Lecture 10: Classic Games

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Outline

Why Study Classic Games?

- Simple rules, deep concepts
- Studied for hundreds or thousands of years
- Meaningful IQ test
- *Drosophila* of artificial intelligence
- Microcosms encapsulating real world issues
- Games are fun!

AI in Games: State of the Art

Program	Level of Play	Program to Achieve Level
Checkers	Perfect	*Chinook*
Chess	Superhuman	*Deep Blue*
Othello	Superhuman	*Logistello*
Backgammon	Superhuman	*TD-Gammon*
Scrabble	Superhuman	*Maven*
Go	Superhuman	*AlphaGo*
Poker[1]	Perfect	*Cepheus*

[1] Heads-Up Limit Texas Hold'em

RL in Games: State of the Art

Program	Level of Play	RL Program to Achieve Level
Checkers	Superhuman	*Chinook*
Chess	International Master	*KnightCap / Meep*
Othello	Superhuman	*Logistello*
Backgammon	Superhuman	*TD-Gammon*
Scrabble	Superhuman	*Maven*
Go	Superhuman	*AlphaGo*
Poker[1]	Superhuman	*SmooCT*

[1] Heads-Up Limit Texas Hold'em

Optimality in Games

- What is the optimal policy π^i for ith player?
- If all other players fix their policies π^{-i}
- Best response $\pi^i_*(\pi^{-i})$ is optimal policy against those policies
- Nash equilibrium is a joint policy for all players

$$\pi^i = \pi^i_*(\pi^{-i})$$

- such that every player's policy is a best response
- i.e. no player would choose to deviate from Nash

Single-Agent and Self-Play Reinforcement Learning

- Best response is solution to single-agent RL problem
  - Other players become part of the environment
  - Game is reduced to an MDP
  - Best response is optimal policy for this MDP
- Nash equilibrium is fixed-point of self-play RL
  - Experience is generated by playing games between agents

$$a_1 \sim \pi^1, a_2 \sim \pi^2, ...$$

  - Each agent learns best response to other players
  - One player's policy determines another player's environment
  - All players are adapting to each other

Two-Player Zero-Sum Games

We will focus on a special class of games:

- A perfect information game is fully observed by all players
  - Chess, Go have perfect information
  - Poker, Scrabble have imperfect information (hidden state)
- A two-player game has two (alternating) players
  - We will name player 1 *white* and player 2 *black*
- A zero sum game has equal and opposite rewards for black and white

$$R^1 + R^2 = 0$$

We consider methods for finding Nash equilibria in perfect information, two-player zero-sum games:

- Game tree search (i.e. planning)
- Self-play reinforcement learning

Minimax

- A value function defines the expected total reward given joint policies $\pi = \langle \pi^1, \pi^2 \rangle$

$$v_\pi(s) = \mathbb{E}_\pi \left[G_t \mid S_t = s \right]$$

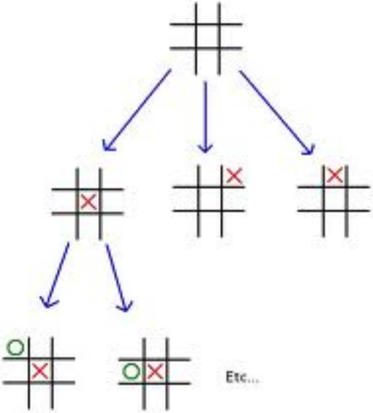
- A minimax value function maximizes white's expected return while minimizing black's expected return

$$v_*(s) = \max_{\pi^1} \min_{\pi^2} v_\pi(s)$$

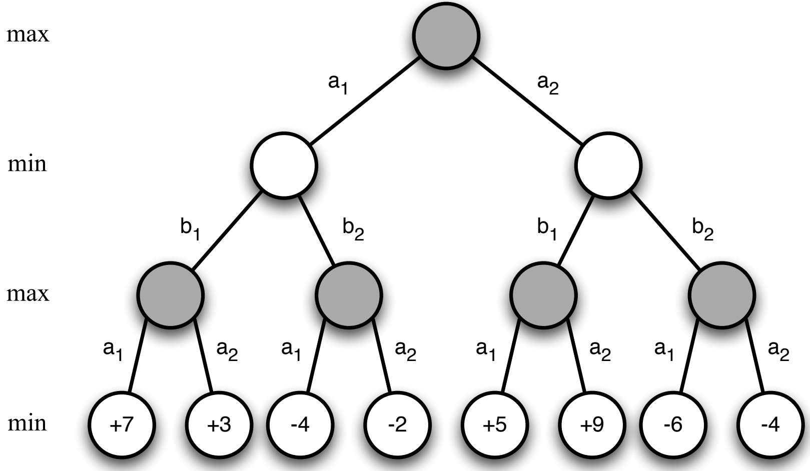
- A minimax policy is a joint policy $\pi = \langle \pi^1, \pi^2 \rangle$ that achieves the minimax values
- There is a unique minimax value function
- A minimax policy is a Nash equilibrium

Minimax Search

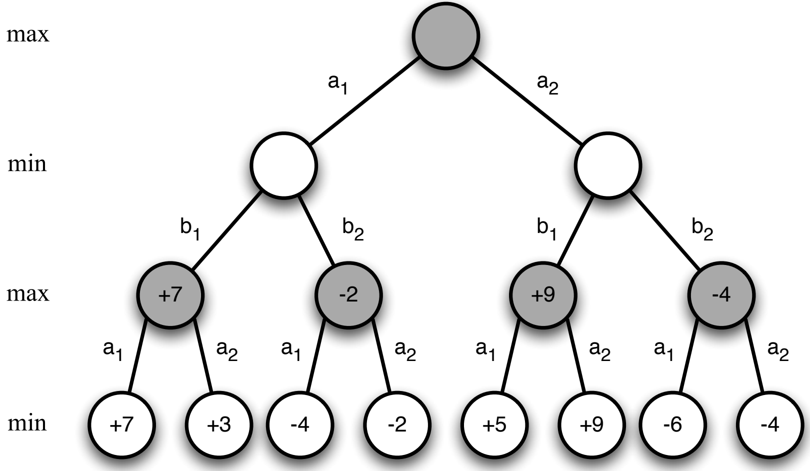
- Minimax values can be found by depth-first game-tree search
- Introduced by Claude Shannon: *Programming a Computer for Playing Chess*
- Ran on paper!



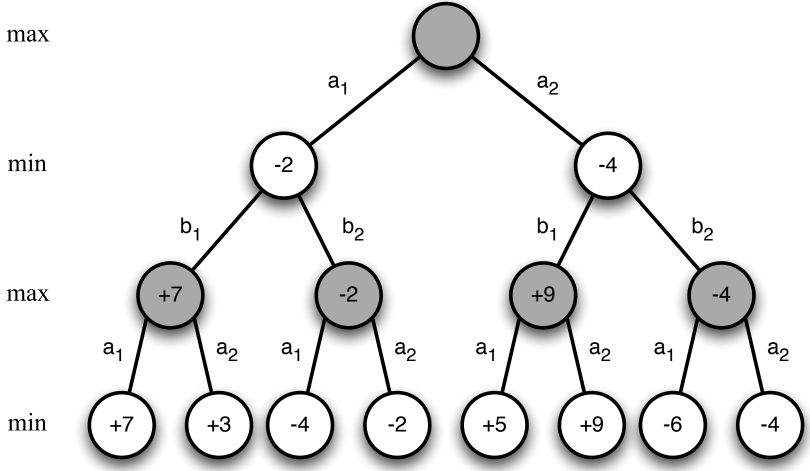
Minimax Search Example



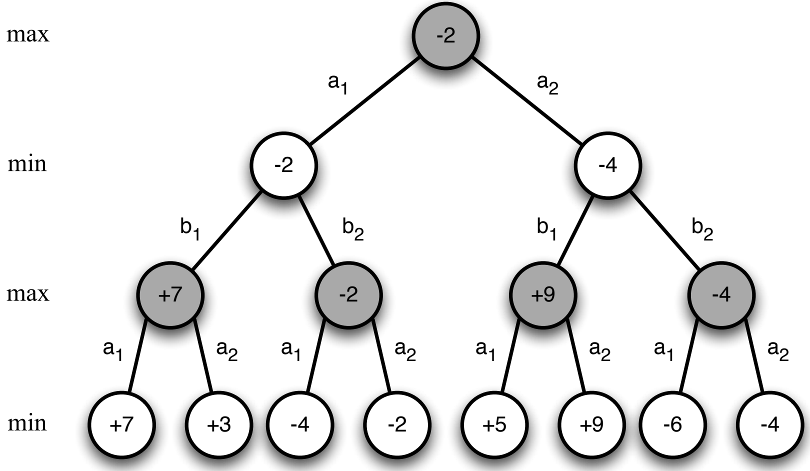
Minimax Search Example



Minimax Search Example



Minimax Search Example

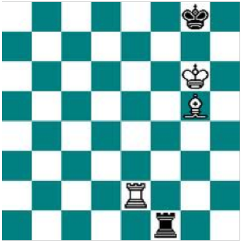


Value Function in Minimax Search

- Search tree grows exponentially
- Impractical to search to the end of the game
- Instead use value function approximator $v(s, \mathbf{w}) \approx v_*(s)$
  - aka *evaluation function*, *heuristic function*
- Use value function to estimate minimax value at leaf nodes
- Minimax search run to fixed depth with respect to leaf values

Binary-Linear Value Function

- Binary feature vector $\mathbf{x}(\mathbf{s})$: e.g. one feature per piece
- Weight vector $\mathbf{w}$: e.g. value of each piece
- Position is evaluated by summing weights of active features



$$v(s,\mathbf{w}) = \mathbf{x}(s)\cdot\mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ \vdots \end{bmatrix} \cdot \begin{array}{c} \begin{bmatrix} +5 \\ +3 \\ +1 \\ -5 \\ -3 \\ -1 \\ \vdots \end{bmatrix} \end{array} \begin{array}{c} ♖ \\ ♗ \\ ♙ \\ ♜ \\ ♝ \\ ♟ \\ \end{array}$$

$$v(s,\mathbf{w}) = 5 + 3 - 5 = 3$$

Deep Blue

- Knowledge
  - 8000 handcrafted chess features
  - Binary-linear value function
  - Weights largely hand-tuned by human experts
- Search
  - High performance parallel alpha-beta search
  - 480 special-purpose VLSI chess processors
  - Searched 200 million positions/second
  - Looked ahead 16-40 ply
- Results
  - Defeated human champion Garry Kasparov 4-2 (1997)
  - Most watched event in internet history

Chinook

- Knowledge
  - Binary-linear value function
  - 21 knowledge-based features (position, mobility, ...)
  - x4 phases of the game
- Search
  - High performance alpha-beta search
  - Retrograde analysis
    - Search backward from won positions
    - Store all winning positions in lookup tables
    - Plays perfectly from last n checkers
- Results
  - Defeated Marion Tinsley in world championship 1994
    - won 2 games but Tinsley withdrew for health reasons
  - Chinook *solved* Checkers in 2007
    - perfect play against God

Self-Play Temporal-Difference Learning

- Apply value-based RL algorithms to games of self-play
- MC: update value function towards the return G_t

$$\Delta \mathbf{w} = \alpha(G_t - v(S_t, \mathbf{w}))\nabla_{\mathbf{w}} v(S_t, \mathbf{w})$$

- TD(0): update value function towards successor value $v(S_{t+1})$

$$\Delta \mathbf{w} = \alpha(v(S_{t+1}, \mathbf{w}) - v(S_t, \mathbf{w}))\nabla_{\mathbf{w}} v(S_t, \mathbf{w})$$

- TD(λ): update value function towards the λ-return G_t^λ

$$\Delta \mathbf{w} = \alpha(G_t^\lambda - v(S_t, \mathbf{w}))\nabla_{\mathbf{w}} v(S_t, \mathbf{w})$$

Policy Improvement with Afterstates

- For deterministic games it is sufficient to estimate $v_*(s)$
- This is because we can efficiently evaluate the afterstate

$$q_*(s, a) = v_*(succ(s, a))$$

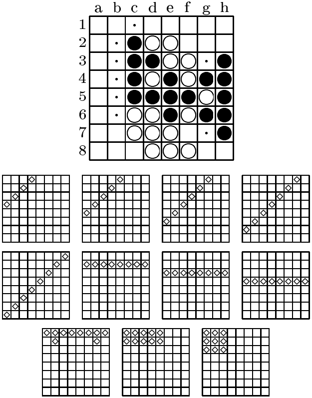
- Rules of the game define the successor state $succ(s, a)$
- Actions are selected e.g. by min/maximising afterstate value

$$\begin{aligned} A_t &= \operatorname*{argmax}_a v_*(succ(S_t, a)) && \text{for white} \\ A_t &= \operatorname*{argmin}_a v_*(succ(S_t, a)) && \text{for black} \end{aligned}$$

- This improves joint policy for both players

Self-Play TD in Othello: *Logistello*

- Logistello created its own features
- Start with raw input features, e.g. "black stone at C1?"
- Construct new features by conjunction/disjunction
- Created 1.5 million features in different configurations
- Binary-linear value function using these features



Reinforcement Learning in Logistello

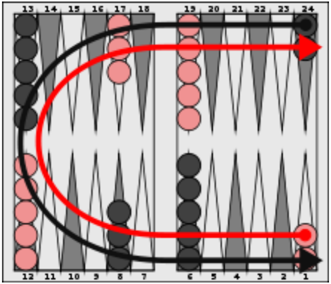
Logistello used generalised policy iteration

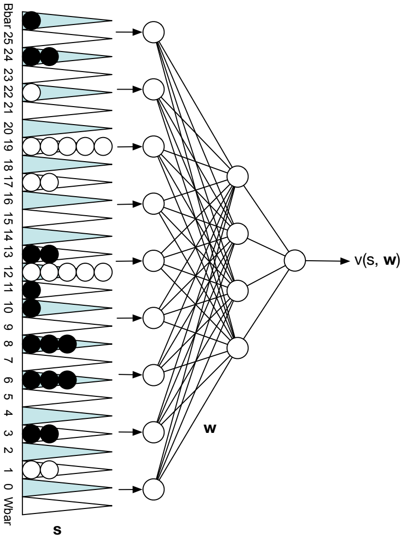
- Generate batch of self-play games from current policy
- Evaluate policies using Monte-Carlo (regress to outcomes)
- Greedy policy improvement to generate new players

Results

- Defeated World Champion Takeshi Murukami 6-0

TD Gammon: Non-Linear Value Function Approximation





Self-Play TD in Backgammon: *TD-Gammon*

- Initialised with random weights
- Trained by games of self-play
- Using non-linear temporal-difference learning

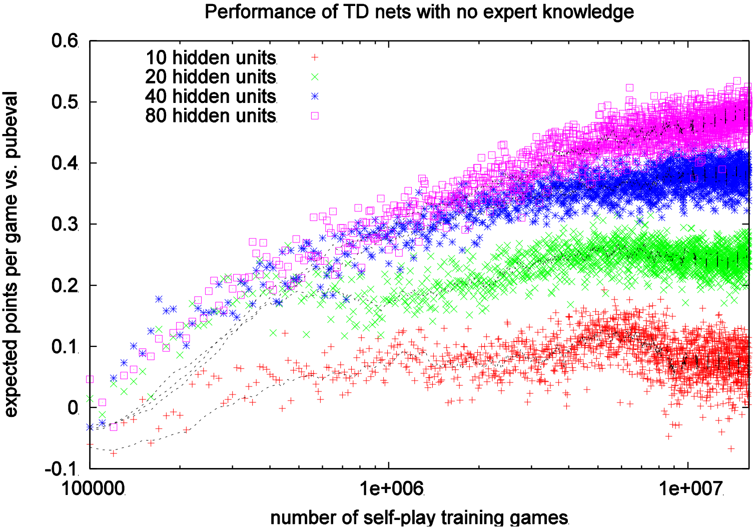
$$\delta_t = v(S_{t+1}, \mathbf{w}) - v(S_t, \mathbf{w})$$
$$\Delta \mathbf{w} = \alpha \delta_t \nabla_{\mathbf{w}} v(S_t, \mathbf{w})$$

- Greedy policy improvement (no exploration)
- Algorithm always converged in practice
- Not true for other games

TD Gammon: Results

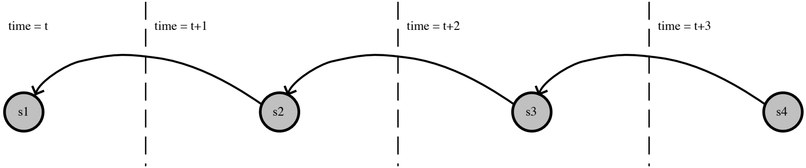
- Zero expert knowledge $\implies$ strong intermediate play
- Hand-crafted features $\implies$ advanced level of play (1991)
- 2-ply search $\implies$ strong master play (1993)
- 3-ply search $\implies$ superhuman play (1998)
- Defeated world champion Luigi Villa 7-1 (1992)

New TD-Gammon Results



Simple TD

- TD: update value towards successor value



- Value function approximator $v(s, \mathbf{w})$ with parameters $\mathbf{w}$
- Value function backed up from raw value at next state

$$v(S_t, \mathbf{w}) \leftarrow v(S_{t+1}, \mathbf{w})$$

- First learn value function by TD learning
- Then use value function in minimax search (no learning)

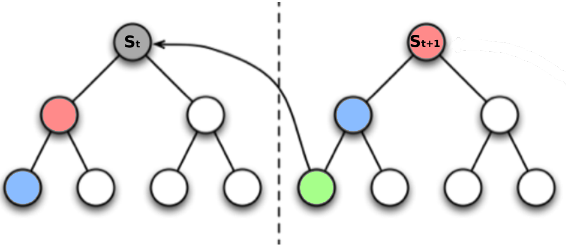
$$v_+(S_t, \mathbf{w}) = \operatorname*{minimax}_{s \in leaves(S_t)} v(s, \mathbf{w})$$

Simple TD: Results

- Othello: superhuman performance in *Logistello*
- Backgammon: superhuman performance in *TD-Gammon*
- Chess: poor performance
- Checkers: poor performance
- In chess tactics seem necessary to find signal in position
- e.g. hard to find checkmates without search
- Can we learn directly from minimax search values?

TD Root

- TD root: update value towards successor search value



- Search value is computed at root position S_t

$$v_+(S_t, \mathbf{w}) = \underset{s \in leaves(S_t)}{\text{minimax}}\ v(s, \mathbf{w})$$

- Value function backed up from *search value* at next state

$$v(S_t, \mathbf{w}) \leftarrow v_+(S_{t+1}, \mathbf{w}) = v(l_+(S_{t+1}), \mathbf{w})$$

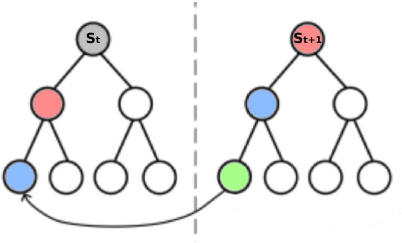
- Where $l_+(s)$ is the leaf node achieving minimax value from s

TD Root in Checkers: *Samuel's Player*

- First ever TD learning algorithm (*Samuel 1959*)
- Applied to a Checkers program that learned by self-play
- Defeated an amateur human player
- Also used other ideas we might now consider strange

TD Leaf

- TD leaf: update search value towards successor search value



- Search value computed at current and next step

$$v_+(S_t, \mathbf{w}) = \underset{s \in leaves(S_t)}{\text{minimax}}\, v(s, \mathbf{w}), \quad v_+(S_{t+1}, \mathbf{w}) = \underset{s \in leaves(S_{t+1})}{\text{minimax}}\, v(s, \mathbf{w})$$

- Search value at step t backed up from *search value* at $t+1$

$$v_+(S_t, \mathbf{w}) \leftarrow v_+(S_{t+1}, \mathbf{w})$$
$$\implies v(l_+(S_t), \mathbf{w}) \leftarrow v(l_+(S_{t+1}), \mathbf{w})$$

TD leaf in Chess: *Knightcap*

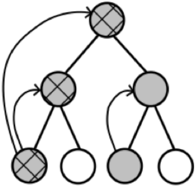
- Learning
  - *Knightcap* trained against expert opponent
  - Starting from standard piece values only
  - Learnt weights using TD leaf
- Search
  - Alpha-beta search with standard enhancements
- Results
  - Achieved master level play after a small number of games
  - Was not effective in self-play
  - Was not effective without starting from good weights

TD leaf in Checkers: *Chinook*

- Original Chinook used hand-tuned weights
- Later version was trained by self-play
- Using TD leaf to adjust weights
  - Except material weights which were kept fixed
- Self-play weights performed $\geq$ hand-tuned weights
- i.e. learning to play at superhuman level

TreeStrap

- TreeStrap: update search values towards deeper search values



- Minimax search value computed at *all* nodes $s \in nodes(S_t)$
- Value backed up from search value, at same step, for all nodes

$$\begin{aligned} v(s, \mathbf{w}) &\leftarrow v_+(s, \mathbf{w}) \\ \implies v(s, \mathbf{w}) &\leftarrow v(l_+(s), \mathbf{w}) \end{aligned}$$

Treestrap in Chess: *Meep*

- Binary linear value function with 2000 features
- Starting from random initial weights (no prior knowledge)
- Weights adjusted by TreeStrap
- Won 13/15 vs. international masters
- Effective in self-play
- Effective from random initial weights

Simulation-Based Search

- Self-play reinforcement learning can replace search
- Simulate games of self-play from root state S_t
- Apply RL to simulated experience
  - Monte-Carlo Control $\implies$ Monte-Carlo Tree Search
  - Most effective variant is UCT algorithm
    - Balance exploration/exploitation in each node using UCB
  - Self-play UCT converges on minimax values
  - Perfect information, zero-sum, 2-player games
  - Imperfect information: see next section

Performance of MCTS in Games

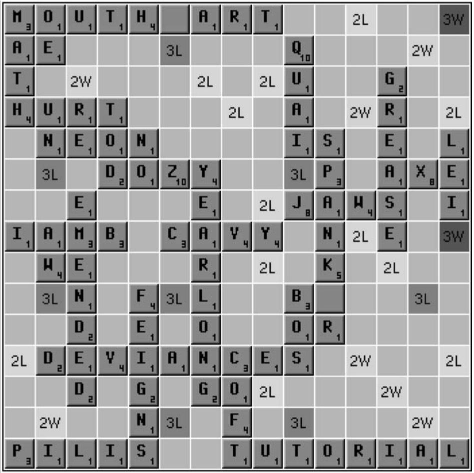
- MCTS is best performing method in many challenging games
  - Go (last lecture)
  - Hex
  - Lines of Action
  - Amazons
- In many games simple Monte-Carlo search is enough
  - Scrabble
  - Backgammon

Simple Monte-Carlo Search in Scrabble (Maven)

- Reinforcement Learning
  - Maven evaluates moves by $score + v(rack)$
  - Binary-linear value function of rack
  - Using one, two and three letter features
  - Q??????, QU?????, III????
  - Learnt by Monte-Carlo policy iteration (cf. Logistello)
- Monte-Carlo Search (MCS)
  - Roll-out moves by imagining n steps of self-play
  - Evaluate resulting position by $score + v(rack)$
  - Score move by average evaluation in rollouts
  - Select and play highest scoring move
  - Specialised endgame search using B*

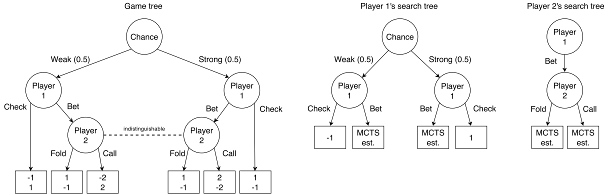
Maven: Results

- Maven beat world champion Adam Logan 9-5
- Here Maven predicted endgame to finish with MOUTHPART
- Analysis showed Maven had error rate of 3 points per game



Game-Tree Search in Imperfect Information Games

- Players have different information states and therefore separate search trees



- There is one node for each information state
  - summarising what a player knows
  - e.g. the cards they have seen
- Many real states may share the same information state
- May also aggregate states e.g. with similar value

Issues in Applying RL to Imperfect Information Games

- Learning dynamics are also more problematic than single agent case
  - Environment depends upon the opponent's current policies
  - Non-stationary environment =¿ potential cycles during learning
- Local search more challenging with hidden state
  - Can apply self-play to history of player's observations
  - Sufficient to learn a best response to opponents
  - and hence a Nash equilibrium
  - But insufficient for local game-tree search from current state
  - Requires an estimate of opponent's hidden state to take opponent's perspective

Fictitious Play

At each iteration j, for each player i

- Compute average policy $\mu^i = \frac{1}{N} \sum_{j=1}^{N} \pi_j^i$
- Compute best response to opponents' average policies, $\pi_j^i = \pi_*^i(\mu^{-i})$

Fictitious play converges to Nash equilibrium for a wide class of imperfect information games

Fictitious Self-Play

At each iteration j, for each player i

- Update average policy μ_j^i by supervised learning
  - Each player predicts their own policy
  - Using previous iterations $1, ..., j$ as training data
- Update best response π^i by reinforcement learning
  - Treating opponents as part of the environment
  - Using games of self-play as training data

Using deep neural networks, FSP achieved superhuman performance in No-Limit Texas Holdem Poker

RL in Games: A Successful Recipe

Program	Input features	Value Fn	RL	Training	Search
Chess *Meep*	*Pieces, pawns, ...*	Linear	TreeStrap	Self-Play / Expert	$\alpha\beta$
Checkers *Chinook*	*Pieces, ...*	Linear	TD leaf	Self-Play	$\alpha\beta$
Othello *Logistello*	*Disc configs*	Linear	MC	Self-Play	$\alpha\beta$
Backgammon *TD Gammon*	*Num checkers*	Neural network	TD(λ)	Self-Play	$\alpha\beta$ / MCS
Go *AlphaGo*	*Stones, ...*	Neural network	MC / Reinforce	Self-Play / Expert	MCTS
Scrabble *Maven*	*Letters on rack*	Linear	MC	Self-Play	MCS
Limit Hold'em *NFSP*	*Cards*	Neural network	TD	Self-Play	-