LEVERAGED EXCHANGE-TRADED FUNDS

A Comprehensive Guide to Structure, Pricing, and Performance

NARAT CHARUPAT & PETER MIU

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Contents

Figures

Tables

1
Introduction

This book is about leveraged and inverse exchange-traded funds (hereafter collectively referred to as leveraged exchange-traded funds [LETFs]). Having debuted in 2006 in the United States, they are relatively new members of the exchange-traded fund (ETF) family. An LETF is a publicly traded fund that promises to provide daily returns that are in a multiple (positive or negative) of the returns on an underlying benchmark index. To meet that promise, the fund uses leverage, which is typically obtained through derivatives such as futures contracts, forward contracts, and total-return swaps.

Ever since its introduction, the LETF has attracted a lot of interest from market participants, as you will find out when we examine some of their trading statistics later in this chapter and in chapter 3. We believe the literature has not been able to keep up with the popularity and rapid development of this new class of investment instrument, which is by design risky given the embedded leverage. LETFs are fundamentally different from traditional nonleveraged ETFs in a few important ways. A lack of a thorough understanding of their characteristics could easily lead to confusion and thus their potential misuse among market participants. For instance, LETFs are not for long-term, buy-and-hold investors. This is because the constant maintaining of their leverage ratios necessitates the constant rebalancing of their exposure to the underlying benchmarks. This rebalancing of exposure will in turn cause their long-term return to deviate from the multiple of the underlying benchmark return over the same period. The magnitude and the direction of the deviation depend on the length of the holding period, the targeted leverage ratio, and the path that the underlying benchmark takes during that period. Market regulators have expressed concern about what they perceived to be considerable misunderstanding on the part of investors about the risk/return patterns of LETFs. For example, in August 2009, the Securities and Exchange Commission (SEC) and the Financial Industry Regulatory Authority (FINRA) issued a joint investor alert to advise investors of the risk of leveraged ETFs and to encourage them to obtain a good understanding of the products before they invest in them. The chairman of BlackRock, the world's largest provider of ETFs, has gone on the record several times as being critical of LETFs (see, e.g., Spence, 2011). Given that there are a number of fundamental differences between LETFs and traditional, nonleveraged ETFs, we believe that a book dedicated to LETFs is required.

In this book, we provide clear and concise explanations of all important aspects of LETFs. These include in-depth coverage of their structure, operation, market, return dynamic, pricing, performance, regulation, taxation, and trading strategies. We also examine the market and pricing of options on LETFs. Our goal is to create a reference text on LETFs by relying on recent findings of relevant academic research. The analytical approach of the book will not only benefit sophisticated market participants by helping them make their investment decisions in an informed fashion but also be useful for regular investors who merely want to have a basic understanding of the operations of LETF markets. In designing this book, we have in mind a targeted readership including both practitioners (e.g., investors in LETFs and related markets, market makers, financial analysts, risk managers, and market regulators) and an academic audience.

Let us start with an overview of the LETF market and the market of LETF options.

Markets of LETFs and LETF options

LETFs may be categorized by their leverage ratios, which may be –1x (i.e., an inverse ETF), +2x, –2x, +3x, or –3x.[1] An inverse ETF is targeted to deliver daily returns that are exactly the opposite of the daily returns of the underlying benchmark index. Suppose you invest in an inverse ETF on the Standard & Poor's (S&P) 500 Index. When the index return on a particular day is 2%, you will earn a return of –2% on that day. However, if the index return is –2%, you will get a 2% return. A +2x (+3x) LETF promises to deliver daily returns that are two (three) times that of the underlying benchmark index. Such LETFs are sometimes referred to as *bull* LETFs, since their returns are positively related to the returns on the underlying benchmark. If you invest in a +3x LETF, when the index return on a particular day is 2%, you can expect to have a return of 6% from the LETF on the same day. Of course, if the index value drops, you will incur a loss that is again three times that of the index return. Finally, a –2x (–3x) LETF promises to generate daily returns equal to two (three) times the inverse of that of the underlying benchmark. Such LETFs are usually referred to as *bear* LETFs, because their payoff is positive when the benchmark value drops. If you invest in a –3x LETF, when the index return on a particular day is 2%, you can expect the LETF's return to be –6% on the same day. However, you can expect a positive return of 6% from the LETF if the index return turns out to be –2%.

The first-ever LETF was issued in the United States in 2006. Since then, LETFs have successfully attracted investor interest. According to BlackRock (2012), in 2012, there were 273 LETFs traded in the United States, representing about 20% of the total number of ETFs trading in the country. Their total assets under management (AUM) were approximately $32 billion, or about 3% of the total AUM of all ETFs in the United States.[2] The underlying benchmarks that the LETFs track range from domestic equity, foreign equity, fixed income, commodities, foreign currency, and alternatives (e.g., the volatility index or the spread between the returns/yields of two different benchmarks). While the amount of assets of LETFs is small relative to the entire ETF market, their trading volume is disproportionately large compared to that of traditional nonleveraged ETFs.

Table 1.1 presents a breakdown of the LETF market in 2012 based on the underlying asset classes. Besides the AUM of each asset class, in the fourth column of the table, you can also find the total average daily volume (ADV) over the last 20 days of the third quarter of 2012. Most of the LETFs are equity based (60% by total number and 65% by total AUM). LETFs on fixed-income, commodity, and foreign currency indices also amassed a substantial amount of investment. Based on the ADV, they are also heavily traded. The average ADV-to-AUM ratio for all equity-based LETFs was 0.15 (see last column of table 1.1), which was comparable with, if not higher than, those of the most popular nonleveraged ETFs. For example, in the same year, the ADV-to-AUM ratios of SPDR S&P 500 ETF and PowerShares QQQ, the two largest nonleveraged US equity-based ETFs by assets, were about 0.145 and 0.061, respectively (BlackRock, 2012). Based on the ADV-to-AUM ratios reported in table 1.1, we note that LETFs tracking alternatives benchmarks were the most heavily traded despite the fact that they represented only a small fraction of

Table 1.1 Total Number, Assets under Management, and Average Daily Volume of LETFs in United States in 2012 by Underlying Asset Classes

	Number	**AUM ($ million)**	**ADV ($ million)**	**ADV/AUM**
Equity	164	20,962	3,150	0.15
US equity	121	18,872	2,941	0.16
Developed international equity	19	836	123	0.15
Emerging markets equity	24	1,255	86	0.07
Fixed income	34	4,984	179	0.04
Commodities	42	3,251	470	0.14
Foreign currencies	12	2,093	79	0.04
Alternatives	21	764	388	0.51
Total	273	32,053	4,265	

Source: BlackRock, 2012.

the total LETF market in terms of their AUM. In chapter 3, we present detailed trading statistics for a number of popular equity-based LETFs tracking broad US equity market indices.

In table 1.2, we list the top ten US LETFs in 2012 in terms of their AUM. The largest LETF was ProShares UltraShort 20+ Year Treasury, which is a –2x LETF that promises to deliver daily returns that correspond to two times the inverse of the daily returns of the Barclays US 20+ Year Treasury Bond Index. In 2012, it had total AUM of about $3 billion and attracted an average daily volume exceeding $100 million. But it was far from the most actively traded. According to the ADV and the ratio of

Table 1.2 Top Ten LETFs in United States in 2012 by Their Assets under Management

	Ticker	Leverage ratio	AUM ($ million)	ADV ($ million)	ADV/AUM
ProShares UltraShort 20+ Year Treasury	TBT	–2x	2,993	102	0.03
ProShares Short S&P 500	SH	–1x	1,987	88	0.04
ProShares UltraShort S&P 500	SDS	–2x	1,924	175	0.09
Direxion Daily Financial Bull 3x	FAS	+3x	1,115	319	0.29
ProShares Ultra S&P 500	SSO	+2x	1,089	318	0.29
Direxion Daily Small Cap Bear 3x	TZA	–3x	1,003	228	0.23
ProShares Ultra Silver	AGQ	+2x	985	92	0.09
ProShares Short 20+ Year Treasury	TBF	–1x	800	9	0.01
ProShares Ultra Financials	UYG	+2x	786	26	0.03
ProShares UltraShort Euro	EUO	–2x	757	29	0.04
Total			13,439	1,386	

Source: BlackRock, 2012.

ADV to AUM, the more heavily traded LETFs in the top ten were still equity-based LETFs. In 2012, six of the top ten LETFs were equity based, and more than a couple of them had average daily trading volume well above $200 million. Among the top ten, besides fixed-income and equity-based LETFs, there were also LETFs tracking commodities (ProShares Ultra Silver) and foreign currency benchmarks (ProShares UltraShort Euro). The LETF market was highly concentrated. These top ten LETFs together represented 42% (32%) of the total AUM (ADV) of all LETFs trading in the United States. As in the overall ETF market, the provider market of LETFs is also highly concentrated. ProShares and Direxion are the two leading fund providers (or fund issuers) in the US LETF market. In 2012, eight of the top ten LETFs were issued and managed by ProShares.

How important is the LETF market outside of the United States? There are well-developed and active LETF markets in Europe and Canada. LETFs have also started getting traction in Asia. The first pair of LETFs started listing on the Taiwan Stock Exchange in October 2014, making Taiwan the third market in the Asia-Pacific region offering LETFs, after Japan and Korea. Let us look at some statistics of the LETF market in Canada. In 2012, there were a total of 42 LETFs in Canada, with a total AUM of $1.43 billion, which was about 3% of the total ETF market (BlackRock, 2012). Similar to the case in the United States, although the amount of assets of LETFs is small relative to the entire ETFs market, their trading volume is disproportionately large compared to that of traditional nonleveraged ETFs. Unlike in the United States, however, only –1x, +2x, and –2x LETFs are available in Canada. Most of them are tracking either commodities or equity market benchmarks. Perhaps not surprisingly, given the importance of commodities to the country, all the top five LETFs by AUM in 2012 were either tracking some kind of commodities indices or equity-based indices representing different commodities-related sectors of the stock market. Actually, seven out of the top ten LETFs were commodities based, tracking crude oil, natural gas, gold, and silver benchmarks. In 2012, the largest LETF in Canada was

Horizons BetaPro NYMEX Crude Oil Bull Plus, which promises daily returns that equal two times that of the light sweet crude oil futures contract trading on New York Mercantile Exchange (NYMEX). In Canada, the leading, if not the only, LETF issuer is Horizons.

In the United States, there is also a well-developed option market on LETFs. After the listing of ProShares Ultra S&P 500 (ticker: SSO; a +2x LETF tracking S&P 500 Index) and ProShares UltraShort S&P 500 (ticker: SDS; a –2x LETF tracking S&P 500 Index) in mid-2006, their call and put options began trading on the Chicago Board Options Exchange (CBOE) starting at the end of 2007. American-style options on different kinds of LETFs have since been actively traded on CBOE. In 2013, among the total of about 500 ETFs, on which there was options trading, 102 were LETFs (CBOE, 2013). Their annual contract volume added up to 14 million in 2013. Not surprisingly, the most popular options are those on equity-based LETFs tracking domestic, foreign, and different industry sectors. They represent the majority of the LETF options on CBOE. In 2013, four of the top five LETF options by contract volume were on equity-based LETFs. The top ten LETF options are listed in table 1.3. As expected, there is a lot of overlap between the top ten LETFs by AUM (table 1.2) and the top ten LETF options by contract volume (table 1.3). Six of the ten in the latter list also appear on the former one. With an annual contract volume of more than two million in 2013, the options on ProShares UltraShort S&P 500 were by far the most popular. The combined annual contract volume of the top ten LETF options was close to 12 million (or about 84% of all the LETF options trading on CBOE). Besides equity-based LETF options, options on commodity- and fixed income-based LETFs have also attracted a lot of trading interest. There are also options on LETFs tracking real estate, foreign currency, and market volatility benchmarks.

By their very nature, LETF options are risky. They provide a second layer of leverage on top of the already leveraged returns on the underlying LETFs. Similar to other options, LETF options

Table 1.3 Top Ten LETFs Based on the Annual Contract Volume of Their Options Trading on the Chicago Board of Exchange in 2013

	Ticker	Leverage ratio	Annual contract volume (in thousand)
ProShares UltraShort S&P 500	SDS	–2x	2,047
Direxion Daily Financial Bull 3x	FAS	+3x	1,685
ProShares UltraShort 20+ Year Treasury	TBT	–2x	1,612
Direxion Daily Small Cap Bull 3x	TNA	+3x	1,428
Direxion Daily Small Cap Bear 3x	TZA	–3x	1,199
ProShares Ultra VIX Short-Term Futures	UVXY	+2x	1,127
ProShares Ultra S&P 500	SSO	+2x	876
Direxion Daily Financial Bear 3x	FAZ	–3x	734
Direxion Daily Gold Miners Bull 3x	NUGT	+3x	597
ProShares Ultra Silver	AGQ	+2x	389
Total			11,694

Source: CBOE, 2013.

allow market participants to profit from their predictions on the changes in the values and/or the volatility of their underlying assets. At the same time, they are also indispensable hedging tools in risk management.

The design of the rest of this book

We use the next three chapters (chapters 2 to 4) to cover the basic characteristics and operations of LETFs. These chapters provide fundamental background information necessary for the appreciation of the in-depth discussions of specific topics covered in the latter chapters of the book. In chapter 2, we provide an overview of the taxation and regulatory environment of the trading of LETFs. In chapter 3, we describe how LETFs are structured so as to deliver the promised leverage returns. We also examine the creation and redemption process, which is crucial to our understanding of the arbitrage argument that governs the pricing efficiency of LETFs. In chapter 4, we explain the compounding effect as a result of the maintaining of the daily targeted leverage ratios. Through the use of numerical examples and simulation analysis, we demonstrate the governing factors of the compounding effect and characterize the implications on the returns of LETFs. For those readers who are well versed on the fundamental background information, they may want to skip the next three chapters and start directly with chapter 5.

In chapter 5, we examine the pricing efficiency of LETFs in detail with the use of a sample of equity-based LETFs. We demonstrate the asymmetric behavior of the deviations of LETF prices from their net asset values (NAVs) for bull and bear LETFs. We show how this asymmetric behavior may be related to the end-of-day rebalancing of the LETFs' exposure. The performance of LETFs can be measured by how close they are tracking the leveraged returns on their underlying benchmark indices. In chapter 6, we describe the different sources of tracking errors. We review different approaches commonly used to measure tracking errors. Central to the discussion is the role played by the compounding effect in defining and dictating LETFs' performance when we are holding them for longer than one day. We turn to LETFs' trading strategies in chapter 7. We study the costs and benefits of shorting LETFs and pair strategies with positions in both bull and bear LETFs. Through a simulation exercise, we identify the market condition under which a particular strategy is preferred. In chapter 8, we discuss the pricing of options on LETFs. We show how we can calculate the price of an LETF option relative to those of other LETF options or options on nonleveraged ETFs that track the same underlying benchmark. With a sample of equity-based LETF options, we demonstrate how the targeted leverage ratio of an LETF can affect the level and behavior of the implied volatility of its options.

2
Regulations and Taxations

In this chapter, we will discuss the regulations governing the issuance and trading of LETFs. We will then look at the tax treatments of LETFs and how they are different from those of traditional ETFs.

Regulations

LETFs are open-ended investment vehicles, the shares of which are listed and traded on exchanges. Most LETFs are organized as investment companies under the Investment Company Act of 1940 and the Securities Act of 1933.[1] Specifically, LETFs must file registration statements with the Securities and Exchange Commission (SEC) under the 1940 act and register their share offerings under the Securities Act. These two acts also govern the issuance and sale of other securities to the public such as open-ended mutual funds (which had been in existence long before ETFs were introduced). In fact, under both acts, LETFs share many similar treatments with mutual funds (e.g., registration procedures, net asset values [NAV] disclosure, and financial reporting). Nevertheless, LETFs (and also traditional ETFs) have certain unique characteristics that are not consistent with several provisions of the acts. As a result, before these vehicles can be issued to the public, fund companies need to apply to the SEC for exemptive relief from these provisions. Examples of exemptive relief include the following:

- The purchase and redemption of "creation units": As will be discussed in the next chapter, LETFs allow certain traders (known as authorized participants) to buy and redeem shares directly with the fund companies. However, the transactions have to be done in lots known as creation units. The size of a creation unit can vary from one fund to another. Typically, each unit consists of 50,000 or 100,000 shares. Exemptive relief is needed because the transactions are not done based on individual shares.
- Shares trading on exchange at prices other than their NAVs: Because LETFs are listed and traded on exchanges, exemptive relief is needed in order for their shares to be purchased and sold at market prices, which are determined by traders and which can be different from the funds' NAVs.
- Sale of shares to other investment companies (e.g., a fund of funds) in excess of the allowable limits.

In addition, because LETFs will be traded on exchanges, they have to comply with the Securities Exchange Act of 1934, which governs trading in the secondary market in the United States. Again, due to some unique characteristics of LETFs, exemptive relief from certain rules is needed. In particular, LETFs must be exempted from Regulation M (Rules 101 and 102) of the 1934 act, which prohibits issuers and distributors of securities from purchasing the distributed securities during a restricted period. This is because LETFs have a redemption provision, which requires fund companies to redeem (i.e., buy back) the shares even though new shares are continuously being offered to the public.

When granting exemptive relief, the SEC imposes certain conditions on LETFs. These conditions are meant to increase LETFs' pricing efficiency and to protect investors. Examples of these conditions include the dissemination of LETFs' intraday NAVs on an interval basis (usually every 15 seconds) and the daily publication on LETF websites of the previous day's closing NAVs, closing market prices, and the premium/discount of the market prices relative to the NAVs.

As mentioned, most LETFs are organized under the Investment Company Act of 1940 and the Securities Act of 1933. It is possible, however, for LETFs to be organized only under the Securities Act of 1933. This can be done for LETFs that are based on commodities or currencies and that use derivatives to generate their returns. Rather than being investment companies (as defined under the Investment Company Act of 1940), these LETFs are structured as commodity pools under the regulations of the Commodity Futures Trading Commission (CFTC). In fact, during the past few years, many LETFs have been established in this manner. The main reason for this is that in March 2010, the SEC issued a release indicating that it would defer consideration of requests for exemptive relief under the Investment Company Act by ETFs that "would make significant investments in derivatives."[2] The release further stated that the SEC staff intended to explore "whether funds that rely substantially on derivatives, particularly those that seek to provide leveraged returns, maintain and implement adequate risk management." The SEC's decision was based on its concern that the use of leverage without proper control and monitoring could have a destabilizing effect on the market. It was also concerned about how funds priced their derivatives holdings and whether the risks created by derivatives were adequately disclosed to investors. The decision effectively put a moratorium on organizing new LETFs under the Investment Company Act of 1940.

Taxation of LETFs

Preliminary—taxation of traditional ETFs

To understand the taxation of LETFs, let us start with how traditional (i.e., nonleveraged) ETFs are taxed. For ETFs that are set up under the Investment Company Act of 1940 (i.e., virtually all *equity* ETFs), their taxation is similar in some aspects to that of mutual funds. In particular, if ETFs and mutual funds satisfy certain requirements set out in the act, such as being diversified and distributing all the dividends and net capital gains that they generate to their investors, the income will not be taxed at the fund level.[3] Instead, it will be taxed at the investor level.

ETFs and mutual funds generate dividends by holding stocks in their portfolios and generate net capital gains by buying and subsequently selling stocks to rebalance their portfolios. In addition, because investors buy and sell mutual funds directly with the fund companies by using cash, mutual funds (and not ETFs) can have net capital gains from selling their holdings to raise cash to meet investors' redemptions. However, most ETF investors buy and sell their shares with one another on stock exchanges, so ETF companies do not have to constantly sell their holdings for redemption purposes. This is one advantage that ETFs have over mutual funds.

As mentioned, LETFs allow selected traders (authorized participants) to buy and redeem shares directly with the fund companies. This is also true with traditional ETFs. Traditional ETFs use an "in-kind" redemption process, in which authorized participants deliver the ETF shares to the fund companies in exchange for baskets of securities of equal value comprising the underlying indices. This creates another advantage for ETFs, in addition to not having to sell their holding to raise cash to meet the redemption requests and incurring capital gains in the process. This advantage is that the fund companies can choose to give the authorized participants securities that have low cost bases. In other words, the fund companies can get rid of securities that would generate large capital gains if they were sold. As a result, the funds are left with securities with high cost bases, which will incur little or no capital gain if they have to be sold.

In summary, any income earned by traditional ETFs is not taxed at the fund level. Rather, it is taxed at the investor level when ETFs distribute dividends and net capital gains to them. However, with respect to capital gains, ETFs are tax efficient for two reasons. First, their shares are traded in the secondary market, so investors do not have to redeem them with the fund companies. This reduces the need to constantly sell securities to raise cash to meet redemption requests, which is what mutual funds have to do. Second, for redemption requests from authorized participants, ETFs use an in-kind process, so they can keep getting rid of securities with low cost bases. This ensures that they will not generate much capital gain when they have to sell those securities. For these two reasons, it is rare for an index ETF to distribute net capital gains to investors.

Taxation of LETFs

Now, what about the taxation of LETFs? Similar to traditional ETFs, income generated by LETFs is not taxed at the fund level. However, LETFs are not as tax efficient as their traditional counterparts because LETFs generate their leveraged returns by using derivatives such as swaps rather than actually borrowing money and investing it in the underlying indices. As a result, their portfolio holding at any point in time consists of the swaps and the investors' money, which they typically invest in a combination of near-cash instruments (e.g., money-market funds, Treasury bills) and the underlying securities. It then follows that LETFs cannot have an in-kind redemption process because only part of their total exposure is in the underlying securities. Rather, LETFs use an "in-cash" process in which authorized participants who redeem the shares will receive cash equal to the value of the redeemed shares (see the next chapter for more discussion on the creation/redemption process).

The in-cash process forces LETFs to come up with cash every time there is a net redemption from authorized participants. The cash may come from their near-cash investments or from selling their holdings of the underlying securities. Accordingly, we can expect LETFs to have more selling activities than traditional ETFs do and thus more possibility of generating capital gains that will be passed on to investors. We should also note two other things. First, because LETFs do not use an in-kind process, they cannot get rid of securities with low cost bases. So, when they have to sell them, the capital gains can be substantial. Second, because many LETFs hold near-cash investments, which generate interest income, the income will be passed to investors.

An extreme example of large capital gains occurred in 2008. In December of that year, one LETF (Rydex Inverse 2X S&P Select Sector Energy [REC]) passed on capital gains to investors in an amount equal to 86% of its NAV! While there is no way to know for sure why such large gains occurred, it has been speculated that they were due to two contributing factors.[4] First, during the months leading up to the financial crisis of 2008, energy prices weakened, and so REC, being a bear (i.e., –2x) LETF, kept gaining from the derivatives that it used to generate its exposure. Second, in September of that year, an authorized participant redeemed a large quantity of shares (close to 50% of all shares outstanding), which forced REC to close out approximately half of its derivatives positions, locking in capital gains from them. Since the shareholder base became much smaller, the capital gains that REC subsequently distributed were spread among fewer shares, thus causing an unusually large distribution per share. We believe that this example is extreme and is unlikely to occur in funds with a larger size and in a normal economy.

Taxation of LETF investors

From the point of view of LETF investors, there are two layers of taxes. First, investors have to pay taxes on the distributions from LETFs. As stated, the distributions can include dividends, interest, and capital gains. Generally, 60% of capital gains from LETFs' derivatives transactions are considered to be long-term capital gains, while the remaining 40% are considered short-term capital gains, regardless of the contract's holding periods. This rule also applies to commodity and currency LETFs. Second, when investors sell their LETF shares in the market, they can have capital gains, which again can be long term or short term depending on how long they held the LETF shares in questions.

3
Mechanics

In this chapter, we will talk about the mechanics of LETFs. We will discuss how they are structured, how they are able to deliver their promised returns, and how much they charge in fees and expenses. We will also look at the trading statistics of selected LETFs.

The structure of LETFs

The idea behind LETFs is straightforward. Fund companies issue securities that promise to generate returns (before expenses) that are in multiples or negative multiples of the returns on some benchmarks over some specified period. For the majority of LETFs in the market, the length of the specified period is one day. That is, the funds attempt to match (the multiples of) the benchmark returns on a daily basis, and so traders can buy or sell these LETFs on any day and get the promised ratios. However, there are now also LETFs that aim to match the returns over a *calendar* month. For these LETFs, the matching period starts on a prespecified day (typically the first business day) of each month. As a result, if the investors buy them on any other day, they will not get the intended leveraged returns. For the discussion in this chapter, we will concentrate on LETFs with a daily matching period. As monthly LETFs follow the same logic, our discussion also applies to them.

To be able to deliver LETFs' promised returns, funds companies use leverage. Leverage can be acquired by borrowing. For example, when a fund company sells a +2x LETF to an investor, it will invest the investor's money in the underlying benchmark. In addition, it will borrow the same amount of money and invest it in the benchmark, so that the total investment (i.e., exposure to the benchmark) is now 200% of the investor's investment. The fund will maintain this exposure *ratio* through time, which means that the dollar amount that it borrows will keep changing depending on how the underlying benchmark moves. To illustrate this point, suppose that today an investor purchases $100 worth of the above +2x LETF. Suppose also that this LETF has a matching period of one day. The fund company will borrow another $100 and invest the total of $200 in the underlying benchmark. Then, if by the end of today, the underlying benchmark goes up by 1%, the total investment will now be worth $202. Out of this total, $102 belongs to the investor (i.e., a return of 2% for the day, which is twice the benchmark's return). In order to maintain the exposure ratio, the fund now needs to have a total of $204 invested in the benchmark, and so it needs to borrow another $2 (and therefore the principal of the loan at the start of tomorrow will be $102). The funds will have to keep doing this exposure adjustment daily in response to the movements of the underlying benchmark. This end-of-day exposure adjustment process is sometimes referred to as "end-of-day rebalancing." While the process is necessary to maintain the leverage ratio from day to day, it will have an effect on the fund's *compounded* return over any holding period longer than one day. The compounded return will typically deviate from the promised ratio. We will discuss this "compounding effect" in detail in the next chapter.

Note that in this example, we assume for ease of exposition that the fund charges no expenses to cover the interest charges on the loan. If it does, then the investor will have slightly less than $102 at the end of today, and the amount of total exposure that the fund needs for tomorrow will have to be adjusted accordingly. We will shortly talk about the fees and expenses that LETFs charge.

While the fund in the above example obtains leverage by borrowing, it is more typical for LETFs to use derivatives such as forward contracts or total return swaps.[1] The counterparties to these contracts promise to give the fund companies returns based on an agreed benchmark, in exchange for premiums and expenses. The use of derivatives to generate returns (i.e., synthetic replication) is usually more precise and cost effective than borrowing money and buying the benchmark's constituent securities (i.e., physical replication). This is because physical replication can involve high transaction costs, and so fund companies may choose not to buy every security in the benchmark. This technique is referred to as "sampling," and results in fund companies buying only the subset of securities that is the optimal representative of the benchmark. In addition, some securities in the benchmark may be illiquid, and it is not easy to buy them in large quantities. This is especially true when the benchmark is a medium- or small-cap index. One downside of using derivatives is counterparty risk. Fund companies have to rely on their derivatives' counterparties to make good on their promises. This risk can be partly mitigated by using collateral and/or by using multiple counterparties.[2]

In practice, some LETFs use a completely synthetic replication technique, which means that all of their exposure is generated by using derivatives (and they invest the investor's money in certain assets such as fixed-income or money-market instruments). Other LETFs use a hybrid approach, in which they invest the investor's money in the benchmark portfolio and synthetically generate the rest of the exposure.

While it is mathematically possible for daily returns on some leveraged ETFs (such as a –2x ETF) to be worse than –100% (e.g., if the underlying benchmark increases by more than 50% in a day), investors will only lose up to the amount of their investment (i.e., limited liability). For underlying benchmarks that are volatile, fund companies typically buy put options to hedge against such an extreme return. For less volatile benchmarks, put options are generally not used, as the daily adjustments to the funds' exposure to reflect daily gains/losses already have an effect of minimizing the chance of large losses (in dollar term).

Fund companies issue units (or shares) of LETFs and sell them to the public. Typically, this is done with the help of market makers. When an LETF is launched, one or more market makers will put up money in order to create the first batch of shares. The process is referred to as "seeding," and the money that is put up is called "seed capital." The seed capital will be the initial investment in the LETF. Once the first batch of shares comes into existence, the market makers can then resell them to the public, after which trading in the secondary market can begin. In exchange for providing seed capital, the market makers will gain certain privileges such as being designated the lead market maker for that LETF (and so will earn commissions from trading with the public). This designation also allows it to earn cash incentives and rebates for maintaining quotes and controlling the trade flows of the LETF.[3]

LETFs have two major categories of expenses. First, there are management fees, which cover various costs such as investment advisory fees, administration expenses, custodial expenses, legal expenses, index licenses, and distribution fees. These fees vary from one fund company to another, and also from one fund to another. Second, the investors are charged with the transaction costs of leverage, investing and hedging. These costs include brokerage fees and interest charges (if physical replication is used) and/or derivatives premiums (which reflect interest expenses and hedging costs that counterparties to the derivatives contracts incur). As a result, they vary according to the level of interest rates and the difficulty in implementing the hedges (which, in turn, depends on the underlying benchmarks and the side of the LETFs [bull vs. bear]). Currently (June 2015), for most LETFs in the US market, the total fees are less than 1% p.a. of the funds' net asset values.[4] For more discussion of the different types of fees, please see chapter 6.

The creation/redemption process

One of the features of all ETFs (including LETFs) is a creation/redemption provision. In fact, it is one of ETFs' selling points. The way it works is that it allows select traders to buy and sell large lots (or prescribed numbers of shares) of funds directly with the fund companies (i.e., ETF issuers) at prices equal to the funds' net asset values (NAVs). That is, these select traders do not have to trade ETF shares in the market at the market prices (but they can still do so if they choose). These traders are known as "authorized participants" (APs) and are typically designated brokers and market makers. To use this feature, APs have to trade in large quantities, usually in lots of 50,000–100,000 shares, depending on the rules of the funds.

For virtually all *traditional* (i.e., nonleveraged) ETFs in the North American market, the purchase and sale under this provision are done "in-kind." For example, to buy shares of an ETF from a fund company, an AP does not pay cash. Rather, it has to deliver a basket of securities. The content of the basket is specified by the fund company, and generally comprises the underlying benchmark of that ETF (but that does not have to be the case). In exchange, the fund company issues new shares of the ETF and gives them to the AP. The price at which this exchange occurs is the NAV per unit of the fund. For example, suppose the NAV per unit is $20 and the AP wants to buy 50,000 shares. The AP will have to deliver the specified basket of securities worth $1 million to the fund company. A sell transaction works in the opposite way. In this case, the AP delivers shares of the ETF to the fund company and receives a basket of securities.[5]

The creation/redemption process for LETFs is slightly different. Instead of doing an in-kind exchange, most LETFs use an "in-cash" procedure, in which transactions are done through cash rather than a basket of securities. Some LETFs use a combination of an in-cash and an in-kind process. The reason for this is that LETFs mainly use derivatives to generate their exposures, and so they do not hold the underlying baskets of securities that they can exchange with the APs.

One major benefit of the creation/redemption provision is that it helps keep market prices of a fund close to its NAV. To see this, suppose the market price of an LETF is below its NAV. An AP can buy shares of the LETF in the market, redeem them immediately with the fund company for the cash, and capture the price difference. This transaction is indeed an arbitrage transaction, and any AP who spots the opportunity will want to exploit it. This action from many APs at the same time will move the LETF's price closer to its NAV and eventually eliminate any exploitable difference.[6]

We note that arbitraging between price and NAV may not be immediate and can involve a certain degree of risk. This is because fund companies may require creation/redemption orders to be submitted prior to the end of a trading day in order for the orders to be executed at that day's NAV (which is determined at the end of a trading day). In such a case, the APs will have to hedge the price risk by using, for example, futures contracts on the underlying index. Still, in well-developed markets such as the North American markets, arbitrage should be effective, and thus the differences between funds' market prices and their NAVs should be small. In other words, the pricing of LETFs should be relatively efficient. We will see in chapter 5 whether this is indeed the case in practice.

Trading of LETFs

As mentioned previously, trading in the secondary market starts once the first batch of shares of an LETF is created by market makers. Investors can trade LETF shares in the same way as they trade stocks. They can buy them outright, buy them on margin, or short sell them. The rule for buying LETFs on margin is different from the rule for regular stocks. This is because LETFs themselves already provide traders with leverage, and the regulators impose a limit on how much further traders can use margin to scale the leverage up. According to the requirements by the Financial Industry Regulatory Authority (FINRA), for the US market, the margin amounts for LETFs depend on the degree of leverage and also whether the LETFs are bull (i.e., those with positive multiples) or bear (i.e., those with negative multiples) funds. For bull funds, the margin is 25% multiplied by the leverage multiples. As such, a +2x LETF requires a margin of 50% (25% x 2), while a +3x LETF requires a margin of 75% (25% x 3). For bear funds, the margin is 30% multiplied by the leverage multiples. Therefore, the margin is 60% for a –2x LETF and 90% for a –3x LETF. These margin levels have to be continuously maintained. The margin requirements for bear funds also apply to short selling of LETFs.

During trading hours, investors who want to buy or sell LETFs may first check the funds' intraday indicative net asset values (iNAVs). As the name suggests, iNAVs are *estimates* of the current value of the funds, as opposed to official NAVs, which are calculated only once a day (typically at the end of the day). They are calculated by a third party based on the most recent trade prices of all the securities in the underlying benchmarks and updated every 15 seconds. In the US market, they are disseminated by the National Securities Clearing Corporation (NSCC) and available in major data feeds (e.g., from Bloomberg or Thomson Reuters) under the LETFs' core ticker symbols followed by the suffix ".IV".

Although iNAVs are a good reference point for investors, it is important to note their limitations. First, because they are disseminated only every 15 seconds, there is a lag between the time they are calculated and the time they are shown to the public. This can be a serious problem in a very volatile market in which prices change very quickly. Second, iNAVs are based on the *trade* prices of the benchmarks' constituent stocks. Some of those stocks (especially illiquid ones) may not trade very frequently, and so their trade prices may come from much earlier points in time. To get around these limitations, market makers and authorized participants usually calculate their own in-house "fair values" of the funds. Their calculations are typically based on the current bid-ask quotes of the constituent stocks, rather than their trade prices. They then use the fair values as the basis for trading with the public, who have no access to this information. Nevertheless, iNAVs are still better estimates of the funds' current values than the funds' official NAVs.

The amount of trading in LETFs is disproportionately large compared to traditional ETFs. While LETFs represents only a small fraction of the whole US ETF market in terms of assets (around 2%), they accounted for over 10% of all ETF trading volume.[7] There are two main reasons for the disproportionate trading activity. First, because of the embedded leverage, LETFs are risky and speculative in nature. Therefore, they attract short-term traders who want to express their views on the underlying benchmarks or to implement short-term hedging strategies. Second, and more importantly, leveraged ETFs are, by design, not intended for long-term investors with a buy-and-hold strategy. Their goal is to provide the specified leveraged returns on a *daily* basis. As we discuss in the next chapter, returns for longer holding periods will generally be different from the promised ratios. Most investors will avoid these deviations by limiting the length of their holding periods. This leads to more trades in the LETFs.

In table 3.1, we present some trading statistics of selected LETFs that are based on three well-known indices: the S&P 500, the Nasdaq 100, and the Russell 2000. For comparison, we also include the nonleveraged ETFs (i.e., +1x and –1x). The first thing to note from the table is that the size of these LETFs was much smaller than the +1x ETF on the same index. With respect to their trading activity, the average number of trades per day and the average dollar values per trade were also smaller for LETFs. However, the magnitude of the differences in the number of trades is less than proportional to the size of the funds. For example, the ProShares Ultra S&P 500 (SSO) had an asset of $1.65 billion, which was about 2% of the size of the SPDR S&P 500 (SPY). However, its number of trades per day was more than 10% of the number for SPY. This indicates that trading in LETFs was very active.

In the last column of table 3.1, we report the average holding periods for these LETFs. The holding periods were generally very short. All of the 2x and 3x LETFs in the table had average holding periods of fewer than six days, with the shortest being only marginally longer than one day. The short holding periods

Table 3.1 Trading Statistics of Selected LETFs

Name	TIC	Multiple	Listing Date	AUM as of Dec 2011 (millions)	Avg. No. of Trades per Day (2011)	Avg. Dollar Value per Trade (2011)	Avg. Holding Period (days)
Panel A: Underlying Index = S&P 500 Index							
SPDR S&P 500	SPY	+1x	29 Jan 93	$87,195	422,656	$64,675	5.19
ProShares Short S&P 500	SH	–1x	21 Jun 06	$2,367	9,744	$20,880	19.76
ProShares Ultra S&P 500	SSO	+2x	21 Jun 06	$1,653	48,165	$16,046	3.27
ProShares UltraShort S&P 500	SDS	–2x	13 Jul 06	$2,014	41,545	$17,547	5.60
ProShares UltraPro S&P 500	UPRO	+3x	25 Jun 09	$343	8,323	$21,252	2.88
ProShares UltraPro Short S&P 500	SPXU	–3x	25 Jun 09	$597	17,251	$9,832	4.08
Panel B: Underlying Index = Nasdaq 100							
PowerShares QQQ (See Note)	QQQ	+1x	10 Mar 99	$25,570	89,993	$41,393	9.87
ProShares Short QQQ	PSQ	–1x	21 Jun 06	$293	1,128	$28,130	13.51
ProShares Ultra QQQ	QLD	+2x	21 Jun 06	$678	19,127	$19,446	3.22
ProShares UltraShort QQQ	QID	–2x	13 Jul 06	$602	22,868	$14,599	3.57
ProShares UltraPro QQQ	TQQQ	+3x	11 Feb 10	$184	2,290	$27,453	4.29
ProShares UltraPro Short QQQ	SQQQ	–3x	11 Feb 10	$95	2,997	$14,056	3.66
Panel C: Underlying Index = Russell 2000							
iShares Russell 2000	IWM	+1x	22 May 00	$14,241	144,772	$35,724	4.56
ProShares Short Russell 2000	RWM	–1x	25 Jan 07	$476	4,007	$13,160	14.05
ProShares Ultra Russell 2000	UWM	+2x	25 Jan 07	$204	8,635	$10,849	3.97
ProShares UltraShort Russell 2000	TWM	–2x	25 Jan 07	$299	18,256	$11,565	3.79
Direxion Daily Small Cap Bull	TNA	+3x	5 Nov 08	$1,025	56,094	$16,622	1.31
Direxion Daily Small Cap Bear	TZA	–3x	5 Nov 08	$699	65,796	$11,913	1.50

Note: This table is reproduced from Charupat and Miu (2013).

were consistent with our arguments above that LETFs attract mainly short-term traders and are not intended for buy-and-hold investors.

It should be noted, however, that the holding periods of the +1x ETFs in our sample were also very short, ranging from 4.56 days (IWM) to 9.87 days (QQQ), which were much shorter than should be expected for a product that appeals to long-term investors. While there is no doubt that these ETFs were held by a lot of buy-and-hold investors, we believe that the short holding periods were caused by arbitrageurs and/or active traders who used the ETFs to speculate on macroeconomic or market/sector-wide trends. The fact that the three +1x ETFs in the table were all based on broad indices and had very liquid markets, makes them appealing to speculators.

Conclusions

There are two ways in which LETF issuers can generate the promised returns. They can borrow money and invest it (together with the investors' money) in the underlying benchmarks. Or they can use derivatives such as swaps. The latter approach is more common, more precise, and more cost effective. Regardless of the approach used, every LETF has to adjust its exposure to the underlying benchmark every day. This is so that it can maintain a constant leverage ratio.

As with any ETF, LETFs are sold to the public through a seeding process in which some market makers put up some capital to create the first batch of shares. In return, these market makers become the lead market makers for the LETFs and earn profits from trading with the public.

All LETFs have a creation/redemption mechanism in which select investors can buy or redeem shares of the funds directly with the issuers. The process is done in cash, rather than in kind, which is what traditional ETFs use. The creation/redemption mechanism helps keep the market prices of LETFs close to their NAVs.

Although LETFs represent only a small fraction of the whole ETF market in terms of assets, their trading activity is disproportionately large. This is because the leverage embedded in LETFs attracts short-term traders who want to speculate or implement short-term hedges. Also, by design, LETFs are not intended for buy-and-hold investors because the daily exposure adjustments will cause the compounded returns over any holding period longer than one day, to deviate from the promised multiple. We discuss the effect of compounding in more details in chapters 4 and 6.

4

Return Dynamics and Compounding Effects

In this chapter, we will take a look at the dynamics of LETFs' returns over different time horizons. In the process, we will discuss in detail how compounding affects LETFs' returns.

As mentioned earlier, the issuers of LETFs promise to generate *daily* returns that are in multiples of the daily returns on the underlying benchmarks (before expenses).[1] For now, we will assume that the issuers are able to do what they promise. That is, except for funds' expenses, there is no "tracking error" on a *daily* basis, and the daily changes in the funds' net asset values (NAVs) correspond proportionately to the changes in the underlying indices. We also ignore any financing costs incurred in delivering the targeted leverage exposure.[2] Formally, consider an LETF with a promised leverage ratio of β. Let I_t be the level of its benchmark index on day t, $i_{t-1,t}$ be the rate of return on the benchmark between day $t-1$ and day t, and k_t be the fund's rate of expenses (prorated to daily). The rate of change in the fund's NAV is then

$$r_{t-1,t} = \beta\left(\frac{I_t - I_{t-1}}{I_{t-1}}\right) - k_t = \beta\, i_{t-1,t} - k_t. \tag{4.1}$$

We will refer to $r_{t-1,t}$ as the rate of return (based on NAV) on the LETF (or "NAV return" for short). To be precise, this rate can be different from the rate of return that an investor will actually get from investing in this LETF. This is because the latter rate will depend on the prices at which the investor buys and subsequently sells the LETF. We will refer to this rate of return as "price return." On any given day, NAV return can be different from price return if the market price of the fund deviates from its NAV. This is the issue of pricing efficiency that we will discuss in the next chapter. But as you will see in the next chapter, deviations of market prices form NAVs are, on average, very small for LETFs on major underlying indices. So, it is not unreasonable to assume that NAV return and price return are approximately the same.

From equation (4.1), the return on an LETF is equal to the return on its underlying index, multiplied by the leveraged ratio, and then minus the rate of daily expenses. It is straightforward to show that the volatility (i.e., standard deviation) of an LETF's return is also equal to the volatility of the index return, multiplied by the leveraged ratio. For example, a +3x LETF will have daily returns that are three times as volatile as that of its underlying index. In other words, the use of leverage magnifies both the return and the risk of the fund.

Compounding effects

Let us now look at the return of an LETF, compounded over a longer period of time. Based on equation (4.1), the compounded return from holding an LETF over N days is

$$r_{t,t+N} = \prod_{j=1}^{N} \left(1+\beta\ i_{t+j-1,t+j} - k_{t+j}\right) - 1 \tag{4.2}$$

To get some intuition on equation (4.2), consider a two-day holding period. We assume for simplicity that the fund's expense, k_t, is zero. The two-day compounded return of an LETF is then

$$\begin{aligned} r_{t,t+2} &= \left(1+\beta\cdot i_{t,t+1}\right)\cdot\left(1+\beta\cdot i_{t+1,t+2}\right)-1 = \beta\cdot i_{t,t+1}+\beta\cdot i_{t+1,t+2} \\ &\quad +\beta^2\cdot i_{t,t+1}\cdot i_{t+1,t+2}. \end{aligned}$$

For example, the two-day compounded return of a +3x LETF is

$$r_{t,t+2} = 3\cdot i_{t,t+1} + 3\cdot i_{t+1,t+2} + 9\cdot i_{t,t+1}\cdot i_{t+1,t+2}.$$

The two-day compounded return depends on the leverage ratio, the underlying index's return on each day, and the cross-product term of the two daily returns. If the two returns are of the opposite signs (i.e., the index goes up one day and down the other day), the cross-product term will be negative, and this will reduce the compounded return.

Now, for comparison, the two-day compounded return on the underlying index is

$$i_{t,t+2} = \left(1 + i_{t,t+1}\right) \cdot \left(1 + i_{t+1,t+2}\right) - 1 = i_{t,t+1} + i_{t+1,t+2} + i_{t,t+1} \cdot i_{t+1,t+2}.$$

With this index's return, an investor who invests in a +3x LETF may mistakenly expect that the two-day return that he is going to get is three times of it, that is,

$$3 \cdot i_{t,t+1} + 3 \cdot i_{t+1,t+2} + 3 \cdot i_{t,t+1} \cdot i_{t+1,t+2},$$

which is what he would get if he used margins (i.e., with no daily rebalancing) to create leverage. The difference between the actual compounded return and the mistaken (or "naive") expected return is in the cross-product term, and is equal to $6 \cdot i_{t,t+1} \cdot i_{t+1,t+2}$. It can be positive or negative depending on the signs of the two returns. It occurs because the fund maintains a constant leverage ratio ($\beta = 3$) from day to day, and so the actual return has a cross-product term of $9 \cdot i_{t,t+1} \cdot i_{t+1,t+2}$.

Let us further explore the difference between an LETF's actual compounded return and the naive expected return. We present in table 4.1 a simple two-day example of a +2x and a +3x LETF under five stylized return paths. Consider first the +2x LETF in Panel A. Under return path #1, the underlying index increases by 10% on day 1, and declines by 5% the next day. Over two days, the index's return is (1.1 x 0.95) – 1 = 4.50%. Some holders of the +2x LETF may mistakenly expect to get twice the index return, or 9%. However, the actual returns on the +2x LETF are a positive 20% for day 1 and a negative 10% for day 2, resulting in a two-day return of (1.2 x 0.9) – 1 = 8%, which is less than twice the index's return. The difference comes from the fact that after

Table 4.1 LETF's Two-Day Compounded Returns under Different Scenarios

This table displays the two-day compounded return of a +2x and a +3x LETFs under different return paths of the underlying index.

Panel A: +2x LETF

Path	**Underlying index return**			**LETF return**			**Difference from stated multiple**
	Day 1	**Day 2**	**2-day compounded**	**Day 1**	**Day 2**	**2-day compounded**	
1	+10%	–5%	+4.50%	+20%	–10%	+8.00%	–1.00%
2	+2.2252%	+2.2252%	+4.50%	+4.4504%	+4.4504%	+9.10%	+0.10%
3	–10%	+5%	–5.50%	–20%	+10%	–12.00%	–1.00%
4	–2.7889%	–2.7889%	–5.50%	–5.5778%	–5.5778%	–10.84%	+0.16%
5	+7.5	–7.5%	–0.56%	+15%	–15%	–2.25%	–1.13%

Panel B: +3x LETF

Path	**Underlying index return**			**LETF Return**			**Difference from stated multiple**
	Day 1	**Day 2**	**2-day compounded**	**Day 1**	**Day 2**	**2-day compounded**	
1	+10%	–5%	+4.50%	+30%	–15%	+10.50%	–3.00%
2	+2.2252%	+2.2252%	+4.50%	+6.6757%	+6.6757%	+13.80%	+0.30%
3	–10%	+5%	–5.50%	–30%	+15%	–19.50%	–3.00%
4	–2.7889%	–2.7889%	–5.50%	–8.3667%	–8.3667%	–16.03%	+0.47%
5	+7.5	–7.5%	–0.56%	+22.5%	–22.5%	–5.06%	–3.38%

the positive return on the first day, the LETF has to increase its (dollar amount) exposure to the underlying index to maintain its +2x leverage ratio. The higher exposure increases the amount of loss when the index drops on the second day.

The less volatile the underlying index's daily returns *over the return path*, the higher the compounded returns on the LETFs will be. This is the case illustrated by return path #2, where the index return on each of the two days was 2.2252% (i.e., zero volatility, but with the same two-day compounded return of the index as in path #1). Here, although the index's compounded return over two days is the same as under path #1 (i.e., 4.50%), the +2x LETF's compounded return is 9.10%, which exceeds twice the index's return.

The above conclusions are equally applicable for cases where the index declines over the two days. Consider return paths #3 and #4. Under both paths, the index's two-day return is –5.50%. However, under path #3 (where there is volatility), the +2x LETF's compounded return is –12%, which is less than (i.e., worse than) twice the benchmark return. Yet, under path #4 (where there is zero volatility), the +2x LETF's compounded return is –10.84%, which is higher than twice the index return.

The effect of volatility is more pronounced in a *sideways* market, where the underlying index moves up and down, but remains around the same level. Consider path #5, which has the same return volatility as paths #1 and #3. However, unlike path #1 (an *up-trending* market) and path #3 (a *down-trending* market), path #5 represents a sideways market with hardly any return on the index over the two-day period. The +2x LETF realizes the most negative deviation from the stated multiple under path #5. Its compounded return is 1.13% lower than twice the return on the index. Given similar realized volatility, a sideways market will result in a more negative deviation from the stated multiple return than either an up-trending or down-trending market.

Next, consider the +3x LETF in Panel B. The effect of volatility and sideways versus trending market on the compounded returns is similar to the case of the +2x LETF. However, it can be seen that the fund's higher leverage magnifies the differences between its compounded returns and the stated multiple returns.

Distributions of holding-period returns

Now that we know about the effect of compounding, we want to have an idea of the possible outcomes that an investor will receive if he/she invests in an LETF for a period of time. That is, we want to see a distribution of all the possible outcomes that he/she will get. We will start with a mathematical representation of the possible outcomes. Then we will perform Monte Carlo simulations to generate those outcomes so that we can draw some inferences from them.

To do so, we first rewrite equation (4.2) as

$$\begin{aligned} 1+r_{t,t+N} &= \prod_{j=1}^{N}\left(1+\beta\, i_{t+j-1,t+j}-k\right) \\ &= \exp\left[\sum_{j=1}^{N}\ln\left(1+\beta\, i_{t+j-1,t+j}-k\right)\right], \end{aligned} \tag{4.3}$$

where, for simplicity, we assume that the fund's daily rate of expenses is constant. Next, we take a second-order Taylor expansion to approximate the right-hand side of equation (4.3) as[3]

$$1+r_{t,t+N} \approx \left(1+\beta\, \bar{i}-k\right)^{N} \exp\left[-\frac{1}{2}\frac{\beta^2}{\left(1+\beta\, \bar{i}-k\right)^2}Ns^2\right], \tag{4.4}$$

where $\bar{i}$ is the arithmetic average of the underlying index's daily return *during the period*, and $s^2 = \frac{1}{N}\sum_{j=1}^{N}\left(i_{t+j-1,t+j}-\bar{i}\right)^2$, is the sample variance of the daily returns.

As $r_{t,t+N}$ is the compounded return on the LETF over N days, we can think of $1 + r_{t,t+N}$ as the Nth-day value (i.e., the payoff) of a \$1 investment in the LETF. Equation (4.4) states that the payoff is equal to the compounded return based on the average daily return on the index over the period, $\bar{i}$, multiplied by an exponential term. The exponential term will be less than one if there is volatility in the return path (i.e., $s > 0$). The higher the volatility, the lower will be the exponential term. Therefore, given an average index's return, $\bar{i}$, the return on an LETF will be lower under more volatile return paths.[4] The return will be especially poor in a sideways market where the benchmark fluctuates but does not change by much over a given period (i.e., $\bar{i} \approx 0$).

There are, of course, an infinite number of return paths that the benchmark can follow. To have an idea about the distribution of the possible payoffs, assume that the daily returns on the underlying index, $i_{t-1,t}$, are independently and identically distributed with mean of μ and standard deviation of σ. Then, by the central limit theorem, the distribution of $1 + r_{t,t+N}$ will approach lognormal, as N becomes large (i.e., for a long holding period).[5]

The expected value of the payoff is $(1+\beta\mu-k)^N$. However, because a lognormal distribution is positively skewed, this expected payoff exceeds the median payoff, and so it will not be what an investor typically receives. Nevertheless, although there are fewer above-average payoffs, they generally exceed the average by a substantial amount.

In addition, since the skewness of a lognormal distribution increases with volatility, these effects will be even more pronounced when $r_{t,t+N}$ is volatile. This occurs when the underlying index is volatile and/or β is large (in absolute value).

Now, when we invest in an LETF, we do not know what path the underlying index will take, and so we do not know how well our investment will do. We can try to get some idea about it by running a Monte Carlo simulation for the underlying index's return paths, and then seeing how the LETF does in each of those paths. And this is what we will do now. We assume that the index's daily returns, or more precisely, log of daily price ratios, are normally distributed. We also assume that daily returns are independently and identically distributed. Identically means that daily returns are drawn from the same distribution day after day. Independently means that today's return and tomorrow's return are not correlated. These assumptions are standard in the finance literature.

Next, we assume that the underlying index has an expected return of 10% per year and volatility of 20% per year. We want to see the possible payoffs we will get after one year from investing \$100 in a +2x (i.e., bull) LETF, and another \$100 in a –2x (i.e., bear) LETF, and the probabilities of those payoffs. In other words, we want to see the distribution of the possible payoffs. We generate 5,000 trials or scenarios, and then plot the outcomes and their frequencies in Figure 4.1a (for the bull LETF) and Figure 4.1b (for the bear LETF).

We can see that the distributions of the outcomes for both the bull and the bear LETFs are skewed. There are small chances of very high payoffs or very low payoffs, but most outcomes occur in the middle. Because the index's expected return is positive, it is not surprising to see that payoffs of the bull LETF are generally higher than the payoffs of the bear LETF. The mean payoffs of

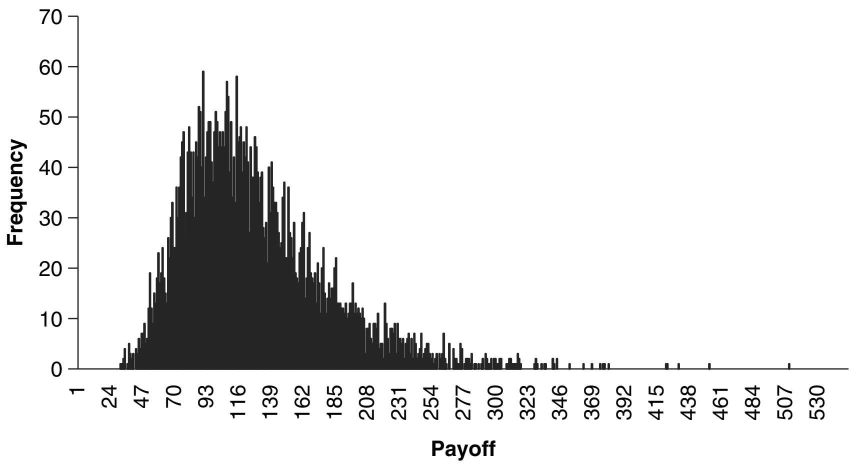

Figure 4.1a Payoffs of a One-Year \$100 Investment in a 2x ETF under Low Volatility.

This figure displays the simulated payoffs of a \$100 investment in a bull (2x) ETF after one year. The underlying benchmark is assumed to follow a geometric Brownian motion with μ = 10% p.a. and σ = 20% p.a. The results are based on a Monte Carlo simulation with 5,000 trials.

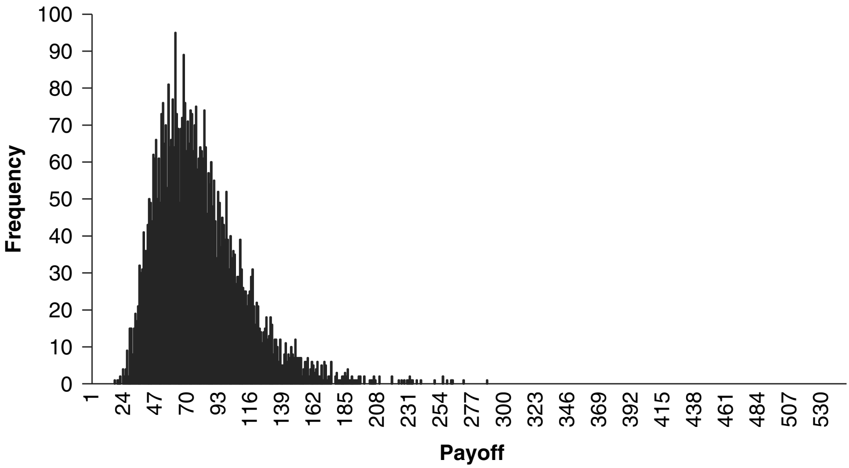

Figure 4.1b Payoffs of a One-Year $100 Investment in a –2x ETF under Low Volatility.

This figure displays the simulated payoffs of a $100 investment in a bear (–2x) ETF after one year. The underlying benchmark is assumed to follow a geometric Brownian motion with $\mu = 10\%$ p.a. and $\sigma = 20\%$ p.a. The results are based on a Monte Carlo simulation with 5,000 trials.

the bull and bear LETFs are $126.98 and $78.58 respectively (see Panel A of table 4.2), while the median payoffs of both LETFs are lower than the means because both distributions are positively skewed (and so there are small chances of very high outcomes). More importantly, the range of possible payoffs of the bull LETF is much wider than that of the bear LETF.

Next, we ran another simulation. But this time we assume that the underlying index is more volatile, with volatility of 40% per year. The results are shown in Figures 4.2a and 4.2b, and Panel B of table 4.2. We can see that the higher volatility causes the payoff distributions to be more dispersed and more skewed. For the bull LETF, the mean payoff is now higher than before ($142.75 vs. $126.98), but the median payoff is lower than before ($103.01 vs. 116.63). This occurs because higher volatility causes the majority of the possible payoffs to be lower, but, at the same time, provides a very small chance of very high payoffs (thus causing the mean payoff in figure 4.2a to be higher than in figure 4.1a).

Table 4.2 Simulation Results

This table displays the results of a Monte Carlo simulation of the payoffs of a 2x ETF and a –2x ETF that are based on an underlying benchmark whose expected return is assumed to be 10% p.a., and whose return volatility is assumed to be either 20% p.a. (low volatility) or 40% p.a. (high volatility). The simulation consists of 5,000 trials.

Panel A: Low Volatility (20% p.a.)

	Benchmark	**Bull (2x) ETF**	**Bear (–2x) ETF**
Mean Payoff	112.71	126.98	78.58
Median Payoff	110.20	116.63	72.98
Minimum Payoff	55.86	30.04	16.72
5th Percentile Payoff	79.51	60.78	37.59
95th Percentile Payoff	153.52	226.59	139.31
Maximum Payoff	229.98	508.20	286.43

Panel B: High Volatility (40% p.a.)

	Benchmark	**Bull (2x) ETF**	**Bear (–2x) ETF**
Mean Payoff	119.58	142.75	69.42
Median Payoff	109.89	103.01	51.16
Minimum Payoff	28.24	6.83	2.65
5th Percentile Payoff	57.21	27.87	13.35
95th Percentile Payoff	213.26	388.22	186.61
Maximum Payoff	478.56	1,957.19	801.75

For the bear LETF, *both* the mean and median payoffs are lower than in the case in which the volatility is 20% per year. Again, higher volatility causes the majority of the possible payoffs to be lower. However, in this case, there are not enough high payoffs to offset those low payoffs, and thus the mean payoff is lower than before.

The simulation results show that over a one-year horizon, the bull LETF performs better (i.e., has return ratios that are closer to its stated ratio) than the bear LETF, regardless of the volatility levels. In addition, higher volatility causes the performance of both the bull and the bear LETFs to deteriorate, with the effect being more pronounced for the bear ETF.

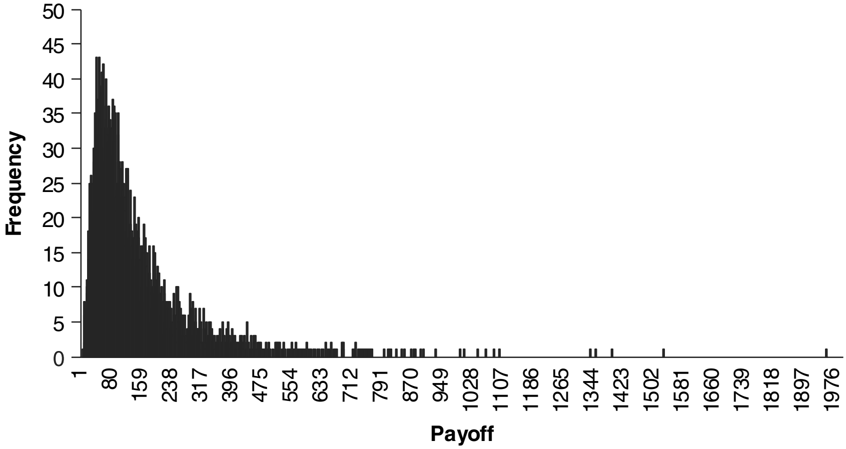

Figure 4.2a Payoffs of a One-Year $100 Investment in a 2x ETF under High Volatility.

This figure displays the simulated payoffs of a $100 investment in a bull (2x) ETF after one year. The underlying benchmark is assumed to follow a geometric Brownian motion with μ = 10% p.a. and σ = 40% p.a. The results are based on a Monte Carlo simulation with 5,000 trials.

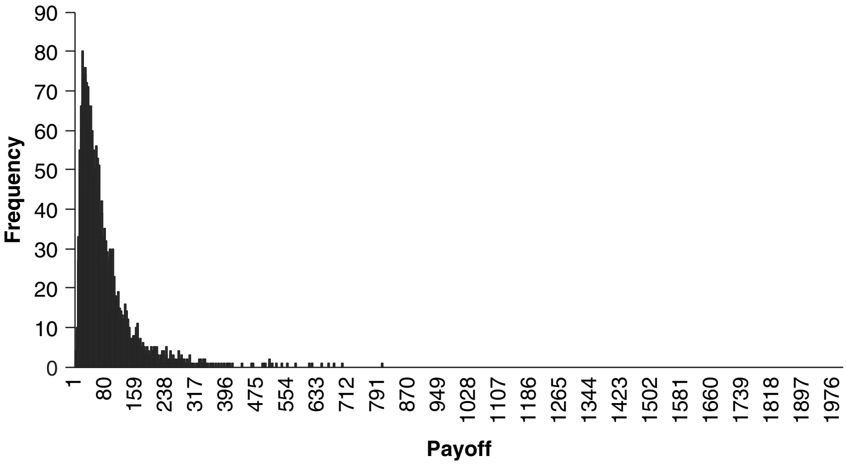

Figure 4.2b Payoffs of a One-Year $100 Investment in a –2x ETF under High Volatility.

This figure displays the simulated payoffs of a $100 investment in a bear (–2x) ETF after one year. The underlying benchmark is assumed to follow a geometric Brownian motion with μ = 10% p.a. and σ = 40% p.a. The results are based on a Monte Carlo simulation with 5,000 trials.

Conclusions

The return on an LETF is negatively affected by the compounding effect when the realized index return is volatile. This phenomenon is sometimes referred to as the *volatility drag* or *volatility decay*. Moreover, the performance will be especially poor in a sideways market as opposed to an up-trending or down-trending market. Everything else being the same, the higher the leverage ratio and the longer the holding period, the stronger is the compounding effect. For a detailed discussion on the effect of compounding on tracking errors and how it may be analytically related to the characteristics of the realized return path of the benchmark index, please refer to chapter 6.

We should add that a recent study by Loviscek et al. (2014) suggests that the compounding effect may not be as serious as our simulation results indicate. In their study, they conduct similar simulations to ours. However, they use real-world, historical data as the basis for their assumptions for the benchmark returns. They show that real-world returns have a distribution that is more leptokurtic (i.e., has a higher peak) than normal distribution. In other words, most of the real-world benchmark returns are concentrated around the mean. In addition, real-world daily returns are positively correlated. That is, if the market goes up today, it is more likely to go up again tomorrow. Both of these two facts help reduce the negative effects of compounding.[6] Their simulation results confirm that the compounding effect does not necessarily exert a significantly negative impact on LETFs' performance even over holding periods of longer than one year. Their findings offer a new perspective on the compounding effects, which, nevertheless, should be interpreted with caution. The dynamics of returns in the market can and do change over time, and so historical returns may not be representative of what will happen in the future.

5
Pricing Efficiency

As we discussed previously, LETFs are traded throughout the day on stock exchanges. You can buy or sell fund units by submitting your order, and the price that you will get is the market price prevailing at the time of your order. Now, market prices are determined by demand and supply, and so it is possible that the market price of a fund at any one time will be different from the fund's net asset value (NAV). If that happens, you will pay more (or less) than what the fund is actually worth. This is the issue of pricing efficiency, where we define the term as the fund's ability to keep its market price close to its NAV so that its price at any time is fair and reflects the fund's underlying value. This is what we will explore in this chapter.

Test of LETFs' pricing efficiency

Let us begin by formally defining price deviations. Let P_t^j and NAV_t^j be the daily closing price and the daily NAV of fund *j* respectively. The price deviation (in percentage term) of this fund on day *t* is

$$\pi_t^j = \frac{P_t^j - NAV_t^j}{NAV_t^j} \times 100. \tag{5.1}$$

If π_t^j is positive, we say that fund *j* trades at a premium. Conversely, if π_t^j is negative, we say that fund *j* trades at a discount.

As discussed in chapter 3, all LETFs have a creation/redemption mechanism that allows authorized participants (typically large institutional traders such as market makers and specialists) to create new shares or redeem existing shares of LETFs. This mechanism helps keep their market prices in line with their NAVs. If prices deviate too far from the NAVs, there will be arbitrage opportunities that the authorized participants can take advantage of. However, these arbitrage opportunities are not always without risk. The risk can come from the difference in timing between the time when creation/redemption orders have to be submitted and the time when the funds' NAVs (which are the transaction prices for creation/redemption orders) are determined. This risk is especially of concern for funds with volatile underlying indices. As a result, it is possible that large premiums/ discounts will from time to time occur.

We will now discuss the results of our test of LETFs' pricing efficiency. We apply equation (5.1) to select LETFs that are traded in the US market. Specifically, we look at three sets of funds. Each set has three pairs of funds that are all based on the same underlying index. The first pair consists of a bull and a bear nonleveraged ETF (i.e., +1x and –1x). The second pair is a +2x and a –2x LETF, while the third pair is a +3x and a –3x LETF. The nonleveraged pair in each set acts as a comparison. The three underlying indices are the S&P 500, Nasdaq 100, and Russell 2000, which are the most commonly followed diversified stock indices and have well-established traditional (+1x) ETFs with large amounts of AUM. More importantly, the three indices have a full set of LETFs (i.e., +2x, –2x, +3x, and –3x) and an inverse (i.e., –1x) on them. This allows us to compare the results to see whether the degrees of leverage affect the behavior of the premiums/discounts.

For each set of funds, the data that we use cover the period that is common to all the funds in the set up to the end of 2011. In other words, the start of the sample period of each set of funds is determined by the ETF or LETF that was most recently introduced in that set. When there are competing ETFs and/or LETFs tracking the same index, we pick the one with the highest AUM (as of the end of 2011). For example, there are three +1x ETFs on the S&P 500 Index (i.e., SPDR, iShares, and Vanguard). We choose SPDR to be in the set because it is the largest of the three. The names and description of the funds (18 in total) are shown in Table 5.1.

We present the results in Panels A–C of Table 5.2. For all ETFs and LETFs in the sample, the premiums/discounts are small. All except two of the average premiums/discounts are less than 0.05% of the funds' NAVs (in absolute term). The two exceptions are TNA (i.e., the +3x LETF on Russell 2000) and UWM (i.e., the +2x LETF on Russell 2000) whose average price deviations are –0.10% and –0.06% respectively. These average premiums/discounts are generally within the bid-ask spreads and transaction costs of conducting an arbitrage trade.[1] Accordingly, we can say that on average, the pricing of LETFs is efficient.

The numbers in Table 5.2 reveal a premiums/discounts pattern that is generally consistent across the three sets of funds. All bull funds (i.e., +1x, +2x, and +3x funds) trade at a discount more often than at a premium. This causes their average price deviations to be negative (i.e., discounts). The higher the bull funds' leverage, the deeper their discounts. This is true for all bull funds, with only one exception (i.e., TQQQ – the +3x LETF on Nasdaq 100). In contrast, six of the nine bear funds trade on average at premiums, while the other three have very small discounts. We perform Wilcoxon tests and find that there are statistically significant differences in the distributions of price deviations between bull and bear funds for all but one pair. The only exception is the TQQQ-SQQQ pair (i.e., the +3x and –3x LETFs on Nasdaq 100).

Another pattern that we can observe is that funds with higher leverage ratios have higher variation in their price deviations. In other words, funds with higher leverage ratios are more prone to having large premiums or discounts than funds with lower leverage ratios. This is evidenced by the fact that for all underlying indices, funds with higher leveraged ratios have more volatile price deviations (i.e., larger standard deviations) and also wider

Table 5.1 Descriptive Statistics

This table provides summary statistics of the funds in the sample. The average holding period is calculated over the 12 months of 2011 in two steps. First, the average number of outstanding shares in each month is divided by the number of fund units traded in that month, and then multiplied by 30 to obtain the holding period (in terms of days) in that month. These holding periods are then averaged to arrive at the figures in the table. The average number of trades per day is calculated by dividing the total number of trades in 2011 by the number of trading days in 2011. The average value per trade is calculated by dividing the total dollar trading value in 2011 by the total number of trades in 2011.

Name	**TIC**	**Multi-ple**	**Listing Date**	**AUM as of Dec 2011 (millions)**	**Avg. No. of Trades per Day (2011)**	**Avg. Dollar Value per Trade (2011)**	**Avg. Holding Period (days)**
Panel A: Underlying Index = S&P 500 Index							
SPDR S&P 500	SPY	+1x	29 Jan 93	$87,195	422,656	$64,675	5.19
ProShares Short S&P 500	SH	–1x	21 Jun 06	$2,367	9,744	$20,880	19.76
ProShares Ultra S&P 500	SSO	+2x	21 Jun 06	$1,653	48,165	$16,046	3.27
ProShares UltraShort S&P 500	SDS	–2x	13 Jul 06	$2,014	41,545	$17,547	5.60
ProShares UltraPro S&P 500	UPRO	+3x	25 Jun 09	$343	8,323	$21,252	2.88
ProShares UltraPro Short S&P 500	SPXU	–3x	25 Jun 09	$597	17,251	$9,832	4.08

Panel B: Underlying Index = Nasdaq 100							
PowerShares QQQ (See Note)	QQQ	+1x	10 Mar 99	$25,570	89,993	$41,393	9.87
ProShares Short QQQ	PSQ	–1x	21 Jun 06	$293	1,128	$28,130	13.51
ProShares Ultra QQQ	QLD	+2x	21 Jun 06	$678	19,127	$19,446	3.22
ProShares UltraShort QQQ	QID	–2x	13 Jul 06	$602	22,868	$14,599	3.57
ProShares UltraPro QQQ	TQQQ	+3x	11 Feb 10	$184	2,290	$27,453	4.29
ProShares UltraPro Short QQQ	SQQQ	–3x	11 Feb 10	$95	2,997	$14,056	3.66
Panel C: Underlying Index = Russell 2000							
iShares Russell 2000	IWM	+1x	22 May 00	$14,241	144,772	$35,724	4.56
ProShares Short Russell 2000	RWM	–1x	25 Jan 07	$476	4,007	$13,160	14.05
ProShares Ultra Russell 2000	UWM	+2x	25 Jan 07	$204	8,635	$10,849	3.97
ProShares UltraShort Russell 2000	TWM	–2x	25 Jan 07	$299	18,256	$11,565	3.79
Direxion Daily Small Cap Bull	TNA	+3x	5 Nov 08	$1,025	56,094	$16,622	1.31
Direxion Daily Small Cap Bear	TZA	–3x	5 Nov 08	$699	65,796	$11,913	1.50

Notes: 1. This table is reproduced from Charupat and Miu (2013).
2. The ticker for PowerShares QQQ Nasdaq 100 ETF was changed from QQQQ to QQQ on March 23, 2011.

Table 5.2 Price Deviations

This table displays summary statistics for deviations between the closing prices of the funds in the sample and their end-of-day NAVs. The data period for each underlying index is the common period for all six funds (i.e., from the inception date of the most recently introduced fund in each set to December 31, 2011). The annualized volatility of NAV returns is calculated by annualizing the standard deviations of the funds' daily NAV returns during the data periods. * denotes significance at the 5% level, while ** denotes significance at the 1% level. The positive and negative percentages do not add up to 100% because there are days when no price deviation occurred.

Fund	**Multiple**	**Price Deviations**							**Annualized volatility of NAV returns (%)**
		N	**Average (%)**	**5th Percentile (%)**	**95th Percentile (%)**	**Std. dev. (%)**	**% Positive (Premium)**	**% Negative (Discount)**	
Panel A: Underlying Index = S&P 500									
SPY	+1x	635	−0.0078**	−0.0893	0.0613	0.0497	40.47	51.18	20.01
SH	−1x	635	0.0017	−0.0737	0.0829	0.0500	39.21	40.63	20.05
SSO	+2x	635	−0.0350**	−0.2156	0.1229	0.1069	28.50	58.74	40.11
SDS	−2x	635	−0.0136**	−0.1526	0.1444	0.0961	31.97	47.09	40.10
UPRO	+3x	635	−0.0400**	−0.3000	0.1921	0.2387	35.96	61.04	60.26
SPXU	−3x	635	−0.0056	−0.2103	0.2234	0.2392	37.17	49.45	60.16

Panel B: Underlying Index = Nasdaq 100									
QQQ	+1x	476	−0.0070**	−0.0739	0.0626	0.0552	36.92	49.16	21.85
PSQ	−1x	476	0.0019	−0.0830	0.0755	0.0504	39.08	36.34	21.75
QLD	+2x	476	−0.0250**	−0.1846	0.1366	0.1330	38.24	55.46	43.50
QID	−2x	476	0.0052	−0.1553	0.1540	0.1160	41.60	40.13	43.51
TQQQ	+3x	476	−0.0180	−0.2660	0.2267	0.2039	40.00	57.68	65.38
SQQQ	−3x	476	−0.0054	−0.3132	0.2757	0.2395	42.44	48.32	65.28
Panel C: Underlying Index = Russell 2000									
IWM	+1x	794	−0.0214**	−0.2280	0.1971	0.1465	38.29	56.30	34.87
RWM	−1x	794	0.0243**	−0.1675	0.2342	0.1301	53.71	35.09	34.81
UWM	+2x	794	−0.0597**	−0.5337	0.3891	0.3232	35.52	59.45	69.77
TWM	−2x	794	0.0184	−0.4164	0.4896	0.3033	48.87	43.45	69.75
TNA	+3x	794	−0.0972**	−0.8543	0.6386	0.4806	36.27	60.83	104.13
TZA	−3x	794	0.0454**	−0.6017	0.7841	0.4533	49.24	43.32	104.81

Note: This table is reproduced from Charupat and Miu (2013).

ranges of deviations (i.e., the gaps between the values of the 5th and 95th percentiles) than do funds with lower leveraged ratios that are on the same side. This pattern is consistent with the findings of a study by Engle and Sarkar (2006). In that study, the two researchers examine the premiums/discounts of nonleveraged ETFs, and report that funds whose NAVs are more volatile have price deviations that are also more volatile. As shown in the last column of Table 5.2, the volatilities of the funds' NAVs are approximately proportional to the funds' leverage ratios. This results in the observed relationship between leverage ratios and the volatility of price deviations.

In summary, we find that both LETFs and nonleveraged ETFs have price deviations that, on average, are small and within transaction costs and bid-ask spreads. However, LETFs are prone to having large premiums and/or discounts. The higher the leverage ratios, the more prone they are. In addition, the behavior of price deviations appears to be different between bull and bear funds, with bull funds trading at a discount more often than bear funds do. We next want to find out what can explain this pattern of behavior.

As a final note, the above results are calculated using closing prices of the funds. To make sure that the results are not due to any bias in the closing prices, we repeat the calculations using the averages between closing bid and closing ask prices (i.e., closing midprices). We find that the average price deviations and their standard deviations are generally slightly lower. However, the observed differences in the behavior of bull and bear funds still exist.

Behavior of price deviations

To further investigate the reasons for the observed patterns of price deviations, we estimate and report the correlations of price deviations of the funds that are based on the same underlying index in Table 5.3. We can see that, for a given underlying index, funds that are on the same side of the market have price deviations that are positively correlated with one another. Bull funds on the same

Table 5.3 Correlations among Price Deviations

This table presents the correlations among the daily price deviations (premiums/discounts) of the funds in the sample. For each underling index, the correlations are calculated over the common data period (i.e., from the inception date of the most recently introduced fund in each set to December 31, 2011). All correlations in the table are significantly different from zero at the 1% level.

Panel A: Underlying Index = S&P 500

	SPY (+1x)	SH (–1x)	SSO (+2x)	SDS (–2x)	UPRO (+3x)	SPXU (–3x)	S&P 500
SPY (+1x)	1.00	–0.58	0.74	–0.71	0.53	–0.53	–0.20
SH (–1x)		1.00	–0.62	0.62	–0.50	0.55	0.15
SSO (+2x)			1.00	–0.74	0.56	–0.54	–0.31
SDS (–2x)				1.00	–0.46	0.49	0.25
UPRO (+3x)					1.00	–0.73	–0.26
SPXU (–3x)						1.00	0.25
S&P 500							1.00

Panel B: Underlying Index = Nasdaq 100

	QQQ (+1x)	PSQ (–1x)	QLD (+2x)	QID (–2x)	TQQQ (+3x)	SQQQ (–3x)	Nasdaq 100
QQQ (+1x)	1.00	–0.49	0.59	–0.75	0.53	–0.49	–0.15
PSQ (–1x)		1.00	–0.39	0.49	–0.39	0.39	0.16
QLD (+2x)			1.00	–0.65	0.51	–0.46	–0.30
QID (–2x)				1.00	–0.56	0.58	0.31
TQQQ (+3x)					1.00	–0.68	–0.47
SQQQ (–3x)						1.00	0.38
Nasdaq 100							1.00

Panel C: Underlying Index = Russell 2000

	IWM (+1x)	RWM (–1x)	UWM (+2x)	TWM (–2x)	TNA (+3x)	TZA (–3x)	Russell 2000
IWM (+1x)	1.00	–0.73	0.82	–0.85	0.86	–0.85	–0.48
RWM (–1x)		1.00	–0.72	0.75	–0.74	0.76	0.40
UWM (+2x)			1.00	–0.85	0.84	–0.83	–0.50
TWM (–2x)				1.00	–0.88	0.86	0.48
TNA (+3x)					1.00	–0.87	–0.56
TZA (–3x)						1.00	0.51
Russell 2000							1.00

Note: This table is reproduced from Charupat and Miu (2013).

underlying index tend to trade at a premium (or a discount) at the same time. The same is true for bear funds. As a result, bull and bear funds (on the same index) have price deviations that move in the opposite directions. When bull funds trade at a premium, bear funds tend to trade at a discount. All of these correlations are significantly different from zero at the 1% level. Many of them are strong, with absolute values greater than 0.50.

It now appears that price deviations are driven by some common factors. To test this conjecture, we also report the correlations between price deviations and the underlying index returns in the last column of each Panel in Table 5.3. Let us look first at bull funds. Their price deviations are all negatively correlated with the underlying index returns. However, many of the correlations are less than 0.30, and so are not particularly strong. The exceptions are the +3x fund on the Nasdaq 100 (in Panel B) and all bull funds that are based on the Russell 2000 Index (in Panel C). We interpret this result to mean that on days when the underlying index does well, bull funds (which also do well on such days) have a slight tendency to trade at a discount; whereas, on days when the underlying index falls, bull funds (which also do poorly on such days) tend slightly to trade at a premium.

For bear funds, their price deviations are positively correlated with the underlying index returns. As before, the correlations are not very strong. We again interpret these results to mean that on days when the underlying index goes up, bear funds (which do poorly on such days) have a small tendency to trade at a premium; whereas, on days when the underlying index drops, bear funds (which do well on such days) tend to trade at a discount.

To further highlight the relationship between price deviations and the underlying index returns, we calculate average price deviations conditional on the direction of the underlying index. The results are shown in Table 5.4. As can be seen, the behavior of price deviations is indeed different between the days when the underlying indices increase and the days they decline. For all bull funds, average price deviations are negative on the "up" days, and

Table 5.4 Price Deviations Based on the Returns on Their Underlying Indices

This table displays the average price deviations of the funds in the sample when the returns on their underlying benchmarks are positive and negative. The data period for each underlying index is the common period for all six funds (i.e., from the inception date of the most recently introduced fund in each set to December 31, 2011). For all funds except QQQ and PSQ, a Wilcoxon-Mann-Whitney test shows that there is a statistically significant difference in the distributions of their premiums between the positive and the negative days at the 1% level. For QQQ, the difference in the distributions is significant at the 5% level. For PSQ, the difference is not significant.

Panel A: Underlying Index = S&P 500

		When underlying benchmark increases		When underlying benchmark decreases	
Fund	**Multiple**	**N**	**Average price deviation (%)**	**N**	**Average price deviation (%)**
SPY	+1x	358	−0.0162	276	0.0030
SH	−1x		0.0082		−0.0066
SSO	+2x	358	−0.0636	276	0.0019
SDS	−2x		0.0028		−0.0345
UPRO	+3x	358	−0.0909	276	0.0260
SPXU	−3x		0.0394		−0.0639

Panel B: Underlying Index = Nasdaq 100

		When underlying benchmark increases		When underlying benchmark decreases	
Fund	**Multiple**	**N**	**Average price deviation (%)**	**N**	**Average price deviation (%)**
QQQ	+1x	264	−0.0117	209	−0.0011
PSQ	−1x		0.0061		−0.0031
QLD	+2x	264	−0.0496	209	0.0058
QID	−2x		0.0260		−0.0214
TQQQ	+3x	264	−0.0785	209	0.0580
SQQQ	−3x		0.0558		−0.0815

Panel C: Underlying Index = Russell 2000

		When underlying benchmark increases		When underlying benchmark decreases	
Fund	**Multiple**	**N**	**Average price deviation (%)**	**N**	**Average price deviation (%)**
IWM	+1x	421	−0.0736	374	0.0376
RWM	−1x		0.0644		−0.0207
UWM	+2x	421	−0.1792	374	0.0736
TWM	−2x		0.1300		−0.1067
TNA	+3x	421	−0.2916	374	0.1208
TZA	−3x		0.2178		−0.1472

Note: This table is reproduced from Charupat and Miu (2013).

positive (or only marginally negative) on the "down" days. It is, however, exactly the opposite for bear funds. All bear funds have average price deviations that are positive (i.e., premiums) on the "up" days, and negative (i.e., discounts) on the "down" days. The pattern is very pronounced for funds on the Russell 2000 Index in Panel C.[2] Finally, the difference in the average price deviations of the funds (on the same side of the market) between the "up" and "down" days increases with the leverage ratio.

A possible explanation for the observed behaviors of price deviations

One possible explanation for the observed behaviors of price deviations is that they are caused by the trading that LETFs have to do at the end of every day in order to adjust their exposures in response to the movement in its underlying index so as to maintain its constant leverage ratio. For example, suppose a +2x LETF is issued today and raises \$100 from investors to invest in a particular index. In order to be able to deliver twice the return on the index, it needs to have an exposure of \$200 in the index. This means that the fund needs to use leverage such as by borrowing another \$100, and so the total investment is \$200 (i.e., twice the amount of the assets under management or the "equity" of the investors). Now, suppose that today the index goes up by 2%. The investment is now worth \$204, and the investors' equity is now \$104. Therefore, in order to maintain the promised leverage ratio, the fund will need to have a total investment of twice the equity amount or \$208. Since the fund now has only \$204, it has to borrow another \$4 and invest it in the index. This process is what we refer to as an "exposure adjustment," and the amount of adjustment for the day is \$4. Normally, the adjustment is carried out at the end of each trading day.

Formally, the dollar amount of exposure adjustment for fund j on day t can be calculated as follows:[3]

$$\Delta_t^j = \left(\beta_j^2 - \beta_j\right) \cdot AUM_{t-1}^j \cdot i_{t-1,t}, \tag{5.2}$$

where β_j is the fund's leverage ratio, AUM_{t-1}^j is the fund's assets under management on day t – 1, and $i_{t-1,t}$ is the return on the fund's underlying index between day t – 1 and day t. For the above example, we can plug the number into the equation to verify the adjustment amount:

$$\left(\beta_j^2 - \beta_j\right) \cdot AUM_{t-1}^j \cdot i_{t-1,t} = \left(2^2 - 2\right) \cdot 100 \cdot 0.02 = 4.$$

Equation (5.2) says that the amount of daily exposure adjustment depends on three factors—the leverage ratio, the fund's AUM at the start of the day, and the underlying index's return for the day. Note that the amount is more than proportional to the fund's leverage ratio (except when β_j = +2). For example, for a –2x LETF, $\beta_j^2 - \beta_j$ is 6 (i.e., 6 times that change in the AUM). In other words, the term $\beta_j^2 - \beta_j$ can magnify the impact of exposure adjustments of LETFs.

In addition, because the term $\beta_j^2 - \beta_j$ will always be positive regardless of whether β_j is positive or negative, the daily exposure adjustments of both bull and bear LETFs (on the same underlying index) are always in the same direction as the index's movement. For example, when the underlying index goes up, bull funds (which have positive exposures to the benchmark) do well and have to increase their exposures. At the same time, bear funds (which have negative exposures to the benchmark) do poorly and have to reduce their negative exposures (i.e., make them more positive). This means both bull and bear funds have to buy the stocks in the benchmark. The reverse is true when the underlying index declines.[4]

The adjustments typically occur at the end of each trading day, which is when the direction and magnitude of the underlying index movement become clear. As a result, their impact on the underlying index will be felt at that time. To see how these adjustments can cause the observed behavior of price deviations reported above, consider the following scenarios. On days when the underlying index declines, fund companies (or, if derivatives such as swaps are used to generate the returns, their swap counterparties) need to sell some of the securities comprising the underlying index, thus putting further downward pressure on the index in the last minutes of those trading days. As a result, the LETFs' closing prices, which are likely to come from an earlier time, do not fully reflect the decline in the index level.[5] This creates an upward bias in the price deviations of bull funds (i.e., closing prices being higher than NAVs), and a downward bias in the price deviations of bear funds (i.e., closing prices being lower than NAVs). Similarly, on days when the underlying index moves up, the exposure adjustments require that the fund issuers buy some of the securities in the underlying index, further increasing the index level at closing. In this case, the LETFs' closing prices do not fully incorporate the increase in the index due to the rebalancing trading activities, creating a downward bias in the price deviations of bull funds and a upward bias in the price deviations of bear funds. Consequently, price deviations of bull funds are negatively correlated with the index returns, while the opposite is true for bear funds.

The effects of exposure adjustments by LETFs are an issue on which there are conflicting views among practitioners in the US market. Some media outlets and market makers have reported anecdotal evidence that exposure adjustments by LETFs have increased market volatility at the end of each day (see, for example, Lauricella et al., 2008; and Zweig, 2009). Regulatory bodies have also been concerned about the impact. For example, in September 2011, the *Wall Street Journal* reported that the Securities and Exchange Commission (SEC) had discussions with firms that trade LETFs to better understand the volatility effect (see Patterson, 2011). In addition, a few researchers have published studies whose results support this point of view. For example, Bai et al. (2012) study LETFs on real-estate indices and find that their end-of-day exposure rebalancing significantly moves the prices of the indices' component stocks and also increases their volatility. Tuzun (2013) reports that the rebalancing trades result in price reaction and higher volatility in various underlying indices in the US market. Finally, according to Shum et al. (2015), the end-of-day volatility is positively correlated with a measure of exposure-adjustment trades. However, they show that the effects are economically significant only on very volatile trading days.

In contrast, there are traders who do not believe that the impact is currently significant (see, for example, Keefe, 2009). Their argument is that LETFs represent only a very small fraction of the market and so their impact is negligible. More recently, Ivanov and Lenkey (2014) suggest that the concern about LETFs' rebalancing effects is exaggerated. They argue that at the end of each day, LETF issuers also have to trade according to the funds' capital flows that day. That is, on a given day, investors may put more money into LETFs, and so LETF issuers will have to create more shares, which means that they will have to buy more stocks in the underlying indices. The reverse is also possible. Ivanov and Lenkey's argument is that the trades (in the underlying stocks) generated by these capital flows can offset the trades generated by the exposure-rebalancing needs, thus reducing the impact of the rebalancing.

Accordingly, whether or not the effects of LETFs' exposure adjustments are significant enough to explain the behavior of price deviations that we observe is indeed an empirical question. This is what we want to find out in the next section.

Tests for the effects of end-of-day exposure adjustments

We run a regression to test for the effects of exposure adjustments. The idea is that, if exposure adjustments have an impact on the underlying indices, we should observe a relationship between LETFs' price deviations and a proxy for the amounts of daily adjustments. We use the following regression equation:

$$\pi_t^j = a^j + b_1^j \frac{\Delta_t}{INDTRADE_t} + b_2^j \pi_{t-1}^j + \varepsilon_t^j, \tag{5.3}$$

where π_t^j is the price deviation (in percentage term) observed on fund *j* on day *t* (see equation [5.1]), Δ_t is the sum of the amounts of day-*t* exposure adjustments of *all* LETFs and inverse ETFs that are based on fund *j*'s underlying index and that are traded in the US market (see equation [5.2]), $INDTRADE_t$ is the total trading value of the fund's underlying index on day t (i.e., sum of the daily dollar trading values across all the index's constituent stocks), and ε_t^j is the residual term.[6]

Accordingly, the first independent variable in the regression is the normalized amount of exposure adjustments (i.e., after taking into account the trading activity in the underlying index), expressed in percentage terms. We normalize the amounts to account for the fact that trading activity in the underlying index varies from one day to the next, and so a given amount of adjustments will have a different impact, depending on how liquid the underlying index is on a particular day. The value of Δ_t is the sum of individual Δ_t^j's, estimated using equation (5.2) above. According to the explanation for the observed pattern of price deviations in the previous section, we expect the coefficient b_1 to be negative (positive) for bull (bear) funds.[7]

The second independent variable in the regression is the previous day's price deviation. We include it in the equation because the data show that the underlying index returns display mild autocorrelations.[8] Since the size and direction of daily exposure adjustments depend on the underlying index return (as seen in equation [5.2]), the moderate predictability of the underlying index returns suggests that lagged price deviations may have some predictive power on current price deviations and thus should be included in the regression. We expect the sign of b_2 to be negative.[9] In addition, out of the three indices, we expect lagged price deviations to have the highest explanatory power for funds based on the Russell 2000 Index (which has the highest autocorrelation in absolute terms).

Before we present the regression results, let us look at Table 5.5, in which we present summary statistics on the total AUM of all the LETFs for each underlying index, the index's market capitalization, the absolute value of daily exposure adjustments ($|\Delta_t|$), and the daily trading value of the index ($INDTRADE_t$). The values shown in the table are the averages of daily values of

Table 5.5 Relative Size of LETFs and Inversed ETFs

This table displays the average aggregate amount of AUM of LETFs and inverse ETFs for each underlying index, together with the average market capitalization of the underlying index. Also displayed is the average of the absolute value of daily exposure adjustments, together with the average daily trading value of the underlying index. For each underlying index, the figures in each column are the averages of daily numbers over the data period, which is the common period for all six funds (i.e., from the inception date of the most recently introduced fund in each set to December 31, 2011).

Underlying Index	**Avg. Total AUM of LETFs and –1x ETFs (million)**	**Avg. Index Market Capitalization (million)**	**Avg. Ratio bet. AUM and Index Market Capitalization**	**Average $\|\Delta_t\|$ (million)**	**Avg. $INDTRADE_t$ (million)**	**$\|\Delta_t\|$ as % of $INDTRADE_t$**		
						Average	**5th Percentile**	**95th Percentile**
S&P 500	\$7,065	\$10,843,175	0.07%	\$270.00	\$29,657.19	0.89%	0.04%	2.53%
Nasdaq 100	\$1,966	\$2,548,461	0.08%	\$80.13	\$9,727.44	0.82%	0.05%	2.23%
Russell 2000	\$1,920	\$1,149,048	0.16%	\$183.50	\$4,197.13	4.30%	0.30%	13.21%

Note: This table is reproduced from Charupat and Miu (2013).

these variables over the sample period. For example, consider the S&P 500 Index. The average total AUM of all the LETFs on it is \$7.07 billion, while the average market capitalization of the S&P 500 is \$10,843 billion. The average of the ratio between the two (henceforth referred to as the AUM-to-index-cap ratio) is only 0.07%, which suggests that the S&P 500 is very large relative to its LETFs. On the other hand, for the Russell 200 Index, the AUM-to-index-cap ratio is larger at 0.16%.

The average amounts of daily exposure adjustments are shown in the fifth column. For the S&P 500 Index, the average daily exposure adjustments are \$270 million,[10] compared to the average value of the index's daily trading activity (*INDTRADE*) of \$29.66 billion. The average of the ratios between these two numbers is 0.89%. This means that on average, the exposure adjustments for all LETFs account for 0.89% of the S&P 500's daily trading value, which is much higher than what the AUM-to-index-cap ratio would suggest (i.e., 0.07%). This is due to the fact that, as mentioned earlier, the amounts of exposure adjustments are not linear in AUM and can be more than proportional to the funds' leverage ratios.

While the average exposure adjustments of 0.89% may not sound significant, remember that these trades come into the market at once near the close of each trading day. Hence, depending on the liquidity at the end of a day, these trades can have an impact on the price and volatility of the index. The numbers for the Nasdaq 100 Index are qualitatively similar to those of the S&P 500 Index. Again, even though the LETFs on it have an average AUM-to-index-cap ratio of only 0.08%, their exposure adjustments account for, on average, 0.82% of the index's daily trading value. Finally, the Russell 200 Index has an average exposure-adjustments ratio of 4.30%, while the number for the 95th percentile is 13.21%. It is clear that exposure adjustments account for a much larger portion of trading in this index at the end of a day than in the other two indices.

The regression estimates are reported in table 5.6. The coefficients b_1's are statistically significant at the 1% level for all funds,

Table 5.6 Regression Estimates

This table displays the results of the following regression:

$$\pi_t^j = a^j + b_1^j \frac{\Delta_t}{INDTRADE_t} + b_2^j \pi_{t-1}^j + \varepsilon_t^j,$$

where π_t^j is the price deviation (in percentage terms) observed on fund *j* on day *t* (see equation [5.1]), Δt is the sum of the amounts of day-*t* exposure adjustments of *all* LETFs and inverse ETFs on the underlying index that are traded in the US market (see equation [5.2]), *INDTRADE*t is the total trading value of the fund's underlying index on day *t*, and ε_t^j is the residual term. Both independent variables are expressed in percentage terms. The data period for each underlying index is the common period for all six funds (i.e., from the inception date of the most recently introduced fund in each set to December 31, 2011). * denotes significance at the 5% level, while ** denotes significance at the 1% level.

Panel A: Underlying Index = S&P 500 Index					
Fund	**Multiple**	a	b_1	b_2	**Adj.** R^2
SPY	+1x	−0.0067**	−0.0084**	0.0419	0.04
SH	−1x	0.0006	0.0070**	0.1664**	0.06
SSO	+2x	−0.0305**	−0.0279**	0.0468	0.10
SDS	−2x	−0.0151**	0.0208**	0.0629	0.07
UPRO	+3x	−0.0337**	−0.0495**	0.0304	0.06
SPXU	−3x	−0.0113	0.0502**	0.0017	0.06
Panel B: Underlying Index = Nasdaq 100					
Fund	**Multiple**	a	b_1	b_2	**Adj.** R^2
QQQ	+1x	−0.0059*	−0.0078**	0.0584	0.02
PSQ	−1x	0.0011	0.0075**	0.1052*	0.03
QLD	+2x	−0.0211**	−0.0346**	0.0359	0.08
QID	−2x	0.0025	0.0308**	0.0146	0.08
TQQQ	+3x	−0.0134	−0.0768**	−0.1605	0.20
SQQQ	−3x	−0.0134	0.0777**	−0.1020	0.13
Panel C: Underlying Index = Russell 2000					
Fund	**Multiple**	a	b_1	b_2	**Adj.** R^2
IWM	+1x	−0.0172**	−0.0098**	0.0710	0.16
RWM	−1x	0.0193**	0.0074**	0.1477**	0.13
UWM	+2x	−0.0489**	−0.0225**	0.1006*	0.18
TWM	−2x	0.0133	0.0198**	0.0768	0.15
TNA	+3x	−0.0761**	−0.0366**	0.1355**	0.22
TZA	−3x	0.0318*	0.0340**	0.1401**	0.21

Note: This table is reproduced from Charupat and Miu (2013).

and are also of the correct signs. For all bull (bear) funds, b_1 is negative (positive). To make sense of the magnitude of b_1, consider, for example, the coefficient for SQQQ (i.e., 0.0777). This value means that every 1% of normalized exposure adjustments is related to a price deviation for SQQQ of +0.0777%.

In addition, b_1 is higher (in absolute terms) for funds with higher leverage ratios. This suggests that end-of-day exposure adjustments have a greater impact on high-leverage funds than on low-leverage funds. This is consistent with the explanation. To see why, suppose that for all funds on the same index, their prices just before the end of a trading day are exactly the same as their NAVs (i.e., no deviations). Then, if end-of-day exposure adjustments cause the index value to move away, the NAVs of those funds will change. Because of their leverage, the NAVs of LETFs will change by more than the NAVs of +1x funds. The higher the funds' leverage ratios, the larger the changes. Therefore, we should expect to see higher values for b_1 for higher-leverage funds.

Next, consider the coefficient estimates for lagged premiums (i.e., b_2). The estimates for b_2 in table 5.6 are generally not significantly different from zero for funds that are based on the S&P 500 and the Nasdaq 100 indices. For the Russell 2000 Index, b_2 is significant for four of the six funds. However, these significant coefficient estimates are positive, rather than negative, which goes against the expectation under the explanation.[11]

Now, consider the intercept term (i.e., a). A majority of the intercepts are statistically different from zero. The sign and magnitude of the estimates of a suggest that, even after controlling for the effects of end-of-day exposure adjustments and lagged price deviations, bear funds still trade on average at a higher premium (or lower discount) than that of their bull counterparts (on the same index). The difference in average premiums between the bull and bear fund in each pair is, however, consistently smaller than that presented in table 5.2 (i.e., before controlling for the end-of-day exposure adjustment effect). For example, consider the +3x and −3x LETFs on the Russell 2000 Index (i.e., TNA and TZA). Before we control for the end-of-day exposure adjustment effect, the difference in the average (unconditional) price deviations between TNA and TZA is 0.1426% (i.e., 0.0454% – (−0.0972%), see table 5.2). After we control for the end-of-day exposure adjustment effect, the difference in the average (conditional) price deviations between TNA and TZA (based on estimated values of a) reduces to 0.1079% (i.e., 0.0318 – (−0.0761%)). In other words, end-of-day exposure adjustments can only roughly explain one-third of the difference in premiums between this particular pair of bull and bear funds.

Finally, the adjusted R-squared of the regressions differs from one underlying index to another. The adjusted R-squareds are generally low, ranging from 0.02 to 0.10, for all funds on the S&P 500 Index (Panel A) and for most funds on the Nasdaq 100 Index (Panel B). That is, end-of-day exposure adjustments cannot explain much of the variation in price deviations of the funds that are based on these two indices.[12] This is not surprising considering that the amounts of daily exposure adjustments are small relative to the index trading for the two indices. In contrast, the adjusted R-squareds are higher for funds on the Russell 2000 Index (Panel C), ranging from 0.13 to 0.22. This is as predicted by the conjecture since exposure adjustments account for a much larger portion of daily trading in the Russell 2000 Index than in the other two indices.

In summary, the regression results provide partial support for the explanation that the observed behavior of price deviations is caused by end-of-day exposure adjustments. For all funds, there is a statistically significant relationship between price deviations and end-of-day exposure adjustments. The explanatory power of exposure adjustments is strongest for the funds on the Russell 2000 Index. This is consistent with the fact that the Russell 2000 Index has the least trading depth relative to the amounts of aggregate exposure adjustments, and thus should be most prone to the effect.

Conclusion

In this chapter, we examine the pricing efficiency of LETFs. On average, the pricing of LETFs is efficient. However, large premiums/discounts are prone to occur. More interestingly, bull LETFs tend to trade at a discount to their NAVs, while bear LETFs tend to trade at a premium. In addition, price deviations of bull LETFs are all negatively correlated with the returns on their own benchmarks; whereas, price deviations of bear LETFs are all positively correlated with their benchmarks' returns. These patterns of price deviation behavior are more pronounced in funds with high leverage ratios. This behavior can be partly explained by the funds' daily exposure adjustments, which have to be done at the end of each trading day in order to maintain their leverage ratios. The explanatory power of exposure adjustments is stronger for funds that are based on indices with less trading depth, such as the Russell 2000 Index.

6
Performance and Tracking Errors

The performance of ETFs is measured by tracking error, commonly defined as the deviation of the returns on net asset values (NAVs) of ETFs from the corresponding returns on the underlying benchmark indices. The deviation tells us how effective and efficient is the fund management in tracking the performance of the benchmark index and delivering the promised return. Unlike price deviations (as discussed in chapter 5), which are typically expected to be within the arbitrage bounds given the creation/redemption process of ETFs, any deviations of the returns on NAV from those of their underlying benchmarks can accumulate over time and thus significantly affect the long-term performance of the ETFs.

Different from traditional nonleveraged ETFs, the tracking ability of LETFs can degenerate substantially as the holding period lengthens due to the daily rebalancing requirement in order to maintain the constant leverage ratios. The long-term compounded return of LETFs can be quite different from the leverage multiple of the underlying index return over the same period. The magnitude and the direction of the deviation depend on the length of the holding period and the path that the underlying benchmark takes during that period. There is a general perception that this effect results in a negative influence on LETFs' performance. As the holding period lengthens, an LETF may deliver compounded returns that are much less than the contemporaneous benchmark returns multiplied by the promised leverage ratio. Under certain circumstances, a bull (bear) LETF may actually deliver a negative holding period return even if the underlying index realizes a positive (negative) compounded return over the same time period. For a given holding period, the higher the multiple (in absolute terms) or the more volatile the underlying index returns (or both), the greater the chance that a LETF's realized return will differ from its stated multiple. The above effect due to the daily rebalancing of exposure is usually referred to as the *compounding effect* of LETFs.

The tracking errors of traditional nonleveraged ETFs are typically estimated by regressing the ETF's NAV returns against the returns of the underlying benchmark index. The standard deviation of the difference between the NAV returns and the returns on the underlying index is also commonly used to gauge the size of the tracking errors. These conventional approaches, however, may lead to ambiguous results if they are directly applied to measure the tracking errors of LETFs. Since the tracking errors of LETFs are dictated not only by factors that are under the control of the fund issuers but also by factors that are outside of their control (e.g., the compounding effect), we cannot easily measure the management efficiency of fund issuers by using these conventional approaches. New regression approaches have been proposed specifically for measuring and analyzing LETFs' tracking errors.

This chapter is organized as follows. We first outline the factors that govern the tracking errors and performance of LETFs. We then examine a number of approaches for analyzing LETFs' tracking errors that are commonly used in the literature. Finally, we present some detailed tracking error analysis results for a sample of LETFs.

Governing factors of tracking errors

In this section, we describe various sources of tracking errors. We group them into those that are, to a certain extent, under the control of the fund management versus those that are not. Note that we will not able to judge the management performance of an LETF based on tracking errors that are attributable to the latter group, which by definition are factors outside the control of its management.

Factors that are under the control of fund management

The first group of factors includes both the explicit (e.g., management fees) and implicit costs (e.g., transaction costs, opportunity costs of cash accrual) of managing LETFs, and also the choice of replication strategies. These sources of tracking errors are applicable to not only LETFs but to all ETFs in general.

Management fees

Similar to traditional nonleveraged ETFs, there are explicit costs of managing the funds that are borne by LETF investors. Investment advisory and management service fees charged by the fund management represent the largest component of these costs. Other explicit costs may include administration fees, custodian fees, licensing fees, and trustee fees. They are typically charged on a daily basis by the fund issuers at annualized rates, usually referred to as the *expense ratios*, which are based on the average daily NAVs. For example, ProShares Ultra S&P 500, the largest +2x LETF tracking the S&P 500 Index, currently has an expense ratio of 0.89% per annum. Ceteris paribus, the higher the expense ratio, the larger will be the tracking errors of an LETF and the more it is expected to underperform the underlying index.

Transaction costs

The transaction costs/fees incurred for entering into and modifying derivative contracts with its counterparties in generating the target returns are not reflected in the fund's expense ratio. Besides using derivative contracts, sometimes (part of) the return may also be derived from direct investment in assets underlying the benchmark index. For example, bull LETFs that track equity indices usually have a portion of their exposure invested in the constituent stocks of the indices. For example, according to ProShares (2014), as of the end of May 2014, ProShares Ultra S&P 500 generated its 200% exposure to the S&P 500 Index by a combination of equity securities mimicking the composition of the index (78%) and swaps and futures contracts (122%). For this kind of LETF, in addition to incurring transaction costs involving derivative contracts, there are also transaction costs for buying and selling the underlying securities. LETFs using the in-kind creation/redemption process tend to incur lower transaction costs, since there is no need to purchase (liquidate) any assets upon a net positive (negative) fund flow. These LETFs nevertheless still incur transaction costs associated with matching the changes in the index weights and compositions. Transaction costs are generally higher for LETFs that (i) track indices that are more volatile or less liquid; (ii) have higher leverage ratios; and (iii) have more frequent creation/redemption of their units.

The above-mentioned transaction costs are part of the costs of the management of the fund. Retail investors of LETFs would likely have to incur additional transaction costs (e.g., commissions and brokerage fees) in buying and selling units of LETFs on the exchanges.

Opportunity costs of cash accrual

Studies on nonleveraged ETFs and index funds (e.g., Elton et al., 2002; Frino et al., 2004) show that tracking errors can occur because of the opportunity costs of not being able to fully reinvest cash dividends from the constituent stocks that are accrued in a cash account held before distributing to unit holders. This cost increases with the dividend yield of the index, the time delay of distribution, and the return on the underlying index. No dividend payments are typically involved if an LETF delivers the leveraged returns by entering into total return swaps. Nevertheless, for those bull equity LETFs that invest a portion of their exposures in the constituent stocks of the benchmark indices, this kind of opportunity cost could potentially be incurred. We do not, however, expect the effect to be economically significant.

First, only a portion of the exposure is generated by holding the constituent stocks. Moreover, since any dividends received by the LETFs are first used to offset the management fees before they are accrued in the cash account, the distributable amounts of dividends are typically much less than the actual dividends received.

Choice of replication strategies

The choice of replication strategy may also dictate a fund's tracking errors. A perfect replication of the daily leveraged underlying index return may not be able to be guaranteed even with the extensive use of derivatives (e.g., futures contracts and total return swaps). There may be an imperfect correlation between the value of the benchmark asset of the LETF and the asset underlying the derivative leading to basis risks or correlation risks, which in turn may result in tracking errors. The choice of replication strategy can become the main determining factor of tracking errors when the benchmark index is much less liquid than a typical domestic equity index. In a recent study, Guo and Leung (2015) find that LETFs that track an *illiquid benchmark index* (e.g., the gold bullion index) tend to have more tracking errors than those that track a more *liquid benchmark index* (e.g., the oil and gas index, of which there are liquid exchange-traded futures contracts). The former LETFs mainly rely on over-the-counter total return swaps to replicate the target leveraged return, which tends to be more costly. The higher costs, together with the fact that the mark-to-market values of these over-the-counter contracts might not be updated frequently can result in substantial tracking errors for these LETFs.

In addition, the use of a portfolio of derivatives (to diversify counterparty credit risks) may further increase the chance of deviation from the promised leveraged returns. For LETFs that invest part of their exposure in the underlying securities, tracking errors may also occur if the fund does not use full replication, and only invests in a subset of the constituent stocks based on some optimization strategies.

Factors that are outside the control of fund management

Here we examine two sources of tracking errors that, to a large extent, are outside the control of the fund management, namely compounding effect and financial costs related to the delivering of leveraged returns. These two factors are specific to LETFs, and thus not relevant to nonleveraged ETFs.

Compounding effect

Because the goal of an LETF is to generate the stated multiple of the return of the underlying index on a daily basis, the fund manager has to rebalance daily its exposure to the index so that the leverage ratio remains constant. The LETF's return over a holding period longer than one day can deviate from the underlying index return multiplied by the promised leverage because of this *compounding effect*. The deviation can be substantial if the holding period is long. The general lack of understanding of the implications of the compounding effect among investors has caused market regulatory bodies to have concern (e.g., Financial Industry Regulatory Authority [FINRA], 2009; Securities and Exchange Commission [SEC], 2009). It is important to emphasize that this effect is outside the control of the fund management. The magnitude/direction of the deviation depends on the nature of the realized return path (both level and volatility) of the underlying benchmark index during the holding period. It also depends on the length of the holding period and the leverage ratio of the LETF.

Chapter 4 provided a detailed discussion on the key characteristics of the compounding effect and outlined its main determinants. To summarize, the return on an LETF is more negatively affected by the compounding effect, thus resulting in larger tracking errors when the realized benchmark index return is more volatile. This phenomenon is sometimes referred to as the *volatility drag* or *volatility decay*. Moreover, the performance will be especially poor in a sideways market as opposed to an up-trending or down-trending market. Ceteris paribus, the higher the leverage ratio and the longer the holding period, the stronger the compounding effect. For a detailed discussion on the compounding effect of tracking errors and how it may be analytically related to the characteristics of the realized return path of the benchmark index, please also refer to Charupat and Miu (2014). It is important to emphasize that the significance of the compounding effect is far from universal. It is highly specific to the nature of the return-generating process of the particular benchmark index and over the particular sample period under consideration. Although the literature is in support of the generally negative influence of the compound effect on the performance of LETFs, and thus cautions against the use of LETFs as a long-term investment vehicle, the simulation analysis performed by Loviscek et al. (2014) using long historical time series of US stock market data shows that the compounding effect does not necessarily exert a negative impact on LETFs' performance even over holding periods of longer than one year. They attribute their finding to the leptokurtic nature of the daily return distribution of the underlying benchmark.

Financing costs

It is quite common for LETFs to use total return swaps contracts to obtain the target leveraged exposure. To deliver the target leveraged returns, the counterparties of these total return swaps will incur positive (negative) financing costs in creating the required positive (negative) leveraged exposures. For example, consider the counterparty of a +2x LETF with a total asset value of \$100. In order to deliver twice the return of the underlying index, the counterparty needs to borrow \$100 in order to have a total exposure of \$200 on the index. The counterparty therefore incurs a positive financing costs on \$100. Now consider the counterparty of a –2x LETF with the same total asset value of \$100. In order to deliver twice the negative of the return of the underlying index, the counterparty needs to have a short exposure of \$200 on the index. The short proceeds of \$200 together with the asset value of \$100 of the LETF amount to \$300, which will be earning interest (i.e., incurring negative financing costs) for the counterparty. Thus, the financing costs are not symmetric between bull and bear funds. The magnitude of the negative financing costs (on \$300) for the above –2x LETF is three times that of the positive financing costs (on \$100) for the +2x LETF. In general, if we use β to denote the target leverage ratio (i.e., –1, –2, –3, +2, or +3), the financing costs are always proportional to (β–1). The financing rate is commonly benchmarked against the London interbank offered rate (LIBOR). The total dollar amount of financing costs therefore equals the total asset value under management multiplied by (β–1) × LIBOR.

In a competitive market, the above positive (negative) financing costs incurred by the counterparties are expected to be passed back to the unit holders of LETFs as additional costs (returns) through the swap contract relation with the fund companies. Financing costs therefore exert opposite effects on the performance of bull versus bear funds. For bull LETFs, the higher the LIBOR and the larger their leveraged ratios, the more negatively their performance will be affected by the financing effect. In contrast, bear LETFs benefit from this financing effect. The higher the LIBOR and the larger the leveraged ratios, the more positively their performance will be affected by the financing effect. Controlling for other factors, the financing effect is therefore expected to result in tracking errors in the sense that bull funds will underperform their benchmarks, while bear funds will outperform their benchmarks.[1] This opposite performance behavior is documented in empirical studies on LETFs (e.g., Charupat and Miu, 2014).

To a large extent, the financing effect is beyond the control of the fund management. As LIBOR increases, the financing effect will be stronger for all LETFs regardless of the issuing fund companies. Any tracking errors as a result of the financing effect therefore do not directly reflect the skill of the fund management.

Different approaches used in analyzing LETFs' tracking errors

To have a better understanding of the magnitude and nature of LETFs' tracking errors, a number of analytical approaches have been developed together with the necessary statistical tools and metrics. Using these tools and metrics, market participants can gauge the performance of LETFs, which are supposed to closely track the leveraged returns of the underlying benchmark indices. If there are any tracking errors, these tools and metrics will allow market participants to find out why that is the case and under what circumstances the errors occur. This information is very important for LETF investors in devising their investment strategy. We examine a number of these analytical approaches used in measuring and analyzing LETFs' tracking errors, namely the regression approach, the decomposition approach, and the simulation approach. We provide the formulations for some of these approaches and highlight some of the main results from previous empirical studies adopting these approaches. In general, even for a holding period as short as three months, the compounding effect is found to be the most important governing factor of tracking errors, followed by management fees and financing costs. Let us start our discussion by formally defining tracking errors.

Definition of tracking errors

In measuring tracking errors of traditional nonleveraged ETFs, researchers typically define tracking errors as the differences between the returns based on the changes in their NAVs and the contemporaneous returns on the benchmark indices they are supposed to track (e.g., Frino and Gallagher, 2001; Elton et al., 2002; Gastineau, 2004). The same definition is also commonly used when studying the tracking errors of LETFs. But, unlike nonleveraged ETFs, here we are comparing the NAV returns on the LETF with the index returns multiplied by the respective promised leverage ratio (β). That is,

$$TE^{NAV}_{t,t+N} = r^{NAV}_{t,t+N} - \beta \cdot I_{t,t+N} \tag{6.1}$$

where $r^{NAV}_{t,t+N}$ is the return on NAV from time t to t+N,

$$r^{NAV}_{t,t+N} = \frac{NAV_{t+N}}{NAV_t} - 1 \tag{6.2}$$

and $I_{t,t+N}$ is the compounded return on the index over the same period of time. The length of the holding period under examination is therefore denoted by N. The NAVs used in this calculation need to be adjusted for dividend and capital-gain distributions that the LETF made to its investors during the sample period. That is, the distributions are added back to the NAVs before we calculate the return in Equation (6.2).

In general, a small $TE^{NAV}_{t,t+N}$ will suggest that $r^{NAV}_{t,t+N}$ is closely tracking $\beta \cdot I_{t,t+N}$, and thus is an indication of *good tracking performance* of the LETF under consideration. This interpretation, however, deserves some clarifications. Given that the objective of LETFs is to deliver a constant leveraged return on a daily basis, the daily rebalancing of exposures and the resulting compounding effect are expected to result in tracking errors over a holding period longer than one day, even if the fund management has done a perfect job and charges minimal management fees. Thus, a large $TE^{NAV}_{t,t+N}$ as defined above does not necessarily imply *poor* management performance because compounding effect is outside the control of management. For a holding period longer than one day, the leveraged index return of $\beta \cdot I_{t,t+N}$ is not an appropriate benchmark against which management performance is to be judged. It is at best a *naive* benchmark. As discussed in chapter 4, the compounding effect can be positive or negative depending on the path of the realized benchmark index returns. Thus, $r^{NAV}_{t,t+N}$ can outperform or underperform the naive benchmark depending on the realized return paths. But in typically volatile market conditions, underperforming the naive benchmark is more the norm, especially when the LETF investment is held for too long. It is a false and unrealistic expectation that an LETF will generate a compounded return equal to the naive benchmark. The potential confusion regarding this naive expectation among some LETF investors has drawn concern from both market regulators and LETF fund issuers. Below in this section, with the objective of addressing these concerns, we introduce different ways to disentangle the compounding effect from other sources of tracking errors.

It is important to note that some researchers choose to define tracking errors differently. They define tracking errors as the differences between the returns based on the changes in the market prices of the LETFs and the contemporaneous leveraged returns on the benchmark indices. That is,

$$TE_{t,t+N}^{Price} = r_{t,t+N}^{Price} - \beta \cdot I_{t,t+N} \tag{6.3}$$

where $r_{t,t+N}^{Price}$ is the return on the market price (P) of LETF from time t to t+N,

$$r_{t,t+N}^{Price} = \frac{P_{t+N}}{P_t} - 1. \tag{6.4}$$

The benefit of adopting the previous definition (i.e., $TE_{t,t+N}^{NAV}$) as opposed to this alternative definition (i.e., $TE_{t,t+N}^{Price}$) is that the latter will be contaminated by any pricing inefficiency of the LETF market (as discussed in chapter 5). To illustrate this point, let us decompose $TE_{t,t+N}^{Price}$ into its two components as follows:

$$\begin{aligned} & TE_{t,t+N}^{Price} = r_{t,t+N}^{Price} - \beta \cdot I_{t,t+N} \\ & \Rightarrow TE_{t,t+N}^{Price} = \left(r_{t,t+N}^{Price} - r_{t,t+N}^{NAV}\right) + \left(r_{t,t+N}^{NAV} - \beta \cdot I_{t,t+N}\right) \\ & \Rightarrow TE_{t,t+N}^{Price} = \left(\frac{P_{t+N}}{P_t} - \frac{NAV_{t+N}}{NAV_t}\right) + TE_{t,t+N}^{NAV} \end{aligned} \tag{6.5}$$

Thus, a high $TE_{t,t+N}^{Price}$ could be the result of a high $TE_{t,t+N}^{NAV}$, but it may well be the result of a large pricing premium or discount (i.e., when P_{t+N} is very different from NAV_{t+N} and/or P_t is very different from NAV_t). By only observing a high $TE_{t,t+N}^{Price}$, it is difficult to tell whether it is because (i) the arbitrage mechanism through the creation/redemption of LETF units that is supposed to align the market price and NAV is not working perfectly or (ii) the fund management is not generating a return on NAV that is closely tracking the benchmark index. Without being able to disentangle these two potentially confounding effects, it is difficult to identify the exact sources of errors and their respective underlying determinants. In the rest of this section, unless otherwise specified, we adopt the definition of tracking errors based on NAV (i.e., $TE_{t,t+N}^{NAV}$) of Equation (6.1). Nevertheless, in the latter sections of this chapter, we also examine a number of decomposition methodologies that involve the definition of tracking errors based on market price (i.e., $TE_{t,t+N}^{Price}$).

In general, the holding period N can be of any length. The choice of N is governed by the investment horizon of interest to the specific researcher or market participant. Repeating the analysis with different values of N will tell us how tracking errors might change with the length of the investment horizon. When we set $N = 1$, we are measuring tracking errors on daily returns. Given that LETFs are rebalanced on a daily basis, tracking errors on daily returns are not subject to the compounding effect. Compounding effect is expected to play a more important role as the holding period becomes much longer than one day.

Of course, the smaller the tracking errors as defined by Equation (6.1), the better the performance of the LETF under examination. We can therefore easily measure the performance by analyzing the distribution of $TE_{t,t+N}^{NAV}$ observed over some historical time period if we have recorded both the historical daily NAVs and the corresponding daily returns of the underlying index. Recently, Rompotis (2013) measured the daily tracking errors (i.e., $N = 1$ day) of 41 pairs of bull and bear ProShares LETFs trading in the United States, tracking domestic/international equity markets, sectors, and commodities from their respective inception dates to September 2012. His analysis revealed that on average the daily tracking errors are quite small (typically less than a couple of basis points) for the LETFs under consideration. Although, over time, large daily tracking errors (a couple of percentage points or more) do sometimes occur, a large majority of them are within the tolerable range of +/– 50 basis points.

Because of the compounding effect, the magnitude of the average tracking errors tends to increase with the holding period (i.e., when N increases). For example, in studying the performance of four leveraged gold ETFs, Leung and Ward (2015) find that these LETFs could on average underperform the naive benchmark (i.e., $\beta \cdot I_{t,t+N}$) by as much as 8 basis points per day if they are held over a 5-day period. This is close to five times the average underperformance if the same LETFs are held for only one day.

A small time-series average value of tracking errors does not necessarily imply a superb tracking performance if there happen to be large positive and negative errors that somehow cancel each other out when we take the average. To understand the nature of tracking errors, besides examining the *average value* of $TE_{t,t+N}^{NAV}$, we should also assess the *variability* of the tracking errors. One commonly adopted variability metric in the traditional nonleveraged ETF literature is the root mean square tracking error. It is also used to measure the variability of the tracking errors of LETFs. It may be formulated as

$$\text{Root Mean Sq. Error} = \sqrt{\frac{\sum\left(TE_{t,t+N}^{NAV} - \overline{TE}^{NAV}\right)^2}{m-1}} \tag{6.6}$$

where $\overline{TE}^{NAV}$ is the mean value of tracking errors over the sample period, and m is the number of N-day periods within the sample period. Note that, by subtracting out the mean value in Equation (6.6), we are focusing our attention on any deviations from the average tracking error. Market participants may therefore judge the performance of LETFs by comparing their root mean square errors. The smaller the root mean square error, the better the performance of the LETF. Tracking errors can therefore manifest themselves in at least two different ways. We may conclude that the performance of an LETF is less than desirable if the magnitude of the average tracking error is large, especially if the average error is negative as opposed to positive (i.e., consistently underperforming as opposed to outperforming the benchmark). However, even though the average tracking error is exactly zero, we may still conclude that the performance is poor if it happens that the root mean square error is large. In the second case, the poor performance is not because of consistent deviation from the benchmark, but because of the undesirability of occasionally realizing large tracking errors that might well be random.

Regression approach

Rearranging Equation (6.1) gives us the following expression:

$$r_{t,t+N}^{NAV} = \beta \cdot I_{t,t+N} + TE_{t,t+N}^{NAV}$$
$$\Rightarrow r_{t,t+N}^{NAV} = \overline{TE}^{NAV} + \beta \cdot I_{t,t+N} + e_t \tag{6.7}$$

where $\overline{TE}^{NAV}$ is the mean value of tracking errors over the sample period, and

$$e_t = TE_{t,t+N}^{NAV} - \overline{TE}^{NAV}$$

are the deviations of the observed tracking errors from their mean. Note that e_t itself has zero mean, and the root mean square tracking error introduced earlier is simply the sample standard deviation of e_t.

$$\text{Root Mean Sq. Error} = \sqrt{\frac{\sum \left(TE_{t,t+N}^{NAV} - \overline{TE}^{NAV} \right)^2}{m-1}} = \sqrt{\frac{\sum e_t^2}{m-1}} \tag{6.8}$$

Equation (6.7) is essentially a *restricted* linear regression of the LETF NAV returns ($r_{t,t+N}^{NAV}$) against the benchmark index returns ($I_{t,t+N}$). It is restricted in the sense that we impose the slope coefficient to be equal to β for the explanatory variable $I_{t,t+N}$. To obtain the two tracking error metrics discussed earlier, namely the average tracking error $\overline{TE}^{NAV}$ and the root mean square tracking error, we can simply run this restricted regression of $r_{t,t+N}^{NAV}$ on $I_{t,t+N}$ using any standard statistical packages. The estimated intercept of the regression will be our $\overline{TE}^{NAV}$, which tells us whether there is any consistent underperformance or outperformance; whereas, the sample standard deviation of the residuals from the regression will be our root mean square error, which tells us the variability of the tracking errors over time.

To assess tracking errors, it is actually more common to run the *unrestricted* version of the regression, which can be expressed as

$$r_{t,t+N}^{NAV} = a + b \cdot I_{t,t+N} + e_t \tag{6.9}$$

Unlike the restricted version, here we allow the estimate of the slope coefficient b to deviate from the target leverage ratio β. This setup allows us also to gauge the LETF performance by examining whether the target leverage ratio is indeed fulfilled in an accurate fashion. To summarize, tracking errors may take three different forms: (i) consistent underperformance or outperformance (as measured by the estimated intercept a); (ii) target leverage ratio not being achieved (as measured by the difference between β and the estimated value of b); (iii) large tracking error variability (as measured by the sample standard deviation of e_t). Regression analysis allows us to individually assess the significance of each of these three possible dimensions of tracking errors. For example, if a statistical test indicates that a is not significantly different from zero, we can conclude that there is no consistent underperformance or outperformance against the benchmark index. However, if a statistical test indicates that b is not significantly different from β, we can conclude that there is minimal error in delivering the target leverage ratio. The fact that an LETF performs well in one dimension does not necessarily imply that its performance is equally desirable when judged based on the other two dimensions. Because of the difference in their investment objectives, different market participants may pursue different dimensions of LETF performance. For example, a long-term investor will be most bothered if a is significantly negative, thus suggesting that the LETF is consistently underperforming when compared with the benchmark.[2] He or she might not be as concerned even if b is quite different from β. However, a trader using the LETF as a hedging tool will be in pursuit of the accuracy of the delivered leverage, and thus cannot afford to have even a slight difference between b and β. Yet this trader may not be bothered by the fact that this LETF tends to underperform the benchmark, given that he or she is likely to be holding the LETF for a very short period of time to fulfill ever-changing hedging needs.

When the holding period N is longer than one day, we may conduct the regression specified in Equation (6.9) in an *overlapping* fashion. For example, if N is five days and we conduct the regression on a daily basis, we will be regressing the NAV compounded returns of Day 1 to 5, Day 2 to 6, Day 3 to 7, and so forth, against the index compounded returns of Day 1 to 5, Day 2 to 6, Day 3 to 7, and so forth. The use of overlapping observations can cause the regression parameter estimates to be inefficient and hypothesis tests biased (Hansen and Hodrick, 1980). To deal with this potential bias, the standard errors of the estimates have to be adjusted before hypothesis tests are conducted. The Newey-West approach of adjustment is commonly adopted.[3]

The above regression has been conducted in a number of empirical studies on LETF tracking errors. For example, Rompotis (2013) performs regression analysis on daily tracking errors of a number of ProShares LETFs tracking a variety of underlying benchmark indices. He conducts the regressions using daily returns. He finds that the average value of a for bull LETFs in his sample is –0.3 basis points. It is both statistically and economically insignificant. The average value of a for bear LETFs is, however, much larger (about –2.0 basis points) and is highly statistically significant. Note that a two-basis-point daily underperformance can easily translate into a few percentage points of annual underperformance. More importantly, all 41 bears in his sample have a negative value of a, and many of them are not only economically significant but also statistically significant.

Regarding the slope coefficient b, he finds that most of them (regardless of whether they are bulls or bears) are very close to the target leverage ratio β. The discrepancy is considered to be negligible, and thus he concludes that the performance is in general quite good. Finally, the high R-squareds of the regressions also suggest that the variability of tracking errors is usually quite small.

Other researchers conduct overlapping regression with holding periods longer than one day. For example, Lu et al. (2009) examine the performance of four pairs of bull and bear funds tracking respectively the Dow Jones Industrial Average, the S&P 500 Index, the Nasdaq 100 Index, and the Russell 2000 Index, using return data up to the end of 2008. They find that tracking errors increase with the length of the holding period. Bull LETFs have difficulty tracking the underlying benchmark indices at the respective target leverage multiples for holding periods of one year or longer. The case for bear LETFs is worse. The slope coefficient b can become very different from the target leverage ratio β for holding periods even as short as three months. Charupat and Miu (2011) also document similar results when they conduct regression analysis on a sample of equity-based LETFs trading in Canada. Specifically, bear LETFs tend to perform worse than bull LETFs in delivering the target leverage as the holding period lengthens. The returns on bears can be very different from the naive benchmark return even for a holding period as short as three months. This lackluster performance can be mostly attributable to the compounding effect, which tends to be stronger for bears than bulls (see chapter 4). As discussed above in this chapter, as the holding period becomes much longer than a few days, we do not expect LETFs to be able to mimic the naive benchmark return of $\beta \cdot I_{t,t+N}$ because of the compounding effect, even if the fund management is in fact doing a perfect job. This highlights one of the shortcomings of the above single-factor regression approach in measuring LETF performance. LETF tracking errors are caused by different factors, as introduced at the beginning of this chapter, that are either within the control of the fund management (e.g., management fees) or outside its control (e.g., compounding effect, financing costs). The use of a single explanatory variable in Equation (6.9) does not allow us to disentangle the different sources of errors. Consequently, the effect of all the different factors may show up in both the intercept and the slope coefficient, making it difficult to interpret the regression results. For example, because of the compounding effect, the estimated value of b may be different from the fund's leverage ratio β even if the fund management is cost effective and can exactly replicate the leveraged return on a daily basis (further on in this chapter we examine a number of examples to further illustrate this point). In order to judge fund management efficiency, we need a way to control for the compounding effect, which is expected to be stronger, the longer the holding period, the higher the target leverage ratio, and for bear LETFs (as opposed to bull LETFs). Below, we introduce a couple of methods for addressing this issue.

By assuming that the underlying index return follows a geometric Brownian motion, Guo and Leung (2015) show that the compounding effect can be captured by the realized variance of daily benchmark index returns. Thus, they propose a method to control for the compounding effect by including this realized variance as the second explanatory variable in the regression analysis. Specifically, they run the following regression:

$$r_{t,t+N}^{NAV} = a + b \cdot I_{t,t+N} + c \cdot V_{t,t+N} + e_t \tag{6.10}$$

where $V_{t,t+N}$ is the realized variance of the daily benchmark index return i between time t and $t+N$, that is,

$$V_{t,t+N} = \sum_{s=t}^{t+N-1} \left(i_s - \bar{i}_{t,t+N} \right)^2$$

where $\bar{i}_{t,t+N}$ is the (arithmetic) average of daily index return between time t and $t+N$. It can also be shown that, if the LETF is doing a good job in replicating the daily leverage return, the second slope coefficient c should be close to $\frac{\beta - \beta^2}{2}$. By explicitly controlling for the compounding effect with the second explanatory variable, the intercept *a* and the first slope coefficient *b* become more direct measures of fund management efficiency. It is important to note that the third term on the right-hand side of Equation (6.10) is expected to be negative, given that $\frac{\beta-\beta^2}{2}$ is always negative for all LETFs (regardless of whether they are bulls or bears). This term in fact captures the amount of *volatility drag*. The higher the realized variance, the lower will be the LETFs' compounded return, and thus the greater will be the underperformance of LETFs when compared with the naive benchmark returns $\beta \cdot I_{t,t+N}$. The magnitude of $\frac{\beta-\beta^2}{2}$ dictates the extent of the volatility drag. Thus, volatility drag grows quadratically in the leverage ratio. Note that the magnitude of $\frac{\beta-\beta^2}{2}$ for a bear LETF is always larger than that of a bull LETF with the same magnitude of β. For example, a –2x LETF has a theoretical value of *c* equals $\frac{-2-(-2)^2}{2}=-3$; whereas, a +2x LETF has a theoretical value of *c* equals $\frac{2-(2)^2}{2}=-1$. The higher magnitude of the coefficient for the –2x LETF reflects the fact that the compounding effect is stronger for bears than for bulls, thus leading to a stronger volatility drag on bears. Haga and Lindset (2012) also use a similar two-variable regression approach to control for the compounding effect when they investigate the performance of a handful of LETFs trading in Norway. Rather than using realized variance as the second explanatory variable, they use realized standard deviation of the benchmark index returns instead. They find that *a* becomes very close to zero, and *b* very close to the target leverage ratio after including the realized standard deviation as the second explanatory variable.

To screen out the compounding effect on LETF returns, Charupat and Miu (2014) propose a different multiple-variable regression approach that not only includes information about the second moment (i.e., realized variance or standard deviation) of the daily return distribution but also higher moments. This approach is free of any distribution assumption of the underlying benchmark index return. Their *full-fledged* regression model may be expressed as the following:

$$r_{t,t+N}^{NAV} = a + b_1 \cdot I_{t,t+N} + b_2 \cdot S_{2,t,t+N} + \ldots + b_m \cdot S_{m,t,t+N} + \ldots + b_N \cdot S_{N,t,t+N} + e_t \tag{6.11}$$

where $S_{m,t,t+N}$ (m = 2 to N) denotes the m-th moment of the underlying benchmark index return over the holding period from time t to $t+N$, which can be expressed as

$$S_{m,t,t+N} = \sum_{1 \le k_1 < k_2 < \ldots < k_m \le N} \left(i_{t+k_1-1,t+k_1} \times i_{t+k_2-1,t+k_2} \times \ldots \times i_{t+k_m-1,t+k_m} \right)$$

The theoretical value of b_1 is the target leverage ratio β. The theoretical value of other slope coefficient b_m is ($\beta^m - \beta$) Thus, b_2 = ($\beta^2 - \beta$), b_3 = ($\beta^3 - \beta$), and so forth. The $N - 1$ higher-order terms (i.e., second moment, third moment, fourth moment terms, etc.) together capture the compounding effect. Once controlled for the compounding effect, the intercept and first slope coefficient b_1 will now serve as cleaner measures of the fund's tracking ability that is mainly attributable to fund management efficiency.

Given that the magnitude of the higher-order term diminishes with its order m, in practice it is suffice to run the regression with only the first three terms. That is,

$$r_{t,t+N}^{NAV} = a + b_1 \cdot I_{t,t+N} + b_2 \cdot S_{2,t,t+N} + b_3 \cdot S_{3,t,t+N} + e_t \tag{6.12}$$

A more careful examination of the second-order term reveals that it measures the autocorrelation of underlying index returns. A *trending* (*sideways*) market tends to have positively (negatively) autocorrelated returns, and thus a positive (negative) second-order term. Together with the positive slope coefficient of the second-order term (i.e., $\beta^2 - \beta > 0$ regardless of whether β is positive or negative), a *trending* (*sideways*) market will likely result in a positive (negative) compounding effect on the returns on both bull and bear LETFs, as illustrated in chapter 4. The slope coefficient of the second-order term represents the exposure of LETF investors to the autocorrelation effect.

Tracking error decomposition and related analysis

To have a more detailed understanding of the underlying drivers of tracking errors, we may also conduct analysis independently for the *compounding* and *noncompounding* components of the total tracking errors. The former captures tracking errors as a result of the compounding effect, while the latter captures those as a result of other factors (e.g., management fees, financing costs). Let us start by restating the definition of the total tracking error introduced earlier:

$$TE_{t,t+N} = r_{t,t+N} - \beta \cdot I_{t,t+N} = r_{t,t+N} - \beta \cdot \left[\prod_{j=1}^{N} \left(1 + i_{t+j-1,t+j}\right) - 1 \right] \quad (6.13)$$

where $i_{t-1,t}$ is the benchmark index return between day t–1 and day t. Here we focus on the tracking errors based on NAV returns (i.e., Equation [6.1]), as opposed to tracking errors based on market price returns (i.e., Equation [6.3]). For the convenience of illustration, we can therefore suppress the superscript *NAV* for variables $TE_{t,t+N}$ and $r_{t,t+N}$ without causing any unnecessary confusion that we are indeed examining tracking errors based on NAV returns. We can decompose the total tracking errors into its two components as follows (Charupat and Miu, 2014; Shum and Kang, 2013; Tang and Xu, 2013):

$$\begin{aligned}
&TE_{t,t+N} = r_{t,t+N} - \beta \cdot \left[\prod_{j=1}^{N} \left(1 + i_{t+j-1,t+j}\right) - 1 \right] \\
&\Rightarrow TE_{t,t+N} = r_{t,t+N} - \left[\prod_{j=1}^{N} \left(1 + \beta \cdot i_{t+j-1,t+j}\right) - 1 \right] \\
&\qquad + \left[\prod_{j=1}^{N} \left(1 + \beta \cdot i_{t+j-1,t+j}\right) - 1 \right] - \beta \cdot \left[\prod_{j=1}^{N} \left(1 + i_{t+j-1,t+j}\right) - 1 \right] \\
&\Rightarrow TE_{t,t+N} = TE_{t,t+N}^{Noncompound} + TE_{t,t+N}^{Compound}, \qquad (6.14)
\end{aligned}$$

where

$$TE_{t,t+N}^{Noncompound} = r_{t,t+N} - \left[\prod_{j=1}^{N} \left(1 + \beta \cdot i_{t+j-1,t+j} \right) - 1 \right] \tag{6.15}$$

$$\begin{aligned} TE_{t,t+N}^{Compound} &= \left[\prod_{j=1}^{N} \left(1 + \beta \cdot i_{t+j-1,t+j} \right) - 1 \right] - \beta \cdot \left[\prod_{j=1}^{N} \left(1 + i_{t+j-1,t+j} \right) - 1 \right] \\ &= \left[\prod_{j=1}^{N} \left(1 + \beta \cdot i_{t+j-1,t+j} \right) - 1 \right] - \beta \cdot I_{t,t+N} \, . \end{aligned} \tag{6.16}$$

Note that $TE_{t,t+N}^{Noncompound}$ is the difference between the LETF NAV returns and the target returns based on the compounding of the leveraged daily index returns. This component of the tracking errors is therefore free from any compounding effect. However, $TE_{t,t+N}^{Compound}$, being the difference between the target returns based on the compounding of the leveraged daily index returns and the naive benchmark returns, becomes a pure measure of compounding effect.

Based on the above decomposition, Shum and Kang (2013) find that $TE_{t,t+N}^{Noncompound}$ (mainly as a result of management fees and financing costs) could sometimes dominate $TE_{t,t+N}^{Compound}$ for their sample of highly liquid LETFs trading in both the United States and Canada in 2008 and 2009. Tang and Xu (2013) have studied the two tracking error components of 12 of the most popular equity-based LETFs trading in the United States from 2006 to 2010. They consider different holding period returns, from 2 days to 40 days. Consistent with the general impression of the effect of volatility drag, they find that the average $TE_{t,t+N}^{Compound}$ is, in general, negative. The magnitude of the average of both $TE_{t,t+N}^{Noncompound}$ and $TE_{t,t+N}^{Compound}$ is found to increase with the length of the holding period. Consistent with the findings of Shum and Kang, the results of Tang and Xu suggest that we should not focus only on tracking errors as a result of the compounding effect and overlook that as a result of noncompounding effects. For the bull LETFs in their sample, $TE_{t,t+N}^{Noncompound}$ tends to contribute more to the overall underperformance of the LETFs than $TE_{t,t+N}^{Compound}$. Conforming to the asymmetric effect of financing costs as discussed earlier, they find that $TE_{t,t+N}^{Noncompound}$ tends to be negative for bulls but positive for bears. Tang and Xu go one step further and conduct a number of regression analyses separately for $TE_{t,t+N}^{Noncompound}$ and $TE_{t,t+N}^{Compound}$. Confirming the significance of the financing effect, they find that the LIBOR market interest rate is the most important driver of the variations of $TE_{t,t+N}^{Noncompound}$ over time. They also detect some influence from the prevailing stock market volatility. For the compounding tracking errors, they find that $TE_{t,t+N}^{Compound}$ is negatively associated with the variance of the underlying index returns, while positively associated with the square of the cumulative underlying index return during the holding period. This implies that investors should invest in bull LETFs during a stable trending bull market and invest in bear LETFs during a stable trending bear market. They should, however, avoid investing in LETFs during volatile and sideways markets.

Simulation-based approach

Given that the first LETF started trading in the United States in 2006, we have a relatively short historical sample period for which we can observe the actual performance of LETFs. The findings of the empirical analysis discussed earlier therefore may not be representative of the performance we may expect LETFs to exhibit under market conditions that are different from what we have experienced in the last few years. For example, how can we be sure that the volatility drag on LETF performance as documented in the empirical literature using the last decade of data will be equally significant in other market conditions that may prevail in the future? Could the poor tracking performance that we witnessed during the financial crisis of 2007 to 2009, which has attracted a lot of attention from market regulators, be specific to that particular time period and thus not representative of their general performance? To answer these questions, we sometimes need to resort to simulation analysis.

One simulation-based approach involves specifying a certain return-generating process for the underlying benchmark index, and from that we can simulate the returns on the LETF, while assuming the daily target leverage return will be delivered exactly as promised by the fund issuer. For example, Mulvey et al. (2013) conduct a simulation analysis by assuming that the underlying benchmark index return follows a geometric Brownian motion. By changing the expected return and volatility parameters of their model, they examine the implication on the deviation of the compounded return of LETFs from their corresponding naive benchmark returns. They find that for a majority of their scenarios, LETFs underperform their naive benchmark. Not surprisingly, the underperformance worsens as volatility increases and the holding period lengthens. In chapter 7, we use a similar simulation-based approach to study the performance of different trading strategies involving LETFs.

Some may question the validity of the specific return-generating model assumed in the above simulation analysis. For example, is the log-normal distribution of the underlying index price as implied by the geometric Brownian model an accurate representation of the actual market price of the index? What if the actual distribution is different? To address these concerns, we may consider replicating LETFs' returns by using the real historical data of the underlying index return but extending the sample period to include the time period well before the inception dates of any LETFs. In doing so, we are essentially assuming history will relive itself in the future. Loviscek et al. (2014) conduct such an analysis using more than a century of historical data of Dow Jones Industrial Average index. By assuming the existence of LETFs throughout history, the authors simulate their performance over holding periods of different lengths. Contrary to the general criticism that LETFs tend to underperform their naive benchmark, the authors' simulation results suggest that LETF performance is more or less on par with the naive benchmark return. They attribute this desirable performance of LETFs to the fact that the distribution of real-world historical index return has a much higher kurtosis and is more leptokurtic than the normal distribution. These distribution characteristics can result in a positive compounding effect, thus enhancing LETFs' performance.

Detailed tracking errors and performance analysis on a sample of LETFs

In this section, we perform a number of the analyses discussed earlier in this chapter on a sample of LETFs with the objective of illustrating the details of the methodologies involved. We will also highlight some of the main empirical results. Our sample consists of six funds that track the Nasdaq 100 Index. One is a traditional nonleveraged ETF (i.e., with a leverage ratio of +1). We also include an inverse (i.e., –1x) ETF and four LETFs (i.e., +2x, –2x, +3x, and –3x). In table 6.1, we present the names of our sample of funds together with their respective inception dates, total AUM, and net expense ratios as of the end of 2012. The expense ratios essentially represent the management fees

Table 6.1 Descriptive Statistics
This table provides summary statistics of the funds in the sample.

Name/Symbol/Multiple	**Inception date**	**AUM as of Dec 2012 (million)**	**Net expense ratio (2012)**
PowerShares QQQ/QQQ/ +1x (See Note 1 below)	10 Mar 99	$30,251	0.20%
ProShares Short QQQ/PSQ /–1x	19 Jun 06	$196	0.95%
ProShares Ultra QQQ/QLD /+2x	19 Jun 06	$675	0.95%
ProShares UltraShort QQQ/ QID/–2x	11 Jul 06	$377	0.95%
ProShares UltraPro QQQ/ TQQQ/+3x	9 Feb 10	$318	0.95%
ProShares UltraPro Short QQQ / SQQQ / –3x	9 Feb 10	$134	0.95%

Note:

1. The ticker for PowerShares QQQ NASDAQ 100 ETF was changed from QQQQ to QQQ on March 23, 2011.
2. This table is reproduced from Table 1 Panel B of Charupat and Miu (2014).

charged by the fund management. They are expressed as percentages of their respective NAVs. The expense ratio of all the LETFs in our sample is 0.95% per year, which is much higher than that of QQQ (the nonleveraged ETF).

Tracking error behavior over time

We first show how the funds' returns under various market conditions deviate from the target leverage ratio, and highlight the effects of compounding, financing, and other management factors on the deviations under those different market conditions. We compute the tracking errors of the funds in our sample over nonoverlapping six-month holding periods starting on July 1, 2010. We compute two versions of tracking errors based on LETF NAV returns: (i) total tracking errors $TE_{t,t+N}$ given by Equation (6.1); and (ii) noncompounding tracking errors $TE_{t,t+N}^{Noncompound}$ given by Equation (6.15). Note that the difference between the two is the compounding tracking errors $TE_{t,t+N}^{Compound}$ introduced earlier. We report these two tracking errors, together with the compound return r of these funds, in table 6.2. In Columns 2 and 3 of the table, we also report the arithmetic mean and standard deviation of the daily returns on Nasdaq 100 Index during the corresponding six-month period.

Consider first the $TE_{t,t+N}^{Noncompound}$, which is free of compounding effect and thus mainly captures tracking errors as a result of management fees and financing costs. It is negative for all the six-month holding periods of all the funds, indicating that all the LETFs in our sample tend to underperform the benchmark index. The underperformance is likely the result of the management fees, which reduce the LETFs' returns relative to their benchmark index. With its lowest net expense ratio among all the funds under consideration, QQQ has the least negative $TE_{t,t+N}^{Noncompound}$.

An interesting pattern is observed in examining the funds' $TE_{t,t+N}^{Noncompound}$. Bull LETFs (i.e., QLD and TQQQ) have more negative $TE_{t,t+N}^{Noncompound}$ than their bear counterparts (i.e., QID and SQQQ), even though they have identical expense ratios. This phenomenon can be explained by the asymmetric effect of financing costs associated with generating the target leverage returns for bull and bear LETFs, as mentioned earlier. This financing effect lowers (enhances) the return on bull (bear) LETFs, thus making their tracking errors more (less) negative.

Compared with $TE_{t,t+N}^{Noncompound}$, the total tracking errors $TE_{t,t+N}$ are much more volatile over time. Remember that $TE_{t,t+N}$ captures tracking errors caused by all factors, including management fees, financing costs, and, most importantly, compounding effect. The magnitude of $TE_{t,t+N}$ is generally much larger than that of $TE_{t,t+N}^{Noncompound}$, indicating that the compounding effect is usually the main factor in governing the sign and magnitude of the overall tracking error.

It is interesting to note that compounding can materially affect the realized returns on the NAVs of LETFs in either a positive or negative fashion. The switching of the sign of $TE_{t,t+N}$ from period to period, as observed in table 6.2, can be explained by the difference in market conditions from one period to the other. As explained in chapter 4, a low-volatility trending market tends to result in a positive compounding effect. It therefore translates into a positive influence on $TE_{t,t+N}$ for both bull and bear LETFs. And that is exactly what we observe during the six-month period from July to December 2010, when the Nasdaq 100 Index assumed an upward trend together with a reasonably low return volatility. During this six-month period, all LETFs in our sample realize their most positive $TE_{t,t+N}$ among all the six-month periods under consideration. Both bull and bear LETFs benefit from this positive compounding effect, and the effect is stronger for LETFs with a higher leverage ratio.

However, we expect a negative compounding effect in a high-volatility sideways market, like what we witnessed for Nasdaq during the six-month period from July to December 2011. The index fluctuated substantially, but ended up with only a slightly positive return. It therefore leads to the realization of the most

Table 6.2 Half-Year Performance (in Percentage Points) of Funds Relative to Underlying Benchmarks

6-month period	**Arith. mean of daily index return**	**Std. dev. of daily index return**	**QQQ (+1x)**			**PSQ (–1x)**		
			r	$TE_{t,t+N}$	$TE_{t,t+N}^{Noncompound}$	r	$TE_{t,t+N}$	$TE_{t,t+N}^{Noncompound}$
2010/07–12	0.199	1.023	27.89	–0.16	–0.16	–23.35	4.70	–0.38
2011/01–06	0.046	0.976	5.14	–0.09	–0.09	–6.57	–1.34	–0.48
2011/07–12	0.006	1.881	–1.62	–0.13	–0.13	–3.46	–4.95	–0.55
2012/01–06	0.120	1.020	15.32	–0.12	–0.12	–14.96	0.48	–0.46
2012/07–12	0.024	0.928	2.41	–0.11	–0.11	–3.91	–1.39	–0.41
Median			**5.14**	**–0.12**	**–0.12**	**–6.57**	**–1.34**	**–0.46**
6-month period	**Arith. mean of daily index return**	**Std. dev. of daily index return**	**QLD (+2x)**			**QID (–2x)**		
			r	$TE_{t,t+N}$	$TE_{t,t+N}^{Noncompound}$	r	$TE_{t,t+N}$	$TE_{t,t+N}^{Noncompound}$
2010/07–12	0.199	1.023	60.70	4.60	–1.05	–41.84	14.26	–0.35
2011/01–06	0.046	0.976	8.68	–1.78	–0.74	–13.44	–2.98	–0.60
2011/07–12	0.006	1.881	–7.86	–4.88	–0.62	–10.49	–13.47	–0.65
2012/01–06	0.120	1.020	30.65	–0.23	–0.89	–28.32	2.56	–0.47
2012/07–12	0.024	0.928	3.31	–1.74	–0.69	–8.41	–3.36	–0.54
Median			**8.68**	**–1.74**	**–0.74**	**–13.44**	**–2.98**	**–0.54**

6-month period	Arith. mean of daily index return	Std. dev. of daily index return	TQQQ (+3x)			SQQQ (–3x)		
			$\boldsymbol{r}$	$\boldsymbol{TE_{t,t+N}}$	$TE_{t,t+N}^{Noncompound}$	$\boldsymbol{r}$	$\boldsymbol{TE_{t,t+N}}$	$TE_{t,t+N}^{Noncompound}$
2010/07–12	0.199	1.023	100.04	15.89	–1.53	–56.48	27.67	–0.29
2011/01–06	0.046	0.976	11.53	–4.16	–0.89	–20.61	–4.92	–0.56
2011/07–12	0.006	1.881	–17.29	–12.81	–0.71	–20.64	–25.12	–0.71
2012/01–06	0.120	1.020	46.77	0.45	–1.19	–40.38	5.94	–0.45
2012/07–12	0.024	0.928	3.52	–4.05	–0.86	–13.54	–5.97	–0.55
Median			**11.53**	**–4.05**	**–0.89**	**–20.64**	**–4.92**	**–0.55**

negative overall tracking errors for all LETFs during this six-month period. Again, the higher the leverage ratio, the stronger the effect.

Regression analysis

We regress the compounded NAV returns for the six funds against the contemporaneous compounded returns on the Nasdaq 100 Index following the single-variable regression approach of Equation (6.9). We run regressions based on one-week, one-month, one-quarter, and one-year holding periods, respectively. We run the regressions from February 2010 to December 2012. We use overlapping weekly returns to generate returns for holding periods of one month (four weeks), one quarter (12 weeks), and one year (52 weeks). To correct for the bias in using overlapping time-series data, we use the Newey-West procedure to calculate the standard errors of the estimated intercept and slope coefficient. The results are reported in table 6.3.

By looking at the results shown in table 6.3, you will notice that the tracking ability of our sample of LETFs degenerates as the holding period lengthens. The performance becomes particularly poor for quarterly and annual holding periods. All of the estimated intercept a are negative, indicating underperformance on average. Moreover, quite a few of the estimated slope coefficients b are both economically and statistically different from the respective target leverage ratios. Let us take SQQQ, the –3x bear LETF, as an example. For the holding period of one quarter. The estimated intercept is –0.0555, suggesting an average return shortfall of 5.55% per quarter as compared with the benchmark. It is also troubling to find out that the estimated slope coefficient is –2.273, which is quite different from the promised leverage ratio of –3. The situation is even worse if you hold this LETF for a year. The average return shortfall becomes 33.76% per year, which is even more than four times the 5.55% per quarter shortfall for the quarterly holding period, as mentioned above. The slope coefficient of –0.969 is far away from

Table 6.3 Single-Variable Regression Results

This table displays the coefficients from the regressions of the NAV returns of the LETFs over various holding periods on the returns of Nasdaq 100 Index (using Equation [6.9]). Overlapping observations are used for holding periods of one month and longer, in which case the standard errors of the estimates are calculated using the Newey-West procedure. The significance for the intercepts is based on whether they are equal to zero. The significance for the slope coefficients is based on whether they are equal to the funds' target leverage ratios.

Fund	Leverage ratio	One week		One month		One quarter		One year	
		a	b	a	b	a	b	a	b
QQQ	+1x	–0.0001**	1.000	–0.0004**	0.999	–0.0010**	1.000	–0.0064**	1.009**
PSQ	–1x	–0.0002	–0.996	–0.0018**	–0.972*	–0.0109**	–0.885**	–0.0785**	–0.611**
QLD	+2x	–0.0003	2.004	–0.0019**	2.020	–0.0110**	2.091**	–0.0951**	2.367**
QID	–2x	–0.0003	–1.987	–0.0043**	–1.908*	–0.0288**	–1.647**	–0.1961**	–0.908**
TQQQ	+3x	–0.0003	3.013	–0.0035*	3.054	–0.0264**	3.261*	–0.2660**	4.104**
SQQQ	–3x	–0.0003	–2.972	–0.0083**	–2.801**	–0.0555**	–2.273**	–0.3376**	–0.969**

** and * indicate statistical significance at the 0.01 and 0.05 levels respectively.

the promised leverage ratio of –3. A similar lackluster performance is also observed for the two 2-time leveraged LETFs QLD and QID, albeit less dramatic than that of SQQQ.

We cannot, however, easily judge the performance of the fund management by just using information from table 6.3, because we cannot separate the compounding effect from other management factors that cause the resulting return shortfall and the deviation of the slope coefficient from the target leverage ratio. The above single-variable regression therefore does not facilitate a comparison of cost effectiveness among different LETFs. For example, by only looking at the quarterly regression results of table 6.3, a market participant may hastily conclude that the management of SQQQ is much less efficient than that of TQQQ given that (i) the estimated intercept of the former (i.e., –0.0555) is more negative than that of the latter (i.e., –0.0264), and (ii) the estimated slope coefficient of the former (i.e., –2.273) is more deviated from its leverage ratio of –3 than that of the latter (i.e., 3.261) from its ratio of 3. This conclusion regarding relative fund management efficiency, however, may be invalid given that the compounding effect is expected to be stronger for bear LETFs than bull LETFs of the same leverage magnitude. And thus the poor performance of SQQQ relative to TQQQ, based on the above regression analysis, might as well be the result of the difference in the impact of compounding effect on these two kinds of LETFs, rather than anything to do with their management efficiency.

To have a more definite conclusion regarding relative management efficiency, we need a way to disentangle the compounding effect from other sources of tracking errors in measuring the tracking errors of LETFs. One way to disentangle the effects is by running the three-variable regression of Equation (6.12). Charupat and Miu (2014) conducted the regression with the same sample of LETFs, tracking the Nasdaq 100 Index under different holding periods from February 2010 to December 2012. They use overlapping, weekly returns to generate returns for holding periods of one month and one quarter. The regression results are reported in their Table 4 Panel B, which is reproduced in table 6.4 here. They test whether the intercept (a) is different from zero, and whether the three slope coefficients (b_1, b_2, and b_3) are different from their theoretical values of β, $\beta^2 - \beta$, and $\beta^3 - \beta$ respectively, according to the fund's leverage ratio.

The estimated intercepts reported in table 6.4 are less negative than their respective values reported in table 6.3. This is because the intercept in table 6.4 is now free from the compounding effect that tends to worsen the return shortfall. The intercepts now mainly capture the management fees and expenses. For example, the estimated intercept of the one-quarter holding period regression of QLD is –0.32%, which is not very different from one quarter of its net expense ratio of 0.95% per year. The first slope coefficients (b_1), reported in table 6.4, now become much closer to the respective target leverage ratios (β) than the slope coefficients reported in table 6.3 after we screen out the compounding effect, which is outside the control of the fund management. Given the reasonably small return shortfall (as measured by the intercept) and the close resemblance of b_1 to β, you may want to conclude that the fund management has in fact been quite effective in accurately delivering the target leverage return.

You will observe an interesting pattern when you compare the intercept terms of bull LETFs with their bear counterparts of the same magnitude of leverage. Bull LETFs tend to have more negative intercepts, thus suggesting larger underperformance with respect to the underlying index, than their bear counterparts. For example, the intercept term in the quarterly return regression of TQQQ is –0.43%, while that of SQQQ is only –0.38%. This observation is consistent with the asymmetric effect of financing costs on bull versus bear funds. The return on the former is negatively affected by the financing costs, while that of the latter benefits from the costs. This effect therefore increases (lessens) the underperformance of bull (bear) LETFs.

After screening out the compounding effect, this regression approach allows us to arrive at a more definite conclusion

Table 6.4 Three-Variable Regression Results

This table displays the coefficients from the regressions of the NAV returns of the LETFs based on equation (6.12). Overlapping observations are used for holding periods of one month and longer, in which case the standard errors are estimated using the Newey-West procedure. The significance for the coefficients is based on whether they are equal to the corresponding theoretical values.

Fund (Leverage ratio)	Coef.	Theor. value	Weekly	Monthly	Quarterly	Fund (Leverage ratio)	Coef.	Theor. value	Weekly	Monthly	Quarterly
QQQ (+1x)	a	0	–0.0000**	–0.0002**	–0.0005**	PSQ (–1x)	a	0	–0.0002**	–0.0008**	–0.0022**
	b_1	1	1.000	0.999*	0.999**		b_1	–1	–1.001	–1.000	–0.999
	b_2	0	–0.007	0.007	0.003		b_2	2	2.051	2.002	2.021*
	b_3	0	5.541**	–0.098	–0.091*		b_3	0	–1.495	–0.381*	–0.130
QLD (+2x)	a	0	–0.0003**	–0.0010**	–0.0032**	QID (–2x)	a	0	–0.0002**	–0.0010**	–0.0030**
	b_1	2	2.000	1.999**	1.998*		b_1	–2	–2.001	–1.998**	–1.992**
	b_2	2	1.957	1.972*	1.925**		b_2	6	6.072	5.976*	5.930**
	b_3	6	4.131	5.857	6.374*		b_3	–6	–3.093	–6.133	–5.210**
TQQQ (+3x)	a	0	–0.0003**	–0.0013**	–0.0043**	SQQQ (–3x)	a	0	–0.0003**	–0.0011**	–0.0038**
	b_1	3	3.000	3.000	3.008		b_1	–3	–3.000	–2.995**	–2.976**
	b_2	6	5.946*	5.882*	5.623**		b_2	12	12.017	11.855**	11.651**
	b_3	24	22.373	23.600	26.522**		b_3	–24	–29.064	–24.520	–20.578**

** and * indicate statistical significance at the 0.01 and 0.05 levels respectively.

Note: This table is reproduced from Table 4 Panel B of Charupat and Miu (2014).

regarding relative management efficiency. Remember, we were saying we cannot draw a conclusion regarding the relative performance of TQQQ and SQQQ simply based on the previous regression results as reported in table 6.3. Now with the benefit of the results reported in table 6.4, we can finally have a verdict regarding their relative cost effectiveness that is free from any contamination of the compounding effect. Based on the quarterly regression results of these two funds as reported in table 6.4, SQQQ actually has a smaller average return shortfall than TQQQ given its less negative estimated intercept (i.e., –0.0038 vs. –0.0043). In other words, SQQQ is considered to be more cost effective than TQQQ.

Conclusion

In this chapter, we examine the tracking errors and performance of LETFs. We discuss in detail the different sources of tracking errors that are either within or outside the control of the fund managers. In particular, we highlight the importance of compounding and financing effects in dictating the fund performance that are specific to LETFs but not traditional nonleveraged ETFs. We then introduce different analytical approaches and statistical metrics used to measure LETFs' tracking errors. Finally, we illustrate the implementation of a number of analytical approaches in assessing the performance of a sample of LETFs.

7
Trading Strategies

The LETF is a relatively new class of investment instruments with limited historical data for judging its performance. The embedded leverage makes LETFs risky and speculative in nature. There has been lots of skepticism among market participants and market regulators since witnessing their poor performance during the recent financial crisis in 2008–2009. Despite this concern, however, the LETF has been a very popular class of ETFs, attracting lots of interest among, in particular, short-term traders, who use LETFs to express their directional views on the underlying benchmarks. LETFs also offer a relatively low-cost solution for retail investors to implement certain trading strategies that were quite costly for them in the past. The volume of LETF trades represents a substantial portion of total ETF trades even though the assets under their management are only a small fraction of the entire ETF markets.

This chapter provides an overview of a number of trading strategies using LETFs. We start with the idea of *optimal* leverage ratio, and how to select an appropriate leverage ratio that satisfies the required risk-return trade-off. We then examine the idea of shorting LETFs. If a long position in LETFs is plagued by volatility drag (as discussed in chapter 6), shorting LETFs should be a rewarding investment strategy. We outline the costs and benefits of shorting LETFs. We then consider a number of *pair strategies* (i.e., trading strategies involving both bull and bear LETFs), emphasizing the risks that can be involved. Finally, we illustrate the effectiveness of some of these strategies based on a number of simulation analyses.

Choice of leverage ratio

We first introduce the idea of an *optimal* leverage ratio. Suppose an investor expects a bullish market condition and would like to use bull LETFs to capitalize on the expected positive changes in the underlying benchmark index value. Picking a higher leverage (e.g., +3x as opposed to +2x or +1x) is supposed to result in a higher return given the higher leverage effect. However, the investment return will be subject to a larger volatility drag, which is the result of the compounding effect due to the expected market volatility, as discussed in previous chapters. The investment performance is therefore the net result of these two offsetting effects. Given that the former positive effect is *linear* in the leverage ratio, while the latter negative effect due to volatility drag grows *quadratically* in the leverage ratio (refer to the coefficient c of the third term on the right-hand side of Equation [6.10]), there exists a unique leverage ratio that will maximize the expected return in optimally trading off these two opposing effects. In selecting the leverage ratio for bear LETFs, a similar trade-off is also applicable. Suppose an investor expects a bearish market condition and would like to use bear LETFs to profit from the expected negative changes in the underlying benchmark index value. Choosing a more negative leverage (e.g., –3x as opposed to –2x or –1x) is supposed to result in a higher return due to the higher leverage effect. But at the same time, the performance will be more negatively affected by the higher volatility drag as a result of the larger magnitude of the leverage ratio. Thus, there also exists a unique optimal (negative) leverage ratio that will maximize the expected return according to the predicted market condition.

Giese (2010) formalizes this idea by assuming the underlying benchmark index return follows a geometric Brownian motion. Based on his setup, the expected continuously compounded return for any bull or bear LETF can be expressed as the following equation:

$$\beta \cdot \mu - (\beta - 1) \cdot r - \frac{1}{2}\beta \cdot (\beta - 1) \cdot \sigma^2 \tag{7.1}$$

where β is the leverage ratio, r is the interest rate (i.e., London interbank offered rate [LIBOR]), μ is the expected annual return on the underlying index, and σ is the annualized volatility of the underlying index. In deriving Equation (7.1), we ignore any management fees and other costs that are incurred in delivering the leveraged returns. Equation (7.1) tells us that the expected return is a quadratic function of the leverage ratio β. The first term of Equation (7.1) captures the leverage effect mentioned above, which is linear in β; whereas, the third term represents the offsetting volatility drag, which is always negative and with magnitude that grows quadratically in β.[1] We can solve for the optimal leverage ratio β^* that will maximize the expected return given specific set of values of μ, σ, and r, which is

$$\beta^* = \frac{1}{2} + \frac{\mu - r}{\sigma^2} \tag{7.2}$$

In table 7.1, we report the optimal leverage ratio β^* under four different market conditions: low-volatility bullish market, high-volatility bullish market, low-volatility bearish market, and high-volatility bearish market. In figures 7.1 to 7.4, we plot the expected return against the leverage ratio β under these four different market conditions. The optimal leverage ratio can be very different depending on the specific market outlook. Besides the expected return on the underlying benchmark index (μ), the volatility of the index (σ) also plays an important role in dictating the optimal strategy. For example, with the same bullish outlook of 10% expected return on the benchmark index, the optimal leverage will be 3.17 if the volatility is 15%, while it will be 1.46 if volatility becomes 25%. In other words, it is optimal

Table 7.1 Optimal Leverage Ratio under Different Market Conditions

	Bullish market		Bearish market	
	Low volatility	**High volatility**	**Low volatility**	**High volatility**
μ	10%	10%	–5%	–5%
σ	15%	25%	15%	25%
r	4%	4%	4%	4%
β^*	3.17	1.46	–3.50	–0.94

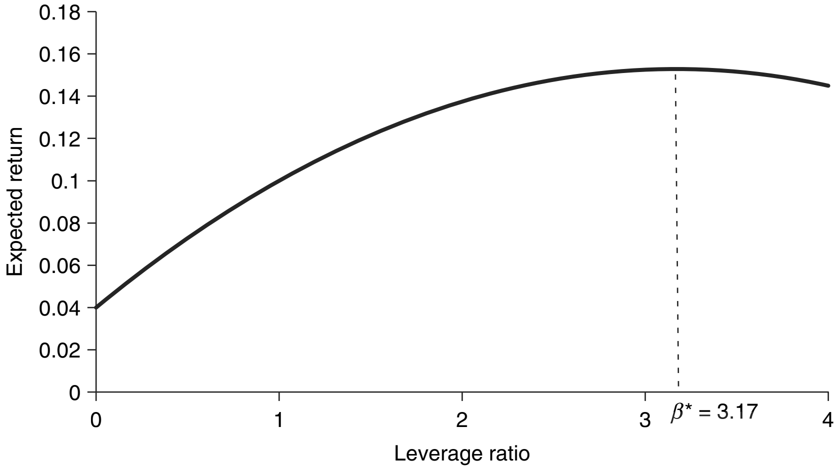

Figure 7.1 Expected Return under Low-Volatility Bullish Market Conditions.

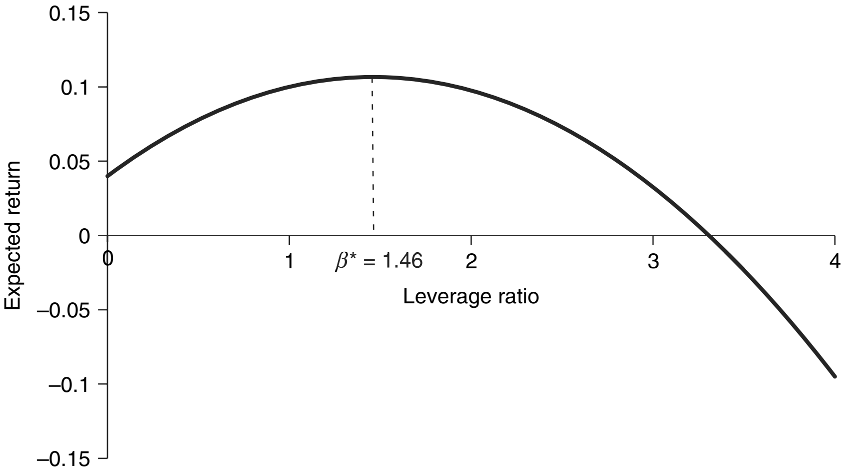

Figure 7.2 Expected Return under High-Volatility Bullish Market Conditions.

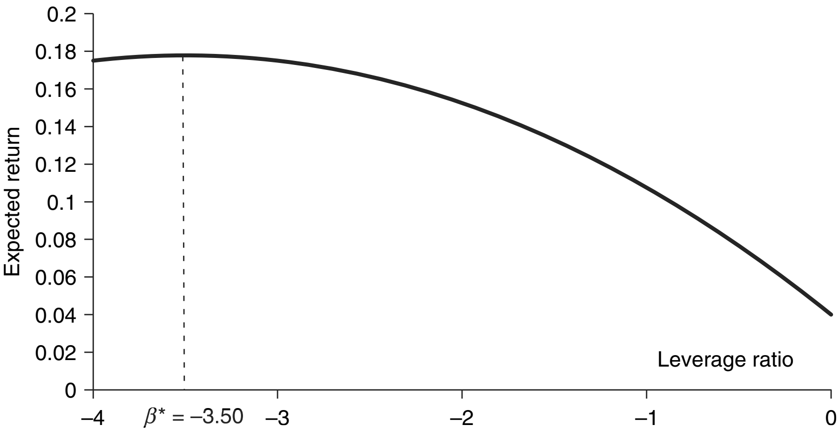

Figure 7.3 Expected Return under Low-Volatility Bearish Market Conditions.

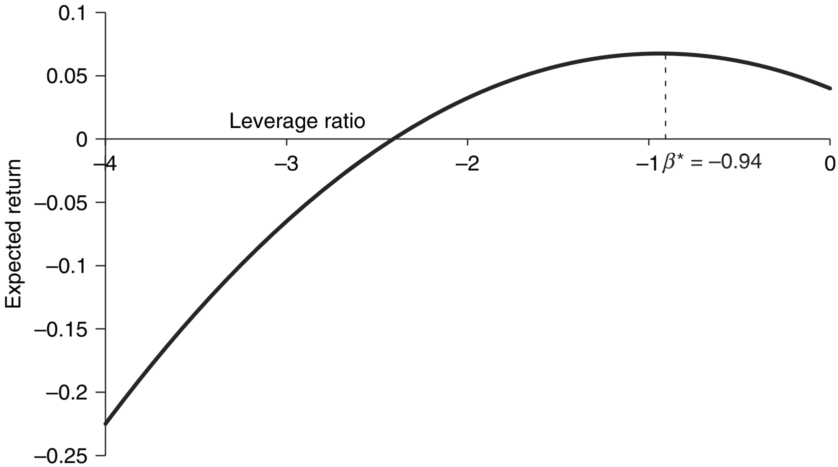

Figure 7.4 Expected Return under High-Volatility Bearish Market Conditions.

to choose a +3x LETF as opposed to a +2x or +1x fund under the former market condition. However, if you expect the latter market condition, it is better to choose a +2x or +1x fund rather than a +3x LETF. It is optimal to reduce the leverage in a high-volatility market given the strong volatility drag. Comparing the low-volatility bearish market plot with that of the high-volatility bearish market, we also witness how higher volatility will significantly lower the magnitude of the optimal leverage ratio.

The pursuit of maximum expected return might not be a wise strategy if risk is not under control. Since, by design, the LETF is a risky investment vehicle, risk concern can easily be a controlling factor in devising LETF trading strategies. In general, the higher the leverage ratio, the riskier the LETF. Moreover, the riskiness of an LETF investment will increase with the length of the holding period (i.e., your investment horizon). How does the risk-return trade-off affect the choice of leverage ratio? You can address this issue, for example, under a mean-variance framework. Suppose you assume the underlying benchmark index return follows a geometric Brownian motion. The expected holding period return on an LETF over investment horizon *T* and the corresponding standard deviation of return can be expressed as

$$E[R] = e^{[\beta(\mu-r)+r]T} - 1 \tag{7.3}$$

$$Std.dev.[R] = e^{[\beta(\mu-r)+r]T}\sqrt{e^{\beta^2\sigma^2 T} - 1} \tag{7.4}$$

where β is the leverage ratio, r is the interest rate (i.e., LIBOR), μ is the expected annual return on the underlying index, and σ is the annualized volatility of the underlying index. Any management fees and other costs that are incurred in delivering the leveraged returns are ignored in deriving Equations (7.3) and (7.4). To ensure that the expected return can cover the risk assumed and thus that we satisfy an appropriate risk-return trade-off, an LETF investor can require that the ratio of $E[R]/Std.dev.[R]$ be higher than some threshold value when he or she chooses the leverage ratio. This idea was first introduced by Leung and Santoli (2012).[2]

Let us illustrate with a numerical example. Suppose the expected return on the underlying index μ equals 10%, its return volatility is 15%, LIBOR is 4%, and the intended length of holding period *T* is two years. Based on this set of parameters, we compute the expected holding period returns and the corresponding standard deviations of returns for different LETFs with hypothetical leverage ratios of +1x, +1.5x, +2x, +2.5x, and +3x, respectively. The results are tabulated in table 7.2 and plotted

Table 7.2 Expected Returns and Standard Deviations of Returns for Different LETFs

Expected returns $E[R]$ and standard deviations of returns $Std.dev.[R]$ over a two-year holding period for different LETFs with hypothetical leverage ratios of +1x, +1.5x, +2x, +2.5x, and +3x, respectively. The ratios of expected returns to standard deviations of returns are reported in the last column.

Leverage ratio	$E[R]$	$Std.dev.[R]$	$E[R]/Std.dev.[R]$
+1.0	0.221	0.262	0.845
+1.5	0.297	0.423	0.701
+2.0	0.377	0.612	0.617
+2.5	0.462	0.833	0.555
+3.0	0.553	1.097	0.504

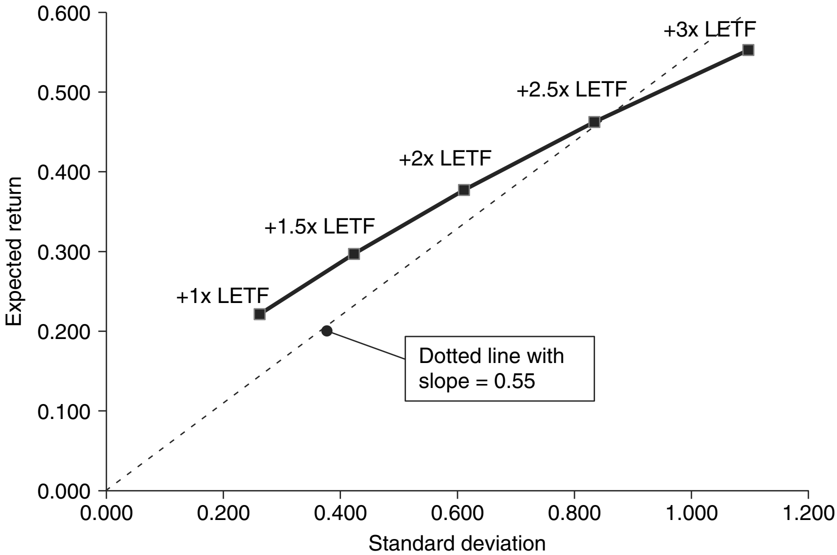

Figure 7.5 A Plot of Expected Two-Year Holding Period Returns against the Corresponding Standard Deviations of Returns for Different LETFs with Hypothetical Leverage Ratios of +1x, +1.5x, +2x, +2.5x, and +3x, Respectively.

in figure 7.5. Note that the return-risk trade-off ratio $E[R]/Std.dev.[R]$ decreases as the leverage ratio increases. In other words, as leverage increases, we need to assume more risks per unit of return we expect to generate. Suppose the minimum acceptable trade-off ratio is 0.55. Then, we should not be holding an LETF with a hypothetical leverage ratio that is higher than +2.5 based on the results reported in table 7.2. In other words, the +3x LETF is not a suitable investment vehicle given the threshold trade-off ratio and under this particular market condition.

Shorting LETFs

In previous chapters, we illustrate how the daily rebalancing of the exposure in the pursuit of a constant leverage ratio results in the compounding effect, which leads to deviations of LETFs' returns from the leveraged returns of their underlying benchmark indices. As demonstrated in chapter 6, the tracking errors as a result of the compounding effect are larger, the longer the holding period and the higher the magnitude of the leverage ratio. Furthermore, bear LETFs are subject to a stronger compounding effect than their bull counterparts of the same magnitude of leverage ratio. Moreover, the nature of the compounding effect is specific to the realized path of returns of the underlying index. A volatile and sideways market tends to result in a *negative* compounding effect where LETFs underperform their leveraged benchmark. In contrast, a calm and trending market tends to result in a *positive* compounding effect where LETFs outperform their leveraged benchmark. Although the simulation analysis conducted by Loviscek et al. (2014), which uses more than a century of Dow Jones Industrial Average Index data, suggests that LETFs' performance can be more or less on par with their leveraged benchmark, the general consensus is that a negative compounding effect (commonly referred to as volatility drag) seems to be more likely to occur in typical market conditions experienced in the last decade.

If long positions in LETFs are more likely to underperform because of volatility drag, shorting LETFs should be on average a better alternative. That is, if you are bullish about the market, then you should be shorting a bear LETF rather than investing in a bull LETF; whereas, if you are bearish, you should be shorting a bull LETF rather having a long position on a bear LETF. As opposed to holding the long position that is more likely to be negatively influenced by the compounding effect, you are more likely to gain from the compounding effect if you hold the short position instead (i.e., like accumulating a *volatility dividend* over time).

Before discussing the costs and benefits of the short strategy in details, let us illustrate the effectiveness of this strategy by considering the returns on ProShares Ultra S&P 500 (ticker: SSO) and ProShares UltraShort S&P 500 (ticker: SDS), respectively the +2x and –2x LETFs tracking returns on the S&P 500 Index. In table 7.3, we present the semiannual returns on these two LETFs from January 2007 to December 2014 together with the contemporaneous semiannual returns on the S&P 500 Index (in Column 2), the arithmetic means (in Column 3), and the standard deviations (in Column 4) of the daily index return within each six-month period.[3]

Suppose an investor would like to use these two LETFs to obtain leveraged returns based on the S&P 500 Index. There are two alternative strategies that this investor can pursue at the beginning of each six-month period. The first is the *long-only strategy*, in which the investor will *buy* the bull LETF SSO if market is expected to go up in the next six months and will *buy* the bear LETF SDS if market is expected to go down in the next six months. The alterative strategy is the *short-only strategy*, in which the investor will *short* the bear LETF SDS if market is expected to go up and will *short* the bull LETF SSO if market is expected to go down. Suppose this investor can always correctly anticipate the directional change of the S&P 500 Index over the next six months. What are the outcomes from these two alternative strategies during our sample period given his or her superior prediction ability? The semiannual returns of these two strategies are reported in Columns 7 and 8 of table 7.3. In this simple illustration, we ignore any costs related to short selling.

In implementing the long-only strategy, by correctly anticipating the negative returns of the S&P 500 Index in 2007/07–12, 2008/01–06, 2008/07–12, 2010/01–06, and 2011/07–12, the investor bought SDS at the beginning of these five six-month

Table 7.3 *Long-Only* versus *Short-Only* Strategy

Semiannual returns on the S&P 500 Index (the underlying benchmark index), the arithmetic means, and the standard deviations of the daily index return are presented in Columns 2 to 4. Semiannual returns of SSO and SDS are reported in Columns 5 and 6. Returns from the *long-only* and *short-only* strategies are reported in Columns 7 and 8. The sample period is from January 2007 to December 2014.

Time period	**S&P 500 Index**			**Semiannual return of SSO**	**Semiannual return of SDS**	**Semiannual return of *long-only* strategy**	**Semiannual return of *short-only* strategy**
	Semiannual return	**Arith. mean of daily index return**	**Std. dev. of daily index return**				
2007/01–06	6.00%	0.050%	0.712%	10.14%	–8.06%	10.14%	8.06%
2007/07–12	–2.33%	–0.011%	1.231%	–8.27%	4.73%	4.73%	8.27%
2008/01–06	–12.83%	–0.101%	1.339%	–25.14%	23.81%	23.81%	25.14%
2008/07–12	–29.43%	–0.215%	3.385%	–57.17%	29.34%	29.34%	57.17%
2009/01–06	1.78%	0.038%	2.203%	0.37%	–22.00%	0.37%	22.00%
2009/07–12	21.30%	0.165%	1.064%	47.11%	–36.64%	47.11%	36.64%
2010/01–06	–7.57%	–0.047%	1.290%	–15.18%	7.45%	7.45%	15.18%
2010/07–12	22.02%	0.168%	0.961%	49.12%	–36.75%	49.12%	36.75%
2011/01–06	5.01%	0.050%	0.810%	10.75%	–13.48%	10.75%	13.48%
2011/07–12	–4.77%	–0.012%	1.908%	–12.08%	–6.45%	–6.45%	12.08%
2012/01–06	8.31%	0.076%	0.849%	18.04%	–19.09%	18.04%	19.09%
2012/07–12	4.70%	0.049%	0.759%	10.76%	–13.24%	10.76%	13.24%
2013/01–06	12.63%	0.099%	0.779%	27.29%	–24.51%	27.29%	24.51%
2013/07–12	15.07%	0.112%	0.611%	33.92%	–27.39%	33.92%	27.39%
2014/01–06	6.05%	0.050%	0.674%	13.01%	–14.23%	13.01%	14.23%
2014/07–12	5.03%	0.041%	0.757%	11.08%	–13.33%	11.08%	13.33%

periods and realized returns of 4.73%, 23.81%, 29.34%, 7.45%, and −6.45% respectively. It is a sad surprise for the investor to realize a negative return from holding SDS in the second half of 2011 even though he or she did accurately predict the negative return of the index. For all the remaining six-month periods, the investor was long in SSO because he or she correctly anticipated a positive return on the index. If the investor had adopted the short-only strategy, he or she should have shorted SSO in 2007/07–12, 2008/01–06, 2008/07–12, 2010/01–06, and 2011/07–12, given the anticipation of a bear market, and would have realized returns of 8.27%, 25.14%, 57.17%, 15.18%, and 12.08%, which are respectively the negatives of the returns on SSO during those five time periods. For all the other remaining six-month periods, the investor should have shorted SDS in correctly anticipating positive index returns, thus realizing the negative of the returns on SDS.

Although the short-only strategy did not always produce a higher return than the long-only strategy, the former did outperform the latter in a majority of the six-month periods (11 out of the total of 16). The five six-month periods (i.e., 2007/01–06, 2009/07–12, 2010/07–12, 2013/01–06, and 2013/07–12), in which the long strategy outperformed the short strategy, are characterized by either a low-volatility market condition (as indicated by the low standard deviations of daily index returns reported in Column 4) or a trending market condition (as indicated by the large magnitude of the arithmetic mean daily index return reported in Column 3), or both. These are the market conditions under which the compounding effect tends to be positive and thus benefits the long strategy at the expense of the short strategy. On the other hand, the short strategy outperformed the long strategy by the most during the darkest times of the global financial crisis (the second half of 2008 and the first half of 2009) and the European sovereign debt crisis (in the second half of 2011) when high volatility prevailed. In figure 7.6, we plot the cumulative investment values of the two strategies, both starting from a notional value of $1 at the beginning of January 2007.

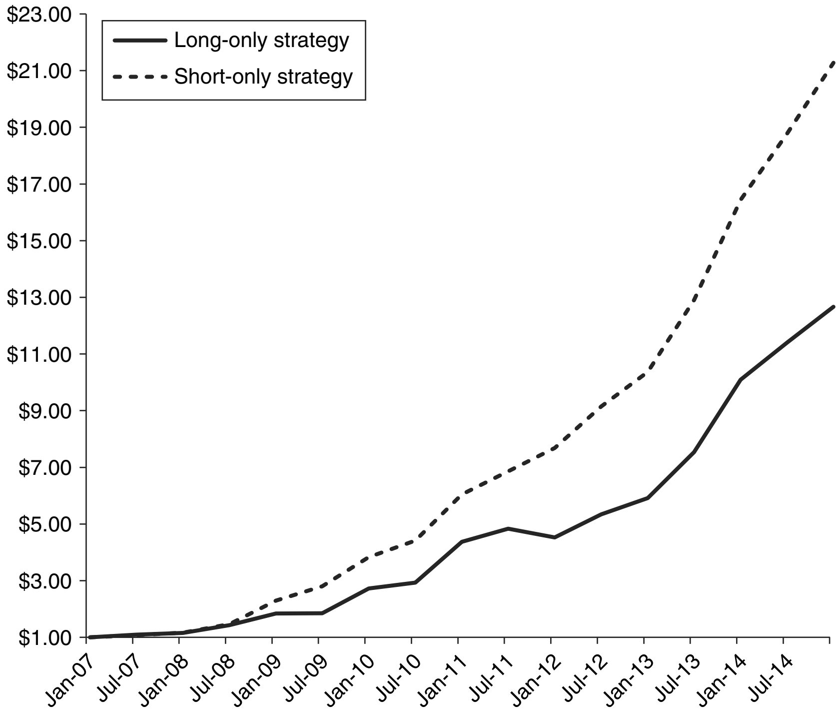

Figure 7.6 Cumulative Investment Value of *Long-Only* versus *Short-Only* Strategy. We plot the cumulative investment value of long-only and short-only strategies using SSO and SDS from January 2007 to December 2014 by starting with $1 at the beginning of January 2007.

The final accumulated investment value of the short-only strategy at the end of this eight-year period is almost twice that of the long-only strategy.

Of course, the assumption of a perfect prediction of directional changes in the market is at best simplistic. But, in reality, we still have the choice of implementing a long or short strategy based on our expected directional changes in the market that might or might not materialize in the way we expected. It is important to understand that the penalty of making the wrong bet is likely to be heavier for the short strategy. Unlike a long position that has an unlimited upside gain but limited downside risk (i.e., losing all of your principal invested), a short position has a limited upside gain (i.e., the full proceed obtained from short selling) but unlimited downside risk. We will quantify the risks of shorting LETFs in the last section of this chapter, in which we conduct a number of simulation exercises to study the risk-adjusted returns of some of the strategies considered in this chapter.

Before we explore other strategies involving a short position in LETFs, let us first summarize the key factors that govern the difference between the long and the short LETF strategy in terms of their return and risk. To be more explicit, suppose a market participant expects a positive return on the S&P 500 Index. Then he or she can either adopt a long strategy by buying SSO or a short strategy by short selling SDS. First, as demonstrated above with our numerical example, the short strategy tends to outperform because of the compounding effect. Unlike the long position that suffers from *volatility drag*, the short position enjoys a *volatility dividend* in volatile market conditions. This is especially true in a sideways market. Second, while the performance of the long position is hurt because of the need to pay management fees based on the expense ratio of the fund, the short position is effectively *earning* the management fees that enhance its return. Third, because of the need for the swap counterparties to rebalance their exposure at the end of each trading day so as to deliver the constant target leverage ratio, there can be price pressure on the underlying index when a large amount of unidirectional market orders arrive at the same time toward the end of the trading session. This may lead to pricing inefficiency and provide a window of opportunity for predatory trading to profit from implementing a front-running strategy in anticipation of the end-of-day rebalancing needs of the swap counterparties. A study by Shum et al. (2015) provides empirical evidence that such a front-running strategy did pay-off handsomely during the volatile periods of 2008–2010. If predatory traders are profiting from end-of-day rebalancing activities, LETF investors must be bearing the costs. The unfavorable transaction prices of the trades executed by the swap counterparties in rebalancing will translate into implicit costs for LETF shareholders, thus lowering their returns. In studying the performance of a large sample of sector LETFs (tracking both domestic and foreign market indices), Dobi and Avellaneda (2013) find that the resultant *slippage* in LETF returns can easily amount to more than 100 basis points per year. If a long position in an LETF is negatively affected by end-of-day rebalancing activities and predatory trading, the performance of a short position in the LETF must be benefiting from such effects.

Having discussed the advantages of adopting a short LETF strategy (as opposed to a long LETF strategy), what are the costs and risks when we short sell LETFs? Short selling LETFs is subject to the same kind of costs and risks encountered by the short sellers of any stocks. First, before you can initiate a short sale, you need to be able to borrow the stock you want to short sell. Although it seems that it is in general not difficult to borrow many, if not most, of the popular LETFs even for retail investors, there is always the risk of being forced to partially or fully close out the short position at the request of the brokerage firm if it becomes difficult to borrow the particular LETF.[4] Second, short sellers need to post margins and they thus incur a funding cost. They also need to set aside capital in preparing for margin calls. Failing to answer a margin call (especially during volatility market conditions) can be costly for the short seller as the position may be closed out at an undesirable price level. Third, short sellers are responsible for any cash dividends distributed by the fund company. Fourth, short sellers have to pay the borrowing costs of the stock that they borrow. Dobi and Avellaneda (2013) find that the stock-loan rates of LETFs are not trivial and can easily be more than 100 basis points (per year), thus almost offsetting the benefit from slippage due to end-of-day rebalancing, as mentioned above. Finally, unlike the long position, of which the maximum loss is limited to the amount of principal invested, short sellers are subject to an unlimited amount of potential loss from their short positions. This is particularly a concern for short sellers of LETFs given the leveraged nature of the returns. As discussed in chapter 4, the return distribution of LETFs is highly skewed where, although it is rare to realize a very high return, a return can be extremely high given the built-in leverage. In other words, for the investors shorting LETFs, although large negative returns are not common, the magnitude of extremely large losses can be overwhelming during a *perfect storm*.

Pair strategies

In the previous section, we examined the viability of the short strategy as an alternative way for market participants to express their views on the future changes in the underlying index. For example, a market participant who expects a positive return on the S&P 500 Index will short sell the bear LETFs (e.g., SDS) on this index; whereas, he or she will short sell the bull LETFs (e.g., SSO) if a negative return is expected. This kind of strategy is of course risky, and the outcome is dictated by the realized market return. You may hedge against the effect of changing index returns while still being able to enjoy the benefits of a volatility dividend, earning the expense ratio, and slippage as mentioned in the previous section by simultaneously shorting both the bull and bear LETFs on the same underlying index. It is important to note that this kind of pair strategy will not be completely *delta neutral* with respect to the underlying index unless you rebalance (i.e., equalize) your short exposures on the bull and bear LETFs in a frequent fashion. If you do not rebalance your pair portfolio, when the market continues to go up, your short exposure to the bull will outweigh your short exposure to the bear, and thus your portfolio will end up with a *negative* delta (i.e., your portfolio returns are negatively associated with the underlying index returns). But if the market assumes a downward trend, your short exposure to the bear will outweigh your short exposure to the bull, resulting in a *positive* delta for your portfolio (i.e., your portfolio returns are positively associated with the underlying index returns). Even if you are willing to incur the transaction costs of frequent rebalancing and equalize your exposures on the bull and bear on a daily basis, you might not be completely free from any price risk since the daily tracking errors of the bull and bear are not necessarily offsetting each other.

How profitable is the pair strategy? And what are the risks involved? Dobi and Avellaneda (2013) examine the profitability from the *daily-rebalanced* pair strategy using sector LETFs tracking indices such as the MSCI Emerging Market, Dow Jones US Financials, Dow Jones US Read Estate, Russell 2000, FTSE China 25, and MSCI Brazil. Their sample consists of a total of 21 pairs of LETFs over the time period from 2009 to 2011. They construct their LETF pairs by matching a bull with its bear counterpart with the same magnitude of target leverage ratio. For example, Direxion Daily Small Cap Bull (ticker: TNA), a +3x LETF tracking Russell 2000, and Direxion Daily Small Cap Bear (ticker: TZA), a –3x LETF tracking the same index, constitute a pair. The pair strategy is initiated by shorting the same amount of the two LETFs. At the end of each trading day, we will short sell more of the one that has dropped in price, while buying back some of the other that has increased in price, so as to rebalance the two short exposures.[5] Based on the analysis of Dobi and Avellaneda, the annualized mean returns of the pair strategy range from 0.7% to 52.5% across their sample of 21 pairs of LETFs. Most of the mean returns are highly statistically significant. The authors attribute these abnormal gains mainly to the slippage as a result of front-running trading related to end-of-day exposure rebalancing. A large proportion of the gains, however, is offset by the costs incurred in borrowing the LETF shares to short sell. Moreover, any transaction costs involved in the daily rebalancing transaction can easily wipe out the remaining profit margin for many of the LETF pairs under consideration. But the profitability of a number of the pairs tracking commodities, real estate, and foreign equities are still promising. The underlying benchmarks for these LETFs tend to be less liquid and thus prone to a higher degree of price inefficiency.

You cannot benefit from the compounding effect by implementing the above pair strategy with daily rebalancing of the exposures. However, if you do not rebalance on a daily basis, your pair portfolio will not be delta neutral, and therefore could subject to significant pricing risk depending on the realized return path of the underlying index. When you lengthen the time interval between rebalancing, you are more likely to realize a higher return because of the stronger compounding effect, while at the same time your pair strategy becomes riskier because it deviates more from delta neutrality and there is a greater chance of realizing larger tracking errors. There is a certain risk-return trade-off in picking an appropriate rebalancing frequency that is specific to the characteristics of the underlying benchmark. In general, the higher the volatility of the underlying benchmark, the more the return of the pair strategy can be enhanced by lengthening the time interval between rebalancing. However, when you rebalance less frequently, the pair strategy will become riskier, the larger the tracking errors of the LETFs, which are more likely to occur when the underlying benchmark is less liquid. Guo and Leung (2015) test the performance of pair strategy with a number of commodity-based LETFs on oil and gas, crude oil, gold, and silver, respectively. These commodity benchmarks can be highly volatile. They are different from equity-based benchmarks in the sense that the underlying physical assets cannot be stored easily, and thus potentially lead to larger LETF tracking errors. Guo and Leung consider pair strategies of rebalancing frequencies ranging from 5 days to 30 days over the sample period from December 2008 to May 2013. They conclude that pair strategies are in general profitable despite the inherent large tracking errors, but these strategies are subject to enormous tail risk. The authors find that the average return from the pair strategy increases with the length of the time interval between rebalancing. Moreover, the pairs tracking energy benchmarks (i.e., crude oil, and oil and gas) perform better than those tracking precious metal benchmarks (i.e., gold and silver). The average return of the 30-day rebalancing pair strategy on oil and gas is almost as high as 2%. Nevertheless, its downside risk can be as much as –20%.

To summarize, it seems that short selling a pair of bull and bear LETFs on the same underlying benchmark can be a profitable proposition, especially when the benchmark is highly volatile. But it can also be highly risky given the unlimited downside risk of short positions. The profitability can be quite different depending on the specific characteristics of the underlying benchmark and the specific market conditions that prevail when the strategy is implemented. With the limited historical pricing data on LETFs, it is difficult to assess the risk-adjusted return on the above-mentioned strategies and to pinpoint the ideal market condition under which the strategies will work best. To address these issues in detail, we have to resort to simulation analysis. In the next section, we conduct a number of simulation exercises to gauge the risk-adjusted performance of pair strategy. Alternatively, one may examine the hypothetical returns on LETFs based on historical return data on the underlying index as if the LETFs existed long before their actual inception date. We can therefore replicate the returns on pair strategies over an extended period of time, covering different kinds of market conditions that we have experienced in the last few decades, so as to offer more definite conclusions on the profitability and risks of pair strategy.

Jiang and Peterburgsky (2013) adopt such an approach to study various pair strategies of LETFs tracking the S&P 500 Index. They replicate the hypothetical returns on a +3x LETF and a –3x LETF as if they existed throughout the 48 years of the sample period from 1963 to 2010. They then calculate the annual performance of shorting the two LETFs at exposure ratios of 1:3, 1:2, 1:1, 2:1, and 3:1. For example, the exposure ratio of 1:3 refers to the pair portfolio consisting of 25% short exposure to the bull and 75% short exposure to the bear; whereas, a ratio of 2:1 refers to the portfolio mix of 66.6% short exposure to the bull and 33.3% short exposure to the bear. Their results suggest that the pair strategies are highly profitable on average, with the Sharpe ratio as much as four times that of the S&P 500 Index. The best-performing pair strategy in terms of the Sharpe ratio consists of holding a 33.3% short position on the bull together with a 66.6% short position on the bear, while rebalancing the exposures when they deviate by more than +/– 20%. The superior performance of this pair strategy is also found to be quite persistent over time. It outperformed the S&P 500 Index in 43 of the 48 years between 1963 and 2010. Nevertheless, the tail risk of the pair strategy could be substantial, especially during a market crisis. Based on their replication results, the one-day loss would be as high as 32% on Black Monday (i.e., on October 19, 1987). The results of Jiang and Peterburgsky reinforce the conclusions of the two other studies examined above in this section. That is, despite the fact that it is risky by its very nature, pair strategy involving shorting bull and bear LETFs can be a rewarding and viable trading strategy.

Simulation analysis

The more volatile the underlying benchmark index, the stronger the compounding effect, and thus shorting an LETF becomes more profitable than taking a long position on the opposite leverage ratio LETF. On the other hand, the larger the magnitude of the expected return on the underlying benchmark, the more persistent the trending in return, and thus shorting an LETF becomes less profitable than taking a long position in the opposite leverage ratio LETF. Suppose you expect a bullish market and are considering whether you should take a long position on a +3x LETF or short sell a –3x LETF. Suppose your investment horizon is one year. With knowledge of the implication of compounding effect, you know the appropriate choice between the two is dictated by the relative magnitude of expected market volatility versus expected return. You will choose the former if you expect volatility is low while return is high; whereas, you will choose the latter if you expect high volatility and low return. What are the threshold values of volatility and return that will lead you to pick the former as opposed to the latter? To answer this question, we conduct an experiment by simulating daily returns of the underlying index over a one-year period by assuming different expected returns (0%, 2%, 4%, 6%, 8%, 10%, 12%, and 14% per annum) and standard deviations of returns (2%, 6%, 10%, 14%, 18%, 22%, 26%, and 30% per annum). We assume the index return follows a geometric Brownian motion, and we simulate a total of 50,000 index return paths over one year for each set of values of expected return and standard deviation of return. For each simulated return paths of the index, we replicate the returns on the +3x LETF and the –3x LETF, and calculate their compounded returns over the year. We record the number of times shorting the –3x LETF will give us a higher compounded return than going long on the +3x LETF over a year across the 50,000 simulations. We report the proportion of times that happens in table 7.4 for each set of expected returns and standard deviation assumed. The shaded cells have values above 50%.

As expected, the higher the volatility, the greater the chance that the compounded return from shorting the –3x LETF will exceed that from going long on the +3x LETF; whereas, the higher the expected return, the lower the chance that the former will outperform the latter. The threshold values of the expected return and the standard deviation dictating the choice between the two strategies can be defined by the line of separation between

Table 7.4 Proportion of Time in Which Shorting the –3x LETF Gives Us a Higher One-Year Compounded Return Than Going Long on the +3x LETF

We simulate 50,000 daily return paths of the underlying index for each set of values of assumed expected return and standard deviation of the index. For each simulated return path, we replicate the returns on the +3x LETF and the –3x LETF, and calculate their respective one-year compounded returns. We tabulate the proportion of time where shorting the –3x LETF gives us a higher compounded return than going long on the +3x LETF across the 50,000 simulations.

Expected return of underlying index	**Standard deviation of return of underlying index**							
	2%	**6%**	**10%**	**14%**	**18%**	**22%**	**26%**	**30%**
0%	68%	68%	68%	69%	69%	70%	70%	71%
2%	47%	66%	68%	68%	69%	70%	70%	71%
4%	16%	58%	65%	67%	68%	69%	70%	71%
6%	2%	47%	60%	64%	67%	68%	69%	70%
8%	0%	36%	54%	61%	65%	67%	68%	70%
10%	0%	25%	48%	58%	62%	65%	67%	69%
12%	0%	16%	41%	53%	59%	63%	65%	68%
14%	0%	9%	34%	48%	56%	61%	64%	66%

the blank cells (below 50%) and shaded cells (above 50%). If we expect the index return, and its standard deviation is above and on the right-hand side of the line of separation, shorting the bear will be the preferred strategy. If they are below and on the left-hand side of the line of separation, simply buying the bull is the preferred strategy. Based on our simulation returns, there is roughly a one-to-one trade-off between the expected return and the standard deviation along the line of separation. For example, the chance for the short strategy to outperform the long strategy is 58% when the expected return in 4% and the standard deviation is 6%. When we increase both the expected return and the standard deviation by 4% (i.e., to 8% and 10% respectively), the chance of outperformance remains essentially the same (at 54%). If we then increase both the expected return and the standard deviation by another 4% (i.e., to 12% and 14% respectively), the chance of outperformance again remains unchanged (at 53%).

From the above simulation results, it seems that shorting the –3x LETF could produce higher returns than simply buying the +3x LETF for a majority of the market conditions under consideration. But how profitable is the short strategy on a risk-adjusted basis? This is not an easy question to answer. We know its return increases with market volatility because of the compounding effect, but so does its degree of riskiness. Can the increase in return sufficiently compensate for the increase in risk as market volatility increases? To answer this question, we compute the Sharpe ratios from shorting the –3x LETF for each set of values of the assumed mean index return and its standard deviation based on the simulated LETF's median compounded return and the standard deviation across the 50,000 scenarios. We assume a risk-free rate of 3% per annum. The resulting Sharpe ratios are reported in table 7.5. Given that you are shorting a bear LETF, of course your risk-adjusted return increases with the expected return of the underlying index for the same level of index volatility (i.e., as you go down each column of table 7.5). What is interesting is tracking the Sharpe ratios reported along each row

Table 7.5 Sharpe Ratio from Shorting the –3x LETF

We simulate 50,000 daily return paths of the underlying index for each set of values of assumed expected return and standard deviation of returns of the index over one year. We then compute the Sharpe ratios from shorting the –3x LETF based on the simulated LETF's median one-year compounded return and standard deviation across the 50,000 scenarios. A risk-free rate of 3% per annum is assumed.

Expected return of underlying index	**Standard deviation of return of underlying index**							
	2%	**6%**	**10%**	**14%**	**18%**	**22%**	**26%**	**30%**
0%	–0.47	–0.07	0.05	0.12	0.18	0.22	0.25	0.27
2%	0.53	0.26	0.24	0.26	0.28	0.30	0.31	0.32
4%	1.59	0.60	0.45	0.40	0.39	0.39	0.38	0.37
6%	2.71	0.98	0.67	0.56	0.51	0.48	0.46	0.43
8%	3.90	1.37	0.90	0.72	0.63	0.57	0.53	0.49
10%	5.17	1.79	1.15	0.89	0.76	0.68	0.61	0.56
12%	6.51	2.23	1.41	1.07	0.90	0.78	0.70	0.63
14%	7.94	2.70	1.69	1.27	1.04	0.90	0.79	0.71

of table 7.5. Although the index volatility strengthens the compounding effect, it is only value adding to the short strategy on a risk-adjusted basis when it is a sideways market with relatively low expected index return (at 0% or 2%, see first two rows of table 7.5). Only along the first two rows do we witness improvement in the Sharpe ratio as volatility increases. For all the other higher levels of expected index returns, the higher the index volatility, the lower the Sharpe ratio from shorting the bear LETF. At these levels of expected index returns, the enhancement of the return of the short strategy as a result of the strengthening of the compounding effect as index volatility increases is not sufficient to cover the increases in the risk of such a strategy as manifested in its higher return standard deviation.

Let us conduct a simulation exercise to discern the profitability of the pair strategy of shorting an equal amount of a +3x LETF and a –3x LETF, and without rebalancing the two short exposures over one year. As discussed in the previous section, it is essentially a volatility play. The return on the pair portfolio is expected to increase with the volatility of the underlying index. This strategy is also expected to be more profitable in a sideways market condition as opposed to a trending market condition. That is when the magnitude of the mean index return is relatively small as compared with the volatility of the index return. At the same time, it is a risky strategy because it is not delta neutral since we do not rebalance in a frequent fashion. The performance of the pair portfolio will be dictated by the realized return path of the underlying index. Table 7.6 reports the median one-year compounded returns of the pair portfolio over 50,000 simulated return paths of the underlying index for different sets of mean index returns and standard deviations of index returns. As expected, the higher the volatility of the underlying index return, the higher the median return from the pair strategy. The simulation results suggest that the pair strategy could be highly profitable under the optimal condition of a sideways market (i.e., mean index return is zero) and high index return volatility. The median annual return could exceed 20%. However, as the mean index return becomes more different from zero and thus as we deviate more from a sideways market condition, the performance of the pair strategy degenerates quickly, especially when the index volatility is not very high. The simulation results as reported in table 7.6 can serve as guidance for investors considering the implementation of the pair strategy. For example, at an expected index return of 8% per annum, the above pair strategy should be a profitable proposition if the standard deviation of the index return is expected to be at least 10% per annum. For the same expected index return of 8%, the investor, however, had better think twice before implementing this strategy if the market volatility is expected to be lower than 10%.

How profitable is the above pair strategy on a risk-adjusted basis? We expect the riskiness of the pair strategy to increase as the volatility of the underlying index increases. Is the enhancement in return as the index volatility increases (as documented in table 7.6) commensurate with the corresponding increase in risk? To address this issue, we calculate the Sharpe ratio of the pair strategy based on its median one-year compounded return

Table 7.6 Simulated Median One-Year Compounded Return on Pair Strategy

We simulate 50,000 daily return paths of the underlying index for each set of values of assumed expected return and standard deviation of returns of the index over one year. We then replicate the returns on the +3x and –3x LETFs, and compute the median one-year compounded return from shorting equal amounts of the +3x and –3x LETFs, and do not rebalance the two short exposures for one year.

Expected return of underlying index	**Standard deviation of return of underlying index**							
	2%	**6%**	**10%**	**14%**	**18%**	**22%**	**26%**	**30%**
–16%	–11.48%	–9.88%	–6.81%	–2.77%	1.53%	6.13%	11.09%	16.39%
–12%	–6.32%	–4.80%	–2.05%	1.14%	4.70%	8.67%	13.23%	18.32%
–8%	–2.69%	–1.27%	0.85%	3.34%	6.49%	10.25%	14.57%	19.45%
–4%	–0.54%	0.50%	2.08%	4.36%	7.33%	10.98%	15.25%	20.14%
0%	0.10%	0.86%	2.39%	4.65%	7.63%	11.28%	15.55%	20.37%
4%	–0.54%	0.48%	2.05%	4.30%	7.29%	10.98%	15.23%	20.07%
8%	–2.70%	–1.31%	0.78%	3.28%	6.39%	10.10%	14.35%	19.34%
12%	–6.34%	–4.85%	–2.14%	1.01%	4.55%	8.51%	13.12%	18.10%
16%	–11.51%	–9.95%	–6.92%	–2.96%	1.30%	5.86%	10.83%	16.18%

Table 7.7 Sharpe Ratio of Pair Strategy

We simulate 50,000 daily return paths of the underlying index for each set of values of assumed expected return and standard deviation of returns of the index over one year. We then replicate the returns of the +3x and –3x LETFs and compute the Sharpe ratio of the pair strategy of shorting equal amounts of the bull and bear LETFs without rebalancing for one year. A risk-free rate of 3% per annum is assumed.

Expected return of underlying index	**Standard deviation of return of underlying index**							
	2%	**6%**	**10%**	**14%**	**18%**	**22%**	**26%**	**30%**
–16%	–4.83	–1.38	–0.59	–0.23	–0.04	0.06	0.13	0.16
–12%	–4.20	–1.10	–0.39	–0.09	0.06	0.13	0.18	0.21
–8%	–3.85	–0.86	–0.22	0.02	0.14	0.20	0.23	0.25
–4%	–4.60	–0.78	–0.12	0.10	0.20	0.24	0.27	0.28
0%	–11.32	–0.93	–0.10	0.13	0.22	0.26	0.28	0.29
4%	–4.64	–0.80	–0.13	0.10	0.19	0.24	0.27	0.28
8%	–3.87	–0.87	–0.23	0.02	0.13	0.20	0.23	0.25
12%	–4.21	–1.12	–0.40	–0.10	0.05	0.13	0.18	0.21
16%	–4.84	–1.39	–0.60	–0.24	–0.05	0.06	0.12	0.16

and its standard deviation of returns across the 50,000 scenarios for each set of assumed mean index return and volatility. Again, we assume a risk-free interest rate of 3% per annum. The results are reported in table 7.7. Based on the Sharpe ratio, the enhancement in return as volatility increases can more than offset the increase in risk involved in the pair strategy. The higher the index return volatility, the higher the risk-adjusted return of the pair strategy. Similar to the pattern observed in table 7.6, the risk-adjusted return degenerates quickly as we start deviating from a neutral (i.e., sideways) market condition.

Conclusion

In this chapter, we examined a number of commonly adopted trading strategies involving LETFs. We started by introducing the idea of the optimal leverage ratio by considering the offsetting effect between leveraged returns and volatility drag. We then considered the risk-return trade-off of LETF investment. Next, we discussed in detail the costs and benefits of shorting LETFs, which is the strategy of short selling a bear (bull) LETF as opposed to going long on a bull (bear) when you predict a bullish (bearish) market. Although the short strategy does not always dominate the simple long strategy, it can be a highly rewarding strategy in a volatile market condition. We also considered the performance of pair strategies involving the simultaneous short selling of either balanced or unbalanced amounts of a bull LETF and a bear LETF tracking the same underlying benchmark. Empirical studies suggest that this kind of pair strategy performs particularly well for LETFs tracking those underlying benchmarks that are illiquid and have a highly volatile return. At the end of the chapter, we conducted a number of simulation exercises so as to pinpoint the market conditions under which these strategies perform the best. Market participants can use this information to help them to implement dynamic LETF strategies conditional on the prevailing market conditions and outlook.

8
Options on LETFs

By their very nature, LETF options are risky. They provide a second layer of leverage on top of the already leveraged returns on the underlying LETFs. Options on different kinds of LETFs are actively traded on the Chicago Board Options Exchange (CBOE).[1] In 2013, among the total of about 500 ETFs, on which there were options trading, 102 were LETFs (CBOE, 2013). The most popular options are those on equity-based LETFs tracking domestic, foreign, and different industry sectors. Additionally, there are options on LETFs tracking commodity, fixed-income, real estate, foreign currency, and market volatility benchmarks. Similar to other options, LETF options allow market participants to profit from the changes in the value and/or volatility of their underlying benchmarks. At the same time, they are also indispensable hedging tools in risk management.

In this chapter, we examine the pricing of LETF options. We start with the use of the basic Black-Scholes option-pricing model (Black and Scholes, 1973) to study the characteristics of their prices, focusing on the role played by the targeted leverage ratio of the underlying LETF. We illustrate the relation between the prices of LETF options and those of options on nonleveraged ETFs tracking the same underlying benchmark through the comparison of their implied volatility. Establishing this relation has important practical implications given that, although LETF options are popular, their market is typically not as liquid and thus not as price informative as that of options on nonleveraged

ETFs. By using the market prices of their nonleveraged counterparts, the relative pricing relation allows us to arrive at more robust pricing for LETF options. We also study the behavior of the implied volatility of LETF options. Unlike ordinary options, of which the implied volatility is fully defined along two dimensions—*moneyness* and *time to maturity* (i.e., term structure), there is a third dimension we need to consider when we define the implied volatility of LETF options. The implied volatility of LETF options is also a function of the targeted leverage ratio of the underlying LETF. We examine the findings from an empirical analysis based on observed market prices of a number of call options on SPDR S&P 500 (ticker: SPY), ProShares Ultra S&P 500 (ticker: SSO; a +2x LETF), and ProShares UltraShort S&P 500 (ticker: SDS; a –2x LETF), which track the S&P 500 Index. We demonstrate how we may relate the implied volatilities of nonleveraged ETF options with those of LETF options by using a *transformed* measure of moneyness. Last but not least, we briefly review the use of stochastic volatility and jump-diffusion models in pricing LETF options.

Pricing LETF options with the Black-Scholes model

Let us start with the standard constant-volatility continuous-time stochastic process of the asset value I_t of the underlying benchmark index (e.g., the S&P 500 Index):[2]

$$\frac{dI_t}{I_t} = \mu_t \cdot dt + \sigma \cdot dz_t \tag{8.1}$$

where μ_t is the instantaneous expected return of the index that may or may not vary over time, dt is a very short period of time over which we observe the change in asset value, σ is the volatility of the index return that is assumed to be constant over time, and dz_t is a stochastic Wiener process. Suppose there is an LETF with a targeted leverage ratio of β tracking this benchmark. For most practical purposes, we will not introduce much error by assuming dt to be exactly one day, and thus the end-of-day rebalancing of the exposure of the LETF happens at the end of each time period dt. Therefore, the return on the asset value of the LETF L_t over time period dt will be exactly the return on the underlying index multiplied by the leverage ratio β, while subtracting any costs incurred in delivering the leveraged return. That is,

$$\frac{dL_t}{L_t} = \beta\frac{dI_t}{I_t} - (\beta - 1)r \cdot dt - k_t \cdot dt \tag{8.2}$$

where r is the risk-free interest rate and $(\beta-1)r$ is the financing costs incurred in generating the leveraged return (as discussed in chapter 6), and k_t is the rate of management fees (i.e., the net expense ratio) of the LETF. Substituting Equation (8.1) into Equation (8.2), we obtain a stochastic process for the LETF price L_t:[3]

$$\frac{dL_t}{L_t} = \left[\beta\mu_t - (\beta-1)r - k_t\right] \cdot dt + \beta\sigma \cdot dz_t \tag{8.3}$$

By looking at the second term of Equation (8.3), we can tell that the volatility of the returns on the LETF equals the magnitude of the leverage ratio $|\beta|$ multiplied by the volatility σ of the returns of the underlying benchmark. From Equations (8.1) and (8.2), we can also solve for the LETF price L_T to be realized at any future time T in terms of the underlying index value I_T to be realized at the same future time T (here we assume $k_t = k$ and is constant over time):

$$\frac{L_T}{L_0} = \left(\frac{I_T}{I_0}\right)^{\beta} \cdot e^{-((\beta-1)r+k)T - \frac{\beta(\beta-1)}{2}\sigma^2 T} \tag{8.4}$$

where L_0 and I_0 are respectively the current price of the LETF and the current value of the underlying index. We will come back to Equation (8.4) later when we discuss the moneyness of LETF options.

Based on Equation (8.3), we can price European call and put options on the LETF by using standard Black-Scholes pricing equations. Specifically, if we ignore any management fees (i.e., setting k_t to zero), the price of a call option c_t on the LETF with a strike price X and maturing at some future time T is

$$c_t = L_t \cdot \Phi(d_1) - Xe^{-r(T-t)} \cdot \Phi(d_2) \tag{8.5}$$

where

$$d_1 = \frac{\ln[L_t / X] + \left(r + \frac{1}{2}\beta^2\sigma^2\right) \cdot (T-t)}{|\beta| \cdot \sigma \cdot \sqrt{T-t}}$$

$$d_2 = d_1 - |\beta| \cdot \sigma \cdot \sqrt{T-t}$$

and $\Phi(\blacksquare)$ is the cumulative standard normal distribution function. The price of the corresponding put option p_t is

$$p_t = Xe^{-r(T-t)} \cdot \Phi(-d_2) - L_t \cdot \Phi(-d_1) \tag{8.6}$$

The moneyness of an option is usually defined by the ratio of the strike price X to the current price of the underlying asset, which is L_t in our case here. Call options are more *in-the-money* (*out-of-the-money*) when X/L_t decreases (increases), and vice versa for puts. By examining Equation (8.5), we notice that the price of a call option on a bull LETF expressed as a fraction of the current LETF price will be exactly the same as the proportional call option price on its bear counterpart (with the same leverage ratio) if the two options have the same moneyness ratio X/L_t. That is,

$$\frac{c_t^{(\beta)}}{L_t^{(\beta)}} = \frac{c_t^{(-\beta)}}{L_t^{(-\beta)}}$$

where $L_t^{(\beta)}$ and $L_t^{(-\beta)}$ denote the current prices of the bull and bear LETFs with leverage ratios β and $-\beta$, and $c_t^{(\beta)}$ and $c_t^{(-\beta)}$ their respective call option prices based on the Black-Scholes model. The same can be said for the puts of a pair of bull and bear LETFs with the same leverage ratio.

Let us present a numerical example to highlight some of the key characteristics of the above Black-Scholes option-pricing equations being applied to LETF options. First, consider call options on a nonleveraged ETF, an inverse ETF (i.e., with leverage ratio of −1), a +2x LETF, a −2x LETF, a +3x LETF, and a −3x LETF, all tracking the same underlying benchmark index. For each ETF or LETF, we consider one-year call options with the moneyness ratios of 0.9, 1.0, and 1.1 respectively. Suppose the return volatility of the underlying index is 20% per annum and the risk-free interest rate is 3% per annum. Table 8.1 shows the call option prices as fractions of the current prices of their underlying ETFs or LETFs. As expected, the more in-the-money (i.e., the lower the moneyness ratio X/L_t), the higher the call option price. As explained previously, according to the Black-Scholes model, each pair of bull and bear LETFs of the same magnitude of leverage ratios has identical call option prices as fractions of its underlying LETFs' prices. As the return volatility of the underlying LETF is magnified by the leverage ratio, the higher the leverage ratio of the underlying LETF, the pricier the call option. This leverage effect on price is particularly salient for out-of-the-money call options. Looking at the last column of table 8.1, which corresponds to the highest moneyness ratio of 1.1 and thus the most out-of-the-money call options, the proportional price of the calls on the 3x LETFs is four times that of the nonleveraged counterparts and almost 80% higher than that of the 2x counterparts.

There is an interesting pattern regarding the price impact of the moneyness ratio across LETFs of different leverage ratios. From table 8.1, it can be observed that, when the moneyness ratio increases from 0.9 to 1.1, the option prices of the nonleveraged ETFs drop by about 66% (=(0.154–0.053)/0.154); whereas those of the 2x LETFs only drop by 40%. The price drop is the least (at about 27%) for options on the 3x LETFs. This observed difference in price sensitivity to the moneyness ratio can be attributable to the difference in return volatility of the underlying assets of the three groups of options. Recall that, the higher the magnitude of the leverage ratio, the higher the volatility of the LETF.

Table 8.1 Call Option Prices as Fractions of Underlying ETF and LETF Prices (i.e., c_t/L_t) Based on Black-Scholes Model for Different Moneyness Ratios

	Moneyness ratio (X/L_t)		
	0.9	**1.0**	**1.1**
+1x ETF	0.154	0.094	0.053
−1x ETF	0.154	0.094	0.053
+2x LETF	0.220	0.171	0.132
−2x LETF	0.220	0.171	0.132
+3x LETF	0.290	0.247	0.211
−3x LETF	0.290	0.247	0.211

Thus, even at a moneyness ratio of 1.1, there is still a reasonable chance that the call options on the 3x LETFs will turn out to be in the money at the maturity date, given the high return volatility. Therefore, the option price at a moneyness ratio of 1.1 is not that different from the price of the at-the-money option with a moneyness ratio of 1.0. The situation is quite different for the options on nonleveraged ETFs. With a return volatility that is only one-third that of the 3x LETFs, it is much harder for the call options to mature in-the-money one year from now if the current moneyness ratio is 1.1. The option price is therefore much lower than that of the at-the-money option. The same argument applies to the in-the-money options. The same moneyness ratio of 0.9 represents different probabilities of exercising, and thus different expected payoffs, for options of different leverage ratios. Given the higher return volatility of the underlying LETF, the higher the magnitude of the leverage ratio, the less chance that the option will remain in the money with the same starting moneyness ratio. That is why we witness a smaller price difference between the at-the-money option with a moneyness ratio of 1.0 and the option at a ratio of 0.9 for the higher leverage LETFs. This finding tells us that the moneyness ratio is not a sufficient measure of the chance of exercising, and thus the expected payoffs, of LETF options across different leverage ratios. In the next section, we introduce a *transformed moneyness measure* that explicitly accounts for the leverage ratio effect. This transformed moneyness measure facilitates the comparison of moneyness across options on LETFs of different leverages, thus allowing us to conduct relative pricing between LETF options and options on its nonleveraged counterparts.

Table 8.2 presents the one-year put options prices as fractions of the current prices of their underlying ETFs and LETFs, which tracks the same underlying index considered above. Again, prices increase with moneyness (i.e., with the moneyness ratio becomes higher). Similar to the observations above regarding the leverage effect on call option prices, the higher the magnitude of the leverage ratio of the underlying LETF, the higher the put option price as a fraction of the underlying LETF price. This leverage effect is again strongest for out-of-the-money puts. Similar to the pattern observed earlier for the call options, the put option prices also exhibit differentiated sensitivity to the moneyness ratio. The higher the magnitude of the leverage ratio of the underlying LETF, the less the put option price is sensitive to the same change in the moneyness ratio. This reinforces the conclusion we drew earlier that the moneyness ratio as it is currently defined is not a sufficient measure of the chance of exercising, and thus the expected payoffs, of LETF options across different leverage ratios.

For those market participants who want to hedge their option exposures using the underlying LETF, they need to calculate the *Greeks* of their options. Standard equations are available to calculate *delta*, *gamma*, *vega*, and *theta* based on the Black-Scholes

Table 8.2 Put Option Prices as Fractions of Underlying ETF and LETF Prices (i.e., p_t/L_t) Based on Black-Scholes Model for Different Moneyness Ratios

	Moneyness ratio (X/L_t)		
	0.9	**1.0**	**1.1**
+1x ETF	0.028	0.065	0.120
–1x ETF	0.028	0.065	0.120
+2x LETF	0.093	0.142	0.200
–2x LETF	0.093	0.142	0.200
+3x LETF	0.163	0.218	0.279
–3x LETF	0.163	0.218	0.279

model (see, e.g., Hull, 2014). Let us focus on *delta hedging* here. To conduct delta hedging, we need to ascertain the delta of our option position. As you know, delta measures the sensitivity of the option price on the change of the current price of its underlying asset (i.e., the price of the ETF or LETF as considered here). Suppose the delta of a call option on an LETF is 0.6. It tells us that, in order to *temporarily* hedge against the risk of underwriting this call option, the underwriter needs to hold a long position on the underlying LETF equal to 60% of the amount of exposure of the call option written. The more in-the-money the call option (i.e., the lower the moneyness ratio), the higher its delta. The higher delta simply reflects the fact that the chance of your counterparty exercising the call option is higher when it is in-the-money, and thus you need to own more of the underlying LETF to prepare for delivery if it is indeed exercised. At the extreme, when your call option is *deep in the money* (i.e., when L_t is much larger than X), delta approaches its maximum value of unity, and thus you need to hold a long position of the underlying LETF that equals the full exposure amount of the call option in performing a delta hedge. If the call option is out-of-the-money, the chance of exercising is not high. The delta of the call option should, therefore, be small, indicating that we do not need to own a lot of the underlying LETF in conducting a delta hedge. At the extreme, we do not need to hold any underlying LETF to conduct a delta hedge if the call option is *deep out of the money* (i.e., when L_t becomes much smaller than X) because the chance of the counterparty exercising is next to nothing. In this extreme case, we therefore expect the delta to approach zero. In summary, we expect the delta of a call option to be an increasing function of its moneyness. In other words, the smaller the moneyness ratio, the larger the delta.

According to the Black-Scholes model, the delta of a call option on an LETF is given by $\Phi(d_1)$, where d_1 is as defined in Equation (8.5) and $\Phi(\blacksquare)$ is the cumulative standard normal distribution function. Let us continue with our numerical example and calculate the deltas of our 18 call options on ETFs and LETFs examined above. They are shown in table 8.3. As expected, the delta

Table 8.3 Deltas of Call Options on ETFs and LETFs for Different Moneyness Ratios Based on Black-Scholes Model

	Moneyness ratio (X/L_t)		
	0.9	**1.0**	**1.1**
+1x ETF	0.781	0.599	0.410
–1x ETF	0.781	0.599	0.410
+2x LETF	0.705	0.608	0.515
–2x LETF	0.705	0.608	0.515
+3x LETF	0.700	0.637	0.576
–3x LETF	0.700	0.637	0.576

increases with moneyness (i.e., when the moneyness ratio X/L_t decreases). Similar to what we observe in tables 8.1 and 8.2, we also witness a differentiated sensitivity of the delta to the moneyness ratio across different leverage ratios. The higher the magnitude of the leverage ratio, the less sensitive the value of the delta to the change in the moneyness ratio. This finding also points to the fact that the moneyness ratio is not a uniform measure across different leverage ratios. For example, at the same moneyness ratio of 1.1, the call option on the +1x ETF has a much lower chance of exercising than that on the +3x LETF, given the lower volatility of the underlying assets of the former. Thus, to conduct delta hedging, we need to hold less of the underlying fund to hedge the risk of the former than that for hedging the latter.

Implied volatility of options on nonleveraged and leveraged ETFs

One of the key assumptions of the analysis performed in this chapter up to now is that the volatility of the underlying index return (σ) is constant over time (see Equation [8.1]). Based on this assumption, we have closed-form solutions for European call and put options on LETFs given by Equations (8.5) and (8.6), respectively. In reality, volatility is anything but constant. Volatility tends to change with market conditions. In the equity market, the level of volatility is generally believed to be negatively related to the prevailing market return. That is, volatility tends to be higher in a down market, and vice versa in an up market. This asymmetric behavior of volatility in the equity market can be explained by the fact that corporations typically borrow money (from banks in the form of loans or by issuing bonds) to finance their operations. As the amount of debts borrowed is relative static, any drop in a corporation's equity value will result in an increase in its financial leverage and thus in turn lead to an increase in its stock price volatility. However, when equity value increases, volatility will decrease as a result of the lowering of the corporation's financial leverage. In the next section, we examine the pricing of LETF options when we relax the assumption of a constant volatility for the underlying index return. In this section, we would like to first illustrate the relation between the prices of LETF options and those of options on nonleveraged ETFs tracking the same underlying benchmark. Establishing this relation allows us to conduct relative pricing for LETF options.

In derivative pricing, we sometimes do not need to come up with a model to price a derivative in an *absolute* sense. In many applications, we do observe market prices of some related assets and/or derivatives. So it will be good enough to adopt a *relative* approach to price a derivative as a function of the observed market prices of the related assets and/or derivatives. This kind of relative pricing approach is particularly relevant in the pricing of LETF options. Although LETF options are popular, their market is typically not as liquid and thus not as price informative as that of options on their nonleveraged ETF counterparts. For example, ProShares UltraShort S&P 500, of which its options on the CBOE are the most popular among all the LETFs tracking the S&P 500 Index, had a total annual option contract volume in 2013 that only amounted to about 1.6% of that of all the options on SPDR S&P 500 ETF, the nonleveraged ETF tracking the S&P 500 Index (CBOE, 2013). Taking the market prices of the options on their nonleveraged counterparts as accurate and given, a relative pricing relation will allow us to arrive at more robust pricing for LETF options that are free from noises as a result of market illiquidity and/or stale market information.

One popular relative pricing approach is through the analysis of the *implied volatility* of the options. We can define implied volatility as the volatility value that allows us to match the observed market price of the option when we plug it into the Black-Scholes formulas of Equations (8.5) or (8.6). There is therefore a one-to-one correspondence between option price and implied volatility via the Black-Scholes formulas. In other words, knowing the implied volatility of an option is as good as knowing its price. If the volatility of the asset underlying the option is indeed constant over time, all call and put options on the same underlying assets but of different moneyness and time to maturity should have the same implied volatility. This situation, however is not what we usually observe in the market. For equity-based options, it is quite typical to observe a negative relation between the implied volatility and the moneyness ratio of an option. This phenomenon is commonly referred to as the *volatility skew*, due to the shape it creates on a graph of plotting implied volatility against the moneyness ratio. The volatility skew therefore manifests the violation of the constant volatility assumption in practice. The volatility skew is in fact consistent with the asymmetric behavior of volatility between up and down market conditions, as mentioned earlier (see Hull, 2014, for details). Besides being a function of moneyness, options with different time to maturity may also have different implied volatility. Implied volatility is commonly used as the key metric in pricing illiquid options relative to the observed market prices of liquid options on the same stock or market index. The first step is to calculate the implied volatility of all the liquid options of different moneyness and time to maturity based on their market prices and compile an implied volatility table like the one in table 8.4. We can then use this table to impute the volatility of an illiquid call option, say, with a moneyness ratio of 0.95 and 90 days to maturity, by interpolating among the implied volatility of the liquid options

Table 8.4 Hypothetical Implied Volatility Table

Moneyness ratio (X / L_t)	**Time to maturity**		
	30 days	**60 days**	**120 days**
0.8	18.5%	20.5%	23.4%
0.9	16.0%	18.2%	21.3%
1.0	14.5%	17.0%	20.1%
1.1	13.5%	16.3%	19.5%
1.2	13.0%	16.1%	19.2%

with the nearest moneyness ratios and time to maturity. In this example, the illiquid call option has a moneyness ratio that is exactly midway between 0.9 and 1.0, and a time to maturity that is again midway between 60 and 120 days. Its imputed volatility is therefore simply the average of the four adjacent implied volatilities. That is,

$$\frac{18.2\% + 21.3\% + 17.0\% + 20.1\%}{4} = 19.15\%$$

We can then use this imputed volatility in Equation (8.5) to calculate the price of this illiquid call option (relative to the observed market prices of the liquid options).

We can implement a similar relative pricing approach to price LETF options based on the observed market prices of heavily traded options on nonleveraged ETFs tracking the same benchmark index. Both LETF options and nonleveraged ETF options are derivatives on the same benchmark index, and thus their prices should be consistent with the same volatility structure. But, unlike in the previous numerical example, of which the implied volatility is fully defined by the moneyness ratio and time to maturity, now we also need to understand how implied volatility is supposed to be related to the targeted leverage ratio of the LETF under consideration. It is important to clarify here that, when we refer to the *implied volatility of an LETF option* in the rest of this chapter, we are in fact referring to the volatility σ of the underlying benchmark index as opposed to the volatility

$|\beta| \times \sigma$ of the LETF itself. Thus, the implied volatility of an option on a +2x LETF should be directly comparable with the implied volatility of an option on a nonleveraged ETF tracking the same underlying benchmark. As demonstrated earlier in this chapter, the moneyness ratio (X/L_t) by itself is not a sufficient measure of prices and deltas of LETF options. Two LETF options with the same moneyness ratio could have different expected payoffs depending on their respective leverage ratios. This is because leverage magnifies the volatility of the underlying LETFs, thus affecting the likelihood of exercising and the eventual payoff of the option. Furthermore, LETFs of different targeted leverage ratios are subject to different magnitudes of compounding effect (as discussed in chapters 4 and 6), which will also affect the implied volatility of their options. Moreover, given the commonly observed asymmetric behavior of volatility in the equity market, mentioned earlier, we expect the implied volatility structure of options on bull LETFs tracking an equity benchmark index to behave differently from that of options on bears tracking the same benchmark. Specifically, options on bulls tend to have a negative relation between the implied volatility and the moneyness ratio (i.e., downward sloping volatility skew); whereas, options on bears tend to have a positive relation (i.e., upward sloping volatility skew). A detailed explanation of this effect is provided in the Appendix. In order to be able to compare the implied volatility of options on LETFs of different leverage ratios and also with options on nonleveraged ETFs, we need a way to control for all of the above-mentioned effects when we conduct relative pricing among different LETF options and nonleveraged ETF options. One way is to transform the moneyness ratio by directly incorporating the leverage effect. Before we introduce a transformation methodology, let us conduct a simple empirical study to see if we indeed observe some of the above-mentioned leverage effects on implied volatility.

We obtained from the CBOE website the market prices of a number of call options on three ETFs tracking the S&P 500 Index: SPDR S&P 500 ETF (ticker: SPY; a nonleveraged ETF), ProShares

Ultra S&P 500 (ticker: SSO; a +2x LETF), and ProShares UltraShort S&P 500 (ticker: SDS; a –2x LETF). The prices were all observed on June 10, 2015. The options have the same expiration date of September 18, 2015, but with different moneyness ratios. We calculate their implied volatility and plot them against the natural logarithms of their moneyness ratios. For ease of comparison, we plot the implied volatility of the options on SPY and SSO on the same graph (figure 8.1). In a separate graph (figure 8.2), we plot those for SPY and SDS. Note that those plots with a positive log moneyness ratio ($Ln(X/L_t)$) are from out-of-money call options where $X > L_t$; whereas, those with a negative log moneyness ratio are from in-the-money call options where $X < L_t$.

Let us first examine figure 8.1. The call options on both the nonleveraged ETF and the bull LETF exhibit the typical volatility skew where implied volatility tends to be negatively related to the moneyness ratio. This is consistent with the asymmetric behavior of volatility in up versus down markets mentioned above (and in detail in the Appendix). Another noteworthy observation is that the slope of the volatility skew is less steep for the bull LETF than for the nonleveraged ETF. In other words,

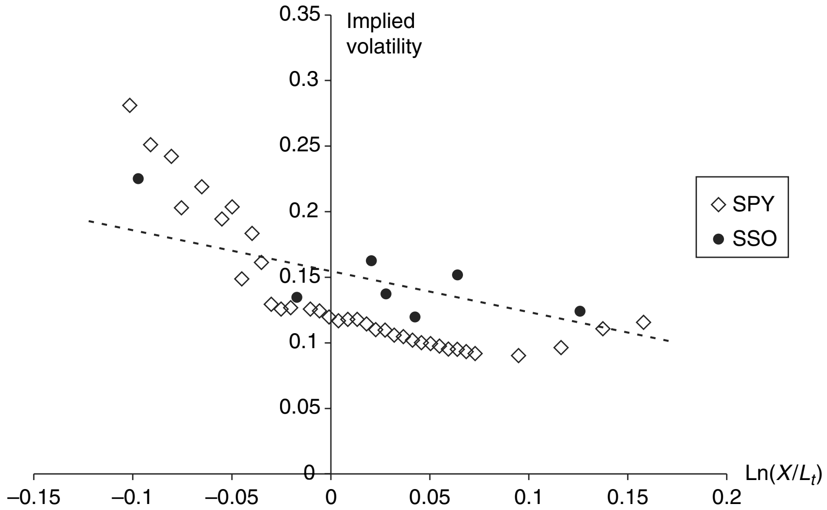

Figure 8.1 A Plot of Implied Volatility against Natural Logarithm of Moneyness Ratio for Call Options on SPY and SSO.

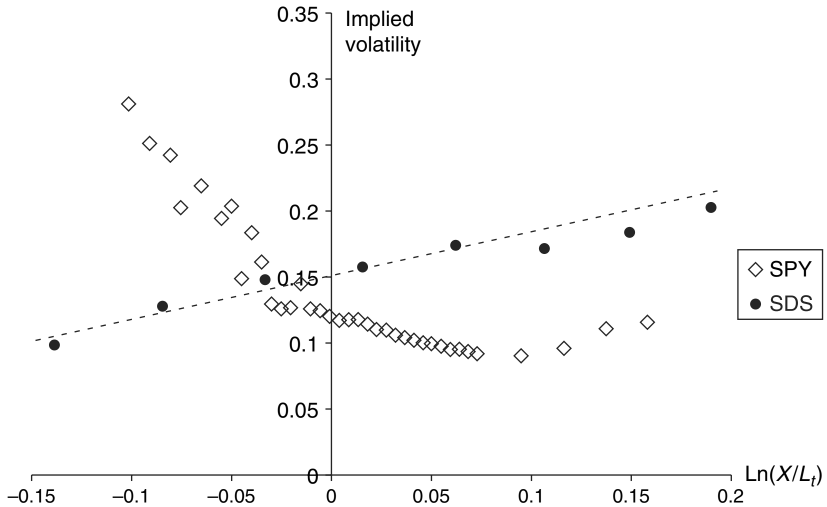

Figure 8.2 A Plot of Implied Volatility against Natural Logarithm of Moneyness Ratio for Call Options on SPY and SDS.

the implied volatility of the bull LETF options tends to decrease by a smaller amount for the same increase in the moneyness ratio. For example, when the log moneyness ratio changes from –0.1 to 0.1, the implied volatility of the options on SPY drops from around 25% to 9%, while that of the options on SSO only changes from around 22% to 12%. This is consistent with our understanding of the leverage effect on volatility as discussed previously, when we examined the results reported in tables 8.1 to 8.3. The lower sensitivity of the implied volatility to $Ln\ (X/L_t)$ for the SSO options simply reflects that the difference in the expected payoffs of the options between $Ln\ (X/L_t)$ of –0.1 and 0.1 is not as much as the corresponding difference in those of the SPY options, given the higher volatility of SSO. It therefore points to the fact that the moneyness ratio by itself cannot fully capture the change in the implied volatility of LETF options of different leverage ratios.

From figure 8.2, we notice the volatility skew of the options on SDS is in the opposite direction. Contrary to that of SSO options, the implied volatility of the SDS options increases with the moneyness ratio.[4] This is again consistent with the asymmetric behavior of volatility in up versus down markets, mentioned earlier (and in detail in the Appendix). Because of the same effect, we also expect the implied volatility of the in-the-money (out-of-money) calls on bull LETFs to be higher (lower) than those on their bear counterparts. This is indeed what we observe when we compare the implied volatility of SSO and SDS, as reported in the two figures. For example, at the log moneyness ratio of –0.1 (i.e., in-the-money), the implied volatility of the options on SSO is about 22%, which is higher than the implied volatility of around 11% for those on SDS. However, at the log moneyness ratio of 0.1 (i.e., out-of-money), the implied volatility of the options on SSO is about 12%, which is lower than the implied volatility of around 20% for those on SDS.

In summary, the results of the simple empirical analysis performed above suggest that the implied volatility from LETF options of different target leverage ratios do not align with each other in terms of both their levels and the shape of their volatility skew. The reason for this is that the moneyness ratio by itself is not a sufficient measure of LETF option prices and the asymmetric implications of the changing volatility behavior as observed in the equity market. With the objective of using the implied volatility as the metric in relative pricing, we need to also capture the leverage effect before we can compare the implied volatility of LETF options with different leverage ratios. Leung and Sircar (2015) propose a transformed moneyness measure based on the relation between the realized price of LETF and the price of its nonleveraged ETF counterpart. Note that, in the absence of tracking errors, the compounded return on a nonleveraged ETF is exactly the same as the compounded return on the underlying benchmark. Thus, according to Equation (8.4), we have

$$\frac{L_T}{L_0} = \left(\frac{S_T}{S_0}\right)^{\beta} \cdot e^{-((\beta-1)r+k)T - \frac{\beta(\beta-1)}{2}\sigma^2 T} \tag{8.7}$$

where S_0 and S_T denote respectively the current price of the nonleveraged ETF and its price to be realized at maturity date T of the option; whereas, L_0 and L_T are respectively the current price of the LETF and its price to be realized at the same maturity date. Taking the natural logarithm of Equation (8.7) gives us

$$\ln\left(L_T / L_0\right) = \beta \cdot \ln\left(S_T / S_0\right) - \left((\beta - 1)r + k\right)T - \frac{\beta(\beta - 1)}{2}\sigma^2 T \tag{8.8}$$

Let X denotes the strike price of an option on the LETF maturing at time T. According to Equation (8.8), if the realized LETF price at time T exactly equals this strike price (i.e., $L_T = X$), the price S_T of its nonleveraged ETF counterpart at time T should satisfy

$$\ln\left(S_T / S_0\right) = \frac{\ln\left(X / L_0\right) + \left((\beta - 1)r + k\right)T + \frac{\beta(\beta - 1)}{2}\sigma^2 T}{\beta}$$

Now suppose there is an option on the nonleveraged ETF also maturing at time T but with strike price X^*. These two options can be considered to be of an *identical degree of moneyness*, and thus of the equal chance of exercising, if the prices of their respective underlying assets attain their respective strike prices at the same time. This will be the case if the following equation is satisfied:

$$\ln\left(X^* / S_0\right) = \frac{\ln\left(X / L_0\right) + \left((\beta - 1)r + k\right)T + \frac{\beta(\beta - 1)}{2}\sigma^2 T}{\beta} \tag{8.9}$$

Equation (8.9) offers a way for us to *normalize* the degree of moneyness of LETF options by knowing their log moneyness ratio (X / L_0), the leverage ratio (β) of their underlying LETFs, the time to maturity T, the risk-free interest rate r, the management fees (k) of the underlying LETFs, and the volatility of the underlying benchmark index (σ). Regardless of the leveraged ratios of their underlying LETFs, two LETF options with the same normalized moneyness measure according to Equation (8.9) will be considered to be of the same degree of moneyness because their underlying funds will attain their respective strike prices at the same time. They are thus expected to have the same implied volatility.[5] With this transformed moneyness measure, we can conduct relative pricing just like what we did earlier in the numerical example based on the implied volatility table presented in table 8.4. The only difference is now we have to tabulate the implied volatility based on the transformed moneyness measure instead of the moneyness ratio. We will then interpolate among the calibrated implied volatility to impute the volatility for an option (and in turn its price), of which the market price is unavailable or is deemed to be too noisy for the particular purpose at hand. In practice, the implied volatility table is constructed and calibrated based on prices of heavily traded options on nonleveraged ETFs. From the calibrated table, we can then impute the volatility of LETF options so as to serve as a pricing benchmark for investment and hedging purposes.[6]

Before we wrap up our discussion on relative pricing through the calculation of the implied volatility, let us continue with our previous empirical study and check whether using the above transformed moneyness measure can indeed allow us to arrive at a better alignment of the level and shape of the volatility skew from the call options on SPY, SSO, and SDS. We repeat the plotting of the implied volatility, but now against the transformed moneyness measure of Equation (8.9). In evaluating Equation (8.9), we ignore the management fees (i.e., setting $k = 0$). We also assume a three-month London interbank offered rate (LIBOR) rate of 30 basis points as our risk-free interest rate, and a volatility (σ) equals the sample average of the implied volatility of all the call options on SPY. Figure 8.3 presents the implied volatility plots for the options on SPY and SSO. Compared with the plots in figure 8.1, the implied volatility of the call options on SPY and SSO are more in line with each other when we define moneyness by the transformed moneyness measure of Equation (8.9). Moreover, the slope of the volatility skew for SSO options is now much steeper and becomes more similar to that of SPY options. Turning to figure 8.4, which presents the implied volatility plots for SPY and SDS options, the implied volatility of the two groups of options is now much more consistent based on the transformed moneyness measure. More importantly, unlike in figure 8.2, the volatility skew of SDS options is now downward sloping, and is quite similar to that of SPY options. These findings

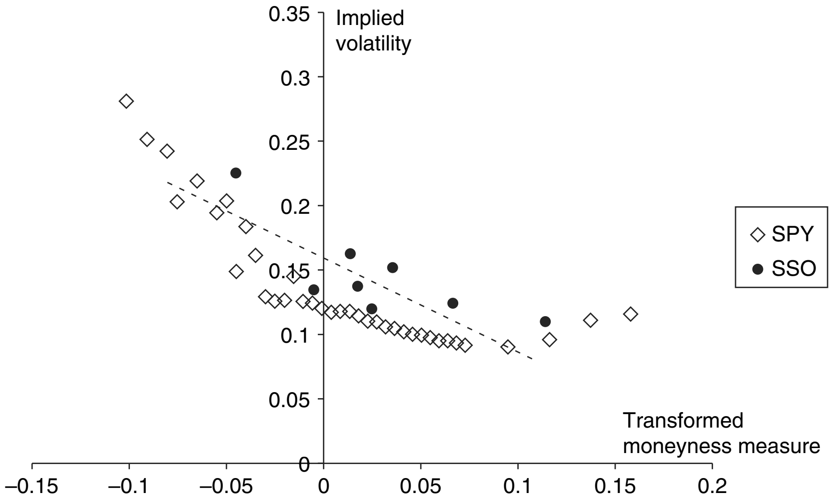

Figure 8.3 A Plot of Implied Volatility against Transformed Moneyness Measure for Call Options on SPY and SSO.

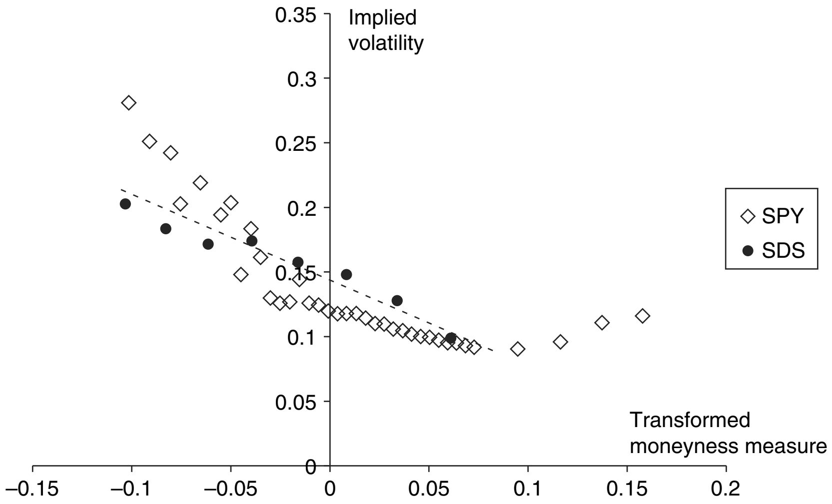

Figure 8.4 A Plot of Implied Volatility against Transformed Moneyness Measure for Call Options on SPY and SDS.

tell us that the transformed moneyness measure can indeed fulfill the objective of more accurately capturing LETF option prices and behavior. The transformed moneyness measure allows us to arrive at an *apple-to-apple* comparison of options across different leverage ratios, which is a crucial criterion in relative pricing.

Beyond Black-Scholes model

The observed volatility skew of LETF options (figures 8.1 to 8.4) is a manifestation of the violation of the constant volatility assumption of the Black-Scholes model. How to price LETF options given the dynamic nature of return volatility of the underlying benchmark? Stochastic volatility models (e.g., Hull and White, 1987; Heston, 1993) and jump-diffusion models (e.g., Merton, 1976) have been commonly used in pricing options on equity, commodity, foreign currency, and so forth. These models can be readily adapted to price LETF options.

Using the pricing data of options on LETFs tracking the S&P 500 Index, Deng et al. (2013) calibrate the Heston stochastic volatility model, of which there exist closed-form solutions for the valuation of European call and put options. They find that the calibrated model can successfully replicate some key observed characteristics of the market data, including the shape of the volatility skew. Their calibrated results also point to a nonzero correlation between the benchmark asset return and its volatility. Also calibrating against the prices of options on LETFs tracking the S&P 500 Index, Zhang (2010) finds that the Heston model performs well in matching the observed implied volatility except for short-dated options. The errors could be substantial for short-dated options that are either deep in-the-money or deep out-of-the-money.

Leung and Sircar (2015) adopt a *multiscale* stochastic volatility model in pricing LETF options by interpreting the LETF as aiming to achieve a prespecified CAPM beta in maintaining its targeted leverage ratio. They conduct a relative pricing exercise by first calibrating their model with pricing data of options on a nonleveraged ETF tracking the S&P 500 Index and then using the calibrated model to compute the implied volatility of options on LETFs tracking the same index. Although in general the model can fit the observed implied volatility well, they notice a negative bias in replicating the slope of the volatility skew of LETF options. It seems that the higher the leverage ratio, the more difficult it is for the model to fit the level of the implied volatility.

Last but not least, Ahn et al. (2014) consider the pricing of LETF options when the return of the underlying benchmark index follows stochastic processes that encompass both stochastic volatility and jump features. In this setting, closed-form solutions for LETF option prices are in general not available. Nevertheless, through a number of Monte Carlo simulations, they show that the LETF option prices obtained from their approximation algorithms can match the true prices quite well.

Conclusion

In this chapter, we examined the pricing of LETF options. We started with the Black-Scholes option pricing model under the assumption of constant return volatility over time. We focused on the roles played by the LETF's targeted leverage ratio and the option's moneyness ratio in governing the LETF option price. We highlighted the importance of the leverage ratio in defining the chance of exercising, and thus in turn the expected payoff, of an LETF option. We then examined the behavior of the implied volatility of options on bull and bear LETFs. With the use of market data of call options on a nonleveraged ETF, a bull LETF, and a bear LETF all tracking the S&P 500 Index, we demonstrated and explained the shape of the observed volatility skew. With the objective of conducting relative pricing among options on nonleveraged ETFs and LETFs, we introduced a transformed moneyness measure that incorporates the leverage ratio, compounding effect, management, and financing costs of the underlying LETF. With this transformed moneyness measure, we can directly compare the implied volatility between options on nonleveraged

ETFs and LETFs, and also between options on LETFs of different leverage ratios. It therefore allows us to estimate the prices of less liquid options based on those that are more liquid and thus expected to be more informative. Finally, we briefly reviewed the use of stochastic volatility and jump-diffusion models in pricing LETF options.

Appendix: Implications of asymmetric behavior of volatility on the implied volatility structure of LETF options

To understand the implications of the asymmetric behavior of volatility on the implied volatility structure of LETF options, let us first consider two out-of-the-money call options with the same (high) moneyness ratio. The first option is on a bull LETF, while the second is on a bear LETF of the same magnitude of leverage ratio and tracking the same underlying benchmark. For illustrative purposes, suppose the market outlook is neutral. If the volatility of the benchmark index is indeed negatively related to the index return, we will expect the chance of exercising the call option on the bull LETF to be lower than that of the bear LETF. This is because the former is benefited from a positive return on the index, which is expected to be accompanied by low return volatility. The low return volatility reduces the chance of realizing a high index value, and therefore limits the chance of the former to mature in the money. In contrast, the latter is benefited from a negative return on the index (i.e., a positive return on the bear LETF), which is usually accompanied by high return volatility. The high return volatility enhances the chance of realizing an even lower index value (i.e., a high bear LETF price), thus increasing the chance of the latter to mature in the money. The asymmetric effect therefore results in a lower price for the former and a higher price for the latter. In other words, the implied volatility of the former will be lower than that of the latter.[7]

The effect is exactly the opposite for in-the-money calls, which are options with low moneyness ratios. To understand this opposite effect, it is easier to consider two out-of-the-money put options instead. Note that out-of-the-money put options also have low moneyness ratios, so the implied volatility effect that we can illustrate with these out-of-the-money put options will also apply to the in-the-money calls. Suppose the first out-of-the-money put is on the bull LETF and the second on the bear. If the volatility of the benchmark index is indeed negatively related to the index return, we will expect the chance of exercising the former to be higher than that of latter. This is because the former is benefited from a drop in the benchmark index value, which is expected to be accompanied by high return volatility. The high return volatility increases the chance of realizing an even lower index value, thus increasing the chance of the former to mature in the money. In contrast, the latter is benefited from a positive return on the index (i.e., a negative return on the bear LETF), which is usually accompanied by low return volatility. The low return volatility reduces the chance of realizing a high index value (i.e., a low bear LETF price), thus limiting the chance of the latter to mature in the money. The asymmetric effect therefore results in a higher price for the former and a lower price for the latter. In other words, the implied volatility of the former will be higher than that of the latter.

To summarize, the asymmetric volatility effect in the equity market results in the implied volatility of options on bull LETFs being higher than that of options on bears when the moneyness ratio is low, and vice versa in high moneyness ratios. The shapes of the resulting volatility skew are therefore exactly the opposite of each other. Options on bulls tend to have a negative relation between the implied volatility and the moneyness ratio (i.e., downward sloping volatility skew); whereas, options on bears tend to have a positive relation (i.e., upward sloping volatility skew).

Notes

1 Introduction

1. Recently, LETFs with leverage ratio of +1.25 have been introduced by Direxion, one of the fund providers in the United States.
2. The market share of LETFs dropped to about 2% as of March 2015.

2 Regulations and Taxations

1. Simply put, the Investment Company Act of 1940 defines and classifies investment companies. It covers, among other things, how investment companies are structured and function, what their investment strategies can be, and how their securities are issued and redeemed. The act attempts to minimize conflicts of interest and ensure that the management of investment companies conducts itself in the best interest of investors. The Securities Act of 1933 regulates public offerings of securities. The act aims to ensure transparency through disclosures of companies' relevant information so that investors can make informed decisions. It also aims to prevent misrepresentation and fraud.
2. See the March 25, 2010, release, available at https://www.sec.gov/news/press/2010/2010-45.htm.
3. The diversification requirement stipulates that no single investment in the fund can exceed 25% of the fund's total value. Also, the aggregate of all positions over 5% of the total fund's value must not exceed 50%.
4. See, for example, "ETF Tax Shocker: Huge Payout for Rydex Inverse Funds" by Matt Hogan, downloadable on www.etf.com.

3 Mechanics

1. A swap is an agreement between two parties to exchange streams of cash flows according to a prespecified formula. Under a total return swap, one party (typically called "total return payer") agrees to pay periodic cash flows based on the return (i.e., dividends and capital gains) of a referenced asset (e.g., a stock index), while the other party (typically called "total return receiver") agrees to pay interest payments (typically London interbank offered rate [LIBOR] plus some spread), plus any capital losses on the referenced asset. Accordingly, the total return receiver gains exposure to the referenced asset without having to own it.
2. When a fund company sells a bear (e.g., –2x) LETF, it will typically invest the money from investors in fixed-income instruments. It will then create short exposure either by shortselling the underlying benchmark or using

derivatives. Daily exposure adjustments work in a similar manner to what we describe for bull LETFs here.

3. We note that it is increasingly difficult for LETFs (in fact, for ETFs in general) to find seed capital. This is because a lot of ETFs have been launched in the past few years, exhausting the available seed capital in the market. In addition, many of these new ETFs have failed to attract enough investor interest, and so their trading activity is small, making them less profitable for market makers who have to maintain quotes on them. The fact that bid-ask spreads have narrowed does not help the matter.
4. As of June 2015, the two largest providers of LETFs in the US market—ProShares and Direxion—capped their total fees at 0.95% p.a.
5. Fund companies typically charge a creation/redemption fee that has to be paid in cash. The fee varies from one fund to the next. Generally, the fee is set as a percentage of the transaction value (e.g., 0.10%) or a fixed dollar amount, whichever is lower.
6. Because of transaction costs in effecting this transaction, price and NAV may not be exactly the same. The difference, if any, should be within the magnitude of the transaction costs.
7. As of March 2015, the total assets of leveraged ETFs in the US market were approximately $32 billion, while the size of the whole US ETF market was estimated at approximately $1.8 trillion.

4 Return Dynamics and Compounding Effects

1. Virtually all LETFs in the market are of the "daily" type.
2. We will discuss tracking errors in detail in chapter 6.
3. This approximation is adapted from Co (2009).
4. This is true regardless of whether β is positive or negative. Therefore, it is possible, for example, for the holding-period return on a –2x bear leveraged ETF to be negative even when the underlying index's return over the same period is negative.
5. This also uses the fact that, as mentioned earlier, daily returns on leveraged ETFs cannot be worse than –100%. Therefore, one plus daily return will be nonnegative.
6. To see how, consider the case of a +2x LETF. Suppose the underlying index goes up today. This LETF will have to increase its exposure in order to maintain its leverage ratio. This increase in exposure will likely lead to further gains because, by positive correlation, tomorrow's return on the benchmark is more likely to be positive as well.

5 Pricing Efficiency

1. The average bid-ask spreads, calculated using daily closing bid and closing ask prices over the sample period and expressed as percentages of closing mid prices, range from 0.0082% (for SPY) to 0.1402% (for TZA).
2. A Wilcoxon-Mann-Whitney test confirms the differences in the distributions of premiums between the "up" and "down" days for all funds except PSQ (i.e., the inverse ETF on Nasdaq 100). When the closing midprices are used, the differences in the distributions of price deviations are significant for all funds, including PSQ.

3. The equation was derived in Cheng and Madhavan (2009).
4. It should be noted, however, that the actual amount of exposure adjustments can be less than that specified by equation (5.2). This is because most funds use derivatives to generate their returns. It is possible that the counterparties to the derivatives may have offsetting positions from their other obligations.
5. It is reasonable to expect the closing price of an underlying index to come from a later time than the closing prices of the funds that are based on it. This is because trading in an index (i.e., its constituents) is typically much more liquid than in the funds that are based on it.
6. To clarify, for each underlying index, Δt includes the amount of exposure adjustments of not only the LETFs and inverse ETF in our sample that are based on that index, but also other LETFs and inverse ETFs traded in the US market that are based on the same index. Therefore, Δt captures the *total* amount of exposure adjustments on that index on day t.
7. To see this, note that price deviations are conjectured to be caused by the impact of exposure adjustments on the underlying index value (and thus on funds' NAVs). The impact is in such a way that price deviations of bull (bear) funds are negatively (positively) correlated with the underlying index returns. As shown in equation (5.2), because the sign of the underlying index returns determines the sign of Δt (and, by extension, the sign of the normalized exposure adjustments), price deviations of bull (bear) funds are negatively (positively) correlated with normalized exposure adjustments. Therefore, the sign of b_1 is expected to be negative (positive) for bull (bear) funds.
8. During the sample period, the autocorrelations of daily returns of the S&P 500, Nasdaq 100, and Russell 2000 indices are –0.08, –0.05, and –0.13 respectively. While these numbers are small, they are significantly different from zero for the S&P 500 and Russell 2000 indices, but not for the Nasdaq 100 Index.
9. To see why, note that, according to the conjecture, price deviations of any LETF (bull or bear) are influenced by the contemporaneous index return. Therefore, if daily returns on the index are autocorrelated in one direction (positively or negatively), so are price deviations in that direction. Hence, b_2 should have the same sign as that of the autocorrelations of index daily returns, which is negative for all three indices in our case.
10. Since the amounts of adjustment (Δt) can be positive or negative depending on the index return on day t, we take absolute value of them first before we calculate the average.
11. The positive sign is, however, consistent with the findings in earlier studies on traditional (i.e., +1x) ETFs by Elton et al. (2002) and Rompotis (2010). Both studies report that lagged price deviations have a positive but very small effect on current price deviations, which is similar to our findings. Elton et al. interpret the results to mean that there is a small degree of persistence in price deviations, but the persistence disappears quickly (over a day) due to arbitrage force.
12. The low adjusted R-squareds could mean that the relationship between the variables is not linear. However, the scatter plots between price deviations and the two independent variables do not indicate a nonlinear relationship. We also repeated the regressions with squared exposure adjustments as the third independent variable (results not shown). The coefficient is

statistically insignificant for 12 of the 18 funds. More importantly, the coefficients for the first two independent variables remain close to their original values in table 5.6. In addition, the adjusted R-squareds also remain approximately the same, suggesting the squared exposure adjustments do not increase the explanatory power of the regression equation.

6 Performance and Tracking Errors

1. By assuming the underlying benchmark index following a geometric Brownian motion, Haga and Lindset (2012) provide the analytical formulation on exactly how LETFs' returns are related to financing costs.
2. Given the *volatility drag*, the LETF is not designed for long-term investors. Nevertheless, it is not uncommon to observe LETF investors holding on to their investment for more than a few days.
3. However, we need to be aware that, in a small sample setting, the use of the Newey-West approach may result in a bias in the sense that the null hypothesis is being rejected too often (Harri and Brorsen, 2009).

7 Trading Strategies

1. The second term captures the financing costs required to generate the target leverage.
2. Leung and Santoli (2012) also examine the choice of leverage ratio based on other risk metrics, namely value-at-risk and conditional value-at-risk.
3. SSO and SDS started trading on June 21, 2006, and July 11, 2006, respectively.
4. For example, it was hard to borrow LETFs during the global financial crisis of 2008–2009.
5. Note that you may also construct a pair portfolio by using bull and bear LETFs of different magnitude of leverage ratios. For example, shorting $3 of a +2x LETF together with shorting $2 of a –3x LETF will also give you a delta-neutral portfolio.

8 Options on LETFs

1. They are American-style options.
2. Without loss of generality, we ignore the distribution of any dividend payment.
3. Here we also assume there is no deviation between the price and net asset value of the LETF.
4. Deng et al. (2013) show that the slope of the volatility skew of the bull LETF options is approximately the negative of that of the volatility skew of the bear LETF options under the stochastic volatility model of Heston (1993).
5. It is important to emphasize that this transformed moneyness measure is only valid under the constant volatility setting. It is at best an approximation if volatility is in fact stochastic.
6. Zhang (2010) offers an alternative way to conduct relative pricing by showing how we can replicate a LETF option by a basket of options on its non-leveraged counterpart of specific strikes price and notional values.

7. Here we are assuming that there is an *objective* expectation and that market participants in both bull and bear LETF option markets have a homogenous belief in the distribution of future returns on the underlying benchmark index. However, this is not necessarily the case in reality. Traders of options on bull LETFs could have very different expectations and risk preferences from traders of options on their bear counterparts. By extracting the implied risk-neutral density of returns from options on bull and bear LETFs, Figlewski and Malik (2014) demonstrate the importance of recognizing investor heterogeneity in pricing these two kinds of options.

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Index