Compound Poisson process

This docoument demonstrates how simulate a compound Poisson process.

- Consider a shop where customers arrive in an exponential distribution with rate $\lambda = 20$.
    - mean of $Exp(\lambda) = 1/\lambda$.
- The number of customers for each arrival follows a geometric distribution with parameter $p = 0.4$.
    - mean of $Geom(p) = 1/p$.
- All random values are independent.

Example of generating exponential random variables:

```
lambda <- 20
rexp(1, lambda)
```

```
## [1] 0.04683933
```

```
lambda <- 20
num_rv <- 5
rexp(num_rv, lambda)
```

```
## [1] 0.064339336 0.034024089 0.008371368 0.066349810 0.180388101
```

From time 0 to 1, how many times will customers arrive?

- Of course, this is random.
- `num_arrivals` represents the total arrival numbers up to time 1.

```
arrivals <- numeric()
while (sum(arrivals)<1){
  arrivals <- c(arrivals, rexp(1, lambda))
}
(num_arrivals <- length(arrivals) - 1)
```

```
## [1] 31
```

The number of cumstomers in each arrival is determined by a geometric distribution.

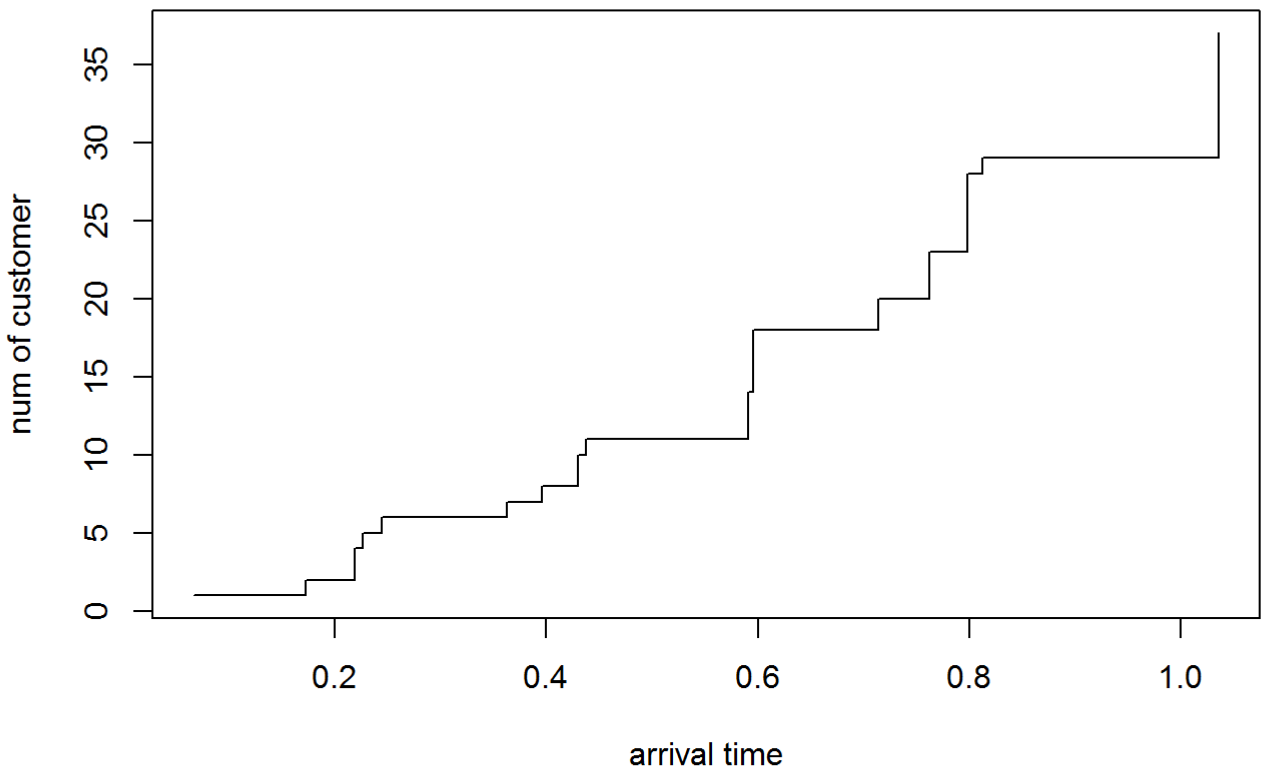
```
p = 0.4
(num_customer <- rgeom(1, p) + 1)
```

```
## [1] 2
```

The following plot represents the customer arrivals:

- To plot stair type graph, use `type='s'`.

```
# Your code shoude be here
```



In this case, the total number of customer up to time 1 is:

```
sum(num_customer[1:(length(num_customer)-1)])
```

```
## [1] 29
```

Now simulate this procedure 100 times.

- first, compute the total number of customers up to time 1 for each simulation.
- next, compute the mean of the total number of customers.

```
# Your code shoude be here
```

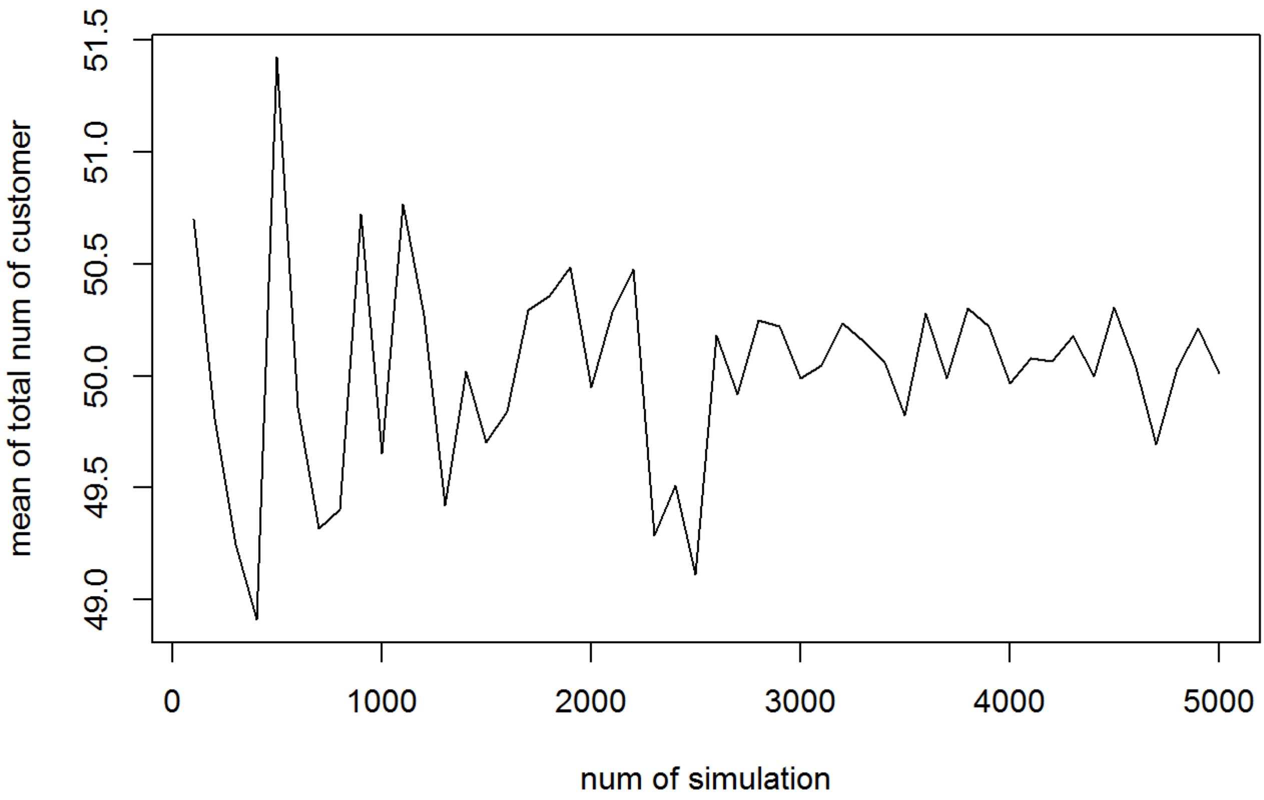
```
## [1] 46.27
```

Increase sequentially the simulation number:

```
num_simuls <- seq(100, 5000, 100)
```

Now plot the mean of the number of total cumstomer with simulation number.

```
# Your code shoude be here
# This takes time.
```



Question :

- Which value does the mean of total num of customer converge?
- Caluate the theoritical value of the mean of total num of customer.