Answers to questions in Lab 1: Filtering operations

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Instructions: Complete the lab according to the instructions in the notes and respond to the questions stated below. Keep the answers short and focus on what is essential. Illustrate with figures only when explicitly requested.

Good luck!

Question 1: Repeat this exercise with the coordinates p and q set to (5, 9), (9, 5), (17, 9), (17, 121), (5, 1) and (125, 1) respectively. What do you observe?

Answers:
The different points represent the frequencies of each component in the Fourier domain. So if we have a spike point only at (p, q) = (5, 9) in the freq. domain, we can observe that the lines will change more in the v-direction (horizontally). The opposite can be observe at (p, q) =(9, 5), where the lines will have the same but shifted changes in the u and v directions respectively.
If the pixel is moved (translation), we will observe that the phase is being changed.
It is the phase that changes in the Fourier domain. It is the same pixel that is being moved in the image. Translation thus only affects the phase.

Question 2: Explain how a position (p, q) in the Fourier domain will be projected as a sine wave in the spatial domain. Illustrate with a Matlab figure.

Answers:
Applying the definition of the inverse Fourier transform (from equation 2 in the instructions) on a Dirac function will result in a complex sinusoid:
$\int_{-\pi}^{\pi} 2\pi\delta(\omega - \omega_0)\, e^{j\omega n}\, \frac{d\omega}{2\pi} \;=\; e^{j\omega_0 n}.$ So in other words will a dirac function in the frequency domain be a (comlex) sinusoid in the spatial domain.
This makes sense sine the Fourier transform is periodis and takes some number and spinns them around the unit circle, which corresponds to cos and sin in x and y axis respectively.

The Fourier transform shows that any signal can be represented as an infinite sum of different sinusoids. If we only have a spike at a particular frequency in the frequency domain, this means that we have a signal that inly has a constant frequency, in other words a sinusoid in spatial domain that is operates at that frequency.

Question 3: How large is the amplitude? Write down the expression derived from Equation (4) in the notes. Complement the code (variable amplitude) accordingly.

Answers: 1/size = 1/N = 1/128, can be seen from

$$F(x) = \mathcal{F}_D^{-1}(\hat{F})(x) = \frac{1}{N} \sum_{u \in [0..N-1]^2} \hat{F}(u)\, e^{\frac{+2\pi i u^T x}{N}}.$$

Question 4: How does the direction and length of the sine wave depend on p and q? Write down the explicit expression that can be found in the lecture notes. Complement the code (variable wavelength) accordingly.

Answers: wavelength: lambda = 1/sqrt((u^2 + v^2)) and direction arctan(v/u) + pi/2. (pi/2 due to that we want to get the directions of the sine waves and not the line).
u and v are the frequencies when we're in Fourier domain.

Question 5: What happens when we pass the point in the center and either p or q exceeds half the image size? Explain and illustrate graphically with MATLAB!

Answers:
The angle (and therefore the imaginary part also) changes sign: cos(wt) + i*sin(wt). sin() only changes sign when we flip the signs of angle.

Question 6: What is the purpose of the instructions following the question *What is done by these instructions?* in the code?

Answers:
The Fourier transformation is being shifted, meaning that the sampling goes from -pi to pi, instead of 0 to 2pi. This is done to get the spikes centered in the figure, and the lines of code makes sure that the pixels are included in this interval when sampled.
It is therefore easier for us to see so that we have lower frequencies in the middle, instead of seeing it at the edge of an image when the lower frequencies are dominant.

Question 7: Why are these Fourier spectra concentrated to the borders of the images? Can you give a mathematical interpretation? Hint: think of the frequencies in the source image and consider the resulting image as a Fourier transform applied to a 2D function. It might be easier to analyze each dimension separately!

Answers:
By convention it goes from 0-2Pi, and the low frequencies will be at the edges. But by shifting it so it goes between -pi and pi, it will be centered in the middle.
The reason we have, for example a vertical line at u=0 in the frequency domain, is because the original picture has no change in the u direction but only changes in the v direction (vertically, due to white line in the horizontal direction).

Question 8: Why is the logarithm function applied?

Answers:
The dynamic range of the Fourier coefficients (i.e. the intensity values in the Fourier image) is too large to be displayed on the screen. It is therefore necessary to apply the logarithmic

function to make sure all the values are included, and the difference could be seen in a reasonable smoothed scale of the frequencies.

Question 9: What conclusions can be drawn regarding linearity? From your observations can you derive a mathematical expression in the general case?

Answers:
It is linear, which can be seen when applying superposition on the different Fourier transforms in the images. Linearity of Fourier (**F**) operation applies: **F**(a*h(x) + b*g(x)) = a***F**(h(x)) + b***F**(g(x)).

Question 10: Are there any other ways to compute the last image? Remember what multiplication in Fourier domain equals to in the spatial domain! Perform these alternative computations in practice.

Answers:

Convolution in one domain is multiplication in the other domain. To obtain the same image, one can take the convolution between the matrices before taking its inverse fast Fourier transform to obtain the same image. (Since MATLAB doesn't divide by N when using the fast Fourier transform, we need to do this ourselves!)

Question 11: What conclusions can be drawn from comparing the results with those in the previous exercise? See how the source images have changed and analyze the effects of scaling.

Answers:
In the previous one, we had a square, which resulted in a symmetric and balanced Sinc shaped function in the frequency domain. But here we instead have rectangle that results in a Sinc function that does not have the same amplitude in the symmetric directions. This is since the rectangle results to higher frequencies (more change) in the u direction relative to the v direction.

Question 12: What can be said about possible similarities and differences? Hint: think of the frequencies and how they are affected by the rotation.

Answers:
If we don't shift the frequency interval before we rotate, the edges (that corresponds to the most information) will be cut out and most of the information will be lost. But if we shift the interval before rotating, the information will be centered at the middle of the image and therefore the rotation does not result in more loss of information,
When we turn the image, certain pixels will be messed up and when we transform it the image will be different. 30 and 60 degrees makes the pixels change the most, compares to 45 and 90 degrees.
When we rotate, we will get different frequencies in each direction.

Question 13: What information is contained in the phase and in the magnitude of the Fourier transform?

Answers:

The phase is the most important part, since it defines how the waveforms are shifted along its direction in the Fourier domain. The phase will therefor correspond to where edges will occur in the image and thus is the most important information in the image.
The magnitude defines how large the waveforms are and will therefor correspond to what grey-levels are on either side of the edges.

Question 14: Show the impulse response and variance for the above-mentioned t-values. What are the variances of your discretized Gaussian kernel for t = 0.1, 0.3, 1.0, 10.0 and 100.0?

Answers:
t = 0.1 → variance 0.2501, t= 0.3 → 0.3192, t=1 → variance = 1, t= 10 → 10, t=100 → 100.

Question 15: Are the results different from or similar to the estimated variance? How does the result correspond to the ideal continuous case? Lead: think of the relation between spatial and Fourier domains for different values of t.

Answers:
Small variance in spatial domain corresponds to big variance in the Fourier domain. This means that information will be lost when we inverse the Fourier transform to go back., because we have discretized (sampeled) the small variance, which is harder to do than the larger variance.

Question 16: Convolve a couple of images with Gaussian functions of different variances (like t = 1.0, 4.0, 16.0, 64.0 and 256.0) and present your results. What effects can you observe?

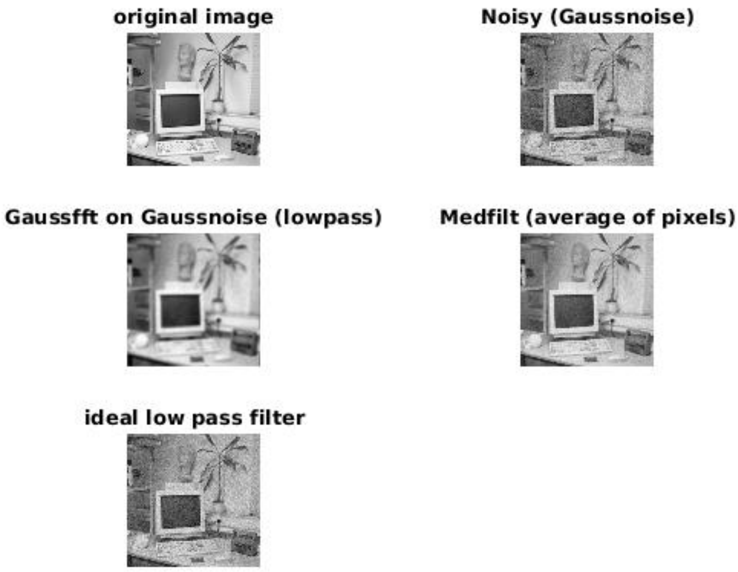
Answers:

For higher variances we get smoother image when we use convolution (through Gaussfft). In other words we get more noise in the picture when we use convolution between the image and the different gaussian nosie.

Question 17: What are the positive and negative effects for each type of filter? Describe what you observe and name the effects that you recognize. How do the results depend on the filter parameters? Illustrate with Matlab figure(s).

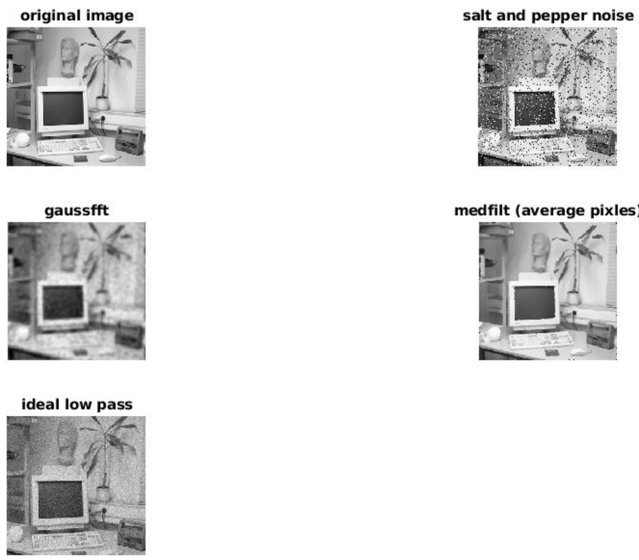
Answers:

In the case of filtering the Gaussian noise:



- The Gaussfft filters out the gaussian noise, which smoothest the picture out. The results become a blurry picture with lower frequencies (small changes in brightness).
- The Medfilt takes the average of a set of pixels and therefor removes some of the Gaussian noise without making the image to blurry, thus lowering the quality of the image.
- The ideal low pass filter doesn't do much difference until it's set to a certain low pass frequency, around 0.3, where it removes some of the gaussian noise, but not all of it.

In the case of filtering the salt and pepper noise:



- The medfilt clearly is the best filter for this case, averaging out the salt and pepper pixel noise. At some places there can still be track of some salt and peper noise, but overall quite good, except at edges and high frequencies of the image where some of the noise can still be seen.
- The Gaussfft filter doesn't work so well here. It manages to remove the salt and pepper random points but lowers the image quality.

- The Ideal low pass filter work quite well here, removing the high frequencies at the parameter 0.3 corresponding to the salt and pepper. The picture becomes a bit greyish and the white parts of the image becomes grey.

Question 18: What conclusions can you draw from comparing the results of the respective methods?

Answers:
That depending on what noise the image contains of, it may be appropriate to use different filters that match the kind of noise that one wishes to remove.

Question 19: What effects do you observe when subsampling the original image and the smoothed variants? Illustrate both filters with the best results found for iteration i = 4.

Answers:
At the last column of the images it can be seen that information is mostly lost when we try to blurry something that is already sampled (lower quality). But when we sample something that is already blurred, we will smooth out the low quality and it's easier to detect (sampel) the image.
The gaussian filter blurred the image quite well when we first sampled and the ideal lowpass filter lowered the quality of the sampled picture.
When we first applied the ideal lowpass filter before sampling, we obtained a better quality than the sampled picture, but it was not as smooth as with the case of the gaussfft.
So Sampling at a high-quality picture means that we will get aliasing and lose information about high frequencies. But if we blurry it out, we will remove some of the high frequencies and there will be no aliasing.

Question 20: What conclusions can you draw regarding the effects of smoothing when combined with subsampling? Hint: think in terms of frequencies and side effects.
Answers:

When sampling we will have more discrete changes in pixels, and therefore most information will be at the high frequencies. So, when we use a lowpass filter to blurry out an image that is sampled, we will filter out the high frequencies and thus most of the information is lost.

On the other hand, if we blurry out an image with a low pass filter we will remove some of the sharp edges and the high frequencies are removed. Therefore, when we sample this image, it will be easier to detect the image since there are no edges. Thus is it easier to sample something that is an smooth image.
