Lab 9 TASK 3

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- Task 3
  - Using myboot() make 95% (alpha=0.05) bootstrap intervals (iter=10000) for the population mean μ and record the plots when the following samples are used:
    - A) `set.seed(39); sam=rnorm(25,mean=25,sd=10)`
    - B) `set.seed(30); sam=rchisq(20,df=3)`
    - C) `set.seed(40); sam=rgamma(30,shape=2,scale=3)`
    - D) `set.seed(10); sam=rbeta(20,shape1=3,shape2=4)`
  - In each of the above cases how close is the point estimate to the population value? HINT: You will need to calculate the population mean.
  - In each of the above cases does the interval contain the population value? (HINT: You will need to calculate the population mean. See MS chapter 5 or use Wikipedia)
  - Using myboot() make 80% (alpha=0.20) bootstrap intervals (iter=10000) for the population variance σ^2 and record the plots when the following samples are used:
    - A) `set.seed(39); sam=rnorm(25,mean=25,sd=10)`
    - B) `set.seed(30); sam=rchisq(20,df=3)`
    - C) `set.seed(40); sam=rgamma(30,shape=2,scale=3)`
    - D) `set.seed(10); sam=rbeta(20,shape1=3,shape2=4)`

Needs attention

Chi square

df = 3

$\mu = 3$

$\sigma^2 = 2 * 3 = 6$

Notation	$\chi^2(k)$ or χ^2_k
Parameters	$k \in \mathbb{N}^*$ (known as "degrees of freedom")
Support	$x \in (0, +\infty)$ if $k = 1$, otherwise $x \in [0, +\infty)$
PDF	$\frac{1}{2^{k/2}\Gamma(k/2)} x^{k/2-1} e^{-x/2}$
CDF	$\frac{1}{\Gamma(k/2)} \gamma\left(\frac{k}{2}, \frac{x}{2}\right)$
Mean	k
Median	$\approx k\left(1 - \frac{2}{9k}\right)^3$
Mode	$\max(k-2, 0)$
Variance	$2k$
Skewness	$\sqrt{8/k}$
Ex. kurtosis	$\frac{12}{k}$
Entropy	$\frac{k}{2} + \ln(2\Gamma(\frac{k}{2})) + (1 - \frac{k}{2})\psi(\frac{k}{2})$
MGF	$(1-2t)^{-k/2}$ for $t < \frac{1}{2}$
CF	$(1-2it)^{-k/2}$ [1]
PGF	$(1-2\ln t)^{-k/2}$ for $0 < t < \sqrt{e}$

Gamma distribution

Shape = 2
Scale = 3

$\mu = 2 * 3 = 6$

$\sigma^2 = 2 * 3^2 = 18$

Parameters	• $k > 0$ shape • $\theta > 0$ scale	• $\alpha > 0$ shape • $\beta > 0$ rate
Support	$x \in (0, \infty)$	$x \in (0, \infty)$
PDF	$f(x) = \frac{1}{\Gamma(k)\theta^k} x^{k-1} e^{-\frac{x}{\theta}}$	$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$
CDF	$F(x) = \frac{1}{\Gamma(k)} \gamma\left(k, \frac{x}{\theta}\right)$	$F(x) = \frac{1}{\Gamma(\alpha)} \gamma(\alpha, \beta x)$
Mean	$k\theta$	$\frac{\alpha}{\beta}$
Median	No simple closed form	No simple closed form
Mode	$(k-1)\theta$ for $k \geq 1$, 0 for $k < 1$	$\frac{\alpha-1}{\beta}$ for $\alpha \geq 1$, 0 for $\alpha < 1$
Variance	$k\theta^2$	$\frac{\alpha}{\beta^2}$
Skewness	$\frac{2}{\sqrt{k}}$	$\frac{2}{\sqrt{\alpha}}$
Ex. kurtosis	$\frac{6}{k}$	$\frac{6}{\alpha}$
Entropy	$k + \ln\theta + \ln\Gamma(k)$ $+ (1-k)\psi(k)$	$\alpha - \ln\beta + \ln\Gamma(\alpha)$ $+ (1-\alpha)\psi(\alpha)$
MGF	$(1-\theta t)^{-k}$ for $t < \frac{1}{\theta}$	$\left(1 - \frac{t}{\beta}\right)^{-\alpha}$ for $t < \beta$
CF	$(1-\theta i t)^{-k}$	$\left(1 - \frac{it}{\beta}\right)^{-\alpha}$
Fisher information	$I(k,\theta) = \begin{pmatrix} \psi^{(1)}(k) & \theta^{-1} \\ \theta^{-1} & k\theta^{-2} \end{pmatrix}$	$I(\alpha,\beta) = \begin{pmatrix} \psi^{(1)}(\alpha) & -\beta^{-1} \\ -\beta^{-1} & \alpha\beta^{-2} \end{pmatrix}$
Method of Moments	$k = \frac{E[X]^2}{V[X]}$ $\theta = \frac{V[X]}{E[X]}$	$\alpha = \frac{E[X]^2}{V[X]}$ $\beta = \frac{E[X]}{V[X]}$

Beta

3,4

$3/(3+4) = 3/7$

$\sigma^2 = 3*4/((3+4)^2(3+4+1)) = 0.03061224$

Parameters	$\alpha > 0$ shape (real) $\beta > 0$ shape (real)
Support	$x \in [0,1]$ or $x \in (0,1)$
PDF	$\frac{x^{\alpha-1}(1-x)^{\beta-1}}{\mathrm{B}(\alpha,\beta)}$ where $\mathrm{B}(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ and Γ is the Gamma function.
CDF	$I_x(\alpha,\beta)$ (the regularized incomplete beta function)
Mean	$\mathrm{E}[X] = \frac{\alpha}{\alpha+\beta}$ $\mathrm{E}[\ln X] = \psi(\alpha) - \psi(\alpha+\beta)$ $\mathrm{E}[X \ln X] = \frac{\alpha}{\alpha+\beta}\left[\psi(\alpha+1) - \psi(\alpha+\beta+1)\right]$ (see section: Geometric mean) where ψ is the digamma function
Median	$I_{\frac{1}{2}}^{[-1]}(\alpha,\beta)$ (in general) $\approx \frac{\alpha-\frac{1}{3}}{\alpha+\beta-\frac{2}{3}}$ for $\alpha,\beta > 1$
Mode	$\frac{\alpha-1}{\alpha+\beta-2}$ for $\alpha, \beta > 1$ any value in $(0,1)$ for $\alpha, \beta = 1$ $\{0, 1\}$ (bimodal) for $\alpha, \beta < 1$ 0 for $\alpha \le 1, \beta > 1$ 1 for $\alpha > 1, \beta \le 1$
Variance	$\mathrm{var}[X] = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$ $\mathrm{var}[\ln X] = \psi_1(\alpha) - \psi_1(\alpha+\beta)$ (see trigamma function and see section: Geometric variance)
Skewness	$\frac{2(\beta-\alpha)\sqrt{\alpha+\beta+1}}{(\alpha+\beta+2)\sqrt{\alpha\beta}}$