Lecture 8: Building agents

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Outline

Recap

- Reinforcement learning is the science of learning to make decisions
- We can do this by learning one or more of:
  - policy
  - value function
  - model
- The general problem involves taking into account time and consequences
- Our decisions affect the reward, our internal knowledge, and the state of the environment

This Lecture

- So far, we mainly discussed learning algorithms
- How do we build a full agent?
- We will talk about which components are needed, and how to combine these
- We will talk about policies and exploration

Interface between agent and environment

- The typical interface to a task is: action goes in, reward and observation come out
- So, our full agent must be a function: $A_t = \text{agent}(R_t, O_t)$
- A policy can be considered a simple agent that only looks at the observation O_t
- We want learning agents, which requires learning from R_t as well
- Simple policies can be embedded inside complex agents

High-level agent components

- Agents may have several components
  - An agent must include a policy $\pi_t : \mathcal{S} \to \mathcal{A}$
    where $\mathcal{S}$ is the set of possible agent states
  - An agent can include a representation $S_t = f(S_{t-1}, O_t)$
  - An agent can include an algorithm to learn the policy
  - An agent can include an algorithm to learn the representation
  - An agent can include an algorithm to learn predictions
  - An agent can include an algorithm to learn a model
  - An agent can include memory of past experiences
  - An agent can include ...
- Different components may or may not share (parts of) the representation

Example: neural Q-learning

- Online neural Q-learning may include:
  - A network q_θ: $O_t \implies (q[1], \ldots, q[m])$ (m actions)
  - An ϵ-greedy exploration policy: $q_t \implies \pi_t \implies A_t$
  - A Q-learning loss function on θ

$$I(\theta) = \frac{1}{2}\left(R_{t+1} + \gamma\left[\max_a q_\theta(S_{t+1}, a)\right] - q_\theta(S_t, A_t)\right)^2$$

  where $[\cdot]$ denotes stopping the gradient, so that the gradient is

$$\nabla_\theta I(\theta) = \left(R_{t+1} + \gamma \max_a q_\theta(S_{t+1}, a) - q_\theta(S_t, A_t)\right) \nabla_\theta q_\theta(S_t, A_t)$$

  - An optimizer to minimize the loss (e.g., SGD, RMSProp, Adam)

Example: TF pseudo-code for Q-learning

```
# Compute Q values Q(S_t, .)
q = q_net(obs)

# Get action A_t
action = epsilon_greedy(q)

# Compute Q(S_t, A_t)
qa = q[action]

# Step in environment
reward, discount, next_obs = env.step(action)

# Get max of values at next state
max_q_next = tf.reduce_max(q_net(next_obs))

# Compute TD-error, do not to propagate into next state value
delta = reward + discount * tf.stop_gradient(max_q_next) - qa

# Define loss
q_loss = tf.square(delta)/2
```

Example: DQN

- DQN includes:
  - A network q_θ: $O_t \implies (q[1], \ldots, q[m])$ (m actions)
  - An ϵ-greedy exploration policy: $q_t \implies \pi_t \implies A_t$
  - A replay buffer to store and retrieve past experiences
  - A target network q_{θ^-}: $O_t \implies (q^-[1], \ldots, q^-[m])$
  - A Q-learning loss function on θ
    (uses replay and target network)

$$I(\theta) = \frac{1}{2}\left(R_{i+1} + \gamma[\max_a q_{\theta^-}(S_{i+1}, a)] - q_\theta(S_i, A_i)\right)^2$$

  - An optimizer to minimize the loss

Example: (Asynchronous) Advantage actor critic

- Advantage actor critic includes:
  - A representation (e.g., LSTM): $(S_{t-1}, O_t) \implies S_t$
  - A network v_η: $S \implies v$
  - A network π_θ: $S \implies \pi$
  - Copies/variants π^m of π_θ as policies: $S_t^m \implies A_t^m$
  - A n-step TD loss on v_η

$$l(\eta) = \frac{1}{2}\left(G_t^{(n)} - v_\eta(S_t)\right)^2$$

    where $G_t^{(n)} = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{n-1} v_{\eta^-}(S_{t+n})$
  - A n-step REINFORCE loss on (each) π_η

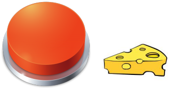
$$l(\theta) = \left[G_t^{(n)} - v_\eta(S_t)\right] \log \pi_\theta(A_t|S_t)$$

  - Optimizers to minimize the losses
- The optimizer can be synchronous (A2C), or asynchronous (A3C)

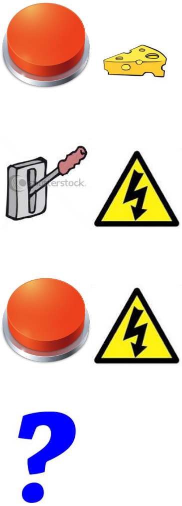
Policies

- We typically agents that learn to improve their behaviour over time
- Good policies trade off exploration and exploitation
- There are many design choices
  - Policies may depend on internal state $S_t \in \mathcal{S}$, or just on observation O_t
  - Policies could be 'blind', e.g., consider uniform random policies
  - Output actions must be compatible with the environment
    e.g., integer for Atari, real-valued vector for continuous control

Rat Example



Rat Example

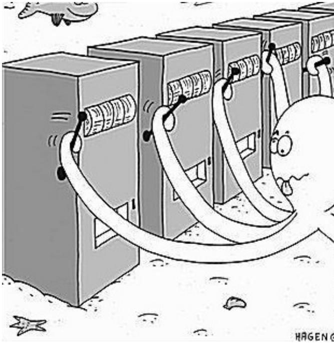


Exploration vs. Exploitation

- Online decision-making involves a fundamental choice:
  - Exploitation: Maximize return given current knowledge
  - Exploration: Increase knowledge
- The best long-term strategy may involve short-term sacrifices
- Gather enough information to make the best overall decisions

Recap: The Multi-Armed Bandit

- A multi-armed bandit is a tuple $\langle \mathcal{A}, \mathcal{R} \rangle$
- $\mathcal{A}$ is a known set of actions (or "arms")
- $\mathcal{R}^a(r) = \mathbb{P}\left[R_t = r | A_t = a\right]$ is an unknown probability distribution over rewards
- At each step t the agent selects an action $A_t \in \mathcal{A}$
- The environment generates a reward $R_t \sim \mathcal{R}^{A_t}$
- The goal is to maximize cumulative reward $\sum_{i=1}^{t} R_i$
- Repeated 'game against nature'



Regret

- The *optimal value* v_* is

$$v_* = \max_{a \in \mathcal{A}} q(a) = \max_a \mathbb{E}\left[R_t \mid A_t = a\right]$$

- Regret is the opportunity loss for one step

$$v_* - q(A_t)$$

- E.g., I might regret going to class, although I might learn by going

Regret

- Trade-off exploration and exploitation by minimizing *total regret*:

$$L_t = \sum_{i=1}^{t} v_* - q(a_i)$$

- Maximise cumulative reward $\equiv$ minimise total regret
- Note: cumulation here extends over termination of 'episode'
- Extends over 'lifetime of learning', rather than over 'current episode'

Counting Regret

- The *gap* Δ_a is the difference in value between action a and optimal action a_*, $\Delta_a = v_* - q(a)$
- Total regret depends on gaps and counts

$$
\begin{aligned}
L_t &= \sum_{i=1}^{t} v_* - q(a_i) \\
&= \sum_{a \in \mathcal{A}} N_t(a)(v_* - q(a)) \\
&= \sum_{a \in \mathcal{A}} N_t(a)\Delta_a
\end{aligned}
$$

- A good algorithm ensures small counts for large gaps
- Problem: gaps are not known...

Exploration

- We need to explore to learn about the values of all actions
- What is a good way to explore?
- One common solution: ϵ-greedy
  - Select greedy action (exploit) w.p. $1 - \epsilon$
  - Select random action (explore) w.p. ϵ
- Used in Atari
- Is this enough?
- How to pick ϵ?

ϵ-Greedy Algorithm

- Greedy can lock onto a suboptimal action forever
- $\Rightarrow$ Greedy has linear expected total regret
- The ϵ-*greedy* algorithm continues to explore forever
  - With probability $1 - \epsilon$ select $a = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\, Q_t(a)$
  - With probability ϵ select a random action
  - Constant ϵ ensures minimum expected regret

$$\mathbb{E}\left[v_* - q(A_t)\right] \geq \frac{\epsilon}{|\mathcal{A}|} \sum_{a \in \mathcal{A}} \Delta_a$$

- $\Rightarrow$ ϵ-greedy with constant ϵ has linear total regret

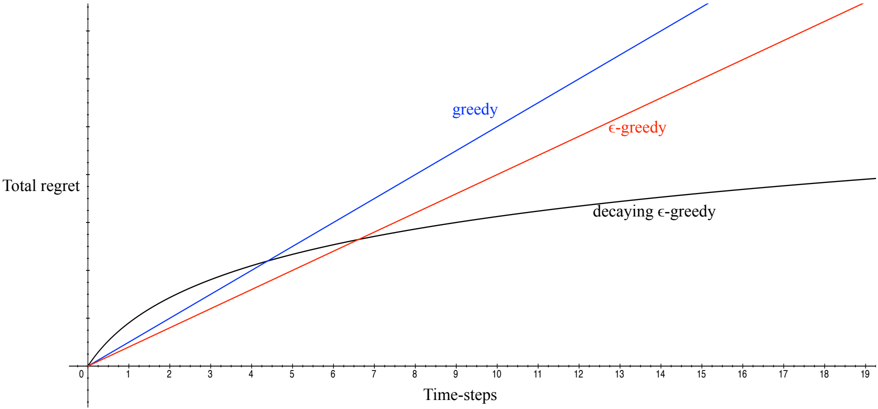
Decaying ϵ_t-Greedy Algorithm

- Pick a decay schedule for $\epsilon_1, \epsilon_2, ...$
- Consider the following schedule

$$\epsilon_t = \frac{c}{t} \qquad \text{where} \qquad c \in (0, 1]$$

- Decaying ϵ_t-greedy has *logarithmic* asymptotic total regret
- Finding proper decay (e.g., c) is not trivial (can depend on problem)

Linear or Sublinear Regret



Lower Bound

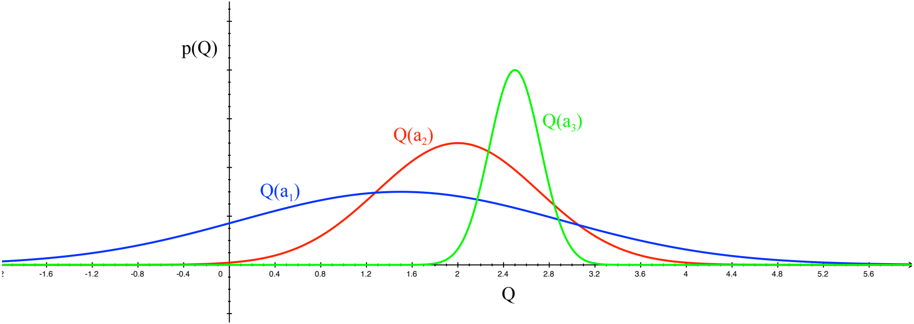
- The performance of any algorithm is determined by similarity between optimal arm and other arms
- Hard problems have arms with similar distributions but different means
- This is described formally by the gap Δ_a and the similarity in distributions $KL(\mathcal{R}^a||\mathcal{R}^a*)$

Theorem (Lai and Robbins)

Asymptotic total regret is at least logarithmic in number of steps

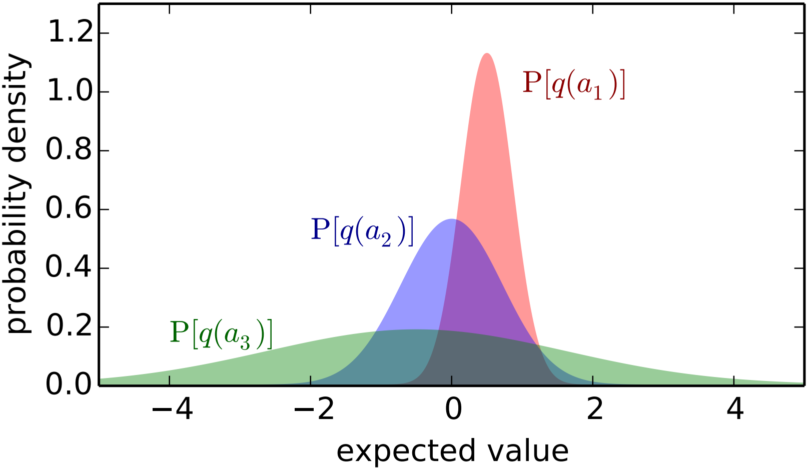
$$\lim_{t\to\infty} L_t \geq \log t \sum_{a|\Delta_a>0} \frac{\Delta_a}{KL(\mathcal{R}^a||\mathcal{R}^{a_*})}$$

Optimism in the Face of Uncertainty



- Which action should we pick?
- More uncertainty: more important to explore that action
- It could turn out to be the best action

Optimism in the Face of Uncertainty



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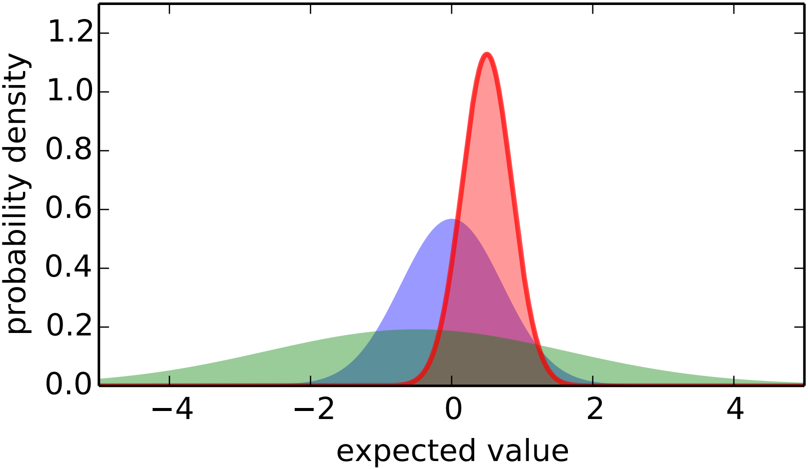

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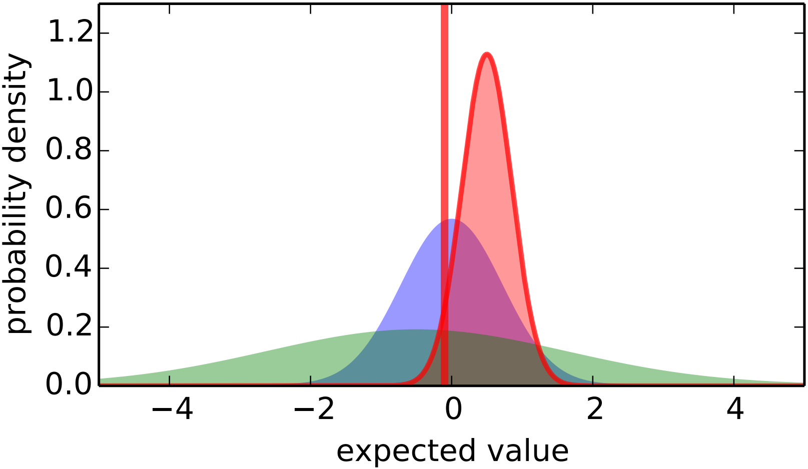

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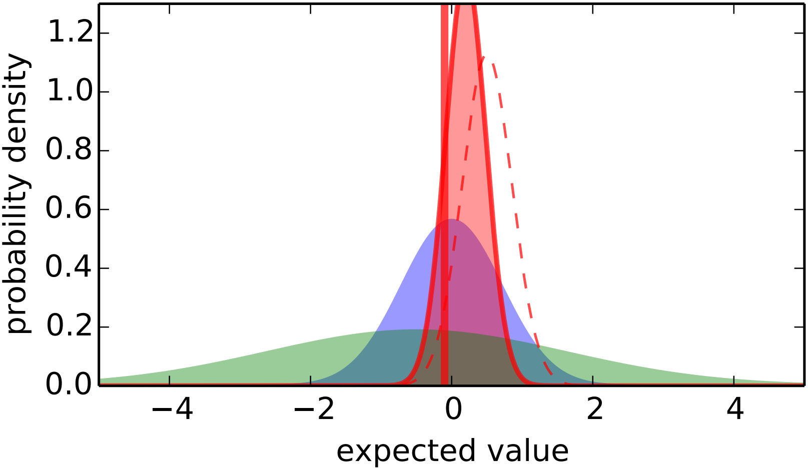

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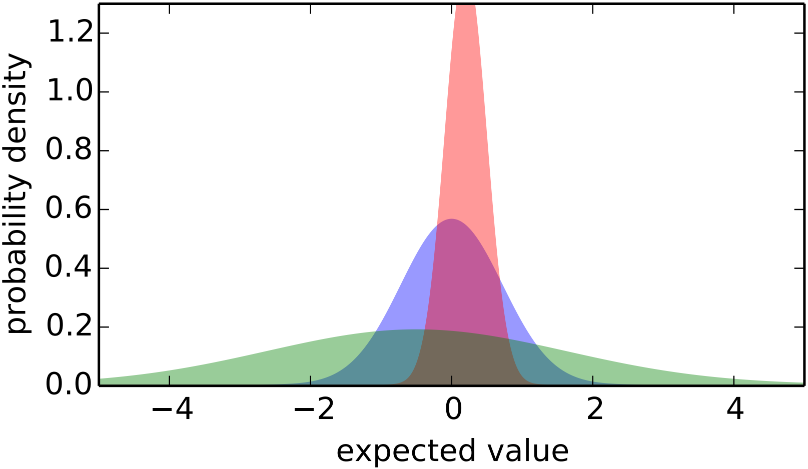

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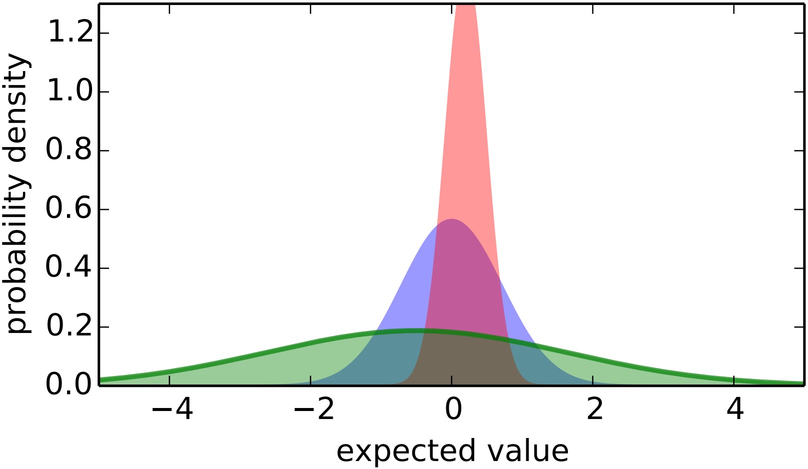

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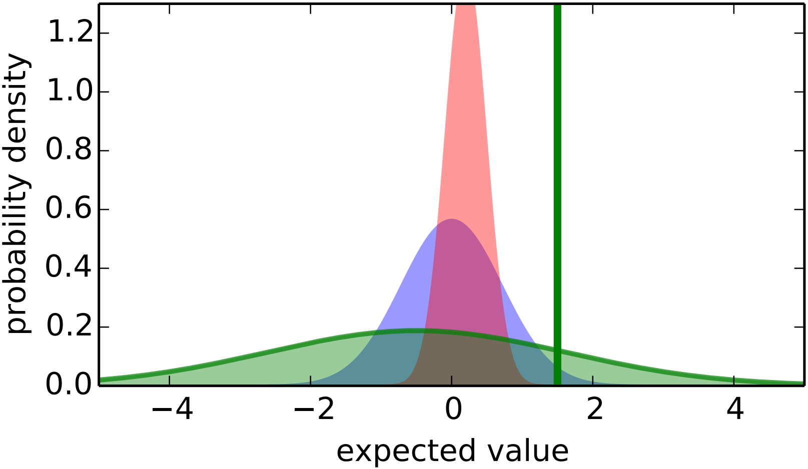

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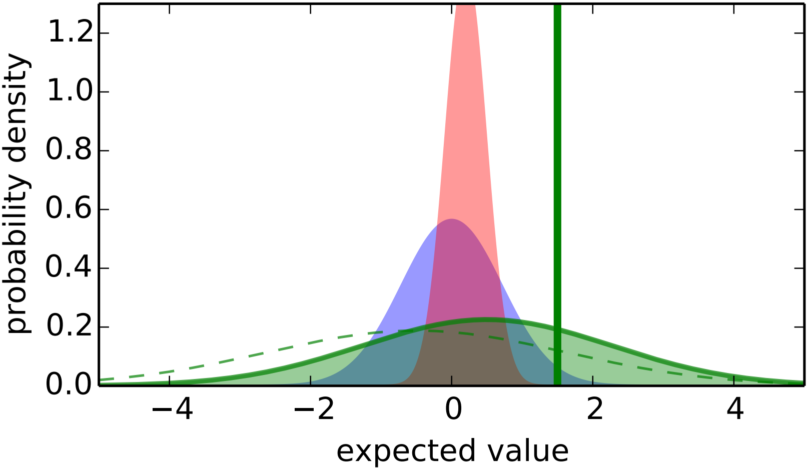

Upper Confidence Bounds

- Estimate an upper confidence $U_t(a)$ for each action value, such that $q(a) \leq Q_t(a) + U_t(a)$ with high probability
- Uncertainty depends on the number of times $N(a)$ has been selected
  - Small $N_t(a) \Rightarrow$ large $U_t(a)$ (estimated value is uncertain)
  - Large $N_t(a) \Rightarrow$ small $U_t(a)$ (estimated value is accurate)
- Select action maximizing Upper Confidence Bound (UCB)

$$a_t = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\; Q_t(a) + U_t(a)$$

Hoeffding's Inequality

Theorem (Hoeffding's Inequality)

Let $\Sigma_1, ..., \Sigma_t$ be i.i.d. random variables in [0,1], and let $\overline{X}_t = \frac{1}{t}\sum_{i=1}^{t}\Sigma_i$ be the sample mean. Then

$$\mathbb{P}\left[\mathbb{E}\left[X\right] > \overline{X}_t + u\right] \leq e^{-2tu^2}$$

- We can apply Hoeffding's Inequality to bandits with bounded rewards
- E.g., if $R_t \in [0, 1]$, then

$$\mathbb{P}\left[q(a) > Q_t(a) + U_t(a)\right] \leq e^{-2N_t(a)U_t(a)^2}$$

Calculating Upper Confidence Bounds

- Pick a probability p that true value exceeds UCB
- Now solve for $U_t(a)$

$$e^{-2N_t(a)U_t(a)^2} = p$$

$$U_t(a) = \sqrt{\frac{-\log p}{2N_t(a)}}$$

- Reduce p as we observe more rewards, e.g. $p = t^{-1}$
- Ensures we select optimal action as $t \to \infty$

$$U_t(a) = \sqrt{\frac{\log t}{2N_t(a)}}$$

UCB1

- This leads to the UCB algorithm

$$a_t = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\, Q_t(a) + c\sqrt{\frac{\log t}{N_t(a)}}$$

with $c > 0$

Theorem (Auer et al., 2002)

The UCB algorithm (with $c = 1/\sqrt{2}$) achieves logarithmic expected total regret

$$L_t \leq 8 \sum_{a|\Delta_a > 0} \frac{\log t}{\Delta_a} + O(\sum_a \Delta_a)$$

for any t

Value of Information

- Exploration is valuable because information is valuable
- Can we quantify the value of information?
- Information gain is higher in uncertain situations
- Therefore it makes sense to explore uncertain situations more
- If we know value of information, we can trade-off exploration and exploitation optimally (in bandits)

Solving information state space bandits

- We can formulate bandits as an infinite MDP over information states
- 'Information state' includes (sufficient statistic) information of what we have learned (e.g., rewards and actions)
- Can be solved by reinforcement learning
- Model-free reinforcement learning
  - e.g. Q-learning (Duff, 1994)
- Bayesian model-based reinforcement learning
  - e.g. Gittins indices (Gittins, 1979)
- Latter approach is known as *Bayes-adaptive* RL
- Finds Bayes-optimal exploration/exploitation trade-off (with respect to prior distribution)

Towards complex environments

- Bandits are simple, in several senses
  - Tabular states, tabular actions, no generalisation
  - No sequential nature within episodes
- Real problems are much harder
- Key insights:
  - Optimism in the face of uncertainty
  - Seek information
- Key question: can we find good proxy for information?
- We need policies that yield information—in some settings this is easy, in some this is hard

Example: Cart Pole

- Imagine balancing a pole on a cart by hitting the cart
- If each episode starts with an almost-balanced pole, exploration is easy

Jittering

- Simplest approach to exploration: jitter
- Example: ϵ-greedy
- Example: soft max policy $\pi(a|s) = e^{p(s,a)} / \sum_b e^{p(s,b)}$
  for some preference function p (e.g., $p(s,a) = q(s,a)/\tau$)
- Used in much current state of the art!

Intrinsic motivation

- Idea: UCB uses counts of states and actions
- These counts capture a notion of uncertainty
- Can we use (pseudo-) counts in complex environments?
- Can we use motivate the agent to go to novel situations?

Video: Montezuma's revenge

https://www.youtube.com/watch?v=0yI2wJ6F8r0

Agent components

- This agent had an additional a component: a density model
- This component was used to improve the behaviour policy
- This component can be (and recently was) swapped out and improved, while keeping the rest of the agent fixed
- But often components interact, so revisiting other components may be important (e.g., retuning)

Auxiliary predictions

- Sometimes we can improve components in interesting ways
- E.g., we can use a representation to make multiple predictions
- For instance: predict immediate rewards with the representation in A2C
- This help performance, even though the signal is quite indirect!

$$\text{Auxiliary predictions} \implies \text{Better representation} \implies \text{Better policy}$$

Models

- We have seen that models can be useful components
- In last lecture, a model was trained, and then used to update to a value function (Dyna)
- In the past, model-based RL was known to be data efficient, while model-free RL was thought to be more computation efficient
- These days: less clear
  - It is hard to learn good models in messy domains
  - Perhaps we are not using them optimally
  - However, replay is essentially a non-parametric model and is useful
  - We probably do not want to model individual pixels—but what then?
  - More to be done!

Questions

Topics and Terms

- Reward (and the reward hypothesis)
- Observation
- Action
- History
- State, environment state, agent state, Markov state
- Markov decision process (MDP)
- Partially observable MDP (POMDP)
- Discount
- Policy
- Value function
- Model-free / model-based
- Actor critic
- Planning
- Prediction
- Control
- Exploration/exploitation
- Multi-armed bandit
- Return
- Bellman equation
- Bellman optimality equation
- Dynamic programming
- Policy evaluation
- Policy improvement
- Policy iteration
- Generalized policy iteration
- Value iteration
- Contraction mappings
- Monte Carlo
- Temporal difference (TD)
- Bootstrapping
- TD error
- n-step returns
- λ-returns, TD(λ)
- Eligibility traces (forward view, backward view)
- On-policy / off-policy
- Sarsa, Q-learning
- Policy gradient
- REINFORCE (with and without baseline)
- Dyna
- Monte Carlo tree search (MCTS)
- Regret
- Optimism in the face of uncertainty