Assignment #1

Summer 2022 REU

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Exercise One:

a) Code:

```
If $f(x) = x^n$,  then
$$
f^\prime(x) = n x^{n-1}.
$$
```

Output: If $f(x) = x^n$, then

$$f'(x) = nx^{n-1}.$$

b)

```
If $n \neq -1$, then
\[ \int x^n dx = \frac{1}{n + 1} x^{x + 1} + C \text{.} \]
```

Output: If $n \neq -1$, then

$$\int x^n dx = \frac{1}{n+1} x^{x+1} + C.$$

c)

```
The derivative of a function $f$ at $x = a$ is
\[ f'(a) = \lim_{h \to 0} \frac{f(a + h) -f(a)}{h} \text{.} \]
```

Output: The derivative of a function f at $x = a$ is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

Exercises Two:

a)

```
The number $e$ is defined by
\[ e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n \text{.} \]
```

Output: The number e is defined by

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n.$$

b)

```
If $f$ is a continuous function, then
\[ \frac{d}{dx} \left[ \int_a^x f(t)dt \right] = f(x) \text{.} \]
```

Output: If f is a continuous function, then

$$\frac{d}{dx} \left[\int_a^x f(t)dt \right] = f(x).$$

Exercise Three:

Code:

```
\begin{center}
\begin{tabular}{|rl|cc|}
\hline
First Name & Last Name & Ice Cream Flavor & Number of Scoops \\
\hline
George     & Ekman     & Butter Pecan     & $\infty$         \\
Alexa      & Leal      & Vanilla          & $4$              \\
Julia      & Maschi    & Chocolate        & $2$              \\
Johnny     & Tran      & Strawberry       & $18$             \\
\hline
\end{tabular}
\end{center}
```

Output:

First Name	Last Name	Ice Cream Flavor	Number of Scoops
George	Ekman	Butter Pecan	∞
Alexa	Leal	Vanilla	4
Julia	Maschi	Chocolate	2
Johnny	Tran	Strawberry	18

Exercise Four:

Code:

```
\[ \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc \text{.} \]
```

Output:

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc \text{.}$$

Exercise Five:

Code:

```
% In preamble:
\newcommand{\pder}[2]{\frac{\partial #1}{\partial #2}}

\newcommand{\Nb}{\mathbb{N}}
\newcommand{\Zb}{\mathbb{Z}}
\newcommand{\Qb}{\mathbb{Q}}
\newcommand{\Cb}{\mathbb{C}}

% Here:
\[ \Nb \subset \Zb \subset \Qb \subset \Rb \subset \Cb \text{.} \]

If $z = x^2 + xy + y^2$, then
\[ \pder{z}{x} = 2x + y \text{.} \]
```

Output:

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}.$$

If $z = x^2 + xy + y^2$, then

$$\frac{\partial z}{\partial x} = 2x + y.$$

Exercise Six:

Code:

```
If $f(x) = x^2$, then
\begin{align*}
f'(a) &= \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} \\
  &= \lim_{h \to 0} \frac{(a + h)^2 - a^2}{h} \\
  &= \lim_{h \to 0} \frac{a^2 + 2ah + h^2 - a^2}{h} \\
  &= \lim_{h \to 0} 2a + h = 2a \text{.}
\end{align*}
```

Output: If $f(x) = x^2$, then

$$\begin{aligned}
f'(a) &= \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} \\
&= \lim_{h \to 0} \frac{(a + h)^2 - a^2}{h} \\
&= \lim_{h \to 0} \frac{a^2 + 2ah + h^2 - a^2}{h} \\
&= \lim_{h \to 0} 2a + h = 2a \text{.}
\end{aligned}$$