Advanced Image Processing: Assignment 3 (Due Mar 22, 2024)

Note: Answer all questions by supporting your answers with suitable plots and observations in a report. You will need to upload code that you may have written to solve the problems. Please provide detailed comments for any such code. The assignment will be evaluated not just based on the final results but also how you obtained them.

Problem 1: MMSE estimation for Laplacian source (10 points)
Consider the noise model $Y = X + Z$, where X is distributed according to a Laplace distribution $f_X(x) = \frac{1}{2\sigma_X}\exp(-\frac{|x|}{\sigma_X})$ with $\sigma_X = 1$ and $Z \sim \mathcal{N}(0, \sigma_Z^2)$ with $\sigma_Z^2 = 0.1$. Compute the minimum mean squared estimate (MMSE) of x given y. If the MMSE estimate can not be written as a closed form expression of y, plot the estimate $\hat{x}$ as a function of y for the given parameters of the system (Ref: E. P. Simoncelli, and E. H. Adelson, "Noise removal via Bayesian wavelet coring," Proc. of IEEE International Conference on Image Processing, 1996).

Problem 2: Image denoising (30 points)
Take the lighthouse image provided to you, convert to greyscale and add white Gaussian noise with variance $\sigma_Z^2 = 100$ to it. Be sure to add noise in the grey scale domain where the range of pixel values is between 0 and 255. Compute and compare (subjectively and using mean squared error) the results of the following denoising methods

1. Low pass Gaussian filter: Vary the filter length in the set $\{3, 7, 11\}$ and standard deviations in the set $\{0.1, 1, 2, 4, 8\}$ to identify the filter with the best mean squared error (MSE).

2. Adaptive MMSE: Compute an adaptive version of the MMSE filter where the estimates are computed for patches of size 32×32 with overlap of 16 in the high pass domain. All pixels belonging to multiple patches need to be assigned the average values arising from the output due to multiple patches. You need to calculate the variance of the high pass coefficients of the original image and the variance of noise in the high pass image given the noise variance $\sigma_Z^2 = 100$ in the pixel domain. You cannot simulate noise in the high pass domain separately and estimate its variance.

3. Adaptive Shrinkage: Shrinkage estimator on the high pass coefficients of the noisy image with the threshold optimized using *SureShrink* for patches of size 32×32. Here it is implicit that you need to the determine the threshold parameter t for every patch. (Ref: D. L. Donoho, and I. M. Johnstone, "Adapting to unknown smoothness via wavelet shrinkage," Journal of the American Statistical Association, vol. 90, no. 432, 1995)