Lemma 1. *The chordarc constant does not to occur with a pair of points where one of the points is a vertex, even when the pair are on nonadjacent edges.*

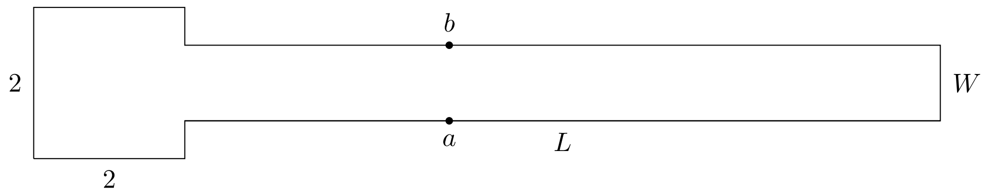

Figure 1: A hammer with $L = 10$, $W = 1$

Proof. The hammer in Figure 1 is a polygon, so the largest possible arc distance is half the perimeter of the polygon, and occurs when the distance is the same measuring in either direction, either through the head or handle of the hammer. The smallest euclidean distance between points a, b on nonadjacent edges is W, when a, b are on the top and bottom of the handle with the same x coordinate.

As long as the length of the handle is strictly greater than the perimeter of the head, this distance will occur along the handle, so we can parameterize the x coordinate of a, b in terms of $\lambda \in (0, 1)$: $2 + \lambda L$. Since $\lambda \in (0, 1)$, neither of a, b is a vertex. Then, the maximum arc distance between a, b of this form will occur when the distance going through the head and going through the end of the handle is the same:

$$\begin{aligned} l_{\text{head}}(a, b) &= l_{\text{handle}}(a, b) \\ 2\lambda L + 8 - W &= 2(1 - \lambda)L + W \\ 4\lambda L &= 2L + 2W - 8 \\ \lambda &= \frac{L + W - 4}{2L} \end{aligned}$$

This formula for λ shows that there is only one pair of points on nonadjacent edges that have both minimum euclidean distance and maximum arc distance.

Now, it remains to show that an adjacent pair of edges do not have a greater chordarc. Since all adjacent edges meet at right angles, the Adjacent Chord Arc gives

$$h(\theta) = \frac{\sqrt{2}}{\sqrt{1 - \cos\theta}} = \sqrt{2}.$$

Let $L = 10$ and $W = 1$. Then, $\lambda = 0.35$, and the chordarc between a, b

$$\frac{l(a, b)}{\|a - b\|} = \frac{2\lambda L + 8 - W}{W} = 14.$$

Therefore, the chordarc constant of the hammer is 14, and is not reached at a pair including a vertex. □