Applied estimation lab2

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1 Preparatory Questions

1.1 Particle Filter

- 1. The particles are a set of samples that represents the posterior distribution of the stochastic process. In other words each particle represent a possible state (hypothesis of where the robot might be). The weights of the particles represents the probability that that particular particle is the right position of the robot.

- 2. The importance weights is increasing the number of particles in the areas of specific interest by a factor. Then we assign inverse weights on these new sampled points in sample weighting so the resolution of the probability distribution remains the same. In other words do we get more detail where it matters more and less where it matters less. The importance weights are ratio between target distribution and proposal distribution evaluated at the particles state.

- 3. Partile deprevation occurs when all particles converge towards an incorrect state, due to when the robot is "kidnapped" to a new state location after the particles have converged to a incorrect position, or when the "good" particles are eliminated due to resampling. The dangers with this is that the algorithm is unable to recover the lost particles (for the true pose) after it has (accidentally) discarded all particles near the correct state during the re sampling.

- 4. Each time we resample, we multiply particles where they seem to be more likely. We resample so we can see which samples survive and which ones are more likely to be the correct one, so we multiply the particles that are more likely and reduce the ones that are less likely. We have to therefore resample after each measurement, so that we take the measurements into account each time, just like the update and predict step in the Kalmsn filter.

- 5. If we take the average of 4 particles that are forming a square (with large distace between each other), the average of the particles will be at the center of the square. The center of the particles might be where a wall is, which is not a valid state for the robot to be in. In other words might the average of the particles represent a state that isn't allowed.

- 6. One could model the states in between particles by fitting Gaussian around the mean and variance of the particle set. It's also possible to create bins and count how many particles are in each bin from the histogram.
- 7. The sampling steps create more randomness to the particle filter, which is noise to the solution. Repeating the sampling will eventually result to a large error. Example of remedies are, a) Use more particles to delay the issue, and b) Use of low-variance sampling.
- 8. With increased spread of the particles, there needs to be more particles to maintain the quality of the estimate.

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2 Warm-up problem: Particle Filter

- Question 1: (5) uses model of constant speed and angle of direction for the velocity, which only at this case depend on the initial θ_0 and v_0. Formula (7) takes into account the change of direction of the target from the previous step, which means that the direction might change of the line of which the target moves at, but the magnitude remains the same. Drawbacks with (5) might be that it's a simple model and doesn't take into account the change of direction, and thus isn't a realistic model of the targets movement. The advantage of (5) might be that the model is simple, and thus doesn't require much computations, or any information except the initial θ_0 and v_0.
- Question 2: We can model circular motions moving in normal circular motion, or moving almost in a spiral way (if the velocities in x and y are not in the same order). We need prior information about the initial angular velocity ω_0, the initial speed v_0 and the initial angle θ_0 for the direction of how it moves in x and y.
- Question 3: Not sure what constant part this question is referring to. But if I understood it correctly, the purpose is to normalize the probability distribution. The denominator in the right side of equation (10) are making sure that the probability will be in the interval $[0, 1]$ meaning that without the denominator it would be a likelihood instead of a probability.
- Question 4: In the multinomial re-sampling we need M (number of particles) number of random number, since we iterate a new one each time in the loop. In the Systematic re-sampling we however only need to use one random number, since we will add to that number each time we iterate in the loop.

- Question 5: The probability for a particle of being re-sampled for particle With weight $w = \frac{1}{M} + \epsilon$: probability of not being re-sampled in in the vanilla method is $[1-(\frac{1}{M}+\epsilon)]^M$ and thus the probability of being surviving the vanilla re-sampling will be $1 - [1 - (\frac{1}{M} + \epsilon)]^M$, where M = number of particles. For the Systematic re-sampling the probability will be 1, hence guaranteed that a particle eventually will be re-sampled with weight $w = \frac{1}{M} + \epsilon$.
  For weights $0 \leq w < \frac{1}{M}$:
  Probability of particle surviving in Vanilla re-sampling method will be $1 - [1 - \frac{1}{w}]^M$.

  Here the probability for the systematic re-sampling, the particle can only survive the re-sampling when $r_0 < w$, $0 \leq w < \frac{1}{M}$. In that case, the probability of survival will be wM. If a particle in the systematic re-sampling has a weight larger than $\frac{1}{M}$, it will eventually be evenlly re-sampled. However, if the weight is smaller than $\frac{1}{M}$, there is still a relative small (depending on the size of its weight) of surviving the re-sampling.

- Question 6: The process noise is modeled through the covariance matrix Σ_R (variable *params.Sigma_R*) and the measurement noise is modeled through the covariance matrix Σ_Q (variable *params.Sigma_Q*).

- Question 7:When we set the process noise to zero we can observe that (almost) all of the particles quickly dissapear in the resampling process, since they don't take into account the process noise and are therefor not good enough to survive the resampling. And when we have one particle that does survive, it will gradually decay from the true position, due to the difference from the process noise.

- Question 8: Without the resamling, the particles are all spread out and there is no precision since we do not filter out the ones that are not accurate and we do therefor not get more of those that are good enough to estimate the right position.

- Question 9: observation Changing in the measurement/observation noise between values 0.001, 100 and 10000. $\Sigma_Q 0.001$ and $\Sigma_Q = 10000$ gave very similar results, spread out particles outside around the space, in chunks together. This gave an underestimate/overestimate of the measurement noise.
  Σ_Q=100 on the other hand gave better precision with regard to the true position and the particles were not spread out in multiple chunks, but instead mostly in one segment quite close to the correct position.

- Question 10: changing the process noise diagonal elements between 0.001, 100 and to 10000. 0.001 gave similar results as when the process noise was removed.
  $\Sigma_R = 100$ for the fixed pose gave more spread out particles, but they were

still around the true position. Using this on the $fixed\_2$ data set we got more outliers detected, and more particles around the true position but they were jumping around a lot in their positions due to increased uncertainty in process noise. In the $fixed_3$ data set, the spread/density of the particles varied/oscillated but remained mostly around its true position. This is because the error in the process was now higher.
$\Sigma_R = 10000$ had fewer particles but more spread out in *dataset_1*. In $fixed_2$ the density between the particles was similar of the from $fixed\_1$, but their position was less certain as they jumped around in a chunk of particles around the true pose. $dataset_3$ got outliers that it take into consideration, and therefor the particles jumped discretely around away from the true pose, but otherwise remained around the true pose.
In other words, a higher process noise co-variance would give a larger spread between the particles and vice versa.

- Question 11: A motion model that is not the same as the true motion will mean that the error will be larger and thus we will need to have a larger process noise to compensate for the error, since we cannot trust the model as much as the measurements.

- Question 12: With a wrong choice of motion model, we will get less accuracy in the results and therefor we will need to have a larger process noise to compensate for this. We will also need to have more particles to cover the target and be able to keep track of it.

- Question 13: For outliers we know that their likelihood will be extreme, so setting a threshold might be a good idea. If the measurement is outside (smaller than) this threshold then we can be confident that it is an outlier.

- Question 14: In the Fixed model, we need to increase the process noise in order to compensate. The fixed model turned out to be more sensitive to process noise and measurement noise than the linear and circular motion model. The values for the best obtained results that I got is in the table below.

	Fixed	Linear	Circular
Q (diagonal matrix)	250	300	250
R (diagonal matrix)	25	5	5
Estimation Error	12.1 $\pm$5.3	9.2 $\pm$3.9	7.5 $\pm$4

- Question 15: Like mentioned before a threshold will affect the detection of outliers. A larger threshold will result in fewer detections of outliers. A smaller threshold will on the other hand result in more outliers that we detect. A small Q (measurement noise) in magnitude will mean that we will be confident about the measurements and thus a narrower shape in the measurement prediction step, thus resulting in a larger likelihood in

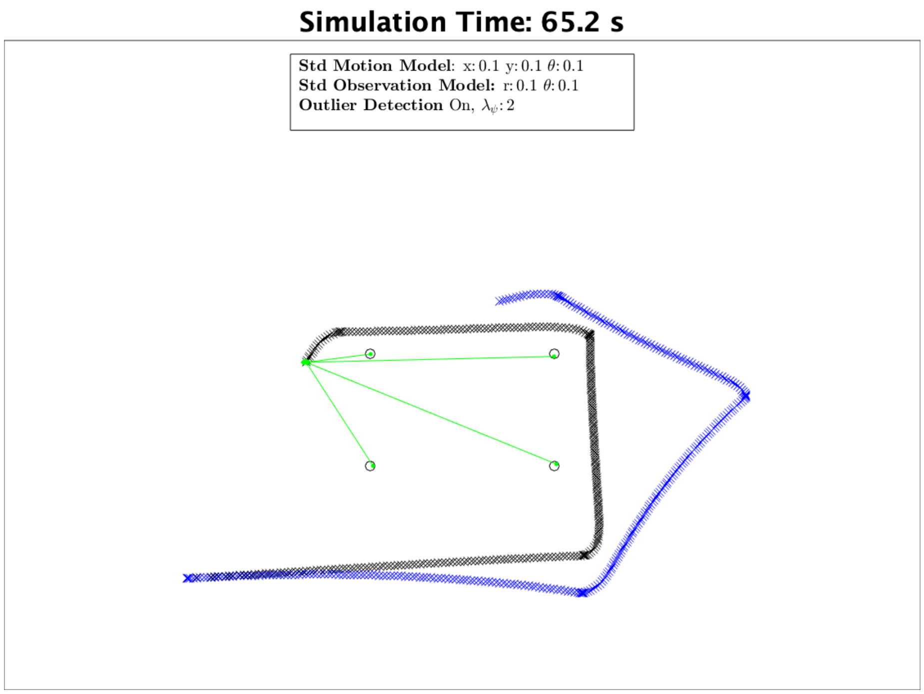

Figure 1: Dataset 4 when running the systematic resampling.

on our particles in average. This means that the measurement will not be as good at detecting the outliers.

- Question 16: If no outliers are detected, the particles that are outliers will be given larger weights, which will be problematic for the convergence of their particles since the outliers will survive the re-sampling easier.

3 Simulation

3.1 Datsaset 4

The landmarks are symmetrically placed, so the map will have symmetry. There are thus 4 hypothesis in the beginning of the simulation. When I use 1000 particles, the particle deprivation does occur more often which is because there are not enough particles to cover all the hypothesises. this means that using 10 000 particles gives a more reliable tracking. The Systematic resampling does tend to perform better than the multinominal resampling, which might be because the multinomial resampling has a larger variance while systematic resampling has a smaller variance.

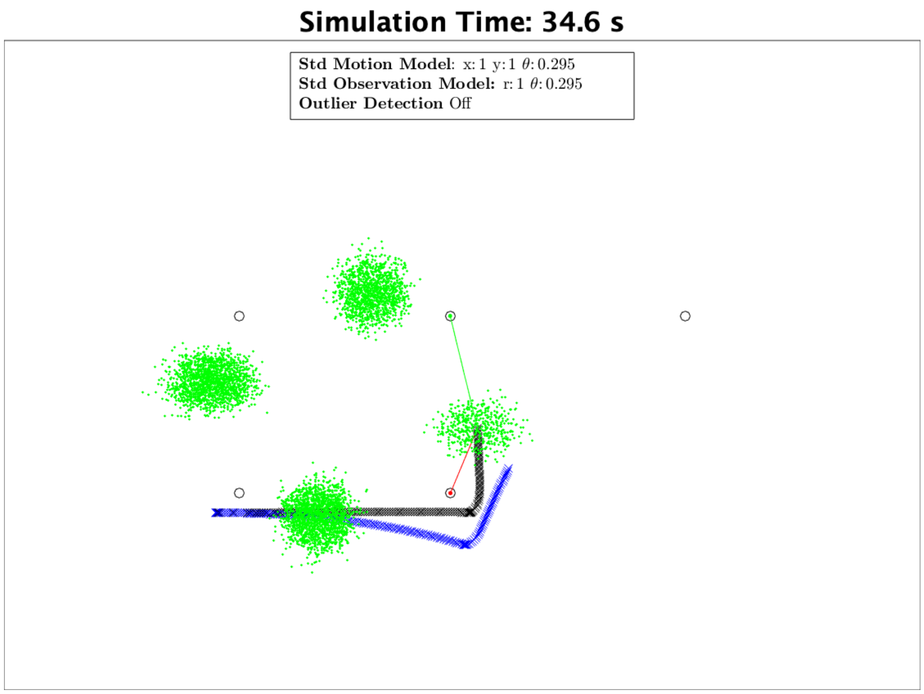

Figure 2: The particle hypothesis just before convergence in the global localisation for Dataset 5.

3.2 Dataset 5

From Figure 2 and 3 we can see that the particles does indeed converge to the correct hypothesis of the robot's position. At first there were 4 hypothesis, but particle deprivation caused them to be reduced. I tried different number of particles, but the ones showed in Figure 3 and 2 there were 10 000 particles and systematic resampling was used here.

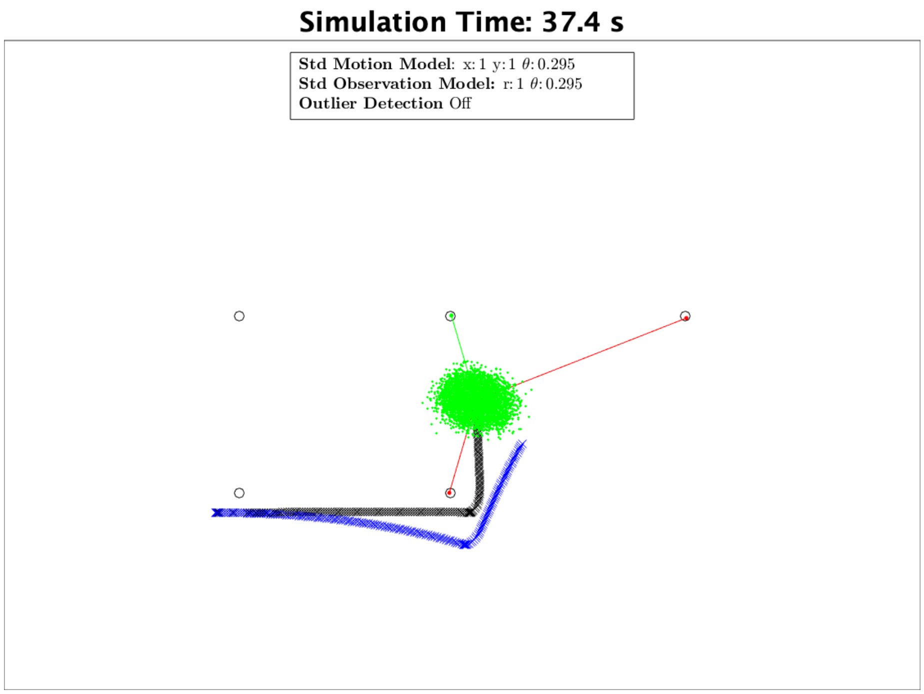

Figure 3: Convergence of particles after symmetry is broken with the fifth landmark for Dataset 5. The particles does indeed converge to the correct hypothesis here.