

Physics HL Internal Assessment

Research Question: *How does change in temperature affect the coefficient of viscosity of honey?*

Session: May 2020

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1:INTRODUCTION

Traffic was heavier than usual on my way to school and there seemed to be a roadblock ahead and I noticed that the roads were blocked in order to apply tar. On heating the tar, it flowed quite smoothly onto the road. This made me think what makes a liquid less dense on heating. Hence I decided to do my experiment on thick liquids and check how viscosity of a liquid varies with temperature.

Viscosity is a physical property of fluids that is the measure of resistance offered against flow. Firstly, I shortlisted the liquids to: castor oil, glycerine, and honey as they are easily available. I wanted to experiment on honey as it's coefficient of viscosity ranges¹ from $2 \text{ kgm}^{-1}\text{s}^{-1}$ to $10 \text{ kgm}^{-1}\text{s}^{-1}$ and it was amongst the most dense liquids. I then decided to calculate the coefficient of viscosity of honey with variations in temperature as viscosity greatly depends on this parameter. As temperature of a liquid increases its density decreases, hence the resistance offered to flow(viscosity) decreases. This paved way for my Research Question: How does change in temperature(in K) affect the coefficient of viscosity of honey(η in $\text{kgm}^{-1}\text{s}^{-1}$) ?

2:INVESTIGATION

2.1: Background Information

Viscosity is a physical property of fluids that retards an object(s) moving through it. The coefficient of viscosity² is the frictional force needed to maintain a difference of velocity of 1 cms^{-1} between parallel layers of a liquid. It is a constant for liquids under standard conditions which decreases with increase in temperature.

An object in motion experiences the resultant of three forces in the liquid namely: its own weight(W), upward thrust or reaction force(F) and viscous drag(U) of the liquid. The object attains maximum speed-terminal velocity- in the liquid(after it has travelled a specific distance d) when $(U+F)-W=0$.

¹ <https://resources.saylor.org/wwwresources/archived/site/wp-content/uploads/2011/04/Viscosity.pdf>

² <https://byjus.com/chemistry/coefficient-of-viscosity/>

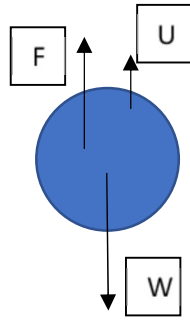


Figure 1: Forces experienced by an object passing through a liquid.

Terminal velocity³ is the maximum velocity attained by an object falling through a liquid when its acceleration is zero. In this case, the sum of the up thrust and drag force offered by the liquid to the spherical objects passing through it is equal to the downward force of gravity acting on the weight of the objects.

Hence, Stoke's Law⁴ gives the equation for finding terminal velocity of small spherical objects passing through a liquid as:

$$(U+F)-W=0$$

$$6\pi\eta rv + \left(\frac{4}{3}\right)r^3\pi\rho g - \left(\frac{4}{3}\right)r^3\pi\sigma g = 0$$

$$\text{Where, Weight of spheres} = \left(\frac{4}{3}\right)r^3\pi\sigma g,$$

$$F = \text{Upthrust offered by liquid} = \left(\frac{4}{3}\right)r^3\pi\rho g,$$

$$U = \text{viscous force on a spherical body} = 6\pi\eta rv$$

Equating for coefficient of viscosity⁵ (η), we get:

$$\eta = \frac{2g(\sigma - \rho)r^2}{9v}$$

where,

- η - coefficient of viscosity($\text{kgm}^{-1}\text{s}^{-1}$)
- v - terminal velocity(in ms^{-1})
- ρ - density of liquid(in kgm^{-3})

³ <https://www.sciencedirect.com/topics/physics-and-astronomy/terminal-velocity>

⁴ <https://byjus.com/physics/stokes-law-derivation/>

⁵ <https://byjus.com/physics/stokes-law-derivation/>

- σ - density of solid spheres(in kgm^{-3})
- r - radius of spheres(in m)
- g - acceleration due to gravity(9.8 ms^{-2})

I performed a trial with castor oil and found its critical velocity to $7.00 \times 10^{-2}\text{m}$.

Experiment performed to find critical velocity can be found in the Appendix for reference.

Hence I concluded that it can be carried out in a test tube. I hence decided to proceed with a larger test tube of length $15.0 \times 10^{-2}\text{m}$.

3: VARIABLES

Independent Variable: Temperature of honey.

Temperature is varied in order to measure its effects on the coefficient of viscosity of honey. The temperature values were varied with an interval of 3K starting from room temperature- 299.65K and ranging to 323.65K .

Dependent Variable: Coefficient of viscosity of honey.

This is the variable whose value is going to be calculated using the formula for coefficient of viscosity of honey(η).

Controlled Variables:

1. Type of honey taken: honey's coefficient of viscosity ranges from $2 \text{ kgm}^{-1}\text{s}^{-1}$ to $10 \text{ kgm}^{-1}\text{s}^{-1}$, pertaining to more than 300 types⁶ of honey. Thus I used honey from the same bottle to ensure similar results
2. Radius of spheres(in m): spheres of 3 different radii are dropped into the honey one at a time and these are kept constant.
3. Height travelled by the spheres(in m): 2 lines- $8 \times 10^{-2}\text{m}$ apart- are marked on the test tube and the time taken by a sphere to pass through these lines is noted. Hence, the distance between the two marked lines must not be changed in order to ensure consistent results.

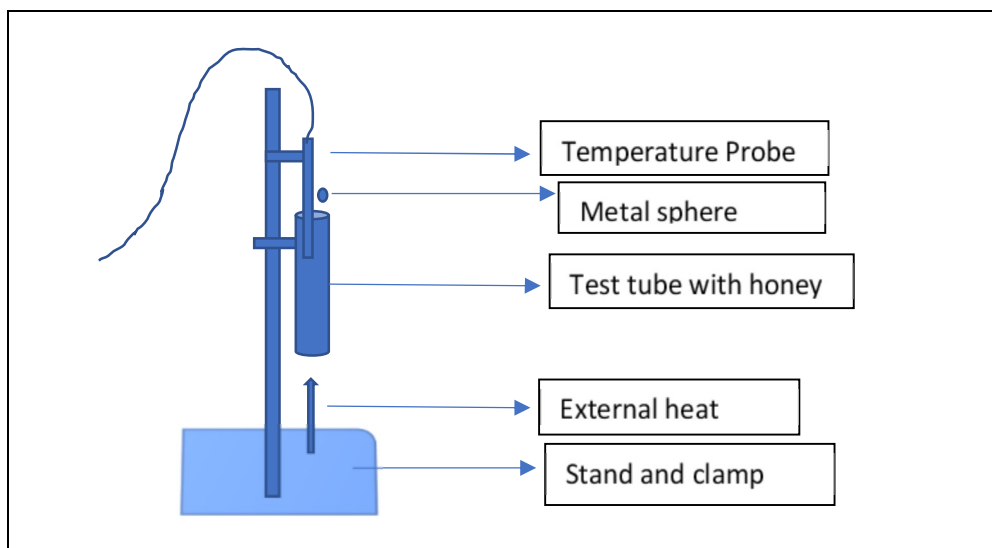
⁶ <https://recipes.timesofindia.com/articles/features/10-different-types-of-honey-and-everything-you-should-know-about-them/photostory/67297711.cms>

4: METHOD

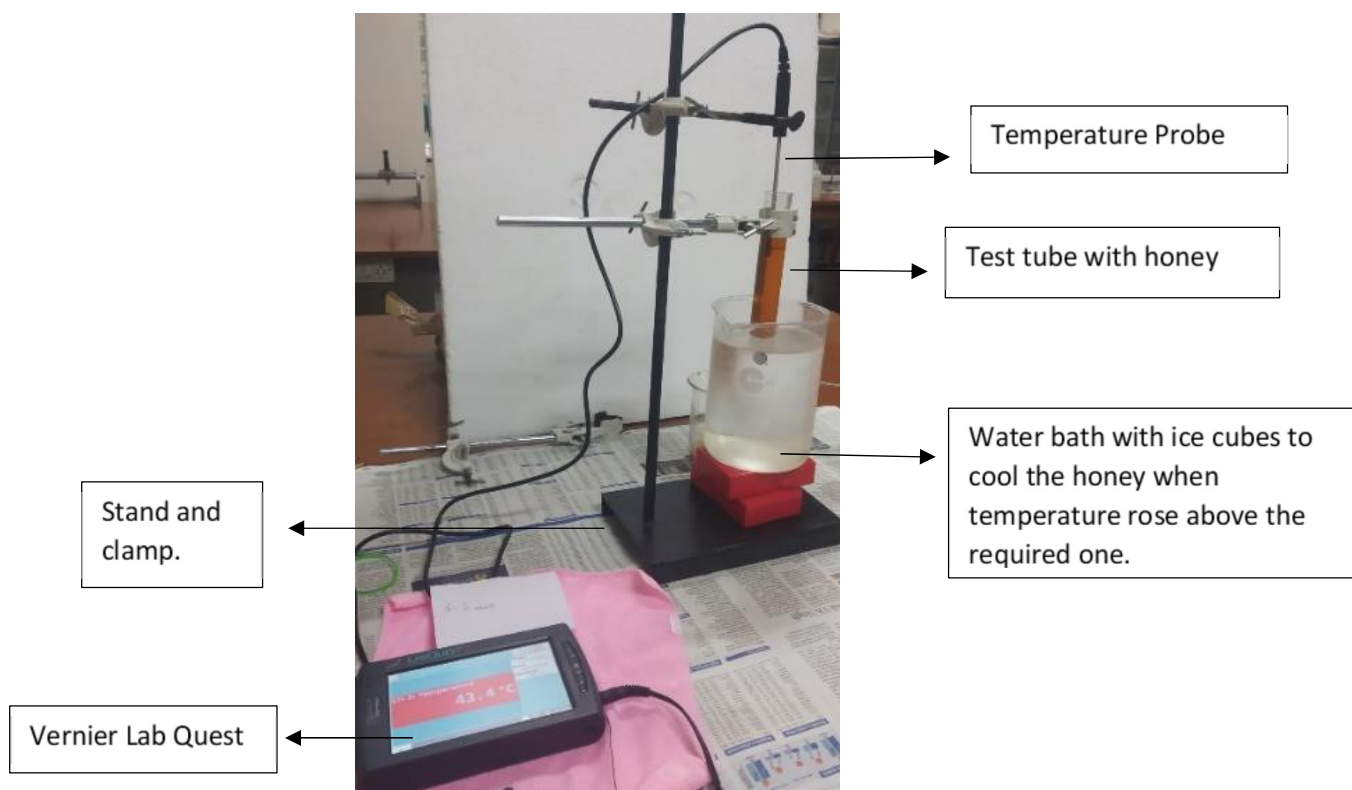
4.1: Apparatus

1. Metal spheres of diameter $3 \times 10^{-2} \text{m}$, $4 \times 10^{-2} \text{m}$ and $6 \times 10^{-2} \text{m}$.
2. Screw gauge of least count $0.01 \times 10^{-3} \text{m}$
3. Clamp and stand
4. Vernier lab quest
5. Temperature Probe of accuracy 0.01K
6. Spirit lamp
7. A test tube capable of withstanding heat
8. Marker pen
9. Glass rod to stir
10. Water bath
11. Stop watch of least count 0.01s
12. 12 MP Camera recorder
13. Logger Pro 3.15
14. Honey
15. Lab Coat, goggles and gloves

4.2: Photograph of set-up



(a) Figure 1: Schematic diagram of the experimental set up



(b) Figure 2: Actual set up of experiment:

4.3: Experimental Procedure

Calculating the density of the metal spheres:

1. I placed a metal sphere on the weighing machine and record its mass(in kg).
2. Then I measured its radius(in m) using a screw gauge, hence its volume.
3. Finally I calculated the density of the metal spheres using the formula: density= mass/volume;

Experiment performed to determine density of metal spheres can be found in the appendix.

To calculate the time taken by each sphere to pass through a distance of 8×10^{-2} m, I:

1. Set up the tripod stand with two clamps- one for the test tube and the other for the temperature probe.
2. Marked two lines on the test tube which are 8cm apart.
3. Poured the honey into the test tube and clamp it along with the temperature probe inserted into it and note the temperature. Make sure the probe is towards the side of the

test tube but does not touch the walls of the test tube either. This leaves space in the middle to drop the metal spheres easily.

4. Took spheres of the 3 different radii and keep a stopwatch ready.
5. Dropped the spheres with the radii $1.50 \times 10^{-3}\text{m}$, $2.00 \times 10^{-3}\text{m}$ and $3.00 \times 10^{-3}\text{m}$ into the test tube one at a time, in that order.
6. Noted the time taken by a sphere to cross the two marked lines using a stopwatch. For video analysis, place a camera which is held firmly and focusses on the spheres.
7. Took 3 trials for each sphere and note down the timings.
8. Then heated the honey in the test tube using a Bunsen burner and stir it so that the heat is distributed evenly. Heat it so that the temperature rises by 3K each time.
9. Showed the test tube to heat for only a small amount of time as honey heats up quickly and unevenly due to its high density. Keep stirring the honey in the test tube to bring down the temperature if required. If the temperature rises too much, place the test tube in an ice bath to cool the honey quickly to the required temperature.
10. Repeated the steps 5 through 8.
11. To find the velocity attained by the spheres through video analysis, I inserted the video in Logger Pro 3.15 and plotted points along the path traced by the sphere, for which the software would give the value of velocity.

4.4: RISK ASSESSMENT

Safety Considerations:

1. Wear gloves at all times to prevent the heated honey from falling onto hands while transferring or stirring.

Ethical Considerations: No ethical considerations as such.

Environmental Considerations: No environmental considerations as such.

Other cautionary measures taken:

1. Expansion of glass on heating is negligible, hence neglected.
2. Honey is heated on the low flame of a spirit lamp and is stirred continuously(using a thin glass rod) for even distribution of heat.
3. I conventionally tested the honey to check how viscous it is by eye. I poured it on to a paper and waited for it to spread. The honey did flow a little more than expected hence confirming that the honey I had bought was not completely pure.

5: RAW DATA

5.1: Qualitative Observations

I found the radius of the spheres used in the experiment with the help of a screw gauge. Pitch and head scale readings are measured to find the diameter of the spheres.

Readings for diameter of the three sets of spheres can be found in the appendix.

Table 1: Measuring time taken by the spheres for the first 3 temperatures to displace $8 \times 10^{-2} \text{m}$:

	Time(in s)		
temperature(in K)	t_1	t_2	t_3
T=299.65			
r(in m)=			
3.00×10^{-2}	8.28	8.29	7.81
2.00×10^{-2}	11.53	11.91	11.87
1.50×10^{-2}	16.41	15.69	16.40
T=302.65			
r(in m)=			
3.00×10^{-2}	7.30	7.00	7.28
2.00×10^{-2}	9.66	9.71	9.77
1.50×10^{-2}	12.72	12.79	12.92
T=305.65			
r(in m)=			
3.00×10^{-2}	3.91	4.37	4.13
2.00×10^{-2}	5.59	5.38	6.00
1.50×10^{-2}	8.16	7.90	7.69

The remaining raw data readings for the time taken by the spheres to displace $8 \times 10^{-2} \text{m}$ for temperatures between: 308.65K to 323.65K can be found in Section 3 of the Appendix.

6: PROCESSED DATA

Table 2(a): showing values of r and r^2 :

r (m)	r^2 (m^2)
3.00×10^{-3}	9.00×10^{-6}
2.00×10^{-3}	4.00×10^{-6}
1.50×10^{-3}	2.25×10^{-6}

$\rho = 1420 \text{ kgm}^{-3}$, $\sigma = 7800 \text{ kgm}^{-3}$, distance between two marked lines= $h = 8.0 \times 10^{-2} \text{m}$;

Table 2(b): showing values of η for different temperatures:

T=(in K)	Average time of t_1, t_2, t_3 respectively	$v = \frac{h}{t}$	$\frac{r^2}{v}$	η
299.65				
	8.13	9.84×10^{-3}	9.14×10^{-4}	
	11.77	6.80×10^{-3}	5.89×10^{-4}	
	16.17	4.94×10^{-3}	4.55×10^{-4}	
Average $r^2/v=$			6.52×10^{-4}	
				9.06
302.65				
	7.19	11.12×10^{-3}	8.09×10^{-4}	
	9.71	8.24×10^{-3}	4.86×10^{-4}	
	12.81	6.25×10^{-3}	3.60×10^{-4}	
Average $r^2/v=$			5.52×10^{-4}	
				7.67
305.65				
	4.14	19.33×10^{-3}	4.65×10^{-4}	
	5.66	14.14×10^{-3}	2.83×10^{-4}	
	7.91	10.11×10^{-3}	2.23×10^{-4}	
Average $r^2/v=$			3.24×10^{-4}	
				4.50
308.65				
	3.25	24.64×10^{-3}	3.65×10^{-4}	
	4.59	17.42×10^{-3}	2.30×10^{-4}	
	3.14	13.04×10^{-3}	1.73×10^{-4}	
Average $r^2/v=$			2.56×10^{-4}	
				3.55
311.65				
	2.46	32.56×10^{-3}	2.76×10^{-4}	
	3.69	21.68×10^{-3}	1.85×10^{-4}	
	4.93	16.13×10^{-3}	1.40×10^{-4}	
Average $r^2/v=$			2.00×10^{-4}	
				2.78
314.65				
	1.44	55.43×10^{-3}	1.62×10^{-4}	
	2.26	35.40×10^{-3}	1.13×10^{-4}	
	3.00	26.67×10^{-3}	0.84×10^{-4}	
Average $r^2/v=$			1.20×10^{-4}	
				1.67

317.65				
	1.28	62.50×10^{-3}	1.44×10^{-4}	
	1.98	40.34×10^{-3}	1.00×10^{-4}	
	3.11	25.72×10^{-3}	0.88×10^{-4}	
Average $r^2/v=$			1.10×10^{-4}	
				1.53
320.65				
	0.81	99.17×10^{-3}	0.91×10^{-4}	
	1.54	52.06×10^{-3}	0.77×10^{-4}	
	1.67	47.81×10^{-3}	0.47×10^{-4}	
Average $r^2/v=$			0.72×10^{-4}	
				0.99
323.65				
	0.66	121.83×10^{-3}	0.74×10^{-4}	
	1.08	73.85×10^{-3}	0.54×10^{-4}	
	1.48	54.06×10^{-3}	0.42×10^{-4}	
Average $r^2/v=$			0.57×10^{-4}	
				0.79

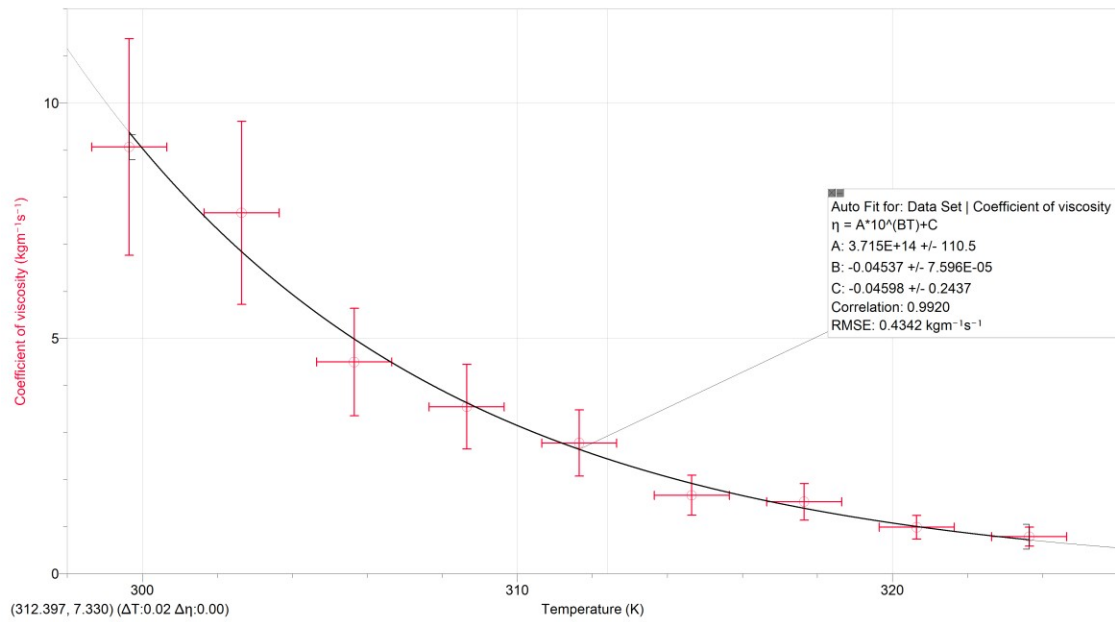


Figure 3: Graph showing the relation of viscosity vs temperature:

Using Video Analysis:

$\rho = 1420 \text{ kgm}^{-3}$, $\sigma = 7800 \text{ kgm}^{-3}$, distance between two marked lines= $h=8.0 \times 10^{-2} \text{ m}$;

Table 3: showing values of η for different temperatures using video analysis:

	v(in ms^{-1})			η
r(in m)=	3.00×10^{-2}	2.00×10^{-2}	1.50×10^{-2}	
T=(in K)				
299.65	4.36×10^{-2}	3.54×10^{-2}	2.59×10^{-2}	1.88
302.65	5.82×10^{-2}	4.93×10^{-2}	3.92×10^{-2}	1.36
305.65	8.62×10^{-2}	7.67×10^{-2}	5.25×10^{-2}	0.92
308.65	1.78×10^{-1}	1.06×10^{-1}	8.35×10^{-2}	0.53
311.65	1.99×10^{-1}	1.66×10^{-1}	1.17×10^{-1}	0.41
314.65	2.60×10^{-1}	2.00×10^{-1}	1.23×10^{-1}	0.34
317.65	3.91×10^{-1}	2.72×10^{-1}	2.00×10^{-1}	0.23
320.65	4.44×10^{-1}	3.39×10^{-1}	2.19×10^{-1}	0.20
323.65	5.15×10^{-1}	3.77×10^{-1}	2.72×10^{-1}	0.17

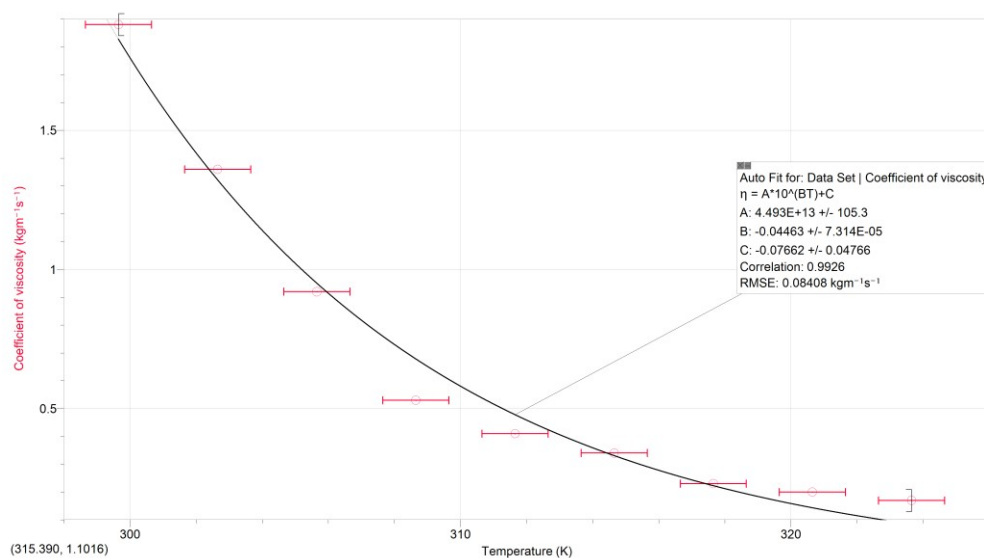


Figure 4: Graph showing the relation of viscosity vs temperature using video analysis:

7: CALCULATION OF RANDOM ERROR

Formula to calculate coefficient of viscosity: $\eta = \left(\frac{2 g(\sigma - \rho)r^2}{9 \times v} \right)$

Example of calculation: Say true $\eta = 9.07$ from the formula: $\eta = \left(\frac{2 g(\sigma - \rho)r^2}{9 \times v} \right)$

Table 4: showing absolute uncertainty

r= (in m)	$r^2/v =$	$\eta =$	Absolute uncertainty(true - experimental value)
3.00×10^{-3}	9.14×10^{-4}	12.7	3.64
2.00×10^{-3}	5.89×10^{-4}	8.18	0.89
1.50×10^{-3}	4.55×10^{-4}	6.32	2.75

$$\text{Mean absolute uncertainty} = \frac{3.64 + 0.89 + 2.75}{3} = 2.43$$

$$\text{Percentage uncertainty} = \frac{2.43}{9.07} \times 100\% = 26.8\%$$

Table 5: Actual error propagation in experiment:

Temperature(in K)	Mean absolute uncertainty(x 10 ⁶)	Percentage uncertainty
299.65	2.42	26.7
302.65	2.38	31.1
305.65	1.31	29.2
308.65	1.01	28.5
311.65	0.07	25.4
314.65	0.39	23.6
317.65	0.31	20.4
320.65	0.02	22.8
323.65	0.16	20.4

Hence, average % uncertainty= 25.4%

8: EVALUATION

The processed and graphical data show that as the temperature of honey was increased, its viscosity decreased exponentially. As temperature increases, the molecules of honey gain kinetic energy and begin to move⁷. This energy along with this movement overcomes the energy binding the complex structure of honey together, allowing for a less resistive flow of honey.

I calculated the percentage error to be 25.4% when I performed the experiment and measured the time taken by the spheres(to cover a distance of 0.08m) using a stopwatch. Hence, to measure the velocity attained by the sphere passing through the honey, I made use of *Logger Pro 3.15* . I took a video of the spheres passing through honey and analysed it on the software, hence using technology to reduce the error drastically.

Attaining and maintaining the required temperature for honey was not an easy task. I kept an ice-cold water bath ready after heating, as a precautionary measure against the honey's temperature rising too quickly and to increase the rate of cooling. Sometimes the temperature decreased at a pace I could not keep up with hence there is an uncertainty of $\pm 3^{\circ}\text{C}$ in temperature as the spheres were dropped into the honey. In my experiment, I have also ignored the expansion of glass on heating was neglected because it is very small.

⁷ <https://sciencing.com/changing-temperature-affect-viscosity-surface-tension-liquid-16797.html>

9: BIBLIOGRAPHY

1. <https://resources.saylor.org/wwwresources/archived/site/wp-content/uploads/2011/04/Viscosity.pdf>. (n.d.). Retrieved from <https://resources.saylor.org/wwwresources/archived/site/wp-content/uploads/2011/04/Viscosity.pdf>
2. Admin. (2019, August 22). Coefficient of Viscosity - Definition, Formula, Videos & Examples. Retrieved from <https://byjus.com/chemistry/coefficient-of-viscosity/>
3. Admin. (2019, October 3). Stokes Law Derivation - Stokes Formula And Terminal Velocity. Retrieved from <https://byjus.com/physics/stokes-law-derivation/>
4. Terminal Velocity. (n.d.). Retrieved from <https://www.sciencedirect.com/topics/physics-and-astronomy/terminal-velocity>
5. Appendix. (2010, August 18). Retrieved from <https://onlinelibrary.wiley.com/doi/pdf/10.1002/9780470423851.app1>
6. Elert, G. (n.d.). Density. Retrieved from <https://physics.info/density/>
7. Times Food. (2018, December 29). 10 different types of honey and everything you should know about them. Retrieved from <https://recipes.timesofindia.com/articles/features/10-different-types-of-honey-and-everything-you-should-know-about-them/photostory/67297711.cms>
8. Rohde, A. (2019, March 2). How Does Changing the Temperature Affect the Viscosity & Surface Tension of a Liquid? Retrieved from <https://sciencing.com/changing-temperature-affect-viscosity-surface-tension-liquid-16797.html>
9. Logger Pro 3.15

10.0 APPENDIX:

Section 1: Prior experimentation on castor oil

$\rho = 960 \text{ kgm}^{-3}$, $\sigma = 7800 \text{ kgm}^{-3}$, distance between two marked lines= $h=7.0 \times 10^{-2} \text{ m}$

r (m)	r ² (m ²)	t (s)	v=h/t (ms ⁻¹)	r ² /v (ms)
3.00x 10 ⁻³	9.00x 10 ⁻⁶	0.63	0.16	5.63x 10 ⁻⁵
2.00x 10 ⁻³	4.00x 10 ⁻⁶	1.13	0.09	4.44x 10 ⁻⁵
1.50x 10 ⁻³	2.25x 10 ⁻⁶	1.53	0.07	7.23x 10 ⁻⁵

Average $r^2/v = 5.80 \times 10^{-5} \text{ ms}$

$\eta = \frac{2g(\sigma-\rho)r^2}{9v} = 2.18 \times 6840 \times (5.80 \times 10^{-5}) = 0.86 \text{ kgm}^{-1}\text{s}^{-1}$. This value is close to the literature value⁹ for $\eta_{\text{castor oil}} = 0.90 \text{ kgm}^{-1}\text{s}^{-1}$.

To get $h=7.0 \times 10^{-2} \text{ m}$ from $r=3.00 \times 10^{-3} \text{ m}$ and literature value of $\eta_{\text{castor oil}}$:

$$v = \frac{2g(\sigma-\rho)r^2}{9\eta} = 2.17 \times 6840 \times (11.25 \times 10^{-6}) = 0.17 \text{ ms}^{-1}$$

$U = 6\pi\eta rv = ma$, where m - mass and a - acceleration of the object;

$$a = \frac{U}{m} = 9.625 \text{ ms}^{-2};$$

$$v^2 - u^2 = 2gx; \quad u^2 = 0 \text{ during free fall};$$

$$x = \frac{v^2}{2(g-a)} = \frac{(0.17)^2}{2(9.8-9.625)} = 7 \times 10^{-2} \text{ m};$$

- The same calculations are used to find the minimum distance needed by a sphere to attain terminal velocity in honey. I have assumed the coefficient of viscosity to be $2 \text{ kgm}^{-1}\text{s}^{-1}$ which is on the lower end of its range between $2 \text{ kgm}^{-1}\text{s}^{-1}$ to $10 \text{ kgm}^{-1}\text{s}^{-1}$.

$\sigma = 7800 \text{ kgm}^{-3}$; Density of honey¹⁰= $\rho=1420 \text{ kgm}^{-3}$;

$$v = \left(\frac{2g(\sigma-\rho)r^2}{9\eta} \right) = 2.17 \times 6380 \times (11.25 \times 10^{-6}) = 0.06 \text{ ms}^{-1};$$

$$U = 6\pi(2)(0.003)(0.06) = 6.8 \times 10^{-3} \text{ kgm}^{-2};$$

$$a = \frac{U}{m} = \frac{(6.8 \times 10^{-3})}{(0.8 \times 10^{-3})} = 8.5 \text{ ms}^{-2};$$

⁸ <https://onlinelibrary.wiley.com/doi/pdf/10.1002/9780470423851.app1>

⁹ <https://onlinelibrary.wiley.com/doi/pdf/10.1002/9780470423851.app1>

¹⁰ <https://physics.info/density/>

$$x = \frac{(0.06)^2}{2(9.8-8.5)} = 1.4 \times 10^{-3} \text{m};$$

Thus the minimum distance needed for an object in honey to attain terminal velocity is $1.4 \times 10^{-3} \text{m}$;

I marked two lines on the test tube $8 \times 10^{-2} \text{m}$ apart, to make it easy to manually calculate the time taken by a sphere to pass through the marked lines.

Section 2: Finding Diameter of metal spheres

Table 2: Diameter of the three sets of metal spheres using a screw gauge:

Trial no.	Pitch scale reading M (mm)	Head scale reading n (div)	Diameter, d= M+(n x L.C) (mm)
1	6	22	6.22
2	6	22	6.22
3	6	22	6.22
4	6	19	6.19

Trial no.	Pitch scale reading M (mm)	Head scale reading n (div)	Diameter, d= M+(n x L.C) (mm)
1	4	21	4.21
2	4	20	4.20
3	4	21	4.21
4	4	21	4.21

Trial no.	Pitch scale reading M (mm)	Head scale reading n (div)	Diameter, d= M+(n x L.C) (mm)
1	3	24	3.24
2	3	23	3.23
3	3	24	3.24
4	3	23	3.23

Section 3: Finding time taken by spheres to displace $8 \times 10^{-2} \text{ m}$

Table 3: Time taken by spheres to cross 2 marked points for a certain temperature:

	Time(in s)		
temperature(in K)	t ₁	t ₂	t ₃
T=299.65			
r(in m)=			
3.00×10^{-2}	8.28	8.29	7.81
2.00×10^{-2}	11.53	11.91	11.87
1.50×10^{-2}	16.41	15.69	16.40
T=302.65			
r(in m)=			
3.00×10^{-2}	7.30	7.00	7.28
2.00×10^{-2}	9.66	9.71	9.77
1.50×10^{-2}	12.72	12.79	12.92
T=305.65			
r(in m)=			
3.00×10^{-2}	3.91	4.37	4.13
2.00×10^{-2}	5.59	5.38	6.00
1.50×10^{-2}	8.16	7.90	7.69
T=308.65			
r(in m)=			
3.00×10^{-2}	3.19	3.25	3.30
2.00×10^{-2}	4.60	4.56	4.62
1.50×10^{-2}	6.13	6.15	6.13
T=311.65			
r(in m)=			
3.00×10^{-2}	2.35	2.62	2.40
2.00×10^{-2}	3.63	3.75	3.69
1.50×10^{-2}	4.97	4.91	5.00
T=314.65			
r(in m)=			
3.00×10^{-2}	1.40	1.44	1.49
2.00×10^{-2}	2.31	2.34	2.13
1.50×10^{-2}	2.90	3.10	3.00
T=317.65			
r(in m)=			
3.00×10^{-2}	1.37	1.20	1.27
2.00×10^{-2}	1.94	2.00	2.01
1.50×10^{-2}	3.09	3.09	3.15
T=320.65			
r(in m)=			

3.00×10^{-2}	0.78	0.81	0.83
2.00×10^{-2}	1.56	1.55	1.50
1.50×10^{-2}	1.66	1.70	1.66
T=323.65			
r(in m)=			
3.00×10^{-2}	0.59	0.68	0.70
2.00×10^{-2}	1.03	1.10	1.12
1.50×10^{-2}	1.44	1.50	1.50