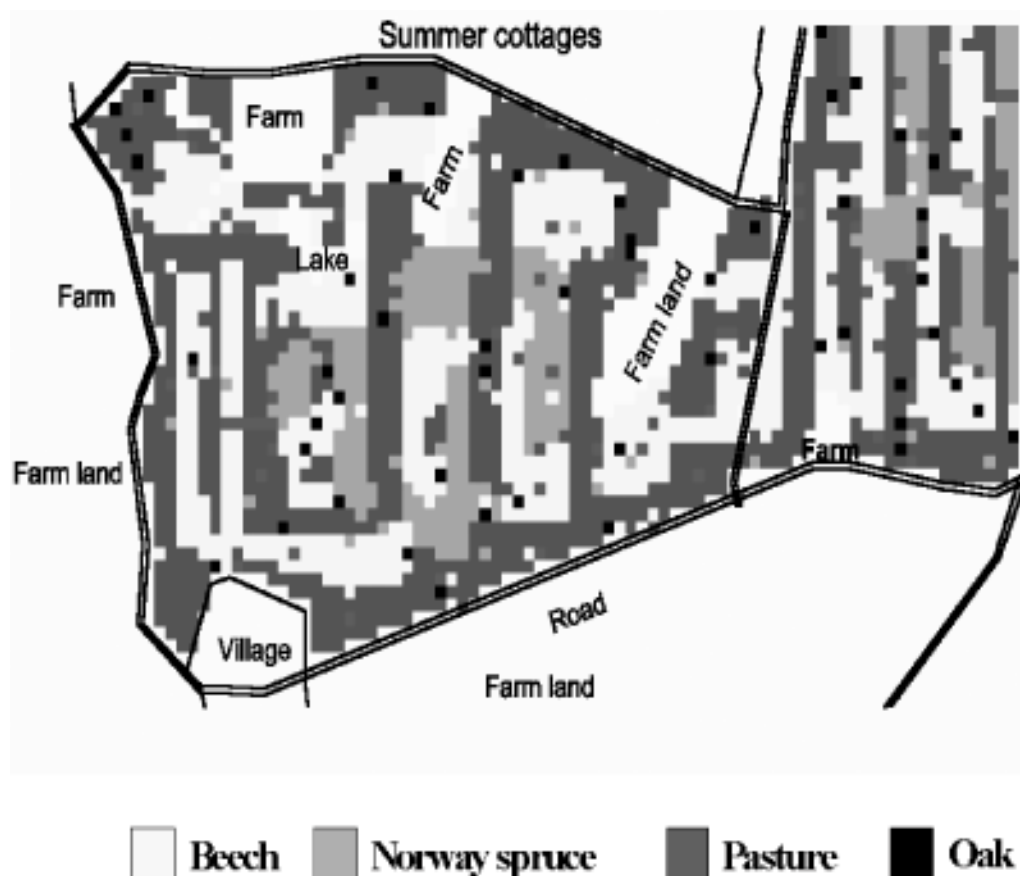

Figure 2. Final solution obtained for an optimization run with 500 iterations. Black lines are roads or boundaries of inhabited areas.
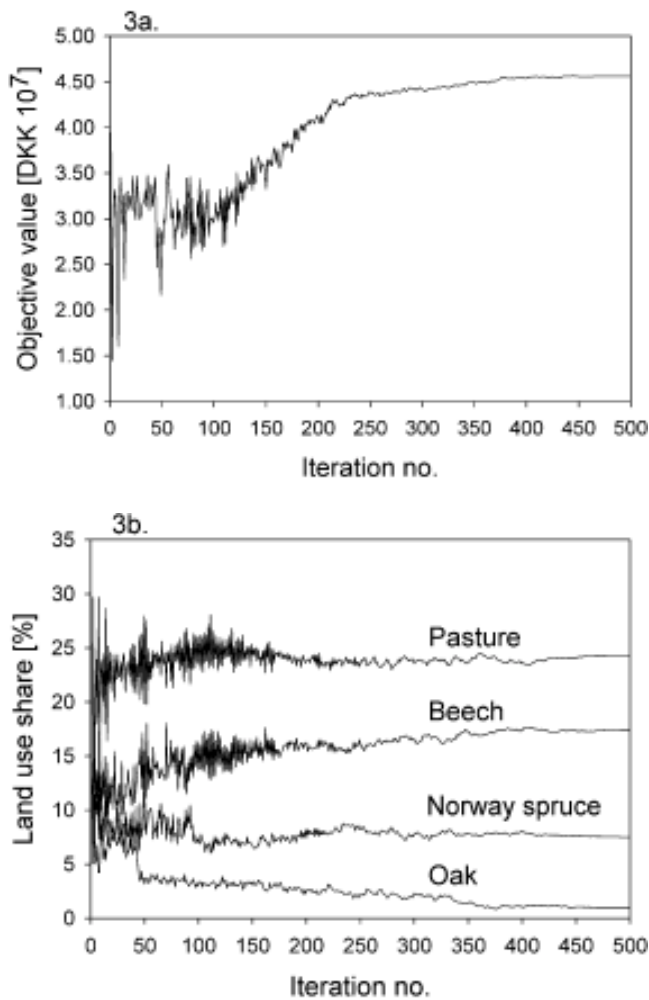
Figure 3. Objective values (a) and land use shares (b) as observed for an optimization run with 500 iterations.

after 100–150 iterations, the system organizes itself towards a more pronounced grouping (larger patches), which reduces the scale-dependent costs without changing the value of the structural variation significantly. It is worth noting that the competition between land uses is very high in the beginning of the optimization process. After only 100 iterations the shares of Norway spruce and oak have almost stabilized. Beech and pasture, however, continue to compete for at least 100 iterations more.

The optimization was performed on a Pentium II 333 Mhz. The computation time was approximately 4 hr and 20 min for an optimization run with 500 iterations.

Stability of the Solution

Even though the optimization process is to some extent guided by geographically fixed structures, i.e., water retention capacity and inhabited areas, the system may have several different attractors, and even though the solutions associated with these attractors might have similar objective values, they are likely to have quite different land use allocation patterns. We therefore investigated whether the solutions found with only 500 iterations per optimization run are structurally stable, i.e., whether different initial configurations converge towards similar solutions. We repeated the optimization process 50 times, each time with

a new random initial configuration. All optimization runs comprised 500 iterations.

The frequency of each land use in cells $c \in C$ at iteration 500 is shown in Figures 4a–d (dark shading corresponds to high frequency). For the current model, the structural stability is high with respect to pasture and beech, whereas the spatial distribution of oak and Norway spruce varies considerably. Between 40 and 50 of all optimization runs find that pasture is the most favorable land use in cells close to the village in the southwestern part of the area, and in the vicinity of the summer cottages in the northern part (Figure 4a). Moreover, the R function implies that pastures and beech are favored near the boundary of the afforestation area. Beech and pasture are associated with the highest recreational values, followed by oak and Norway spruce, and this is clearly reflected in the frequency pattern of Figures 4b–d. In Figure 4b a zone with a high frequency of beech is found "inside" the outer pasture zone, followed by high frequencies of oak and Norway spruce inside the beech zone. The more distant from inhabited areas, the more Norway spruce is chosen as the most appropriate land use. In the center and in the eastern part of the afforestation area Norway spruce is chosen in more than 50% of all solutions (Figure 4d). Outside these regions, the short distance to inhabited areas increases the "fitness" of other tree species comparatively more and reduces the frequency of Norway spruce to approximately 0 to 10% of all solutions. The strongest impact of G is experienced in areas that are located far from any inhabited areas. Since Norway spruce has the highest G of all land uses, irrespective of the water retention capacity, it is the main tree species in the center and in the far eastern part of the area.

The distributions of the objective values at iteration 1, 10, 50, 100, 200, 400, and 500 are presented in Figure 5. All objective values (50) at iteration 500 are higher than those obtained below 400 iterations. The average objective value decreases from iteration 1 towards 100 but, hereafter, the system generally finds a trajectory that increases the objective value (see also Figure 3). The average values (circles), are $DKK\ 3.94 \cdot 10^7$ at iteration 1, $DKK\ 2.99 \cdot 10^7$ at iteration 10, $DKK\ 3.12 \cdot 10^7$ at iteration 50, $DKK\ 3.06 \cdot 10^7$ at iteration 100, $DKK\ 4.09 \cdot 10^7$ at iteration 200, $DKK\ 4.52 \cdot 10^7$ at iteration 400, and $DKK\ 4.56 \cdot 10^7$ at iteration 500.

The relationship between the objective value and the land use composition is presented in Figures 6a–d at iterations 1, 10, 50, 100, 200, 400, and 500. The figures are based on 50 independent optimization runs. The ellipses confine regions corresponding to the central 90% of the estimated distributions, and the lines that connect the centers of the ellipses illustrate the movement of the system through solution space. Generally, the objective value increases with iteration number, i.e., the system is in fact optimizing. However, as mentioned earlier, the objective values of iterations 10–100 are lower than the objective value at iteration 1. Generally, the most favorable land use distribution is approached shortly after 100 iterations. The shape and size of the ellipses clearly show that, with respect to the share of oak and Norway spruce, many different solutions yield almost the same final objective value. As regards the share of pasture, however, all
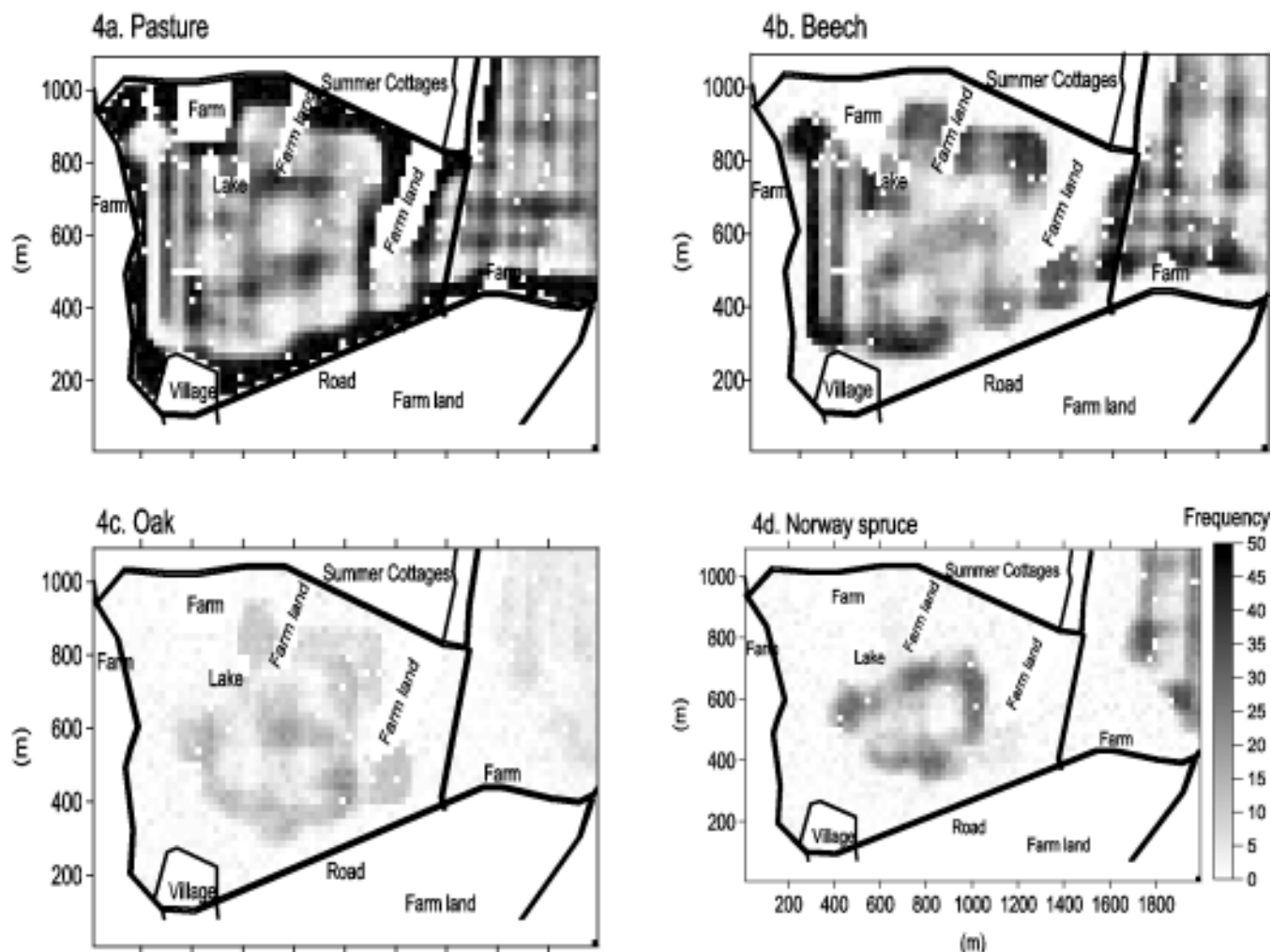
Figure 4. Number of final solutions with pasture (a), beech (b), oak (c), and Norway spruce (d), respectively, in each of the cells. Based on 50 optimization runs with 500 iterations each.

final solutions are similar. This result corresponds to the conclusions we made on the basis of Figure 4 a–d. The trajectory from the initial configuration towards the final solution may, however, lead through configurations with shares of pasture ranging from less than 15% to more than 30% (iteration 10).
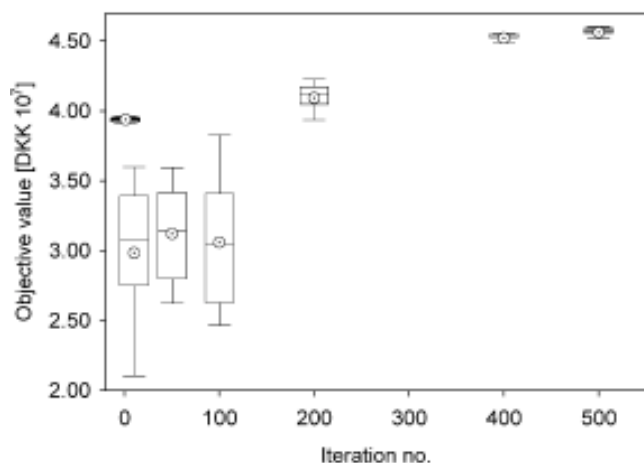

Figure 5. Box plot of objective values at iterations 1, 10, 50, 100, 200, 400, and 500. Circles indicate the mean objective value. Based on 50 optimization runs with 500 iterations each.

Discussion

It is recognized in other studies that spatial optimization problems easily get very complicated. If the number of sites is large and the mutual dependence between different sites is high, it may not be possible to find optimal or good solutions by means of traditional optimization methods. We have demonstrated that appropriate modifications of cellular automata may be a useful supplement to traditional methods. In fact, it seems that high-quality solutions can be found comparatively fast. Application of cellular automata for land use optimization has the added advantage of being a natural extension of grid-based geographic information systems, and they may work well with data in vector format. In the future, when multiprocessor computers will enable massive parallel computation, simple parallel (self-organizing) algorithms will be easier to implement and operate faster than centralized algorithms, based on serially working, centralistic decision rules.

In the evolutionary algorithm applied here, the activity of the cellular automaton gradually decreases. In the first iteration, every interior cell is—on average—surrounded by equal numbers of all land uses, $d(c) \in D$. This implies
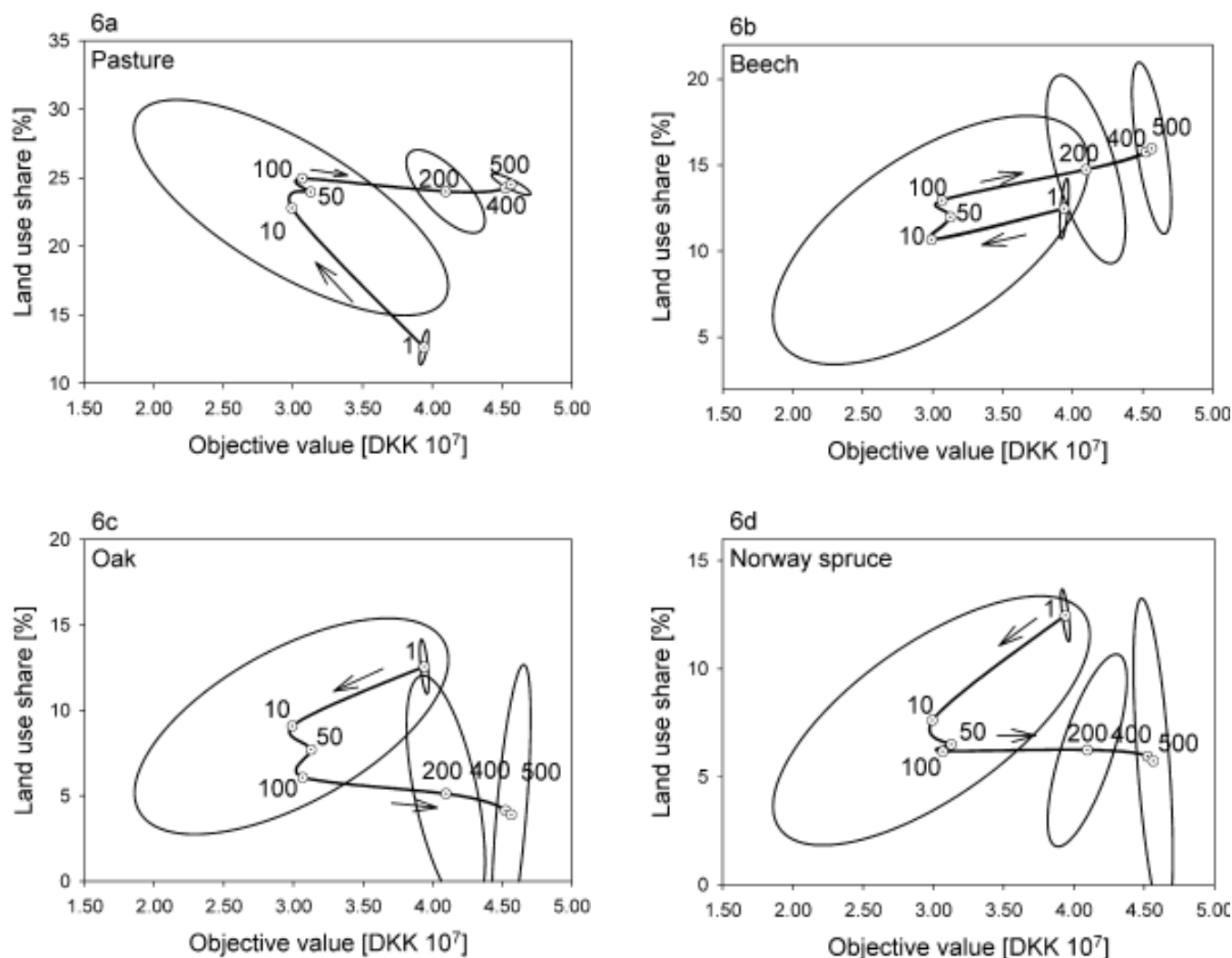
Figure 6. Distribution of final objective values and percentages of pasture (a), beech (b), oak (c), and Norway spruce (d) at iterations 1, 10, 50, 100, 200, 400, and 500. Based on 50 optimization runs with 500 iterations each. Ellipsoidal contours confine those regions where 90% of the solutions are found.

that in the very first iteration the values of G and R usually determine the choice of land use in those cells that are evaluated. This further implies that the G and R functions dominate the system from the outset. In later iterations the A and F functions compete to form patches which are large enough for scale-dependent costs to be low and small enough to maintain a certain amount of variation within the radius r.

We have suggested that spatial optimization problems where the biological and economic interactions decrease rapidly with distance may be particularly adaptable for self-organizing algorithms such as the one applied here. As examples of local interactions, we have used a scale-dependent cost function and the value of structural variation within a given neighborhood. Moreover, we have introduced the value of recreation, which is presumed to decrease with increasing distance to inhabited areas. As regards the scale-dependent cost function, we have assumed that the costs per cell are reduced linearly with increasing number of similar neighbors within a neighborhood comprising eight cells. This is motivated by the economies of scale characterizing timber harvesting as well as other silvicultural activities.

To test the efficiency of the self-organizing algorithm, we use a simplified version of the afforestation model (Appendix). This simple problem shows that the self-organizing algorithm is actually capable of finding good and sometimes optimal solutions. Accordingly, it is likely that this is also the case for the larger problem presented in Section 3. Furthermore, the simplified model showed that while simulated annealing was superior to the self-organizing algorithm for a very small problem, the self-organizing algorithm became increasingly superior to simulated annealing with increasing problem size. Intuitively, this result appears quite reasonable. Due to the fact that the self-organizing algorithm is based on local optimization only, the processing time required to obtain results of a given quality is likely to increase at a low rate with increasing number of cells, as compared with more centralized heuristics.

Potentially, spatial economic models like the one presented here may be applied for land use planning and policy evaluation. Ideally, the computation time should be so short that various policies can be tested interactively. Whether this requirement can be met or not depends on the complexity of the economic model, the spatial resolution, and the computa-

tional power available. The visualization tools of a geographic information system may be used for displaying the achievements of the optimization process. Potentially, this allows the policy maker to suggest an economic model and watch the land use setup evolving from random initial states towards a good solution. The solution will be characterized by a structural order that reflects the suggested economic rules and the variation of site conditions. Moreover, the solution represents one of the best allocations of land use, provided that the suggested economic rules are in fact relevant in every part of the area evaluated. Thus, the result can be used for adjusting the rules in such a way that a politically acceptable high-quality solution is obtained. Insight acquired through this sort of iterative adaptation of economic rules may act as decision-support.

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APPENDIX. A Test Problem

In the following, we present a small 0-1 integer problem which is solved using the self-organizing algorithm, simulated annealing, and Extended Lindo Version 6.1. The aim of the study is to test the performance of the self-organizing algorithm by comparing the quality of its solutions with those of simulated annealing.

Consider an afforestation area, where for each location we want to choose the economically optimal land use. The economic objective is to maximize a linear combination of the gross margin of the mean annual production of the chosen land use at each location, and a scale-dependent cost component similar to the one described in the main text. The test problem includes two land use alternatives and 25 decision units (locations).

The area is defined as an $I \times J = 5 \times 5$ lattice of square cells, each with an area of 1 ha. Let c denote a specific cell and C a set including all cells i.e., $C = \{c = (c_1, c_2) \mid (c_1 = 1,...,I, c_2 = 1,...,J)\}$. Let $D = \{A,B\}$ denote the possible land use alternatives in each cell, and $d(c) \in D$ the specific alternative chosen for cell c. A specific land use configuration can be denoted $d(c) = [d(c), c \in C]$. The number of such configurations is $2^{25} = 3.355 \cdot 10^7$.

A given cell, c, is characterized by an expected gross margin of land use A, $g^A(c)$, and land use B, $g^B(c)$. The assumed values of $g^A(c)$ are shown in Table A1. The gross margin of land use B is estimated on the basis of the gross margin of land use A: $g^B(c) = 0.6335 g^A(c) + 596.55$.

In small cellular automata, edge effects may have global influence. To avoid such effects and the formulation of edge specific rules, the automaton is folded and joined (like the inner tube of a car tire). We define the neighborhood $\Psi(c)$ around c as those four cells that share a full border with c, i.e., the Von Neumann neighborhood (Wolfram 1984). We introduce a neighborhood function similar to Equation (2), expressing the number of neighbors in $\Psi(c)$ with the same use as the cell in question. Similar to Equation (7), the scale-dependent cost function is estimated as a linear function which decreases with the number of neighbors with similar land uses. The overall value of the land use configuration is estimated as in Equation (5), and the value generated in cell c, $v_c(d(C))$, is the value of the gross margin $G(c,d(c))$ minus the value of the scale-dependent cost function $F(c,d(C))$.

The simulated annealing procedure, as first suggested by Kirkpatrick et al. (1983) and Cerny (1985), is formulated as in Arts and Korst (1989, p. 16). Based on several tests we set the initial temperature at $5 \cdot 10^5$, the cooling ratio at 0.8, and the number of transitions at each tempera-

Table A1. Experimental data set on gross margin of land use A, $g^A(c)$, in each cell of the torus (DKK ha^{-1} yr^{-1}).

$j \backslash i$	1	2	3	4	5
1	2,591.9	1,947.0	1,491.7	1,835.5	2,879.3
2	1,129.8	882.9	1,469.8	1,300.5	1,994.6
3	2,199.3	1,176.9	1,944.1	2,281.7	1,825.8
4	951.0	2,547.7	1,661.4	1,715.4	2,555.9
5	1,818.8	1,634.5	2,671.5	931.6	1,220.8

ture level at 50. The stop criterion is fulfilled when the improvement of the objective function is less than $1 \cdot 10^{-6}$ or when no improvements have been found at a given temperature level.

The self-organizing algorithm is formulated as presented in Figure 1 and in Equations (10) and (11). Numerous optimization runs with different parameter settings were executed to find the best possible setup, which turned out to be when the initial probability of innovation, P_I^0 was 0.9, the initial probability of mutation, P_M^0, was 0.01, and when $T = 500$.

The 0-1 integer model formulation of the 5×5 cell problem is as follows:

$$\max \sum_{i=1}^{5} \sum_{j=1}^{5} \sum_{p \in \{A,B\}} \left[(g_{ijp} - \omega_1) X_{ijp} + \omega_2 (N_{ijp} + E_{ijp} + S_{ijp} + W_{ijp}) \right]$$

subject to

$$\sum_{p \in \{A,B\}} X_{ijp} \leq 1$$
$$\text{for } i = 1,2,...,5 \text{ and } j = 1,2,...,5$$

$$X_{ijp} + X_{n(ij)p} - 2N_{ijp} \geq 0$$
$$\text{for } i = 1,2,...,5 \text{ and } j = 1,2,...,5, \text{ and } p \in \{A,B\}$$

$$X_{ijp} + X_{e(ij)p} - 2E_{ijp} \geq 0$$
$$\text{for } i = 1,2,...,5 \text{ and } j = 1,2,...,5, \text{ and } p \in \{A,B\}$$

$$X_{ijp} + X_{s(ij)p} - 2S_{ijp} \geq 0$$
$$\text{for } i = 1,2,...,5 \text{ and } j = 1,2,...,5, \text{ and } p \in \{A,B\}$$

$$X_{ijp} + X_{w(ij)p} - 2W_{ijp} \geq 0$$
$$\text{for } i = 1,2,...,5 \text{ and } j = 1,2,...,5, \text{ and } p \in \{A,B\}$$

$$X_{ijp}, N_{ijp}, E_{ijp}, S_{ijp}, W_{ijp}, \in \{0,1\},$$

where X_{ijp} is a binary variable that takes a value of 1 if cell ij is assigned land use p, and 0 otherwise. g_{ijp} is the gross margin earned if cell ij is assigned land use p. $\omega_1 = 300$ is the per-cell cost if all of the neighboring cells are assigned a different land use. $\omega_2 = 30$ is the per-cell cost savings of assigning the same land use to two adjacent cells. The four variables N_{ijp}, E_{ijp}, S_{ijp}, and W_{ijp} are binary and take a value of 1 if land use p is assigned both to cell ij and the adjacent cell to the north, east, south, and west, respectively. Otherwise, they take a value of 0.

The four functions $n(ij)$, $e(ij)$, $s(i,j)$, and $w(i,j)$ return the coordinates of cells to the north, east, south, and west of cell ij, respectively. If cell ij is at the northernmost or southernmost edge of the lattice, the coordinates of the cell at the opposite edge of the lattice (in the same column) are returned. Similarly, if cell ij is at the easternmost or westernmost edge, the coordinates of the cell at the opposite end of the same row are returned.

The linear programming model was solved using Extended LINDO Version 6.1. The self-organizing algorithm

Table A2. Objective values and computation time characterizing various optimization methods.

Method	No. of repetitions	Minimum objective value	Maximum objective value	Average objective value	Standard deviation	Average computation time (seconds)	No. of optimal solutions
I = J = 5:							
Simulated annealing	1,000	40,160	40,352	40,335	31.5	280	668
Self-organization	1,000	40,015	40,352	40,307	32.9	280	296
0–1 *IP*			40,352			1	
I = J = 10:							
Simulated annealing	1,000	159,400	161,302	160,724	285.8	4,500	0
Self-organization	1,000	160,582	161,409	161,228	90.9	4,500	6
0–1 *IP*			161,409			5	

and the simulated annealing algorithm were both implemented in software written in Borland Turbo Pascal. All three were run on a Pentium II 333 MH.

As regards the self-organizing algorithm, preliminary experiments showed that different initial configurations may certainly lead to different final solutions. In fact, for the 25 cell problem, the self-organizing process turned out to have two main attractors, the objective values of which were, however, almost similar. One of these solutions was the optimal configuration. To compare the properties of the self-organizing algorithm with simulated annealing we repeated the two procedures for 1,000 random initial configurations. Minimum and maximum objective values, mean objective value, standard deviation, average computation time needed for optimization, and the number of globally optimal solutions are reported in Table A2. To enable comparison between the two heuristics, the parameters of the simulated annealing algorithm were optimized subject to the restriction that the average computation time should be similar to that of the self-organizing algorithm.

We first tested the optimization procedures on the 25 cell problem. The branch and bound algorithm succeeded in finding the optimum solution within approximately 1 sec. For this solution, land use *A* covers 76% of the area, and the objective value is 40,352.

For the heuristics, the following results were obtained. The optimal solution was found in 66.8 and 29.6% of the optimization runs with simulated annealing and the self-organizing algorithm, respectively (Table A2). Based on the relative performance of the two methods, it appears that the simulated annealing algorithm is superior to the self-organizing algorithm as it finds the true optimum more frequently and has a slightly higher average objective value. The hypothesis of identical mean objective values was rejected at the 1 per mille level.

To test the sensitivity of the optimization procedure to problem size, a 100 cell problem was created by copying the data structure in Table A1 to each of the four 5 by 5 corner blocks of a 10 by 10 cell matrix. The 0-1 integer programming solver found the optimal solution to this problem within 5 sec. For both simulated annealing and the self-organizing algorithm 1,000 optimization runs were performed. Simulated annealing did not find the optimal solution in any of the optimization runs. The self-organizing algorithm found the optimal solution 6 times, which is actually very close to the expected $1,000 \cdot 0.296^4 \sim 8$ times. As the self-organizing algorithm is based on local optimization, its performance is not influenced by problem size to the same extent as simulated annealing. It should be noticed that the average performance of the self-organizing algorithm is unchanged, as the average objective value is actually four times the value achieved for the 25 cell problem. This also implies that the ratio of the average objective value to the global maximum is not influenced by the increased number of cells.

In contrast to this, the performance of the simulated annealing algorithm clearly depends on problem size. Again, we find that the hypothesis of simulated annealing and the self-organizing algorithm having identical mean objective values can be rejected at the 1 per mille level. Thus, for the 100 cell problem, the self-organizing algorithm outperforms simulated annealing in number of hits on the optimum as well as on mean objective value. Results for even larger problems not presented here support that for similar computational effort, the self-organizing algorithm becomes increasingly superior to simulated annealing with increasing number of cells.