Solving the Return Deviation Conundrum of Leveraged Exchange-traded Funds

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Abstract

The large deviation of the actual return of a Leveraged Exchange-Traded Fund (LETF) from the leveraged multiple of the underlying index return has drawn considerable attention from investors, regulators, and the financial media. Despite this attention, the sources and fundamental determinants of the LETF return deviation remain unidentified. This study constructs a clear, unified, objective, and executable framework that addresses the behaviors, sources, and determinants of the LETF compounding and non-compounding deviations. Our theoretical predictions and empirical results hold the promise of guiding investors, regulators, financial advisors, and portfolio managers toward a thorough understanding of the return behavior of LETFs.

Keywords: Leveraged ETFs; Return Deviation; Compounding Effect; Non-compounding Effect; Tracking Error; Market Inefficiency

JEL Classifications**:** G11, G14

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I. Introduction

A Leveraged Exchange-Traded Fund (LETF) is a derivative-based, publicly-traded fund that seeks to deliver a daily return that is a multiple or inverse multiple of the return on an underlying index.[1] LETFs allow investors to execute hedge fund-like strategies with the liquidity and convenience of an ETF. They employ swaps and futures to achieve their target leverage and shorting multiples on a daily basis. The integration of constant daily leverage, shorting, derivatives, and indexing into one retail financial product has made LETFs, since their inception in 2006, one of the most innovative and controversial financial innovations for individual and institutional investors.

As of 2010, 174 leveraged ETFs were traded in the U.S. with total net assets under management (AUM) of $31.6 billion and annual dollar volume of $2.4 trillion, accounting for 18.9% of the total number, 3.2% of the total AUM, and 13.4% of the dollar volume of all ETFs traded in the U.S. Appendix Table A2 illustrates the evolution of the market size of LETFs relative to the total size of all ETFs from 2006 to 2010. A significant amount of inflow from 2006 to 2008 can be attributed to the perception that an LETF will be able to deliver the target multiple times the cumulative return on the underlying index over multiple holding days. This perception, however, deviates from reality. Media attention and regulatory reports indicate that some investors have not been fully aware of the constant leverage and daily rebalancing nature of LETFs and their compounding deviations. In fact, when the holding period is more than a day, the compounding effect will lead to a divergence of the LETF target return from the naïve

[1] See Appendix Table A1 for detailed definitions on the key terms and variables used in this paper.

expected return. These effects were most evident during the turbulence of the global financial crisis of 2007-09, when the return performance of LETFs negatively and dramatically deviated from the naïve expected return.

Computed as the LETF target return (i.e., the daily leveraged target return compounded over the holding period) less the naïve expected return (i.e., the product between the daily target multiple and the cumulative return of the underlying index), the LETF compounding effect, or compounding deviation, has drawn the displeasure of many investors since 2009. As a result, LETF providers have been the recipients of class action lawsuits that charge them with misleading investors regarding the stated performance objectives of LETFs. In the summer of 2009, both the Financial Regulatory Industry Authority (FINRA) and the Securities and Exchange Commission (SEC) formally warned buy-and-hold investors of the extra risks associated with LETFs over the long term.[2] In March 2010, the SEC launched a comprehensive review to evaluate the use of derivatives by investment funds, especially leveraged ETFs, to ensure that "regulatory protections keep up with the increasing complexity of these instruments and how they are used by fund managers."[3] With increased awareness of the potential risks associated with LETFs and tighter regulatory scrutiny of them, the growth rate of LETF AUM slowed to 1.59% in 2010, a figure well below the 27.7% growth in overall ETF AUM during the same period.

Because LETFs are relatively new and complex financial instruments, academic research on them is still in its infancy. Along with the growing industry and regulatory attention on

[2] See the regulatory notices 09-31 and 09-53 issued by the FINRA in June 2009 and August 2009 and the investor alert jointly issued by the FINRA and SEC in August 2009 about LETFs.

[3] See the news release 2010-45 issued by the SEC in March 2010.

LETFs, several recent studies have examined the return behavior and compounding effect of them. Cheng and Madhavan (2009), for example, demonstrate the impact of daily LETF rebalancing on the underlying market volatility, and derive the relationship between an LETF return and an underlying index return using continuous time assumptions. Avellaneda and Zhang (2009) construct a model on the LETF return after factoring in financing costs and an expense ratio, and provide strong empirical support for their theoretical model. Lu, Wang, and Zhang (2009) examine the performance of LETFs and conclude that over holding periods of one month or less, LETF returns are near their naïve expected returns, but not over longer holding periods. Although these studies have examined the LETF compounding effect, none of them has provided an executable framework to address the fundamental determinants of such deviation.

Other than the compounding effect, the deviation of the LETF return from the naïve expected return could also be attributed to other effects, such as management tracking error, market frictions, or inefficiency. Two recent studies have started to pay attention to the non-compounding deviation of actual LETF returns from their target returns. Charupat and Miu (2011) examine the characteristics, trading behavior, price deviation, and tracking errors of a set of Canadian LETFs, with special attention on the daily premiums/discounts of LETFs relative to their NAVs. However, their study does not explicitly identify the potential deviation of NAV returns from the LETF's target returns. Shum (2011) also investigates a set of Canadian LETFs by distinguishing the compounding effect from the management effect and the trading premium/discount. However, the study neither provides complete decomposition of the total return deviation, nor addresses its fundamental determinants.

Five years after the birth of the LETFs, their return deviations largely remain a conundrum, especially across different multiples and holdings periods. Their unpredictability and potential value destruction caused by the "constant leverage trap" are thought to be too onerous for investors. As a result, regulators and some financial firms have advised investors to avoid LETFs. This stance, however, is without the provision of clear guidance on the behaviors, sources, and determinants of LETF return deviations because research has yet to provide it.

From an investor's perspective, because LETFs are designed to track their underlying indices with daily target multiples, it would be insightful and instructive to examine their ability to deliver their daily return objectives and the underlying drivers of their daily return deviations. Equally, if not more important, investors need to know the implications of a lengthened holding period, including the behavior and determinants of various return deviation components that emanate from the compounding effect, management tracking error, and market inefficiency.

We construct a clear framework to solve the return deviation conundrum associated with LETFs. In addition, we conduct a comprehensive empirical study on the most popular LETF families that track or magnify the long or short performance of the S&P 500 (SPX), Dow Jones Industrial Average (INDU), NASDAQ 100 (NDX), and S&P Midcap 400 (MID) indices. This sample represents all twelve LETFs across various target multiples that were initially launched in 2006 and includes complete daily data from inception to December 2010, the longest period to date over which LETFs have been studied.[4]

Two questions guide our research. First, how well do LETFs achieve their daily performance objectives and what are the determinants of the daily return deviations? Addressing it, we examine the daily deviation between the actual LETF return and target return and find that LETFs show an underexposure to the index that they seek to track. To further examine whether this daily deviation is due to management tracking error or due to market inefficiency, we decompose it into an NAV deviation component and a residual deviation component and

[4] See Appendix Table A3 for detailed information on the LETF sample used in our study.

examine the determinants of each component. Our results show that the holding-period LIBOR interest is the main determinant of the daily NAV deviation. We also find that the daily residual deviation is driven by its own mean reversion and the underlying index return, showing that the observed underexposure is actually due to the market frictions and inefficiency but not due to the management tracking error. The results are robust before and after controlling for the market-wide volatility factor, as well as fund-specific liquidity and fund flow factors.

Second, to what extent do actual LETF returns deviate from their naïve expected returns over various holding periods, and what are the sources and determinants of these cumulative return deviations? Addressing this question, we expand the holding period from one trading day to forty trading days. In turn, we study the cumulative compounding deviation, which is the difference between the target return and the naïve expected return, and the non-compounding deviation, which is the difference between the actual return and the target return, over time and across different target multiples. We find that both deviations accumulate over time, and that the size of non-compounding deviation is at least as significant as that of the compounding deviation. We further decompose this non-compounding deviation into a management tracking error component, as captured by the NAV return deviation, and a market inefficiency component, as accounted for by the residual deviation. Our results clearly show that the management tracking error, rather than market inefficiency, is the main source of the non-compounding deviation. We also uncover a swap-related, holding-period LIBOR interest factor as the key determinant of the cumulative NAV deviation and construct a framework to show that the NAV deviation is highly predictable across various multiples over various holding periods. Building on the literature, we also construct a unified framework of the LETF compounding deviation by using the squared cumulative index return, daily return variance on the underlying index, length of the holding period, and daily target multiple. We provide evidence that the historical behavior of LETF compounding deviations can be fully explained by this framework.

This study contributes to the literature by providing a clear, unified, objective, and executable framework that addresses the behaviors and sources of LETF return deviations. Using daily data over the longest sample period to date, it also uncovers their fundamental determinants. Not only may the framework and findings guide investors, regulators and financial advisors on the return behavior of LETFs, but they also can assist portfolio managers in formulating more effective tactical asset allocation strategies than they have to date.

The rest of the article proceeds as follows. Section II develops the framework for the determinants of the compounding and non-compounding deviations on LETFs. Section III describes the data and defines key variables. Section IV presents the empirical results. Section V draws the conclusions and discusses the implications.

II. Determinants of the Return Deviations on LETFs

To guide our analysis, we broadly define LETFs as all exchange-traded funds designed to track the return on an underlying benchmark index with a daily target multiple, such as (2 ×), (3 ×), (-1 ×), (-2 ×), (-3 ×), using swaps and other derivatives. These new, innovative financial instruments are in contrast to the regular ETFs that are designed to track (1 ×) the underlying index. The (2 ×) or (3 ×) funds, often referred to as the narrowly defined LETFs, long ETFs, or bull LETFs, are designed to provide double or triple long exposure to the underlying index. The (-1 ×), (-2 ×) or (-3 ×) funds, often referred to as the inverse LETFs, short LETFs, or bear LETFs, seek to deliver single-short, double-short or triple-short exposure to the underlying index, respectively.

While attention has focused on the compounding deviation, which is tied to the daily rebalancing of LETFs, the deviation of an LETF's actual return from the multiple of its underlying index return could also be due to non-compounding effects, such as fund management tracking errors, market frictions, and inefficiency. To examine the sources and determinants of the return deviation between an LETF's actual market return and the naïve expected return, we decompose the total return deviation into a compounding component, which is the difference between the target return and the naïve expected return, and a non-compounding component, which is the difference between the actual market return and the target return.

A. Determinants of the Non-compounding Deviation of LETFs

Because an LETF is designed to track the underlying index with a daily target multiple, it is important to understand how well these LETFs achieve their daily performance objectives. The daily return deviation, computed as the difference between an LETF's actual market return and the target return, is essentially the non-compounding deviation. This deviation may be decomposed into two components: the NAV deviation between the NAV return and the target return (due to management tracking error) and the residual deviation between the actual market return and NAV return (due to market frictions and inefficiency).

The NAV deviation shows the degree to which fund management achieves the target it intends to deliver. Because LETF management firms use swaps and other derivatives to deliver the target leveraged performance exposure to the underlying index, we expect the NAV deviation to be sizable and accumulative due to the swap-related interest payout/receipt. The residual deviation shows the accuracy with which the LETF market reflects the fundamental value of the fund. The availability of the creation-redemption feature and the ease of arbitrage between the actual price and NAV lead us to expect the residual deviation between the actual LETF return and NAV return to be small, mean-reverting, and noncumulative.

We regress each of these two daily return deviation components on the following possible determinants: holding period LIBOR interest, underlying index return, lagged return deviation, VIX, fund turnover, relative fund bid-ask spread, and net fund flows. These variables are identified based on the swap-related floating rate payout/receipt, fundamental return, volatility, and liquidity factors that may contribute to an LETF's NAV and residual deviations. The following discusses the rationales for the use of these variables as possible determinants in the NAV and residual return deviation regressions.

Holding-period LIBOR Interest: According to the fund prospectus, LETFs mainly use equity swaps to achieve their intended daily leveraged exposure to the underlying index. In general, an equity swap involves the exchange of equity return (such as the S&P 500 return) for a floating interest rate (such as the LIBOR). The equity return receiver (such as bull LETFs) pays the floating rate while the equity return payer (such as bear LETFs) receives the floating rate. The payout (receipt) of the floating rate implies that the NAV of bull (bear) LETFs will have a negative (positive) NAV deviation from the target return. For an LETF, to receive the target multiple (m) times the underlying index return, the fund needs to pay (m-1) times the floating rate.[5] In addition, the expense ratio will be a negative drag on LETF returns. The combined

[5] We assume that a bull LETF uses the existing fund value to "long" the underlying index and uses equity swaps to increase the exposure to deliver the target multiple of m. Alternatively, the fund can also lend out the fund value to earn the LIBOR interest and use swaps to leverage the exposure to deliver the target multiple of m, which will lead to the same conclusion. For an inverse LETF, as long as the fund can lend out the fund value to earn the LIBOR interest and use equity swaps to achieve the target multiple of m, the fund will receive a total interest of (-m+1), or equivalent to paying (m-1), times the LIBOR interest.

effect of the swap-related interest rate payout/receipt and the expense ratio implies that the LETF NAV return will deviate from the target return, and that this NAV deviation will accumulate over time. In particular, this NAV deviation can be captured by the following equation:

$$R_{LNAV} - R_{LT} = -(m-1)\int_{s=0}^{s=t} r_s ds - f \times t \quad (1)$$

where R_{LNAV} is the NAV return, R_{LT} is the LETF target return, m is the LETF's daily target multiple (such as 2 ×, 3 ×, -1 ×, -2 ×, -3 ×, etc.), $\int_{s=0}^{s=t} r_s ds$ is the holding period LIBOR interest,[6] and f is the annual LETF expense ratio divided by the number of trading days in a year. This framework predicts that the holding period LIBOR interest should have negative (positive) effect on the *NAV return deviation* of bull (bear) ETFs, and the regression coefficient should be close to -1, 2, 3 for the (2 ×), (-1 ×), and (-2 ×) LETFs, respectively. In contrast, we do not expect the LIBOR interest to affect the *residual return deviation*.

<u>Underlying Index Return</u>: To maintain effectively an LETF's daily target exposure to its underlying index, the fund manager needs to rebalance its derivative positions for the coming day near the end of each trading day. Cheng and Madhavan (2009) show that an LETF's daily re-balancing need is positively related to the underlying index return on that day and the LETF's multiple function (m^2- m), which is equal to 2, 2, and 6 for (2 ×), (-1 ×), and (-2 ×) funds, respectively. For any LETF, there is always a need for increased exposure to (i.e., buying) its underlying index whenever the index experiences a positive return and always a need for decreased exposure to (i.e., selling) the underlying index whenever the index return is negative.

[6] In particular, the LIBOR interest earned over one trading day is calculated using the actual number of calendar days between previous trading day and current day, divided by 360 and multiplied by the 3-month annualized LIBOR rate from the previous trading day. For multiple holding days, this measure is accumulated over each day during the holding period.

Because all LETFs of the same family need to decrease exposure to their underlying index when it falls, a strong "short" need emerges. Similarly, a "long" need emerges when the underlying index rises. The LETFs' "short" need on "down" days and "long" need on "up" days may increase the cost of daily re-hedging. With the closing market volatility and the difficulty of achieving the target multiple at the market's close, the fund management company may prefer underexposure to overexposure to save on hedging costs. If the fund management consistently underexposes the LETF to its underlying index, the NAV return deviation of bull (bear) LETFs will be negatively (positively) affected by the underlying index return. We therefore include the underlying index return as a possible determinant of the daily LETF *NAV return deviation* to see if the underexposure is due to management tracking error.

However, as argued by Charupat and Miu (2011), non-synchronization between the LETF closing price and its underlying index and the short-term trading behavior from LETF investors could lead to a downward (upward) bias on the bull (bear) LETF returns on an underlying index "up" day and an upward (downward) bias on the bull (bear) LETF returns on an underlying index "down" day. On the other hand, since LETF trading is dominated by short-term investors who tend to sell (buy) when LETF prices move up (down), making the closing transaction price more likely to occur on the bid (ask) for bull ETFs and on the ask (bid) for bear LETFs on a underlying index "up" ("down") day. The existence of non-synchronization and short-term trading behavior is expected to lead to a negative (positive) effect of the underlying index return on the residual deviation of bull (bear) LETFs. We therefore include the underlying index return as a possible determinant of the daily LETF *residual return deviation*.

<u>Lagged Autoregressive Term</u>: A major attribute of ETFs is the creation and redemption in block size of creation units (e.g., 50,000 shares for the (1 ×) SPX fund, SPY)). Ackert and

Tian (2000) and Curcio, Lipka, and Thornton (2004) study the price and NAV of SPY and QQQQ and show that the creation-redemption feature is indeed highly effective in aligning the price of an ETF back to its NAV from any temporary deviation. The creation-redemption mechanism allows for easy arbitrage if the ETF price moves away from its NAV. However, the need to rebalance their investment portfolios during the last hour of trading motivates fund management companies to prohibit the creation or redemption of LETF shares between 3:00 P.M. and 4:00 P.M. Because the daily LETF closing price may occur either at the bid or ask, it may differ from the NAV. It may also differ because closing market volatility and the unavailability of the creation-redemption feature during the last trading hour may prevent closing market arbitrage. On the next day, with the reopening of the creation-redemption feature, arbitrageurs will tend to push the price back to its NAV. This temporary inefficiency due to market frictions and subsequent adjustment could lead to the mean reversion of the residual deviation. To account for the mean reversion property of this deviation, we include a one-day autoregressive term as a potential determinant of the daily LETF *residual return deviation*.

On the other hand, because the mark-to-market value of derivative contracts typically is not as accurate as that of the underlying index, and because some hedging activities may continue even after the market closes, the closing NAV that a fund company reports may differ from the fundamental value of a fund. The possibility of fund management's inaccurate assessment at the market closing, followed by a correction on the next day, could potentially lead to a mean reversion in the NAV deviation. We therefore include the lagged one-day autoregressive term as a possible determinant of the daily LETF *NAV return deviation*.

In addition, we include the VIX variable to control for market-wide volatility in both daily deviation regressions. Finally, we also include the liquidity and fund flow factors of the regular ETF in the daily NAV deviation regression, and the LETF-specific liquidity and fund flow factors in the daily residual deviation regression. The rationales for the inclusion of these variables are summarized in Appendix Table A4.

Following the daily regressions, we examine the behavior of the cumulative non-compounding deviation, and its NAV and residual deviation components, for holding periods up to 40 days. Because our results show that the NAV deviation is the dominating non-compounding deviation component, we test for the determinants of the NAV return deviations across different holding periods and target multiples.

B. Compounding Effect of LETFs

The LETF compounding effect, which is also called the LETF compounding deviation, is the cumulative target return of an LETF less the product between the daily multiple of an LETF and the cumulative return of the underlying index. Building on the work of Avellaneda and Zhang (2009) and Cheng and Madhavan (2009), we obtain the relation between the LETF's target return and its underlying index return:[7]

$$Ln\left(\frac{L_t}{L_0}\right) = mLn\left(\frac{I_t}{I_0}\right) + \frac{(m-m^2)\sigma^2 t}{2} \tag{2}$$

where m is the daily target multiple of an LETF, t is the length of the holding period, σ^2 is the variance of the underlying index return, and I_0 (I_t) and L_0 (L_t) are the underlying index level and the LETF target price level at the beginning (end) of the holding period, respectively.

We denote the cumulative return on the underlying index and the cumulative target return on the LETF as $R_I = \frac{I_t}{I_0} - 1$ and $R_{LT} = \frac{L_t}{L_0} - 1$, respectively.

When the cumulative return is small, using a second-order Taylor expansion, we have:

[7] A detailed derivation of equation (2) is available upon request.

(3) $$R_{LT} - \frac{R_{LT}^2}{2} = m\left(R_I - \frac{R_I^2}{2}\right) + \frac{(m-m^2)\sigma^2 t}{2}$$

By rearranging equation (3), we have the following:

(4) $$R_{LT} - mR_I = \frac{R_{LT}^2}{2} - m\frac{R_I^2}{2} + \frac{(m-m^2)\sigma^2 t}{2}$$

Using a first-order approximation, we can substitute R_{LT} with mR_I on the right hand side to derive the following equation for the compounding deviation of an LETF:

(5) $$R_{LT} - mR_I = \frac{(m^2-m)}{2}R_I^2 - \frac{(m^2-m)\mathrm{t}}{2}\sigma^2$$

According to the model, we hypothesize that the compounding deviation ($R_{LT} - mR_I$) should be positively related to the squared cumulative return on the underlying index (R_I^2), and negatively related to the variance (σ^2) of the underlying index return. In addition, we hypothesize that the strength of the compounding deviation should be positively related to the multiple function (m^2- m), which equals 2, 6, 2, 6, 12 when the multiple (m) is 2, 3, -1, -2, -3, respectively. We expect the compounding deviation to be similar for the long double (2 ×) and for the single inverse (-1 ×) LETFs tracking the same index, and the compounding deviation for the short double (-2 ×) LETFs to be three times that of the single inverse (-1 ×) and long double (2 ×) LETFs. We also expect the strength of the negative impact of the underlying index volatility to be positively related to the length of the holding period (t).

III. Data and Variable Definitions

Our sample consists of the four families of ETFs. They include the (1 ×) regular ETFs and the (2 ×), (-1 ×), (-2 ×) leveraged ETFs that use the S&P 500 (SPX), Dow Jones Industrial Average (INDU), NASDAQ 100 (NDX), and S&P Midcap 400 (MID) as their underlying benchmark indices, respectively. The twelve leveraged ETFs included in this study represent a complete list of the LETFs that were created in 2006. The (1 ×) funds are regular, non-leveraged ETFs for the purpose of comparison with the LETFs tracking the same underlying index. Because neither the triple nor inverse triple LETFs were available prior to June 2009, they were excluded from our study. Appendix Table A3 describes our sample LETFs in detail, including the daily target multiple, underlying benchmark index, inception date, expense ratio, total assets under management of year-end 2010, and average number of holding days during the sample period.

Our sample LETFs have expense ratios ranging from 0.90% to 0.95%, while their regular ETF counterparts have far lower expense ratios, ranging from 0.095% to 0.20%. The AUM for the LETFs is well below the AUM for their regular ETF counterpart. The combined AUM of the (2 ×), (-1 ×), and (-2 ×) LETFs in our sample is only 6.2% of the AUM of their regular (1 ×) ETF counterparts.

Daily data on the price, dividend, NAV, bid-ask spread, trading volume, and number of outstanding shares on these LETFs, along with the total return on the four underlying benchmark indices (S&P 500, Dow Jones Industrial Average, NASDAQ 100 and S&P Midcap 400), were obtained from Bloomberg. Because the first set of the LETFs (i.e., the (2 ×) funds and the (-1 ×) funds) were created in June of 2006, and the second set (i.e., (-2 ×) funds) were launched in July 2006, we use a common sample period from July 17, 2006 to December 31, 2010, which amounts to 1125 trading days. When calculating the returns for ETFs and their underlying indices, we always include the effects of both price changes and dividend payments.

To calculate the daily target return for an LETF, we use the product between the multiple of the LETF and the return on the underlying benchmark index. To calculate the LETF target return over multiple holding days, we compound the LETF daily target return geometrically.

This is the target return that the LETF is designed to achieve. The non-compounding deviation is calculated by using the actual LETF return less the target return.

The naive expected return of an LETF is calculated as the product between the daily multiple of the fund and the cumulative return of the underlying benchmark index. Although this is what an LETF intends to track on a daily basis (as stated in the prospectus), there is no promise or evidence that an LETF will be able to deliver the naive expected return over a holding period longer than one day. The compounding effect is calculated by using the target return less the naive expected return. This compounding effect is due to the product design of LETFs.

To examine further the non-compounding deviation, we break it down into an NAV deviation and a residual deviation. The NAV deviation of an LETF is calculated by using the NAV return less the target return. If the NAV is an accurate measure of the fundamental value of the fund, the NAV deviation reflects management's ability to achieve its target. Because the target return is computed by using the sum of the price return and dividend yield, we also adjust the NAV by the LETF's dividend payments. The residual deviation is calculated by using an LETF's actual return less its NAV return. This residual deviation reflects the deviation of an LETF actual market price from its NAV.

IV. Results

A. Daily Return Deviation and Beta Estimation of LETFs

Table 1 shows the daily returns and betas of the LETFs during the sample period in Panels A and B, respectively. The daily average return results from Panel A across the SPX, INDU, NDX, and MID ETF families consistently show that regular ETFs (i.e., the (1 ×) funds) track the underlying indices very well. The actual returns of bull LETFs (i.e., the (2 ×) funds) are lower than their target returns. In contrast, the actual returns of bear LETFs (i.e., the (-1 ×) and (-2 ×) funds) are higher than their target returns. Succinctly, all LETFs offer an average daily return that is smaller in magnitude than their target returns.

To enhance our understanding of the exposure of LETFs to their underlying indices, we apply regression analysis to the daily returns of LETFs on their underlying index returns. Panel B of Table 1 shows the regression coefficients of the single-index model for the SPX, INDU, NDX, and MID ETF families, respectively. The coefficient on the underlying index return is referred to as the daily beta. From an investor's point of view, this actual daily beta should be equal to the stated target multiple promised by the fund sponsor. As shown in the first row of Panel B, the beta for the (1 ×) SPX fund is 0.9848, which is not statistically different from its target multiple of 1.00. The actual daily betas for the (2 ×), (-1 ×), and (-2 ×) SPX funds are 1.8943, -0.9775, and -1.9125, which have significantly smaller magnitudes than their target multiples of 2.00, -1.00, and -2.00, respectively. As shown in the rest of Panel B, the results from the INDU family, NDX family, and MID family confirm those from the SPX family. This finding raises the question of whether this observed underexposure is due to management tracking error or market frictions and inefficiency, which will be examined later. Because it is difficult and costly to hedge perfectly toward market closing, the fund management company may prefer underexposure to overexposure to save hedging costs. However, this observed underexposure may not only be due to management issues, but also market frictions and inefficiency.

B. Determinants of the Daily NAV and Residual Deviations

At a daily interval, the return deviation is solely due to the non-compounding effect. Why would the daily return of an LETF deviate from its target return? Is this deviation due to management issues or due to the market factors? As explained in Section II, we decompose the daily return deviation into two components: the NAV return deviation that reflects the fund management's inaccuracy in achieving the target leveraged exposure to the underlying index and the residual return deviation that reflects the LETF market frictions and inefficiency.

Panel A of Table 2 presents the summary statistics for the daily NAV deviation and residual deviation of the LETFs. The residual deviation consistently shows a smaller mean and larger standard deviation relative to the NAV deviation. For example, the average NAV deviation for the (2 ×) SPX fund is -1.45 basis points, while its average residual deviation is only -0.45 basis points. On the other hand, the standard deviation of the NAV deviation for the (2 ×) SPX fund is 2.47 basis points, while that of its residual deviation is 45.65 basis points. These findings suggest that the NAV deviation is the dominant component of the observed daily return deviation of LETFs, and that the drivers of the NAV deviation are more stable and persistent than those affecting the residual deviation.

What are the determinants of the daily NAV and residual deviations? As explained in Section II and Appendix Table A4, we include the following variables as possible determinants of the daily NAV and residual return deviations: the holding period LIBOR interest to capture the swap-related interest payout/receipt; the return on the underlying index to capture the performance of the underlying index; an autoregressive term to capture possible mean-reverting behavior of the return deviation; the VIX index to capture the market volatility and sentiment; the regular (1 ×) ETF's fund-specific liquidity and excess demand factors for the NAV return deviation regression; and the LETF's fund-specific liquidity and excess demand factors for the residual deviation regression. The summary statistics of these fund-specific and market-wide explanatory variables are shown in Panel A and Panel B of Table 2, respectively. During our sample period, the holding-period LIBOR interest has an average value of 1.1 basis points for each trading day, with a standard deviation of 1.2 basis points.[8] The average VIX is 24.63 with a standard deviation of 12.05, indicating a high level of market volatility and the wide variation associated with it. The average net fund flows are positive in all cases, showing an increase in fund demand during the sample period.

Table 2, Panels C and D, present the respective regression results for the daily NAV and residual return deviations for the SPX LETF fund family. As shown in Panel C, the holding-period LIBOR interest is indeed the single most important determinant of the LETF NAV deviations. As shown in Columns (4), (7) and (10), the holding-period LIBOR interest has a highly significant coefficient of -0.93, 1.68, and 2.59 for the (2 ×), (-1 ×), and (-2 ×) SPX funds, respectively. These estimates are consistent with the predicted value of -1.00, 2.00, and 3.00 from equation (1) of Section II.A. The rest of Panel C confirms that the coefficient of the holding-period LIBOR interest remains very significant and close to its predicted value even after we add various control variables.

After controlling for the holding-period LIBOR interest, as shown in Columns (4), (7) and (10), the intercepts are -0.46, -0.39, and -0.51 basis points for the (2 ×), (-1 ×), and (-2 ×) SPX funds, respectively. This negative drag on the performance is consistent with the expense ratio of these LETFs. In particular, if the annual expense ratio is distributed evenly into each

[8] We perform a Dickey-Fuller unit root test on the *holding period LIBOR interest* and reject the hypothesis of a unit root.

trading day, an expense ratio of 0.95% will account for a drag of 0.38 basis points on the LETF's daily NAV deviation.

As shown previously in Panel B of Table 1, there is a beta deviation of -0.106, 0.023, and 0.088 for the (2 ×), (-1 ×), and (-2 ×) SPX funds, respectively. Panel C of Table 2 shows that the coefficient of the SPX daily return in the NAV return deviation regression is actually much smaller, registering values of -0.001, 0.002, and 0.002 for the (2 ×), (-1 ×), and (-2 ×) SPX funds, respectively. These findings suggest that the observed LETF underexposure to the underlying index is not due to fund management issues. For the autoregressive term on the NAV return deviation, we observe some small mean-reversion for the [deleted (1 ×) and] (2 ×) SPX funds, and no significant mean reversion for the (-1 ×) and (-2 ×) SPX funds.

Panel D of Table 2 shows the regression results for the daily residual return deviations for the SPX LETF family. As shown in Columns (4), (7), and (10), the holding-period LIBOR interest is not related to the residual deviation. As shown in Columns (5), (8), and (11), the SPX daily return registers strongly significant coefficients of -0.091, 0.017, and 0.074 for the (2 ×), (-1 ×), and (-2 ×) SPX funds, respectively. These coefficients on the underlying index return are consistent with the two explanations offered by Charupat and Miu (2011); namely, the non-synchronization between the closing market prices of LETFs and their NAVs, and the trading behavior caused by short-term LETF investors. These findings also confirm that our previously observed under-exposure of LETFs is indeed due to market frictions and inefficiency, as captured by the residual deviation component, rather than fund mismanagement. In addition, there is a strong mean-reversion in the residual return deviation for every fund. This is consistent with the effectiveness of the ETF creation-redemption feature at pushing the fund prices back to their NAVs and reversing any temporary residual deviations.

Overall, our daily analysis shows that the holding-period LIBOR interest and the expense ratio are the two key determinants of the NAV return deviation, and that the lagged residual deviation and underlying index return are two key determinants of residual deviation. These findings are robust before and after controlling for the market-wide volatility and fund specific liquidity and excess demand factors.

C. Cumulative Return Deviations: Compounding and Non-compounding Effects

When we move from one trading day to multiple holding days, the target return of an LETF will also deviate from its naive expected value, in addition to the deviation between the LETF actual return and its target return. As mentioned, the negative sentiment, both publicly and academically, surrounding LETFs is centered on this compounding effect. How significant is it? How important is this compounding effect compared to the cumulative non-compounding deviation over multiple holding days?

To address these questions, we compare the compounding deviation to the non-compounding deviation over various holding periods up to forty trading days, on a rolling basis. We present the averages along with the Newey-West (1987) robust standard errors and statistical significance in Table 3.[9] The average compounding effects of the (2 ×), (-1 ×), and (-2 ×) funds are generally negative, showing that the cumulative target returns are lower than the naïve expected returns during our sample period. This finding is consistent with the sentiment in the literature, financial media reports, and the warnings from fund providers, the SEC, and FINRA.

[9] We thank the editor for suggesting the presentation of standard errors and statistical significance that account for the degree of overlaps in our rolling sample.

As the number of holding days increases from 2 to 40, the compounding effect for the (2 ×) SPX fund increases from -0.7 to -22.5 basis points, while the non-compounding effect increases from -3.3 to -56.8 basis points. The cumulative effect is statistically significant for all holding periods for the non-compounding effect, and significant up to 20 days for the compounding effect. Compared to the compounding effect, the non-compounding effect for the (2 ×) SPX fund is higher in magnitude and more stable in statistical significance. These findings show that the overwhelming media and regulatory attention on the compounding effect runs the significant risk of overlooking the non-compounding effect.

For the (-1 ×) and (-2 ×) SPX fund, we find similar results. First, the compounding deviation increases as the holding period gets longer. Second, the non-compounding deviation also accumulates over time. However, the non-compounding deviation for the (-1 ×) and (-2 ×) SPX funds is positive. This positive deviation for the (-1 ×) and (-2 ×) SPX funds and the negative deviation for the (2 ×) SPX fund are consistent with the fact that the bear LETFs receive floating rates while the bull LETFs pay floating rates in equity swaps. In addition, the compounding effect of the (-2 ×) SPX fund is about three times that of the (2 ×) or (-1 ×) SPX funds across various holding periods, which is consistent with the theoretical predictions from equation (5) of Section II.B. As a result, the size of the compounding effect of the (-2 ×) SPX is stronger than the non-compounding effect for periods longer than five days.

Figure 1 illustrates the cumulative effects of the compounding, non-compounding, and total deviations for the SPX family of LETFs up to 40 days. Overall, the results in Table 3 and patterns in Figure 1 show that (1) both compounding and non-compounding deviations tend to increase in size as the number of holding days increases; (2) the non-compounding deviation is at least as important as, and sometimes dominates, the compounding deviation; (3) the compounding deviation is negative during the sample period; and (4) the non-compounding deviation is negative for the bull LETFs and positive for the bear LETFs. These findings are generally confirmed by evidence from the other LETF families, as shown in Table 3.

D. Cumulative Non-compounding Deviation: Management Tracking Error or Market Inefficiency?

As previously discussed, the non-compounding deviation has two components: NAV deviation and residual deviation. How do these two components accumulate over multiple trading days? Table 4 presents the mean and Newey-West adjusted standard error of the NAV and residual return deviations over various holding periods, from two to forty trading days, on a rolling basis. Figure 2 illustrates the cumulative NAV and residual deviations for the SPX family of LETFs up to 40 days.

Table 4 and Figure 2 show that the NAV deviation is much larger than the residual deviation over multiple holding days. For the LETFs in our sample, the residual deviation does not accumulate over time and is statistically insignificant across various holding periods. This can be explained by the creation-redemption feature that prevents the residual deviation from accumulating, promoting market efficiency. However, the NAV deviation accumulates as time increases and remains statistically significant across various holding periods. For example, the two-day average NAV deviation for the (2 ×) SPX fund is -2.89 basis points, while the two-day residual deviation is only -0.42 basis points. When extended to 40 trading days, the average cumulative NAV deviation is -57.2 basis points, but the average cumulative residual deviation is only 0.39 basis points. These findings suggest that management tracking error is the dominant reason behind the observed cumulative non-compounding deviations. Market inefficiency, as measured by the return deviation between the actual price of an LETF and its NAV, registers a much smaller magnitude than management tracking error.

As demonstrated in equation (1) of Section II.A, the cumulated NAV deviations over multiple holding days should be driven negatively (positively) by the swap-related floating rate payout (receipt) for bull (bear) LETFs and negatively by the expense ratio for all LETFs. Both the floating rate payout and expense ratio effects on a bull LETF will lead to a negative accumulation of the NAV deviation over multiple holding days. The average three-month LIBOR is 2.63% during our sample period, which will have a negative impact of 41.7 basis points on the 40-trading-day cumulative non-compounding deviation. In addition, the expense ratio will lead to an additional drag of 15.1 basis points on the 40-day non-compounding deviation. These two factors predict a 40-day non-compounding deviation of -56.8 basis points, which approximates the -57.2 basis points we observe from Table 4.

For an inverse LETF, however, the receipt of the floating rate in equity swaps will have a positive impact on the cumulated NAV deviation over multiple holding days. Because the positive effect of the floating rate receipt dominates the negative effect of the expense ratio for a bear LETF, we should observe a positive cumulated NAV deviation, which is consistent with the results in Table 4. Because the floating rate receipt of the (-2 ×) fund is more than that of the (-1 ×) fund in equity swaps, we observe a substantially larger positive NAV deviation for the (-2 ×) fund relative to the (-1 ×) fund. In addition, because the effects of the floating rate payout/receipt and expense ratio are both linear functions of the length of the holding period, we observe larger NAV deviations as the holding periods lengthen.

The INDU, NDX, and MID LETF families show a similar pattern with respect to the cumulative NAV and residual deviations as those for the SPX LETF family. The findings on the sizable and cumulative NAV deviations and the framework demonstrated by equation (1) suggest that an LETF's target return is not achievable after taking into consideration the swap-related floating rate payout/receipt and expense ratio. While the resulting management tracking error may not indicate mismanagement by the fund provider, the result is a significant, sizable and accumulative NAV return deviation from the target return. Although the residual deviation is small and noncumulative, attributable to the availability of the creation-redemption feature, the existence of a large and accumulative NAV deviation leads to a sizable and accumulative non-compounding deviation that is an important component of an LETF's total return deviation.

E. Determinants of the Cumulative NAV Deviation

As shown in the previous subsection, the cumulative non-compounding deviation is mainly composed of the NAV deviation. In this subsection, we examine the determinants of this cumulative NAV deviation using regression analysis. As we have previously shown, the holding-period LIBOR interest is a key determinant of daily NAV deviation. We therefore regress the cumulative NAV deviation of the (2 ×), (-1 ×), and (-2 ×) SPX LETFs on the cumulated LIBOR interest during the holding period. To avoid the large number of overlapping days between consecutive observations, we conduct the analysis by using non-overlapping observations for 10-day and 40-day holding periods to approximate the bi-weekly and bi-monthly holding periods, respectively.[10] Table 5 reports the regression results in Panels A and B, along with the descriptive statistics for the dependent variable in Panel C and the explanatory variables in Panel D.

[10] We find similar results when using overlapping observations on a rolling basis and when using other holding periods, such as five and twenty days.

Consistent with the rolling sample statistics from Table 4, the non-overlapping sample statistics from Panel C of Table 5 show an average 10-day NAV deviation of -14.5, 14.0, and 22.5 basis points for the (2 ×), (-1 ×), and (-2 ×) SPX LETFs. In addition, there is a large variation in the cumulative NAV deviation. For example, the 10-day NAV deviation of (2 ×) SPX fund ranges from -33.7 to 8.4 basis points. This large variation justifies the importance of our regression analysis. As shown in Panel D of Table 5, the ten-day cumulative LIBOR interest is 10.6 basis points and has a large variation from 28.3 to 0.97 basis points, reflecting the sharp decrease in the LIBOR rate during the sample period.

As shown in Panel A, for the 10-day holding period, the cumulative LIBOR interest alone explains the majority of the variation in the return deviations of the NAV. As shown in Column (3), (5) and (7), the LIBOR interest variable alone can explain 86.3%, 95.7%, and 95.1% of the NAV deviation for the (2 ×), (-1 ×), and (-2 ×) SPX funds, respectively. This confirms the prediction from equation (1) of Section II.A and demonstrates the importance of the holding period LIBOR interest payout/receipt factor in driving the cumulative NAV return deviation.

Interestingly, as shown in Column (3), for the (2 ×) SPX fund, the coefficient on the cumulative LIBOR interest is -1.01, which is almost the same as our predicted value of -1.00 from equation (1), confirming the importance of it in explaining the cumulative NAV return. After controlling for this LIBOR interest, there is an intercept of -3.72 basis points, which is very close to the predicted value of -3.75 basis points based on an annual expense ratio of 0.95%. As shown in Column (5), for the (-1 ×) SPX fund, the coefficient on the cumulative LIBOR interest is 2.01, which virtually equals our predicted value of 2.00. Similarly, as shown in Column (7), for (-2 ×) SPX fund, the coefficient on the cumulative LIBOR interest is 3.05, which approximates the predicted value of 3.00. As shown in the rest of Panel A, the coefficient on the cumulative LIBOR interest remains strongly significant and at the same level even after we add other control variables. For the control variables of the inverse funds, there is a negative coefficient on the volatility of the SPX daily return during the holding period, which is consistent with the interpretation that daily hedging costs increase as market volatility rises.

As shown in Panel B, for the holding period of 40 trading days, the results are similar. In summary, our cumulative NAV deviation analysis shows that the holding-period cumulative LIBOR interest and fund expense ratio are two key determinants of the cumulative NAV deviation over multiple holding days.

F. Determinants of the Compounding Deviation

To test for the determinants of the compounding deviation, we regress the compounding deviation of the (2 ×), (-1 ×), and (-2 ×) SPX LETFs on the squared cumulative return of the SPX and the variance of the daily SPX return, using non-overlapping observations for ten-day and forty-day holding periods. Table 6 reports the regression results in Panel A, along with the descriptive statistics for the dependent variable in Panel B and the explanatory variables in Panel C.

Consistent with the rolling sample statistics from Table 3, the non-overlapping sample statistics from Panel B of Table 6 confirm that the size of the compounding deviation varies according to the multiple function ($m^2 - m$). Panel C shows that the variance of the SPX daily return is averaged at 2.84 and 2.75, based on the ten-day and forty-day holding periods, respectively. This reflects the highly volatile market during the sample period, which includes the financial crisis of 2007-09. The squares of the cumulative SPX return during the ten-day and forty-day holding periods are 15.97 and 57.22, respectively.

Consistent with the predictions from equation (5), the regression results in Panel A of Table 6 show that the variance of the daily SPX return has a negative coefficient and the squared cumulative SPX return has a positive coefficient, each significant at the 1% level. For the same holding period, the sizes of the coefficients are similar for the (2 ×) SPX and (-1 ×) SPX funds, but tripled for the (-2 ×) SPX fund, confirming that the strength of the compounding effect is positively related to the multiple function (m^2-m). For the same target multiple, the coefficients on the SPX return variance for the forty-day holding period is about four times those of the ten-day holding period, confirming that the effect of the daily SPX return variance on the compounding effect is positively associated with the length of the holding period. The R^2's range from 93.5% to 99.8%, reflecting the highly mechanical nature of the compounding effect and the strong explanatory power of our model. In sum, the results in Table 6 strongly confirm the theoretical predictions from equation (5) about the determinants of the compounding deviation.

Based on this framework, the compounding effect will be positive when the squared cumulative return on the underlying index outweighs the product of the holding period and the daily index return variance, and negative otherwise. This implies that investors should invest in bull LETFs during a stable bull market and invest in bear LETFs during a stable bear market, and avoid investing in LETFs during volatile, see-sawing markets.

G. Determinants of the Total Cumulative Return Deviation

To help investors understand the ultimate drivers for the observed deviation between the actual LETF return and the leveraged multiple of the underlying index return, we test for the determinants of the total cumulative return deviation. Based on the previous empirical results and equations (1) and (5), we hypothesize that the holding period LIBOR interest, the variance of the daily SPX return, and the squared cumulative SPX return will determine the total cumulative return deviation of an LETF actual return from its naive expected return. Panel A of Table 7 displays the cumulative return deviation regressions for the SPX LETF family across various multiples for the ten-day and forty-day holding periods. The descriptive statistics for the dependent variables are in Panel B.

As shown in Columns (2) through (4) of Panel A, for ten-day periods, these three variables can explain 50% of the variation of total cumulative return deviation for the (2 ×) SPX fund, 91% for the (-1 ×) SPX fund, and 94% for the (-2 ×) SPX fund. The coefficient of the holding period LIBOR interest is close to -1, 2 and 3 for the (2 ×), (-1 ×) and the (-2 ×) SPX fund, respectively. In addition, the variance of the daily SPX return has a negative coefficient and the squared cumulative SPX return has a positive coefficient. As for the forty-day holding periods, the explanatory power is even higher, ranging from 84% to 99%. These findings show that the total cumulative return deviation of LETFs are indeed mainly driven by the holding period LIBOR interest, the variance of the underlying index daily return, and the squared cumulative index return during the holding period.

V. Conclusions and Implications

LETFs have experienced outsized growth since their inception in 2006. For multiple holding days, there is a large deviation between an LETF's actual market return and the product of the LETF's target multiple with the underlying index return. This total return deviation could be attributed to three sources: the compounding effect due to daily rebalancing, fund management tracking error, and market inefficiency. Current industry, regulatory and academic attention has centered on the LETF's compounding effect. To date, however, the LETF return deviation remains largely unexplained and a unified and executable framework for understanding the sources and fundamental drivers of various components of LETF return deviations is nonexistent.

To solve the return deviation conundrum of LETFs, we develop a clear, unified, objective, and executable framework that addresses the sources and determinants of the return deviations on LETFs. We also conduct a comprehensive empirical analysis on the most popular LETF families that track or magnify the long or short performance of the S&P 500, Dow Jones Industrial Average, NASDAQ 100, and S&P Midcap 400 indices. This sample represents all twelve LETFs across various target multiples that were initially launched in 2006 and includes complete daily data from their inception through December 2010.

The daily LETF return deviation is solely due to two components that are not related to the compounding effect: an NAV deviation component that is due to management's tracking error in achieving a fund's target return and a residual deviation component that is due to market frictions and inefficiency. Our results show that the swap-related holding period LIBOR interest is the main driver of the daily NAV deviation, while the lagged one-day residual deviation and the underlying index return are two key determinants of the daily residual deviation.

For multiple-day holding periods, the return deviation includes both the compounding deviation and the non-compounding deviation. Our results show that both the compounding and non-compounding deviations tend to increase in size as the holding period increases. They also indicate that the non-compounding deviation is at least as important as, and sometimes even dominates, the compounding deviation.

We further decompose the cumulative non-compounding deviation into the NAV deviation and the residual deviation. As the number of holding days increases, the NAV deviation accumulates in size and overwhelmingly dominates the mean-reverting and non-cumulative residual deviation. Because the long (short) equity swaps are associated with the payout (receipt) of the floating rate, the result will be a substantial and cumulative negative (positive) NAV deviation from the target return for bull (bear) LETFs. Based on the fact that LETFs mainly rely on equity swaps to deliver their target exposure to the underlying index, our framework shows that an LETF's cumulative NAV deviation is mainly driven by the swap-related holding period LIBOR interest, and that the strength of this effect is positively related to the difference between the target multiple and 1 (i.e., target multiple -1) and the length of the holding period. Our findings further confirm the theoretical predictions and pronounced high explanatory power of this framework. While LETF management firms have warned investors that the compounding effect may lead to a deviation of the LETF return from the naïve expected return, there is little disclosure or warning on the potential management tracking error attributable to the swap-related floating rate payout/receipt. Our framework and findings reveal the holding period LIBOR interest as the primary driver of the cumulative and sizable management tracking error, an outcome that cannot be attributed to the compounding effect.

Within a unified framework to explain the determinants of the compounding deviation of LETFs, we demonstrate theoretically and empirically that the deviation is positively driven by the squared cumulative return of the underlying index and negatively driven by the variance of the return on the underlying index. In addition, the strengths of the compounding effect are related to the difference between the target multiple squared and the target multiple as well as the length of the holding period. Our framework also shows that the compounding effect will be positive when the squared cumulative return on the underlying index outweighs the product of the holding period and the daily index return variance, and negative otherwise. This framework suggests that investors should invest in bull LETFs during a stable bull market and invest in bear LETFs during a stable bear market, and avoid investing in LETFs during volatile see-sawing markets.

Emerging out of a short history that has been dominated by media and regulatory attention on the negative aspects of the compounding effect, LETFs need to be understood in a more objective and systematic fashion than they have to date. The theoretical framework and empirical results from this study can be used by investors, regulators, financial advisors, and portfolio managers to understand the behaviors, sources, and determinants of the return deviations of LETFs. They can also help sophisticated, alpha-seeking investors calibrate tactical asset allocation strategies that generate absolute returns from both a magnified exposure to the underlying index and a positive return deviation.

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Table 1: Daily Return and Beta Deviations of Leveraged ETFs

Panel A: For all the LETFs in the sample, this panel shows the mean and standard deviation (in *Italics)* of the target and actual daily returns, as well as the daily deviation between these two returns. Returns are denoted in %. The sample period is from July 17, 2006 to December 31, 2010, for a total of 1125 trading days. See Appendix Table A3 for details on the names and specifications on the LETFs tracking the four indices (SPX, INDU, NDX and MID). See Appendix Table A1 for variable definitions for all tables.

		(1 ×) fund return			(2 ×) fund return			(-1 ×) fund return			(-2 ×) fund return		
LETF family	Statistics	Target	Actual	Actual - target	Target	Actual	Actual - target	Target	Actual	Actual - target	Target	Actual	Actual - target
SPX	mean	0.0236	0.0234	-0.0002	0.0471	0.0281	-0.019	-0.0236	-0.0101	0.0135	-0.0471	-0.0284	0.0188
	Std Dev	*1.6447*	*1.6397*	*0.2556*	*3.2894*	*3.145*	*0.4629*	*1.6447*	*1.6211*	*0.2112*	*3.2894*	*3.1722*	*0.4356*
INDU	mean	0.0287	0.0286	-0.0001	0.0574	0.0385	-0.0189	-0.0287	-0.016	0.0127	-0.0574	-0.0406	0.0168
	Std Dev	*1.4997*	*1.5276*	*0.2577*	*2.9994*	*2.8665*	*0.5227*	*1.4997*	*1.4617*	*0.2268*	*2.9994*	*2.8873*	*0.4231*
NDX	mean	0.0542	0.0524	-0.0019	0.1085	0.0876	-0.0209	-0.0542	-0.0427	0.0115	-0.1085	-0.0953	0.0132
	Std Dev	*1.7207*	*1.6484*	*0.2251*	*3.4414*	*3.2684*	*0.525*	*1.7207*	*1.6528*	*0.3261*	*3.4414*	*3.2684*	*0.5335*
MID	mean	0.0419	0.0412	-0.0007	0.0838	0.0691	-0.0147	-0.0419	-0.0307	0.0112	-0.0838	-0.0664	0.0175
	Std Dev	*1.8107*	*1.7816*	*0.2093*	*3.6214*	*3.6136*	*0.4425*	*1.8107*	*1.7639*	*0.2404*	*3.6214*	*3.5872*	*0.4305*

Panel B: This panel shows beta estimation results by regressing LETF returns on its underlying index returns using the single-index model. The dependent variable is the LETF daily return and the independent variable is the return of the underlying index with a constant term. Robust standard errors in parentheses; ** p<1% and * p<5% for the difference between the estimated beta and the LETF's target multiple.

		(1 ×) fund return			(2 ×) fund return			(-1 ×) fund return			(-2 ×) fund return		
LETF family	Statistics	Target	Actual	Actual - target	Target	Actual	Actual - target	Target	Actual	Actual - target	Target	Actual	Actual - target
SPX	Beta	1	0.9848	-0.0152	2	1.8943	-0.1057**	-1	-0.9775	0.0225**	-2	-1.9125	0.0875**
	Std Err.		(0.015)	(0.015)		(0.017)	(0.017)		(0.007)	(0.007)		(0.013)	(0.013)
INDU	Beta	1	1.004	0.004	2	1.883	-0.1170**	-1	-0.9636	0.0364**	-2	-1.9068	0.0932**
	Std Err.		(0.018)	(0.018)		(0.027)	(0.027)		(0.008)	(0.008)		(0.018)	(0.018)
NDX	Beta	1	0.9503	-0.0497**	2	1.8787	-0.1213**	-1	-0.9433	0.0567**	-2	-1.8779	0.1221**
	Std Err.		(0.010)	(0.010)		(0.012)	(0.012)		(0.008)	(0.008)		(0.012)	(0.012)
MID	Beta	1	0.9774	-0.0226**	2	1.9807	-0.0193	-1	-0.9656	0.0344**	-2	-1.967	0.0330**
	Std Err.		(0.005)	(0.005)		(0.017)	(0.017)		(0.007)	(0.007)		(0.010)	(0.010)

Table 2: LETF Daily NAV and Residual Return Deviations

This table shows the summary statistics of daily return deviation related variables (Panels A and B), as well as the determinants of the LETF daily NAV return deviation (Panel C) and residual return deviation (Panel D) for the SPX LETF family. The dependent variable is the LETF daily NAV return deviation from its target return for Panel C and the LETF daily residual return deviation between the actual LETF market return and the NAV return for Panel D. For Panels C and D, Columns (1)-(3) show the deviation of the (1 ×) SPX fund (SPY). Columns (4)-(6) show the deviation of the (2 ×) fund (SSO). Columns (7)-(9) show the deviation of the (-1 ×) fund (SH). Columns (10)-(12) show the deviation of the (-2) × fund (SDS). Robust standard errors in parentheses; ** p<1% and * p<5%.

Panel A: Summary statistics for the fund specific variables

	(1 ×) fund			(2 ×) fund			(-1 ×) fund			(-2 ×) fund		
	Mean	Std Dev	N	Mean	Std Dev	N	Mean	Std Dev	N	Mean	Std Dev	N
Daily NAV deviation	-0.0003	0.0068	1125	-0.0145	0.0247	1125	0.0139	0.0302	1125	0.0224	0.0426	1125
Daily residual deviation	0.0001	0.2551	1125	-0.0045	0.4565	1125	-0.0004	0.2040	1125	-0.0036	0.4250	1125
Turnover of the fund	0.340	0.171	1125	0.438	0.293	1125	0.101	0.095	1125	0.584	0.560	1125
Relative bid-ask spread	0.030	0.063	881	0.650	2.818	889	1.174	2.927	868	0.589	1.431	872
Net fund flows	0.079	2.683	1125	0.465	3.208	1125	0.375	2.573	1125	0.648	4.599	1125

Panel B: Summary statistics for common variables

	Mean	Std Dev	N		Mean	Std Dev	N		Mean	Std Dev	N
Holding-period LIBOR interest	0.011	0.012	1125	VIX	24.630	12.045	1125	SPX index daily return	0.024	1.645	1125

Panel C: Determinants of daily NAV return deviation

	Dep. Var. = Daily NAV Return Deviation of LETF											
	(1 ×) fund			(2 ×) fund			(-1 ×) fund			(-2 ×) fund		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Holding-period	0.0033	-0.0059	-0.0050	-0.9268**	-1.0339**	-0.9690**	1.6770**	1.7838**	1.6629**	2.5946**	2.6320**	2.4987**
LIBOR interest	(0.013)	(0.010)	(0.012)	(0.047)	(0.055)	(0.056)	(0.054)	(0.082)	(0.083)	(0.065)	(0.095)	(0.092)
Lagged NAV		-0.0888*	-0.1260**		-0.2755**	-0.1926*		-0.1172	-0.1238		-0.0243	-0.0353
deviation of LETF		(0.038)	(0.047)		(0.090)	(0.080)		(0.066)	(0.079)		(0.056)	(0.048)
SPX daily return		-0.0031**	-0.0031**		-0.0009	-0.0014		0.0020	0.0021		0.0023	0.0021
		(0.000)	(0.000)		(0.001)	(0.001)		(0.001)	(0.001)		(0.001)	(0.001)
VIX			-0.0000			0.0001			-0.0005**			-0.0007**
			(0.000)			(0.000)			(0.000)			(0.000)
Turnover of SPY			0.0009			-0.0061			0.0124**			0.0211**
			(0.002)			(0.004)			(0.005)			(0.006)
Relative bid-ask			-0.0003			-0.0015			-0.0106			-0.0041
spread of SPY			(0.002)			(0.006)			(0.010)			(0.010)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Net fund flows of SPY			0.0000			-0.0000			-0.0000			0.0001
			(0.000)			(0.000)			(0.000)			(0.000)
Constant	-0.0003	-0.0002	-0.0001	-0.0046**	-0.0075**	-0.0071*	-0.0039**	-0.0035**	0.0063	-0.0052**	-0.0051**	0.0070
	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.003)	(0.001)	(0.001)	(0.003)	(0.001)	(0.001)	(0.004)
No. of observations	1125	1125	881	1125	1125	881	1125	1125	881	1125	1125	881
R-squared	0.000	0.574	0.572	0.201	0.281	0.315	0.440	0.467	0.503	0.530	0.539	0.588
F Statistics	0.0691	70.54	25.40	388.6	194.5	84.54	969.7	367.2	169.0	1616	547.7	270.5

Panel D: Determinants of daily residual return deviation

	Dep. Var. = Daily Residual Return Deviation of LETF											
		(1 ×) fund			(2 ×) fund			(-1 ×) fund			(-2 ×) fund	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Holding-period LIBOR interest	0.8100			1.2158			-0.1203			-0.2571		
	(0.769)			(1.037)			(0.461)			(0.964)		
Lagged residual deviation of LETF		-0.4191**	-0.4121**		-0.4554**	-0.4467**		-0.3818**	-0.3762**		-0.4491**	-0.4392**
		(0.074)	(0.077)		(0.063)	(0.065)		(0.054)	(0.061)		(0.049)	(0.053)
SPX daily return		-0.0094	-0.0091		-0.0913**	-0.1044**		0.0165**	0.0164*		0.0738**	0.0766**
		(0.013)	(0.017)		(0.014)	(0.018)		(0.006)	(0.007)		(0.011)	(0.014)
VIX			0.0006			-0.0002			0.0003			-0.0040
			(0.002)			(0.003)			(0.001)			(0.003)
Turnover of LETF			-0.1708*			-0.1347			0.0979			0.1165
			(0.073)			(0.087)			(0.206)			(0.069)
Relative bid-ask spread of LETF			-0.2288			0.0006			-0.0003			-0.0013
			(0.172)			(0.002)			(0.001)			(0.006)
Net fund flows of LETF			0.0032			-0.0046			-0.0034			-0.0013
			(0.003)			(0.004)			(0.003)			(0.002)
Constant	-0.0085	0.0004	0.0501	-0.0174	-0.0044	0.0579	0.0009	-0.0010	-0.0108	-0.0009	-0.0070	0.0287
	(0.008)	(0.007)	(0.036)	(0.015)	(0.011)	(0.040)	(0.007)	(0.006)	(0.019)	(0.013)	(0.011)	(0.041)
No. of observations	1125	1125	881	1125	1125	889	1125	1125	868	1125	1125	872
R-squared	0.001	0.181	0.185	0.001	0.347	0.356	0.000	0.172	0.168	0.000	0.309	0.308
F Statistics	1.108	20.66	7.912	1.375	42.74	13.43	0.0682	30.72	8.472	0.0712	72.88	20.36

Table 3: Cumulative Effect of Compounding and Non-compounding Deviations

This table shows the compounding and non-compounding return deviations accumulated over various holding periods on a rolling basis. Newey-West robust standard errors in parentheses; ** p<1% and * p<5%. All figures are in %.

Number of holding days	Average compounding effect	Average non-compounding deviation	Average compounding effect	Average non-compounding deviation	Average compounding effect	Average non-compounding deviation
	(2 ×) SPX fund		(-1 ×) SPX fund		(-2 ×) SPX fund	
2	-0.0070**	-0.0330*	-0.0070**	0.0275**	-0.0210**	0.0416**
	(0.002)	(0.014)	(0.002)	(0.007)	(0.007)	(0.013)
3	-0.0192**	-0.0452**	-0.0195**	0.0416**	-0.0590**	0.0656**
	(0.006)	(0.013)	(0.006)	(0.008)	(0.018)	(0.014)
5	-0.0363**	-0.0735**	-0.0371**	0.0692**	-0.1120**	0.1095**
	(0.012)	(0.014)	(0.012)	(0.010)	(0.038)	(0.016)
10	-0.0849**	-0.1444**	-0.0933**	0.1391**	-0.2889**	0.2231**
	(0.030)	(0.016)	(0.031)	(0.019)	(0.096)	(0.029)
20	-0.1584*	-0.2851**	-0.2009*	0.2781**	-0.6457*	0.4488**
	(0.071)	(0.027)	(0.082)	(0.049)	(0.259)	(0.073)
40	-0.2252	-0.5678**	-0.3712	0.5569**	-1.2652*	0.8979**
	(0.164)	(0.071)	(0.192)	(0.132)	(0.631)	(0.198)
	(2 ×) INDU fund		(-1 ×) INDU fund		(-2 ×) INDU fund	
2	-0.0058**	-0.0339*	-0.0058**	0.0257**	-0.0175**	0.0363**
	(0.002)	(0.015)	(0.002)	(0.007)	(0.006)	(0.014)
3	-0.0159**	-0.0472**	-0.0162**	0.0387**	-0.0488**	0.0575**
	(0.005)	(0.014)	(0.005)	(0.009)	(0.016)	(0.015)
5	-0.0286**	-0.0792**	-0.0294**	0.0643**	-0.0889**	0.0961**
	(0.011)	(0.015)	(0.011)	(0.011)	(0.034)	(0.017)
10	-0.0691*	-0.1533**	-0.0757**	0.1304**	-0.2340**	0.1977**
	(0.027)	(0.017)	(0.028)	(0.020)	(0.085)	(0.031)
20	-0.1287	-0.3049**	-0.1602*	0.2610**	-0.5107*	0.3944**
	(0.067)	(0.029)	(0.077)	(0.049)	(0.241)	(0.073)
40	-0.1966	-0.6075**	-0.3117	0.5196**	-1.0438	0.7838**
	(0.138)	(0.072)	(0.182)	(0.132)	(0.601)	(0.198)
	(2 ×) NDX fund		(-1 ×) NDX fund		(-2 ×) NDX fund	
2	-0.0067**	-0.0352*	-0.0067**	0.0242*	-0.0200**	0.0317
	(0.003)	(0.016)	(0.003)	(0.010)	(0.008)	(0.016)
3	-0.0179**	-0.0472**	-0.0186**	0.0374**	-0.0566**	0.0534**
	(0.006)	(0.016)	(0.006)	(0.011)	(0.019)	(0.016)
5	-0.0325**	-0.0783**	-0.0342**	0.0624**	-0.1042**	0.0902**
	(0.012)	(0.016)	(0.013)	(0.012)	(0.038)	(0.019)
10	-0.0665*	-0.1509**	-0.0775**	0.1254**	-0.2448**	0.1865**
	(0.028)	(0.017)	(0.030)	(0.021)	(0.091)	(0.030)
20	-0.0997	-0.2986**	-0.1451	0.2499**	-0.4872	0.3768**
	(0.071)	(0.027)	(0.078)	(0.051)	(0.249)	(0.074)
40	-0.0500	-0.6043**	-0.1707	0.4946**	-0.6721	0.7436**
	(0.220)	(0.070)	(0.212)	(0.137)	(0.644)	(0.199)
	(2 ×) MID fund		(-1 ×) MID fund		(-2 ×) MID fund	
2	-0.0046	-0.0286*	-0.0046	0.0237**	-0.0138	0.0357*
	(0.003)	(0.014)	(0.003)	(0.007)	(0.009)	(0.014)
3	-0.0137*	-0.0428**	-0.0139*	0.0360**	-0.0420*	0.0553**
	(0.007)	(0.014)	(0.007)	(0.008)	(0.021)	(0.015)
5	-0.0294*	-0.0726**	-0.0292*	0.0608**	-0.0876*	0.0937**
	(0.014)	(0.013)	(0.014)	(0.009)	(0.043)	(0.017)
10	-0.0744	-0.1470**	-0.0763*	0.1214**	-0.2323*	0.1902**
	(0.039)	(0.015)	(0.038)	(0.019)	(0.112)	(0.030)
20	-0.1233	-0.2953**	-0.1440	0.2442**	-0.4582	0.3812**
	(0.103)	(0.027)	(0.102)	(0.049)	(0.299)	(0.074)
40	-0.1557	-0.5907**	-0.2219	0.4889**	-0.7619	0.7496**
	(0.278)	(0.071)	(0.257)	(0.135)	(0.722)	(0.201)

Table 4: Cumulative Effect of NAV and Residual Deviations

This table shows the NAV and residual return deviations cumulated over various holding periods on a rolling basis. Newey-West robust standard errors in parentheses; ** p<1% and * p<5%. All figures are in %.

Num. of holding days	Average NAV deviation (Std. Err.)	Average residual deviation (Std. Err.)	Average NAV deviation (Std. Err.)	Average residual deviation (Std. Err.)	Average NAV deviation (Std. Err.)	Average residual deviation (Std. Err.)
	(2 ×) SPX fund		(-1 ×) SPX fund		(-2 ×) SPX fund	
2	-0.0289**	-0.0042	0.0278**	-0.0003	0.0448**	-0.0032
	(0.001)	(0.013)	(0.002)	(0.007)	(0.003)	(0.013)
3	-0.0432**	-0.0020	0.0417**	-0.0001	0.0672**	-0.0016
	(0.002)	(0.013)	(0.003)	(0.007)	(0.004)	(0.013)
5	-0.0719**	-0.0016	0.0696**	-0.0004	0.1120**	-0.0025
	(0.003)	(0.013)	(0.006)	(0.007)	(0.009)	(0.013)
10	-0.1439**	-0.0005	0.1393**	-0.0003	0.2249**	-0.0017
	(0.009)	(0.013)	(0.017)	(0.009)	(0.025)	(0.014)
20	-0.2873**	0.0022	0.2788**	-0.0007	0.4506**	-0.0018
	(0.024)	(0.012)	(0.047)	(0.011)	(0.072)	(0.015)
40	-0.5717**	0.0039	0.5577**	-0.0008	0.9019**	-0.0040
	(0.068)	(0.014)	(0.132)	(0.013)	(0.199)	(0.020)
	(2 ×) INDU fund		(-1 ×) INDU fund		(-2 ×) INDU fund	
2	-0.0307**	-0.0031	0.0262**	-0.0006	0.0400**	-0.0037
	(0.009)	(0.017)	(0.005)	(0.009)	(0.009)	(0.015)
3	-0.0450**	-0.0022	0.0390**	-0.0003	0.0594**	-0.0019
	(0.010)	(0.016)	(0.006)	(0.009)	(0.010)	(0.016)
5	-0.0680**	-0.0112	0.0616**	0.0027	0.0921**	0.0040
	(0.009)	(0.016)	(0.008)	(0.010)	(0.012)	(0.015)
10	-0.1443**	-0.0090	0.1273**	0.0031	0.1929**	0.0047
	(0.012)	(0.016)	(0.017)	(0.011)	(0.027)	(0.018)
20	-0.2958**	-0.0090	0.2578**	0.0031	0.3920**	0.0023
	(0.025)	(0.016)	(0.047)	(0.011)	(0.072)	(0.019)
40	-0.5967**	-0.0108	0.5171**	0.0026	0.7841**	-0.0003
	(0.068)	(0.017)	(0.132)	(0.009)	(0.200)	(0.022)
	(2 ×) NDX fund		(-1 ×) NDX fund		(-2 ×) NDX fund	
2	-0.0298**	-0.0054	0.0251**	-0.0008	0.0384**	-0.0067
	(0.001)	(0.016)	(0.002)	(0.010)	(0.003)	(0.016)
3	-0.0444**	-0.0028	0.0377**	-0.0002	0.0578**	-0.0044
	(0.002)	(0.016)	(0.003)	(0.010)	(0.005)	(0.016)
5	-0.0741**	-0.0042	0.0628**	-0.0005	0.0963**	-0.0061
	(0.003)	(0.016)	(0.006)	(0.011)	(0.009)	(0.016)
10	-0.1482**	-0.0027	0.1252**	0.0002	0.1916**	-0.0051
	(0.008)	(0.015)	(0.017)	(0.012)	(0.026)	(0.014)
20	-0.2977**	-0.0009	0.2494**	0.0004	0.3813**	-0.0044
	(0.024)	(0.013)	(0.048)	(0.015)	(0.073)	(0.013)
40	-0.6002**	-0.0040	0.4946**	-0.0000	0.7510**	-0.0074
	(0.069)	(0.014)	(0.133)	(0.021)	(0.201)	(0.014)
	(2 ×) MID fund		(-1 ×) MID fund		(-2 ×) MID fund	
2	-0.0302**	0.0017	0.0250**	-0.0013	0.0384**	-0.0027
	(0.001)	(0.013)	(0.002)	(0.007)	(0.003)	(0.014)
3	-0.0452**	0.0025	0.0375**	-0.0015	0.0576**	-0.0023
	(0.002)	(0.013)	(0.003)	(0.007)	(0.005)	(0.015)
5	-0.0751**	0.0025	0.0625**	-0.0017	0.0958**	-0.0021
	(0.003)	(0.013)	(0.006)	(0.007)	(0.010)	(0.014)
10	-0.1498**	0.0027	0.1249**	-0.0035	0.1910**	-0.0009
	(0.009)	(0.012)	(0.017)	(0.009)	(0.026)	(0.014)
20	-0.2985**	0.0031	0.2493**	-0.0052	0.3803**	0.0009
	(0.024)	(0.013)	(0.048)	(0.010)	(0.073)	(0.015)
40	-0.5944**	0.0037	0.4961**	-0.0072	0.7525**	-0.0029
	(0.069)	(0.017)	(0.135)	(0.011)	(0.201)	(0.016)

Table 5: Determinants of the Cumulative NAV Return Deviation of LETFs

This table examines determinants of the cumulative NAV return deviation over the holding period of 10 trading days (Panel A) and 40 trading days (Panel B) using OLS regression for the SPX LETF family. The dependent variable is the cumulative NAV return deviation of an LETF over the holding period on a non-rolling basis. White's (1980) heteroskedasticity-consistent standard errors in parentheses; ** p<1% and * p<5%.

Panel A: Determinants of cumulative NAV return deviation over the holding period of 10 trading days

	Dep. Var. = Cumulative NAV Return Deviation of LETF over 10 Trading Days							
	(1 ×) fund		(2 ×) fund		(-1 ×) fund		(-2 ×) fund	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Cumulative LIBOR interest during	0.0007	-0.0154	-1.0099**	-1.0242**	2.0130**	1.9756**	3.0528**	2.9865**
holding period	(0.014)	(0.009)	(0.024)	(0.025)	(0.028)	(0.024)	(0.044)	(0.037)
Volatility of SPX daily return during		-0.0020		0.0008		-0.0251**		-0.0399**
holding period		(0.001)		(0.005)		(0.004)		(0.005)
Cumulative SPX return during the holding		-0.0033**		-0.0038*		-0.0006		-0.0027
period		(0.000)		(0.001)		(0.001)		(0.002)
Constant	-0.0028	0.0022	-0.0372**	-0.0361**	-0.0742**	-0.0365**	-0.0992**	-0.0381**
	(0.003)	(0.002)	(0.003)	(0.007)	(0.005)	(0.006)	(0.008)	(0.009)
No. of observations	112	112	112	112	112	112	112	112
R-squared	0.000	0.626	0.863	0.890	0.957	0.976	0.951	0.970
F Statistics	0.00245	18.27	1722	864.1	5340	3376	4816	2936

Panel B: Determinants of cumulative NAV return deviation over the holding period of 40 trading days

	Dep. Var. = Cumulative NAV Return Deviation of LETF over 40 Trading Days							
	(1 ×) fund		(2 ×) fund		(-1 ×) fund		(-2 ×) fund	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	0.0034	-0.0015	-1.0141**	-1.0714**	2.0013**	1.9153**	2.9965**	2.7935**
Cumulative LIBOR interest during holding period	(0.012)	(0.009)	(0.042)	(0.034)	(0.045)	(0.027)	(0.086)	(0.073)
		0.0054		-0.0069		-0.1436**		-0.2790**
Volatility of SPX daily return during holding period		(0.004)		(0.021)		(0.019)		(0.051)
		-0.0018**		-0.0139**		-0.0064**		-0.0217**
Cumulative SPX return during the holding period		(0.001)		(0.005)		(0.002)		(0.008)
Constant	-0.0119	-0.0159	-0.1439**	-0.1003*	-0.2901**	-0.0511	-0.3719**	0.1141
	(0.007)	(0.009)	(0.021)	(0.047)	(0.032)	(0.030)	(0.052)	(0.090)
No. of observations	28	28	28	28	28	28	28	28
R-squared	0.003	0.592	0.896	0.969	0.970	0.991	0.954	0.981
F Statistics	0.0793	10.46	594.9	548.9	2005	1795	1220	656.0

Panel C: Summary statistics of the dependent variable for 10-day and 40-day non-rolling holding periods

	Summary Statistics for Cumulative NAV Return Deviation							
Number of holding days	10-day				40-day			
Fund multiple	(1 ×)	(2 ×)	(-1 ×)	(-2 ×)	(1 ×)	(2 ×)	(-1 ×)	(-2 ×)
Mean	-0.0028	-0.1445	0.1396	0.2251	-0.0104	-0.5748	0.5603	0.9013
Standard Deviation	0.0159	0.0942	0.1783	0.2712	0.0234	0.3723	0.7061	1.0662
Number of observations	112	112	112	112	28	28	28	28
Minimum	-0.0622	-0.3374	-0.0705	-0.1773	-0.0502	-1.1859	-0.1522	-0.1655
First Quartile	-0.0083	-0.2463	-0.0268	-0.0285	-0.0258	-1.0059	-0.0999	-0.1010
Median	-0.0040	-0.1320	0.1128	0.1973	-0.0137	-0.4503	0.4414	0.6620
Third Quartile	0.0026	-0.0582	0.3472	0.5396	0.0062	-0.2262	1.3704	2.0620
Maximum	0.0785	0.0840	0.5078	0.7900	0.0399	-0.1788	1.6060	2.4122

Panel D: Summary statistics of explanatory variables

Variable name		# of Obs.	Mean	Std Dev	Minimum	1st Quartile	Median	3rd Quartile	Maximum
Holding-period LIBOR	10-Day	112	0.1062	0.0866	0.0097	0.0162	0.1060	0.2083	0.2830
interest	40-Day	28	0.4249	0.3475	0.0404	0.0646	0.4388	0.8417	0.9240
Volatility of SPX daily return	10-Day	112	1.3414	1.0249	0.2928	0.6970	1.1123	1.5415	6.2170
during the holding period	40-Day	28	1.3780	0.9402	0.4530	0.7757	1.2116	1.5149	5.0590
Cumulative SPX return	10-Day	112	0.1834	4.0095	-15.0882	-1.8847	0.8233	2.5830	10.6469
during the holding period	40-Day	28	0.7053	7.6697	-19.2935	-2.6347	1.2944	6.0912	13.8734

Table 6: Determinants of the Compounding Deviation of LETFs

Panel A: This panel examines the determinants of LETF compounding effect using OLS regression for the SPX LETF family. The dependent variable is the LETF compounding deviation over various holding days on a non-rolling basis. White's (1980) heteroskedasticity-consistent standard errors in parentheses; ** p<1% and * p<5%.

	Dep. Var. = Compounding Deviation					
Number of holding days	10-day			40-day		
Fund multiple	2 ×	-1 ×	-2 ×	2 ×	-1 ×	-2 ×
	(1)	(2)	(3)	(4)	(5)	(6)
Variance of SPX daily return during the holding period	-0.0830**	-0.0921**	-0.2828**	-0.2508**	-0.4252**	-1.4018**
	(0.004)	(0.001)	(0.010)	(0.022)	(0.009)	(0.048)
Square of cumulative SPX return during the holding period	0.0097**	0.0094**	0.0278**	0.0101**	0.0100**	0.0291**
	(0.000)	(0.000)	(0.002)	(0.001)	(0.001)	(0.002)
Constant	-0.0190**	-0.0015	0.0118	-0.2385**	0.0407*	0.3565**
	(0.005)	(0.003)	(0.016)	(0.042)	(0.016)	(0.088)
No. of observations	112	112	112	28	28	28
R-squared	0.988	0.998	0.993	0.935	0.997	0.994
F Statistics	440.7	2025	436.6	62.54	1429	519.8

Panel B: Summary statistics of the dependent variable for 10-day and 40-day non-rolling holding periods

	Summary Statistics for Compounding Deviation					
Number of holding days	10-day			40-day		
Fund multiple	2 ×	-1 ×	-2 ×	2 ×	-1 ×	-2 ×
Mean	-0.100	-0.112	-0.347	-0.350	-0.557	-1.837
Standard Deviation	0.467	0.503	1.538	0.863	1.530	5.202
First Quartile	-0.135	-0.138	-0.404	-0.566	-0.595	-1.784
Median	-0.020	-0.021	-0.063	-0.231	-0.275	-0.853
Third Quartile	0.019	0.016	0.041	0.168	0.190	0.626

Panel C: Summary statistics of the explanatory variables

Variable name		# of Obs.	Mean	Std Dev	1st Quartile	Median	3rd Quartile
Square of cumulative SPX return	10-Day	112	15.97	31.82	1.34	4.62	17.74
during the holding period	40-Day	28	57.22	93.90	4.79	16.48	67.08
Variance of SPX daily return	10-Day	112	2.84	5.55	0.49	1.24	2.38
during the holding period	40-Day	28	2.75	4.88	0.60	1.47	2.30

Table 7: Determinants of the Total Cumulative Return Deviation of LETFs

Panel A: This panel examines the determinants of total cumulative return deviation using OLS regression for the SPX LETF family. The dependent variable is the total cumulative return deviation over various holding days on a non-rolling basis. White's (1980) heteroskedasticity-consistent standard errors in parentheses; ** p<1% and * p<5%.

	Dep. Var. = Total Cumulative Return Deviation							
Number of holding days	10-day				40-day			
Fund multiple	1 ×	2 ×	-1 ×	-2 ×	1 ×	2 ×	-1 ×	-2 ×
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Cumulative LIBOR interest during holding period	-0.0175	-1.0330**	1.9525**	3.0199**	0.0512	-0.7717**	1.9427**	2.6092**
	(0.231)	(0.320)	(0.151)	(0.398)	(0.073)	(0.212)	(0.063)	(0.233)
Variance of SPX daily return during the holding period	0.0095	-0.0822**	-0.0933**	-0.2876**	0.0371**	-0.2538**	-0.4354**	-1.4441**
	(0.010)	(0.019)	(0.003)	(0.012)	(0.013)	(0.021)	(0.011)	(0.051)
Square of cumulative SPX return during the holding period	-0.0011	0.0085**	0.0083**	0.0275**	-0.0009	0.0113**	0.0096**	0.0289**
	(0.002)	(0.003)	(0.001)	(0.003)	(0.001)	(0.001)	(0.001)	(0.003)
Constant	-0.0074	-0.0349	-0.0479	-0.0634	-0.0789	-0.5544**	-0.1789**	0.2697
	(0.048)	(0.060)	(0.026)	(0.068)	(0.044)	(0.107)	(0.043)	(0.152)
Observations	112	112	112	112	28	28	28	28
R-squared	0.048	0.498	0.914	0.935	0.398	0.839	0.994	0.992
F Statistic	0.365	10.95	354.5	228.9	4.763	68.68	2076	417.6

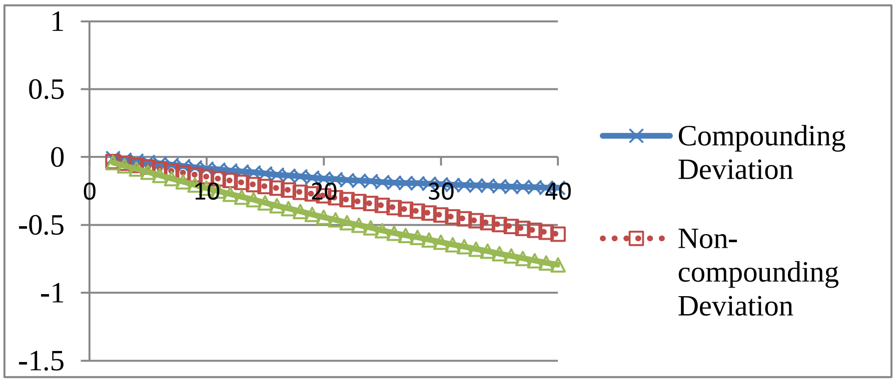
Panel B: Summary statistics of the dependent variable for 10-day and 40-day holding periods

	Summary Statistics for Total Cumulative Return Deviation							
Number of holding days	10-day				40-day			
Fund multiple	1 ×	2 ×	-1 ×	-2 ×	1 ×	2 ×	-1 ×	-2 ×
Mean	-0.0003	-0.2418	0.0269	-0.1204	-0.0083	-0.9364	-0.0006	-0.9394
Standard Deviation	0.2416	0.6569	0.5418	1.6112	0.2095	0.9855	1.7452	5.5078
Number of observations	112	112	112	112	28	28	28	28
Minimum	-1.2905	-3.4831	-3.1958	-9.9834	-0.4421	-3.0919	-6.8244	-25.4145
First Quartile	-0.0732	-0.4003	-0.1245	-0.2796	-0.1189	-1.5257	-0.5092	-1.6313
Median	0.0070	-0.2230	0.0777	0.0864	-0.0130	-0.7081	0.3767	0.3281
Third Quartile	0.0903	0.0556	0.2976	0.5262	0.0622	-0.3707	1.0837	1.7582
Maximum	1.0435	2.2917	1.3243	4.1155	0.7202	1.1876	1.8672	2.9452

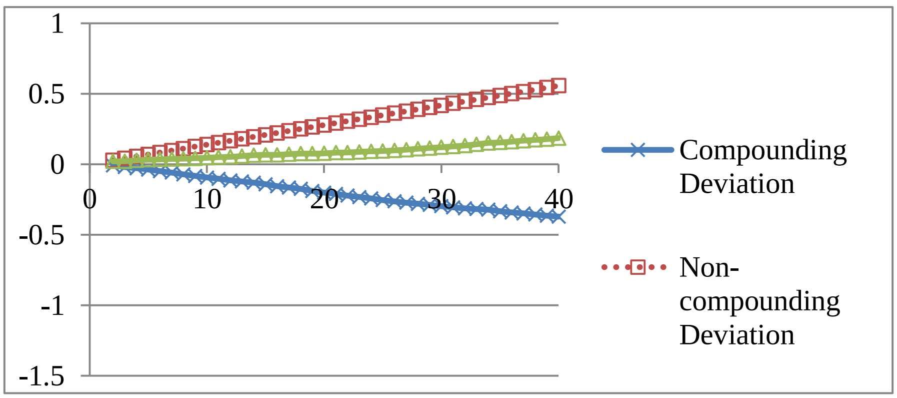
Figure 1. Plot of the Compounding, Non-compounding, and Total Return Deviations of LETFs over the Number of Holding Days

Figure 1 illustrates the cumulative effects of the compounding, non-compounding, and total deviations for the SPX family of LETFs up to 40 days. The horizontal axis shows the number of trading days in the holding period, while and the vertical axis shows the return deviations in %.

Graph A. Return deviations for the (2 ×) SPX fund



Graph B. Return deviations for the (-1 ×) SPX fund



Graph C. Return deviations for the (-2 ×) SPX fund

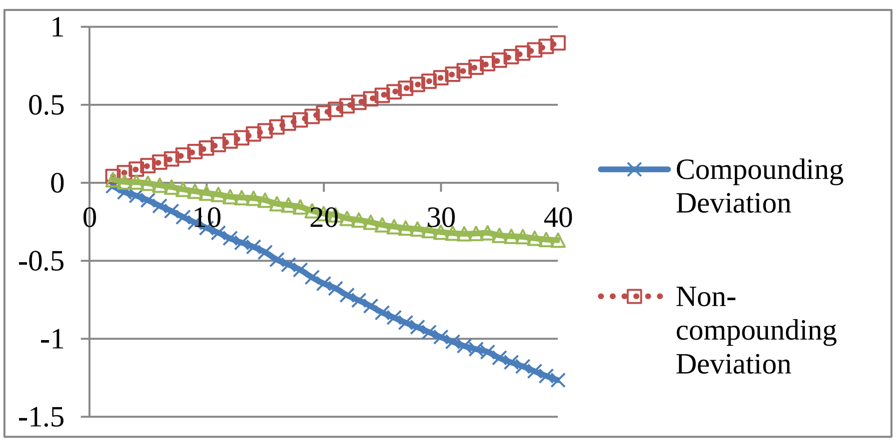
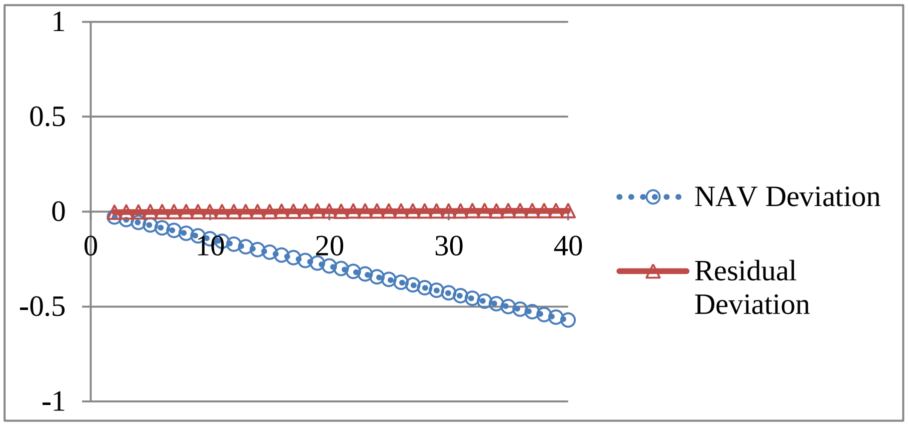


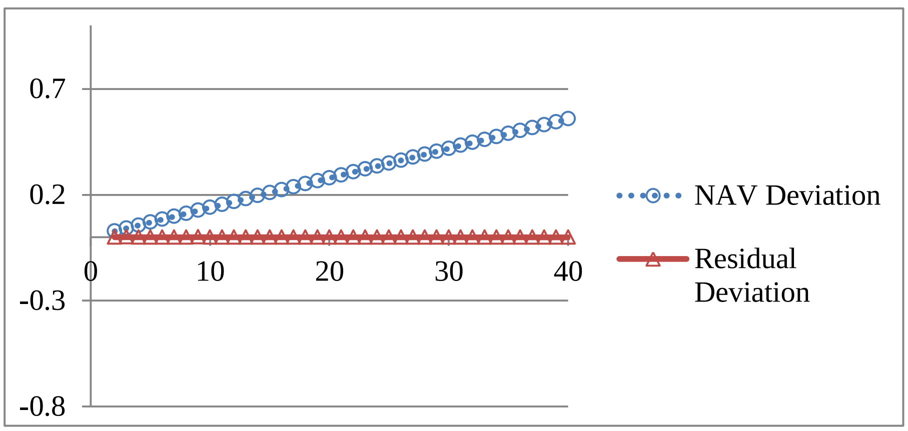
Figure 2. Plot of the NAV and Residual Deviations of LETFs over the Number of Holding Days

Figure 2 illustrates the cumulative effects of the NAV and residual deviations for the SPX family of LETFs up to 40 days. The horizontal axis shows the number of trading days in the holding period, while the vertical axis shows the return deviations in %.

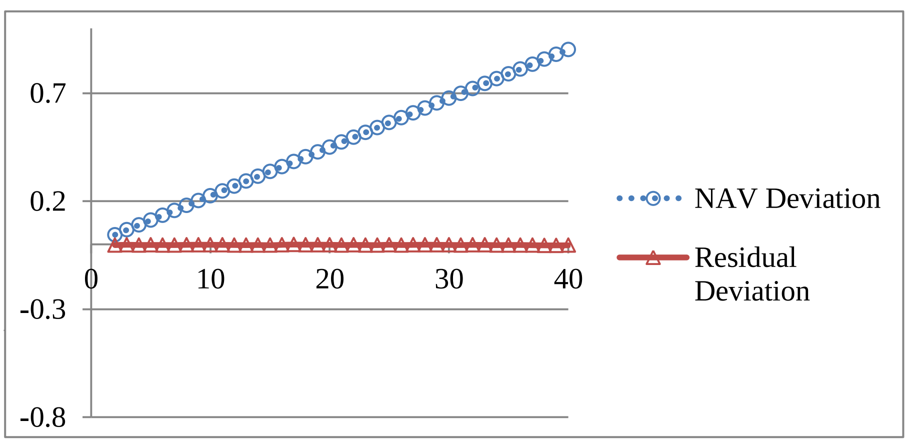
Graph A. NAV and residual deviations for the (2 ×) SPX fund



Graph B. NAV and residual deviations for the (-1 ×) SPX fund



Graph C. NAV and residual deviations for the (-2 ×) SPX fund



Appendix

This appendix includes four tables. Table A1 defines the terms and variables used in the paper. Table A2 summarizes the number and total assets under management (AUM) for all ETFs and leveraged ETFs (LETFs) from 2006 to 2010. Table A3 describes the leveraged ETFs included in this study, along with their regular ETF counterparts. Finally, Table A4 summarizes the expected signs and rationales for the use of various independent variables in the daily NAV and residual deviation regressions in Table 2.

Table A1: Definitions of Terms and Variables

This table defines the terms and variables used in the paper.

Names	Definitions
Actual return	Also called the actual market return of the LETF. Refers to the return of an LETF based on actual percentage price change, adjusted for dividend yield. All return variables in this study are in percentage, unless noted otherwise.
Beta	Slope coefficient from regressing daily return of an LETF on the daily return of the underlying index with a constant term.
Compounding deviation	Also called compounding effect. Computed as the cumulative target return of an LETF less the naïve expected return, which is calculated as the product between the daily target multiple of an LETF and the cumulative return of the underlying index. Compounding deviation occurs only when investors hold an LETF for more than one trading day.
Creation/redemption feature	If the ETF price is lower than its NAV, qualified investors can redeem ETF shares to get the underlying basket of stocks and realize the difference between the higher NAV and lower ETF price. On the other hand, if ETF price is higher than its NAV, qualified investors can deposit the underlying basket of stocks to create ETF shares and realize the difference between higher ETF price and lower NAV. LETFs have the creation/redemption feature except for the last hour of the trading day.
Holding-period LIBOR interest	The LIBOR interest accumulated over the holding period. In particular, the LIBOR interest earned over one trading day is calculated using the actual number of calendar days between the previous trading day and the current trading day, divided by 360, and then multiplied by the LIBOR rate from the previous trading day. For multiple holding days, this measure is accumulated over each day during the holding period.
LETFs	Leveraged Exchange-Traded Funds (LETFs) are broadly defined as ETFs that track the return on an underlying index with a daily target multiple. The most popular multiples are 2 ×, 3 ×, -1 ×, -2 ×, and -3 ×.
LETF family	The series of LETFs that track the same underlying index with various multiples such as 2 ×, -1 ×, and -2 ×. We also include the 1 × regular ETF as part of the family for comparison. The 3 x and -3 x LETFs are not included in this study due to their short history of available data.
LETF provider	Also called LETF management firm. Refers to the firms that issue and manage the leveraged exchange-traded funds.
Multiple	Also called leveraged multiple, target multiple, or daily target multiple. Reflects the LETF's daily target exposure to the underlying index. The multiple can be positive (for bull LETFs) or negative (for bear LETFs). Examples of multiple for LETFs include (2 ×), (3 ×), (-1 ×), (-2 ×), and (-3 ×).
Naïve expected return	The return expected by a naïve investor is calculated as the product between the daily target multiple of an LETF and the cumulative return of the underlying index. This naïve expected return differs from the target return for LETFs over multiple-day holding periods.
NAV	Net asset value of an LETF
NAV deviation	Also called NAV return deviation. It is computed as the NAV return of an LETF less its target return.
NAV return	Return of an LETF based on percentage change in NAV, adjusted for dividend yield.
Net fund flows	100 times the change in shares outstanding from the previous trading day, scaled by shares outstanding from the previous trading day. This measures the daily net fund flows and excess demand from investors.
Non-compounding deviation	Actual return of an LETF less target return of the LETF based on its product design. Non-compounding deviation includes the NAV deviation and residual deviation.
Relative bid-ask spread	100 times the bid-ask spread over the midquote.
Residual deviation	Also called residual return deviation. It is computed as actual Return of an LETF less its NAV return.
Return deviation	Also called total return deviation. Actual Return of an LETF less the daily multiple times the return on the underlying index. On a daily basis, the return deviation is equal to the non-compounding deviation. For a holding period greater than one trading day, the return deviation can be decomposed into compounding and non-compounding deviations.
Target return	Return target that an LETF is designed to achieve based on its product design. The daily target return equals the daily target multiple times the underlying index return. When the holding period is longer than one day, the target return of an LETF equals the daily LETF target return compounded over the holding period.
Turnover	Daily trading volume of an LETF over its shares outstanding.
Underlying index return	Total return (including price return and dividend yield) of the underlying benchmark index that an LETF is tracking.
VIX	Near-term volatility implied by S&P 500 stock index option prices.

Table A2: Market Size of Leveraged ETFs from 2006 to 2010

This table summarizes the number and total assets under management (AUM) for all ETFs and leveraged ETFs (LETFs) from 2006 to 2010.

Year	2006	2007	2008	2009	2010
No. of LETFs Listed in the U.S.	12	66	123	153	174
Total No. of ETFs Listed in the U.S.	359	629	728	797	923
Number of LETFs as a percentage of All ETFs	3.34%	10.49%	16.90%	19.20%	18.85%
Total AUM of LETFs in billion USD	0.96	4.56	22.58	31.07	31.57
Total AUM of All ETFs in billion USD	423	608	531	777	992
AUM of LETFs as a percentage of All ETFs	0.23%	0.75%	4.25%	4.00%	3.18%
Total Dollar Volume for LETFs in billion USD	23.84	517.94	2,907.86	3,614.73	2,436.76
Average Annual Turnover for LETFs	9.47	20.45	67.14	56.78	38.84

Sources: Investment Company Institute; Bloomberg.

Table A3: List of Leveraged ETFs in the Sample

This table describes the leveraged ETFs included in this study, along with their regular ETF counterparts.

ETF Name	Ticker Symbol	Daily Multiple	Index Symbol*	Inception Date	Expense Ratio (%)	AUM (in million USD) in 2010	Average Number of Holding Days**
S&P 500	SPY	1 ×	SPX	1/22/1993	0.095	89,875.03	2.9
Ultra S&P500	SSO	2 ×	SPX	6/21/2006	0.950	1,589.54	2.3
Short S&P500	SH	-1 ×	SPX	6/21/2006	0.950	1,553.77	9.9
UltraShort S&P500	SDS	-2 ×	SPX	7/13/2006	0.910	2,052.69	1.7
Dow30	DIA	1 ×	INDU	1/14/1998	0.170	8,721.08	4.9
Ultra Dow30	DDM	2 ×	INDU	6/21/2006	0.950	314.22	3.2
Short Dow30	DOG	-1 ×	INDU	6/21/2006	0.950	246.20	7.5
UltraShort Dow30	DXD	-2 ×	INDU	7/13/2006	0.950	379.73	1.9
QQQ	QQQQ	1 ×	NDX	3/10/1999	0.200	22,060.72	3.1
Ultra QQQ	QLD	2 ×	NDX	6/21/2006	0.950	842.08	1.6
Short QQQ	PSQ	-1 ×	NDX	6/21/2006	0.950	200.33	9.2
UltraShort QQQ	QID	-2 ×	NDX	7/13/2006	0.950	629.97	1.2
MidCap400	IJH	1 ×	MID	5/26/2000	0.200	9,315.33	85.6
Ultra MidCap400	MVV	2 ×	MID	6/21/2006	0.950	137.94	7.1
Short MidCap400	MYY	-1 ×	MID	6/21/2006	0.950	32.38	15.7
UltraShort MidCap400	MZZ	-2 ×	MID	6/21/2006	0.950	19.65	3.4

Note: In this paper, we broadly refer to all leveraged and/or inverse ETFs as leveraged ETFs (LETFs) since they seek to deliver a daily target multiple (such as 2 ×, -1 ×, or -2 ×) of the underlying index return. This sample represents a complete list of all twelve LETFs that were launched in 2006. The (2 ×) funds are bull LETFs. The (-1 ×) and (-2 ×) funds are inverse or bear LETFs. The (1 ×) funds are regular, non-leveraged ETFs for the purpose of comparison with the LETFs.

* Index symbol in Bloomberg. SPX, INDU, NDX and MID refer to the S&P 500, Dow Jones Industrial Average, NASDAQ-100, and S&P MidCap 400, respectively.

** Average number of holding days for each ETF during the sample period from July 2006 to December 2010.

Source: Bloomberg.

Table A4: Explanatory Variables for the Daily NAV and Residual Return Deviations

This table summarizes the expected signs and rationales for the use of various independent variables in the daily NAV and residual deviation regressions in Table 2.

Panel A: Independent variables for the daily NAV return deviation regression

Variable Name	Expected Signs and Rationales
Holding-period LIBOR Interest	- for bull LETFs and + for bear LETFs. Coefficient should be equal to (1-target multiple) according to equation (1) in Section 2.A. This factor is due to the floating rate interest payout (receipt) associated with long (short) equity swaps used by bull (bear) LETFs in delivering the leveraged target exposure to the underlying index.
Underlying Index Return	- for bull LETFs and + for bear LETFs if the underexposure is due to management tracking error; 0 otherwise.
Lagged NAV Deviation of The LETF	- if there is mean reversion in the NAV return deviation. The possibility of fund management's inaccurate assessment of the LETF's fundamental value at the market closing, followed by a correction on the next day, could potentially lead to mean reversion in the NAV deviation.
VIX	Higher market volatility makes it more costly to rehedge toward the target exposure to the underlying index. See Whaley (2009) for a full explanation of the VIX index and its applications.
Fund Turnover of the (1×) Fund	Higher turnover (better liquidity) may lead to smaller deviation, as it is less costly to rehedge. For example, Brennan and Subrahmanyam (1996) and Elton and Green (1998) suggest that volume and turnover ratio can be used to measure market depth and liquidity of a security.
Bid-ask Spread of the (1×) Fund	Larger spread (worse liquidity) may lead to larger deviation, as it is more costly to rehedge. For example, Amihud and Mendelson (1986, 1989), Eleswarapu (1997), and Hasbrouck and Seppi (2001) demonstrate that the bid-ask spread can be used to capture the tightness and liquidity of the security market.
Net Fund Flows of the (1×) Fund	Greater net fund flows may affect the difficulty of rehedging. For example, Kraus and Stoll (1972) and Keim and Madhavan (1996) show that large excess demands may have significant price impacts on stocks.

Panel B: Independent variables for the daily residual return deviation regression

Variable Name	Expected Signs and Rationales
Holding-period LIBOR Interest	Zero
Underlying Index Return	- for bull LETFs and + for bear LETFs. As conjectured by Charupat and Miu (2011), non-synchronization between the LETF closing price and its underlying index and the short-term trading behavior from LETF investors could lead to a downward (upward) bias on the bull (bear) LETF returns on an underlying index up day and a upward (downward) bias on the bull (bear) ETF returns on an underlying index down day.
Lagged Residual Deviation of the LETF	- if there is mean reversion in the residual return deviation. Any departure of the LETF market return from the NAV return due to market frictions and inefficiency will subsequently be corrected due to the availability of the creation/redemption feature, leading to the observed mean reversion in residual return deviation.
VIX	Higher market volatility is often associated with negative investor sentiment, which may lead to lower demand for LETFs.
Turnover of the LETF	Higher turnover (better liquidity) may lead to smaller deviation. For example, Brennan and Subrahmanyam (1996) and Elton and Green (1998) suggest that volume and turnover ratio can be used to measure market depth and liquidity of a security.
Bid-ask Spread of the LETF	Larger spread (worse liquidity) may lead to larger deviation. For example, Amihud and Mendelson (1986, 1989), Eleswarapu (1997), and Hasbrouck and Seppi (2001) demonstrate that the bid-ask spread can be used to capture the tightness and liquidity of the security market.
Net Fund Flows of the LETF	Greater net fund flows show higher excess demand for the LETF and may lead to more upside deviation. For example, Kraus and Stoll (1972) and Keim and Madhavan (1996) show that large excess demands may have significant price impacts on stocks.