Assignment 1

UW-Madison MATH 421

January 28, 2021

LaTeX is a very useful tool for writing up and communicating mathematics. This assignment will introduce you to some of the things you can do with this tool.

First set up an environment where you can compile a LaTeX file. There are (at least) two options:

1. Follow the instructions on `https://guides.nyu.edu/LaTeX` to download the (free) applications necessary to compile a LaTeX file on your computer.

2. Set up a (free) account at `Overleaf.com`. This provides a browser based environment for using LaTeX.

Next download the file `HW1_template.tex` on the Canvas site and compile it. You should produce a pdf that is identical to `HW1_template.pdf` on the Canvas site

Each of the prompts below will ask you to produce some LaTeX code to perform a particular task. In the `HW1_template.tex` file you should include your LaTeX code using the **verbatim** environment as well as the output of that code. As a model I've answered Exercise 1 part (a) for you. Your submission should consist of **only** the pdf output.

In the beginning using LaTeX may seem overwhelming, but it is widely used in the scientific community and there are many helpful resources:

1. On the Canvas site (under files) you can find `LATEX_tutorial.pdf` which contains lots of helpful information.

2. The NYU libraries LaTeX website, `https://guides.nyu.edu/LaTeX`, is another useful resource

3. If you are having trouble finding commands, formatting issues, etc., you might try a google search of the form "How do you do [BLAH] in LaTeX." Chances are someone else had the exact same problem and the solution can be found online.

Exercise One (equations):

There are two ways to input mathematical text/symbols. "Inline", which makes use of the $ symbols, and "display mode" which makes use of the double dollar sign $$ (there is also an open/closed bracket notation). Note that inline should be reserved for short collections of symbols or notations and display mode should be reserved to highlight an important detail or to deal with a lengthy collection of symbols. You should also use display mode to deal with "tall" objects which can mess up line spacing: vectors, matrices, rational expressions, and so on.

Exercise: Recreate the following statements (remember to include the code!):

a) If $f(x) = x^n$, then

$$f'(x) = nx^{n-1}.$$

b) If $n \neq -1$, then

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C.$$

c) The derivative of a function f at $x = a$ is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

(Hint: take a look at pages 10-12 of `LATEX_tutorial.pdf` on Canvas to learn how to write fractions, $\neq$, $\int$, and $\lim_{h \to 0}$.)

Exercises Two (fences):

Here is some code that produces silly looking output:

code: `$$(\frac{1}{x})$$`

output: $$(\frac{1}{x})$$

This can be improved by using the `\right` and `\left` commands as follows:

code: `$$\left( \frac{1}{x} \right)$$`

output: $$\left(\frac{1}{x} \right)$$

The `\right` and `\left` commands placed before a "fence" tell LaTeX to adjust the height to match whatever is inside.

Exercise: Using the `\right` and `\left` commands, recreate the following statements (remember to include the code!):

a) The number e is defined by

$$e = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n.$$

b) If f is a continuous function, then

$$\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x).$$

(Hint: take a look at pages 10-13 of `LATEX_tutorial.pdf` on Canvas.)

Exercise Three (tables):

Tables can be constructed as follows:

```
\begin{center}
  \begin{tabular}{c|cc}
   Name & Ice Cream Flavor & Number of Scoops \\
   \hline
   Alex  & Vanilla    & $4$ \\
   Susan & Chocolate  & $2$ \\
   Tran  & Strawberry & $18$
  \end{tabular}
\end{center}
```

First Name	Ice Cream Flavor	Number of Scoops
Alexa	Vanilla	4
Julia	Chocolate	2
Johnny	Strawberry	18

Let's take it apart and talk about what each piece does: The `\begin{center}...\end{center}` centers the entire table. The `tabular` environment expects you to describe the number of columns as well as other information. Here we have `{c|cc}` which indicates three columns each with center justification and a vertical line between the first and second column. We can change the justification by using `r` for right justification and `l` for left justification. Finally the `&` symbols separate the columns, the `\\` symbol indicates a new row, and the command `\hline` adds a horizontal line (This is all a bit of an oversimplification, but no big deal.)

Exercise: Construct a new table from the above as follows (remember to include the code!):

1. Insert a new column between "First Name" and "Ice Cream Flavor." This column will be for last names. The last names for Alexa, Julia, and Johnny are (respectively) Leal, Maschi, and Tran.

2. Justify the First Name Column to the right and the Last Name Column to the left. Leave the other columns centered.

3. Insert a new row which has your first name, last name, and a flavor of ice cream (or none, if you do not like ice cream!), and use the symbol for infinity for your number of scoops. You should insert your row so that it fits alphabetically by last name.

4. Add a "box" around the entire table. That is: horizontal lines on the top and bottom and vertical lines to the left and the right.

Exercise Four (matrices):

Matrices are common enough that they have their own environment. For example the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

is given by

```
$$
 \begin{pmatrix}
  1 & 2 & 3 \\
  4 & 5 & 6
 \end{pmatrix}
$$
```

Note that you do not need to specify the number of columns anymore. Also the **p** in `pmatrix` indicates that the matrix should be surrounded by **p**arentheses.

Exercise: Recreate the following using a display mode equation (remember to include the code!):

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc.$$

Hint: take a look at page 12 of `LATEX_tutorial.pdf` on Canvas to learn how to write det.

Exercise Five (commands):

Some commands in LaTex can be lengthy, might not fit your style, are used over and over again, or otherwise might benefit from simplification or customization command. You can do this using the `\newcommand`. These are typically placed near the beginning of your LaTeX file and you can see some examples around line ten of the source file `HW1_template.tex`. Here are some other examples and exercises.

1. The standard symbol for the real numbers is $\mathbb{R}$. The code which makes this is `\mathbb{R}`. Since this is a symbol you might use over and over and over again, you might want to make it simpler to reduce key strokes. Near the top of `HW1_template.tex` you will find:

   ```
   \newcommand{\Rb}{\mathbb{R}}
   ```

   Now we can use the command `\Rb` to create the symbol $\Rb$. (Note: All commands must begin with the forward slash `\`. Also since this command is for a mathematics symbol, it needs to be in a math mode. But this is not true for all commands.)

Exercise: Construct custom commands for the natural numbers $\mathbb{N}$, integers $\mathbb{Z}$, rationals $\mathbb{Q}$, and complex numbers $\mathbb{C}$. Place them at the top of `HW1_template.tex` in an appropriate place. Then recreate the following as a display mode equation:

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}.$$

(Hint: to write $\subset$ use `\subset`.)

2. You may want your custom commands to accept input. You can find an example where I have created a custom command for a derivative symbol near the top of the `HW1_template.tex`:

```
\newcommand{\der}[2]{\frac{d #1}{d #2}}
```

Now we can use the command `\der{y}{x}` and `\der{f}{t}` which gives

$$\frac{dy}{dx} \text{ and } \frac{df}{dt}$$

As you can see, if we change the input symbols then we can change the output.

Let's break down the code real quick before the exercises begin: Note that the command name is in braces `{\der}` and is immediately followed by the number of inputs in brackets: `[2]`. If a command does not have any inputs then we do not need the brackets. Next, the definition of the command is placed in its own braces with the symbols `#1` and `#2` in place of the inputs as written in order: `\der{#1}{#2}`.

Exercise: Make a partial derivative command. The command should be named `\pder` and you will need the `\partial` command. Then use your command to recreate:

If $z = x^2 + xy + y^2$, then

$$\frac{\partial z}{\partial x} = 2x + y.$$

Exercise Six (multiline equations):

Multiline equations can be created using the `align*` environment. For instance, the code

```
\begin{align*}
\int \frac{2}{1-x^2}dx & = \int \frac{1}{1-x} + \frac{1}{1+x} dx \\
& = -\log(1-x)+\log(1+x) +C \\
& = \log \left( \frac{1+x}{1-x} \right)+C.
\end{align*}
```

produces

$$\begin{aligned}\int \frac{2}{1-x^2}dx &= \int \frac{1}{1-x} + \frac{1}{1+x}dx \\ &= -\log(1-x) + \log(1+x) + C \\ &= \log\left(\frac{1+x}{1-x}\right) + C.\end{aligned}$$

Notice that `\\` produces a new line and the `&` is where the lines are aligned.

Exercise: Using the `align*` environment, recreate the following:

If $f(x) = x^2$, then

$$\begin{aligned} f'(a) &= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \\ &= \lim_{h \to 0} \frac{(a+h)^2 - a^2}{h} \\ &= \lim_{h \to 0} \frac{a^2 + 2ah + h^2 - a^2}{h} \\ &= \lim_{h \to 0} 2a + h = 2a. \end{aligned}$$

(Hint: remember to include the code!)