PHY 101: **GENERAL PHYSICS I**

Mechanics and Rigid Body

By

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COURSE OUTLINE

Space and time,
Units and dimensions.
Kinematics.
Fundamental laws of Mechanics,
Statics and Dynamics.

•OBJECTIVES OF THE STUDY

Students should be able to

- Differentiate between Fundamental and derived quantity
- Give two examples of each of these quantities
- Explain Dimension and differentiate it from Unit of fundamental derived quantity
- Apply Dimension to validate a given equation
- Use dimension to determine the relationship among quantity
- Solve problems on dimension
- Define and distinguish between velocity and acceleration
- Explain instantaneous velocity and instantaneous acceleration
- Solve problems on instantaneous velocity and acceleration
- Explain Newton's laws of motion
- Explain the concept Momentum and Impulse
- State and identify areas where Newton's second law is applied in life
- Define Rigid body
- State conditions necessary for rigid body to be in equilibrium
- Solve problems associated with rigid body

QUANTITIES AND THEIR DIMENSIONS

Quantity is a term used when referring to the measurement of a scalar ,vector , number of items.

- There are two district kinds of measurement in physics.
- First kind is called fundamental quantity, examples are mass, time, length, temperature, amount of substance, luminous intensity, current.
- The second kind is called derived quantities, examples are density, force, resistance, energy, displacement, acceleration, density, weight, pressure, area, volume, Resistance

- **DIMENSIONS**
- The dimension of a physical quantity is the expression which shows how the quantity is related to the fundamental units from which it has been derived.
- In Mechanics the three fundamental quantities are Length , Mass and Time
- Their respective dimensions are L, M and T
- Dimensions of all other quantities are derived from these three examples include
- Area = Volume = L^3 Density = ML^{-3} Velocity = ML^{-1} Acceleration = ML^{-2}

- **APPLICATIONS OF DIMENSIONS**

(1) Checking the validity of equations: The principle is that all terms in a particular equation must have the same dimension

For example $V^2 = U^2 + 2as$

$$(LT^{-1})^2 = (LT^{-1})^2 + (LT^{-2})(L)$$

$$L^2T^{-2} = L^2T^{-2} + L^2T^{-2}$$

APPLICATION OF DIMENSION CONTINUED

Since each term of equation has the same dimension of L^2T^{-2}, the equation is homogeneous
and hence dimensionally valid

- The **second application** is to determine the relationship between physical quantities

For Example, The period of a simple pendulum depends on

(i) The length l of the pendulum,
(ii) The mass m of the bob and
(iii) The acceleration due to gravity g ,find the actual equation

$$T \propto l^x m^y g^z$$

$$T = K\ L^x M^y (LT^{-2})^z$$

$$T = L^{x+z} M^y T^{-2z}$$

$$-2x = 1$$

$$x + z = 0$$

$$y = 0$$

Solving

$$x = -\frac{1}{2}, \quad z = \frac{1}{2}, \ y = 0$$

$$T = K\left(l^{\frac{1}{2}} m^0 g^{-\frac{1}{2}}\right)$$

$$T = K\sqrt{\frac{l}{g}}$$

APPLICATION OF DIMENSION CONTINUED

The value of K can be determined from experiment or theory as 2π

KINEMATICS

Kinematics is the study of motion of objects

Displacement

Assume a body moves from one point (x_1, y_1, z_1) in space to another point (x_2, y_2, z_2)

The displacement S is given by $S = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$

In two dimension i.e a plane, the displacement from point P(2,2) to P(6,3) is

S = (6 − 2, 3 − 2)

= (4, 1)

$$|S| = \sqrt{4^2 + 1^2}$$

$$= 4.12$$

Average Velocity during the motion is define as $\bar{v} = \frac{s}{t}$ where t is the time taken

The time rate of change of displacement at any point is called the **instantaneous velocity**

$v = \frac{\Delta s}{\Delta t}$ with $\Delta t \rightarrow 0$ then $v = \frac{ds}{dt}$. If S is plotted against t , the instantaneous velocity is equal to the slope at any point

Example: A particle moving in a plane has its motion described by $x = 12t + 15$ and $y = 6t^2$ where distances are in m and time in seconds. Find the magnitude and direction of its velocity at the instant when t =3s

Solution: The velocity at an instant has both x and y components. That is

KINEMATICS CONTD

$$V_x = \frac{dx}{dt} = 12, \quad V_x = 12 \text{ m/s}$$

$$Vy = \frac{dy}{dt} = 12t \quad V_y = 36 \text{ m/s at } t = 3s$$

Since V_x and V_y are perpendicu lar, then thei r resultant R is given by

$$R = \sqrt{12^2 + 36^2} = 37.96 \text{ m/s and}$$

$$\theta = \tan^{-1} \frac{36}{12} = 71.56^o \text{ to x axis}$$

When the velocity of the body is increasing either in magnitude or direction or both, the body is said to be undergoing **Acceleration.** The acceleration is the time rate of increase in velocity i.e

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

Instantaneous Acceleration is given by and zero when velocity is constant
The motion is uniform when acceleration is constant. In complex motion acceleration may not be uniform and each of such motions can be treated uniquely with its own equation. For all uniform motions , the uniform acceleration is

$$a = \frac{v - u}{t} \text{ where } t = \text{time for the change}$$

$$v = u + at \qquad ,,,1$$

The average velocity over a distance S is also constant and it is given by

$$\bar{v} = \frac{u + v}{2} \text{ and}$$

$$\text{Distance } S = \bar{v}\, t$$

$$\Rightarrow S = \left(\frac{u + v}{2}\right) t$$

Eliminate v from (1)

$$S = ut + \frac{1}{2} at^2$$

From (1) eliminate t

$$v^2 = u^2 + 2as$$

NEWTON'S LAWS OF MOTION

First Law: States that a body at rest will remain at rest and a body in motion will maintain the motion with a constant speed in a straight line, as long as no net force acts on it.
This law implies that all objects have property which is a tendency to remain at rest when at rest or to continue moving when in motion. This property is called its **INERTIA** and it is measured by the mass of the body

Second Law: This law state that when a body is acted upon by a force , the body will be accelerated, the magnitude of the acceleration being proportional to the magnitude of the force and its direction, being the same as that of the force.
Before we discuss this law let us discuss the concept called **Momentum**

Momentum is define as the product of mass and velocity

P = m v

With this concept, it means that the net force is necessary to produce a change in momentum, either in magnitude or in direction or both.
Second law implies that when a change in momentum is produced by a net force, the magnitude of the change is proportional to the net force and the change is in the direction of the force

$$F = \frac{k\ dP}{dt} = \frac{kmdv}{dt} = kma$$

But in S.I. Units, 1 newton is that force that will produce an acceleration of 1m/s2 on a body of 1kg
When the applied force is not steady as that on a ball hitting a wall, the effect of the force F can be studied by considering the force and the time interval Δt for which it acts
The product of F and Δt is called the **Impulse (**Δj). That is

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$\Delta j = F\,\Delta t$

$$F = \frac{\Delta P}{\Delta t}.\Delta t$$

$$\Delta J = \Delta P$$

$$\int dJ = \int_{Pi}^{Pf} d\,P$$

$$J = P_f - P_i$$

When an impulsive force acts on a body, the impulse is equal to the difference in the final momentum and initial momentum

It is easier to observer changes in momentum than to measure impulse

Example

A man of mass 80kg jumps from a height of 4 m unto the ground. Compare the forces that act on the man if (i) he lands barefooted and the landing takes place in 0.01s (ii) he lands on a cushion such that the landing is delayed to 0.08s (g = 10mls2)

Solution:

Velocity on landing $v = \sqrt{2gh} = 8.85m/$

Momentum on landing, $P_i = 80 \text{ x } 8.85 = 708.35 \text{ kg m/s}$

After landing, momentum $P_j = 0$

Impulse $J = P_f - P_i$

$= -708.35 kgm/ \; s$

(i) $F_1 = \frac{J}{\Delta t_1} = -70835 \text{ N}$

(ii) $F_2 = \frac{J}{\Delta t_2} = -8854.4 \text{ N}$

The minus sign implies that the impulse opposes a change in momentum caused by the upward reaction of the ground

Newton Third law of motion state that whenever a body exerts a force on another, the second body exerts a force equal in magnitude and opposite in direction on the first body

SOME ILLUSTRATIONS OF SECOND LAW OF MOTION

Suppose two bodies were suspended by a rope passing over a frictionless fixed pulley. We can apply second law to determine both the acceleration and the tension in the rope

$$T_1 - m_1 g = m_1 a_1$$

$$T_2 - m_2 g = m_2 a_2$$

where $T_1 = T_2 = T$

$$a_1 = - a_2 = a$$

Hence

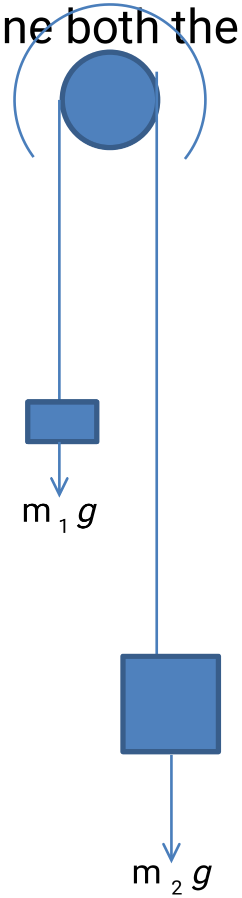
$$T - m_1 g = m_1 a$$

$$T - m_2 g = - m_2 a$$

From where we get

$$m_2 g - m_1 g = m_2 a - m_1 a$$

$$a = g\left(\frac{m_2 - m_1}{m_2 + m_1}\right)$$



Show that

$$T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right)$$

Example: In the diagram below, find the acceleration of the system and the tension in the rope if $m_1 = 3kg$ and $m_2 = 2kg$, assuming that the pulley and the plane are frictionless $\theta = 30°$

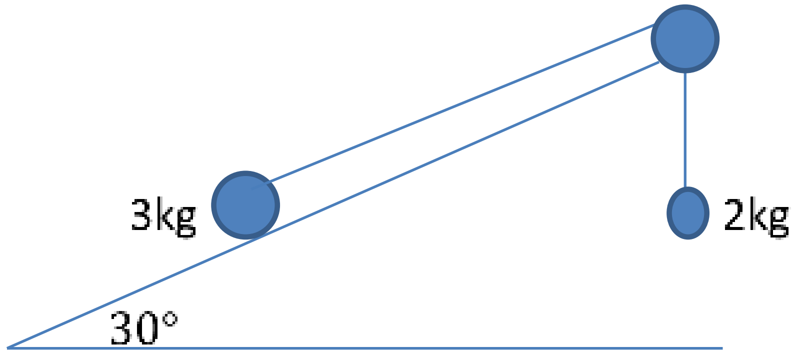
$$T - m_2 g \ Sin \ \theta = m_1 a$$

$$-T + m_2 g = m_2 a$$

$$a = g\left(\frac{m_2 - m_1 g \sin 30}{m_1 + m_2}\right)$$

$$a = 9.8\left(\frac{2 - 1.5}{5}\right)$$

$$= 0.98 \ m/s^2$$



T= 17.64 N and is obtain when the value of a is substituted in either equation.

A passenger in a lift

Another instance when one can apply second law of motion.
When inside the lift the passenger will experiences three possibilities
Let R be the reaction on the passenger from the floor of the lift and W is the weight

(i) When the lift is at rest

$$R - W = 0$$

$$R = W$$

The passenger feels the real weight

(ii) When the lift is ascending with an acceleration a,

$R - W = ma$ i.e $R = mg + ma$

This represent the apparent weight of the man and he feels heavier than his normal weight.

A passenger in a lift continued

(iii) When the passenger is descending with an acceleration a,
The force of the motion $mg - R = ma$ i.e. $R = mg - ma$
In this case the passenger feels lighter than his real weight
A special case may arise when the lift moves down with an acceleration $a = g$
In this case $R = 0$ and the man feels weightless

FRICTIONAL FORCE

Friction is the force that opposes the relative motion between any two surfaces in contact. This force is parallel to the two surfaces and its magnitude is proportional to the normal reaction between the two surfaces

$$F \propto N$$

$$F = \mu N$$

μ is the constant of proportionality called Coefficient of Static Friction. μ depend on the nature of the two surfaces in contact and unaffected by the size of areas in contact

(a) For a body of Mass M lying horizontally on another surface, the normal reaction is the weight Mg of the body

$$\mu = \frac{F}{Mg}$$

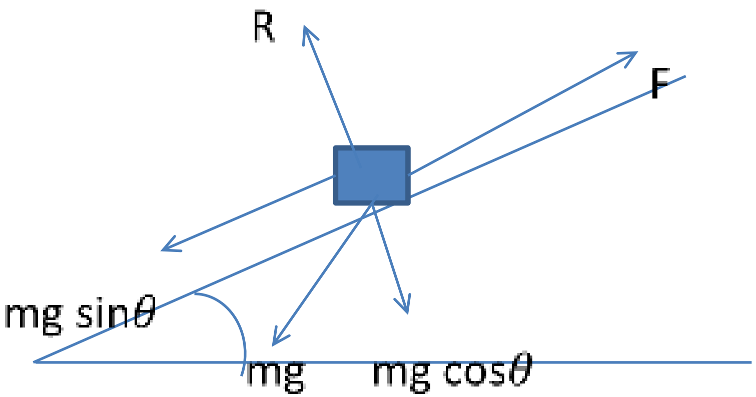
F is the limiting frictional force or the force that will just overcome the friction and set the body for motion

(b) When the surface between the two bodies is an inclined plane
At the limiting point, when $mg \sin\theta$ = frictional force,

$$\mu = \frac{mg \sin\theta}{mg \cos\theta} = \tan\theta$$

Where θ is the angle of inclination when the body is just set to move

Where θ



Example : Calculate the acceleration and the tension of the system below if μ = 0.3 between m_1 and the plane. The masses m_1 and m_2 are 15kg and 10kg respectively

Solution: For masses m_1 and m_2

$$F_r - F_g = m_1 a$$

$$-T + W_2 = m_2 a$$

$$T - mg\sin\theta - \mu m_1 g\cos\theta = m_1 a$$

$$-T + mg = m_2 a$$

Putting in the given values

$$T - 15 \times 9.8 \sin 30 - 0.3 \times 15 \times 9.8 \cos 30 = 15a$$

$$-T + 10 \times 9.8 = 10a$$

From where $a = 1.89\ m/s^2$ and $T = 79.07N$

Kinetic Friction is the friction that exists between bodies moving relative to each other.

EQUILIBRIUM OF A RIGID BODY

A rigid body is a body in which the distance between any two of its constituent particles is constant

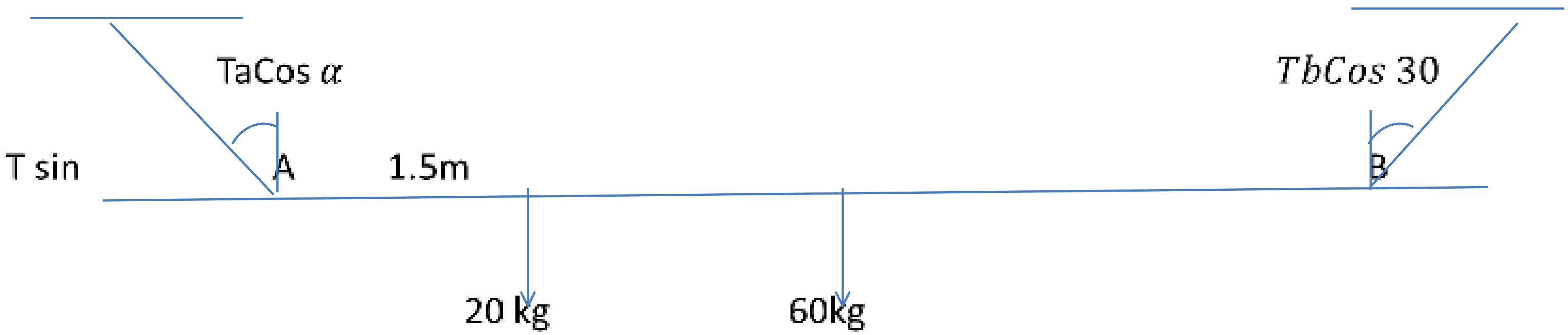
The study of a rigid body in a state of equilibrium under the action of many forces is referred to as statics. Two conditions are required

(i) The sum of all the forces must be zero (translational equilibrium) $\sum_i Fi = 0$

(ii) The sum of all Torque in relation to any axis must be zero (for rotational equilibrium) $\sum_i \tau = 0$

These two conditions are very important in solving problems relating to equilibrium of forces . Another thing is that one must be able to recognizes all the forces acting on the body and their components along the two perpendicular axis

Example: A uniform bar AB 5m long and weighing 60kg is supported horizontally by two cords fastened to two ends. The cord at B makes angle 30° with the vertical. A mass of 20kg is suspended from a point 1.5m from A on the bar. Find the tension in the cord at A and angle it makes with the vertical.



Along y : $\sum F_y = T_A \, Cos \, \alpha + T_B \, Cos \, 30 - 20 - 60 = 0$

i.e $T_A \, Sin \, \alpha + T_B \, Cos \, 30 = 80$ ..i

Along x : $\sum F_x = T_A \, Sin \, \alpha - T_B \, \text{Sin} \, 30 = 0$

i.e $T_A \, \text{Sin} \, \alpha = \frac{1}{2} T_B$ ..ii

Taking moments about A (it could have been about B)

$\sum T_A = 5T_B \, Cos \, 30 - 60 \text{ x } 2.5 - 20 \text{ x } 1.5 = 0$

i.e $T_B = \frac{207.8}{5} = 41.56\text{kgf.}$

Subt. T_B in eqn i and ii

From eqn i, $T_A \, Cos \, \alpha = 80 - 41.56\text{Cos } 30 = 44\text{kg}$ ..iii

From eqn ii, $T_A \, \text{Sin} \, \alpha = \frac{1}{2} \text{ x } 21.56 = 20.78$ ..iv

Dividing iv by iii

$\text{Tan } \alpha = \frac{20.78}{44} = 0.472$

i.e $\alpha = 25.3°$

From aquation ii,

$$T_A = \frac{\frac{1}{2} T_B}{Sin \; 25.3} = 48.66\text{kgf.}$$

END OF
STUDY