We prove 2 lemmas about the Chord Arc Quantity between points on adjacent edges, which will lead to an efficient "Adjacent Chord Arc Quantity" calculation algorithm.

In figure 1, e_1 and e_2 are two adjacent edges connected by vertex $\mathbf{v}$, and θ is the angle between e_1 and e_2. They form a "corner" of any polygon. We pick an arbitrary point $\mathbf{x}$ on e_1 and the distance between $\mathbf{x}$ and $\mathbf{v}$ is l_1. Similarly, we pick an arbitrary point $\mathbf{y}$ on e_2 and the distance between $\mathbf{y}$ and $\mathbf{v}$ is l_2. We assume $l_1 + l_2$ is less than half of the total length of the polygon's edges.

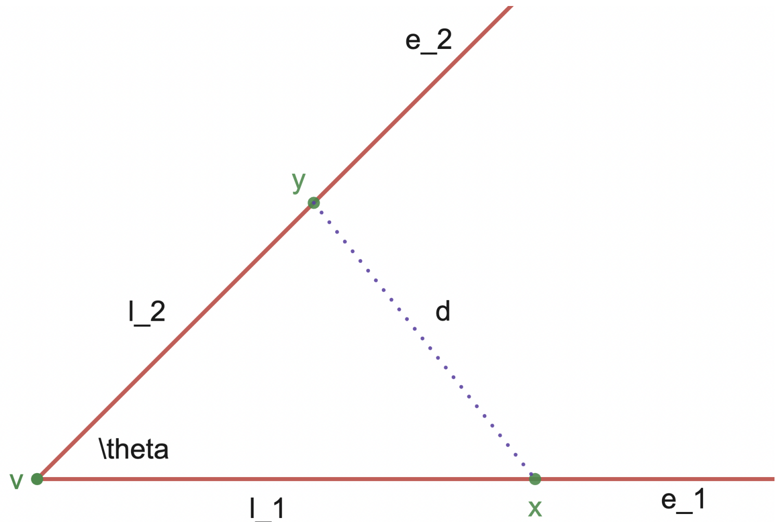

Figure 1: A Corner of Any Polygon

Lemma 1: For a given "corner" of a polygon, the Chord Arc Quantity between $\mathbf{x}, \mathbf{y}$ is maximized when $l_1 = l_2$, i.e., $\mathbf{x}$ and $\mathbf{y}$ are equidistant from $\mathbf{v}$.

Proof: The Chord Arc Quantity between $\mathbf{x}$ and $\mathbf{y}$ is

$$\frac{l_1 + l_2}{d},$$

and we are trying to maximize it. Suppose we fix $\mathbf{x}$, then l_1 is fixed and what we want to maximize is

$$\max f(l_2) = \frac{l_1 + l_2}{d} \qquad \Longleftrightarrow \qquad \max g(l_2) = \Big(f(l_2)\Big)^2 = \frac{(l_1 + l_2)^2}{d^2}.$$

We can express d in terms of l_1 and l_2 since they form a triangle:

$$d^2 = l_1^2 + l_2^2 - 2l_1 l_2 \cos(\theta).$$

For g to attain its extremum, we need

$$\begin{aligned}
\frac{d}{dl_2} g(l_2) &= 0 \\
\frac{d}{dl_2} \frac{(l_1 + l_2)^2}{l_1^2 + l_2^2 - 2l_1 l_2 \cos(\theta)} &= 0 \\
\frac{4l_1(l_1^2 - l_2^2)\cos^2(\frac{\theta}{2})}{\Big(l_1^2 + l_2^2 - 2l_1 l_2 \cos(\theta)\Big)^2} &= 0 \\
4l_1(l_1^2 - l_2^2)\cos^2(\frac{\theta}{2}) &= 0 \\
l_2^* &= l_1,
\end{aligned}$$

which implies that $\mathbf{x}$ and $\mathbf{y}$ are equidistant from $\mathbf{v}$. Now we use the second derivative test to make sure this extremum of g is indeed a maximum. Let $l_2 = l_1$,

$$
\begin{aligned}
\frac{d^2}{dl_2^2} g(l_2) &= \frac{8l_1 \cos^2(\frac{\theta}{2})\Big(2l_1^3\cos(\theta) - 3l_2 l_1^2 + l_2^3\Big)}{\Big(l_1^2 + l_2^2 - 2l_1 l_2 \cos(\theta)\Big)^3} \\
&= \frac{8l_1 \cos^2(\frac{\theta}{2})\Big(2l_1^3\cos(\theta) - 2l_1^3\Big)}{d^6} \\
&= \frac{8l_1 \cos^2(\frac{\theta}{2})\Big(2l_1^3 \overset{<0,\ \text{since}\ \theta\in(0,\pi)}{\big(\cos(\theta) - 1\big)}\Big)}{d^6} \\
&< 0
\end{aligned}
$$

Therefore, $l_2^* = l_1$ is a maximizer of g. □

Lemma 2: For a given "corner" of a polygon, the maximum Chord Arc Quantity between $\mathbf{x}$ and $\mathbf{y}$ is only dependent on θ.

Proof: When $l_2 = l_1$, the maximum Chord Arc Quantity between $\mathbf{x}$ and $\mathbf{y}$ achieved is

$$
\begin{aligned}
\frac{l_1 + l_2}{d} &= \frac{2l_1}{\sqrt{l_1^2 + l_2^2 - 2l_1 l_2 \cos(\theta)}} \\
&= \frac{2l_1}{\sqrt{2l_1^2 - 2l_1^2\cos(\theta)}} \\
&= \frac{\sqrt{2}}{\sqrt{1 - \cos(\theta)}},
\end{aligned}
$$

which only depends on θ. □

Implication: We can simply use $h(\theta) = \frac{\sqrt{2}}{\sqrt{1-\cos(\theta)}}$ to calculate the "Adjacent Chord Arc Quantity" of a polygon's each "corner".

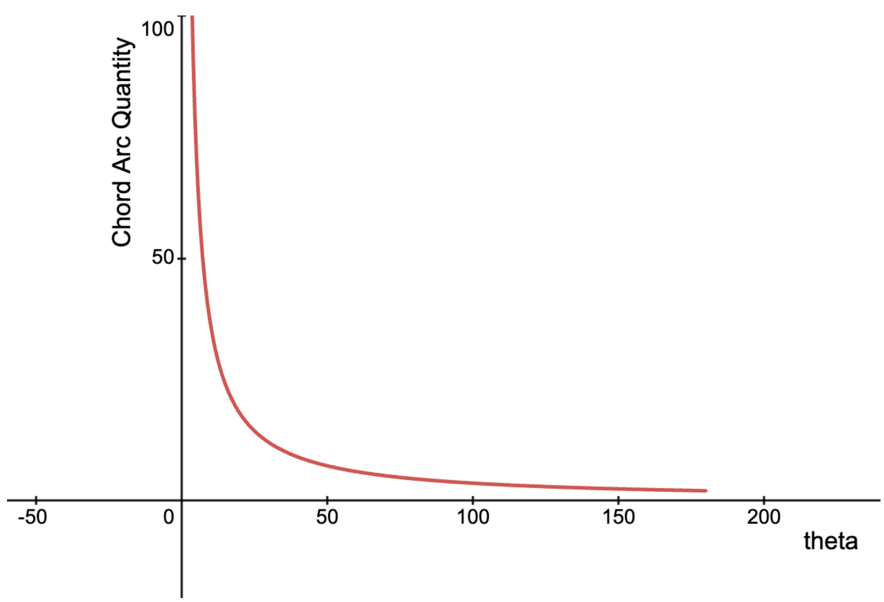

Figure 2: Plot of $h(\theta)$:
Maximum "Adjacent Chord Arc Quantity" vs θ (in degree)