Problem 1. Consider the following familiar scenario: a signal $s(n)$ is corrupted by additive noise $v(n)$ to form the observations $z(n)$,

$$z(n) = s(n) + v(n)$$

However, suppose that $R_{sz}(k)$ is not known. Instead, a sensor located away from the signal source measures $v'(n)$, which is correlated with the measurement noise $v(n)$ but not with the signal. Then it is possible to estimate $v(n)$ from $v'(n)$ and produce an estimate of the signal via

$$\hat{s}(n) = z(n) - \hat{v}(n).$$

This approach forms the basis for noise cancellation and is diagrammed in Figure 4.5. Rather than estimate the signal, we attempt to estimate the noise and remove it. Such a system might be employed by military aircraft radio systems or manufacturing plants near residential areas.

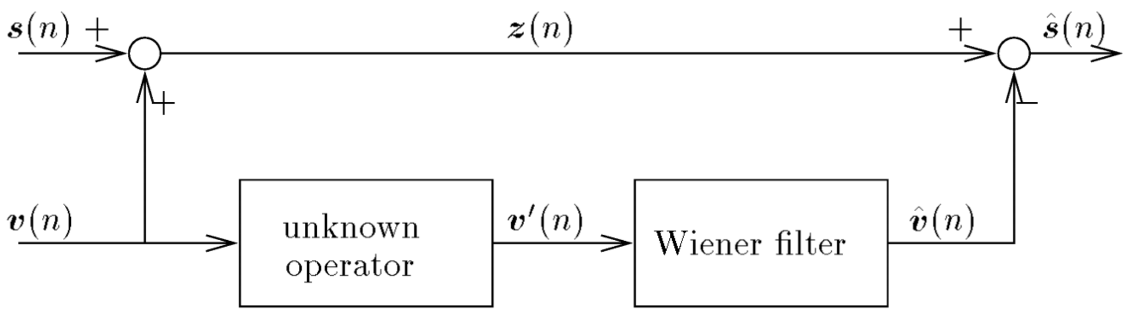

Figure 4.6. A simple noise cancellation system. Signal $s(n)$ is corrupted by additive white noise $v(n)$ to produce measurements $z(n)$. Another sensor measures $v'(n)$, which is related to $v(n)$ in some unknown manner. A Wiener filter estimates $v(n)$ from $v'(n)$ and the estimate of the noise $\hat{v}(n)$ is removed from $z(n)$.

This problem uses an FIR Wiener filter to estimate the noise. The Wiener-Hopf equation (4.17) becomes

$$R_{v'}h = r_{vv'}$$

(a) Assume $v'(n)$ is zero-mean and uncorrelated with $s(n)$. Show that

$$R_{vv'}(k) = R_{zv'}(k),$$

so that it is possible to set up the Wiener-Hopf equation strictly from $\boldsymbol{z}(n)$ and $v'(n)$

The remainder of this problem develops a MATLAB simulation. Let the signal be given by

$$s(n) = \cos(0.04\pi n).$$

Let the noise processes $v(n)$ and $v'(n)$ be produced via

$$\boldsymbol{v}(n) = 0.7\boldsymbol{v}(n-1) + \boldsymbol{y}(n)$$

and

$$\boldsymbol{v}'(n) = -0.5\boldsymbol{v}'(n-1) + 0.36\boldsymbol{v}'(n-2) + \boldsymbol{y}(n)$$

where $\boldsymbol{y}(n)$ is zero-mean white Gaussian noise with unit variance.

(b) Use MatLaB to generate 100-sample realizations of $s(n)$ and $\boldsymbol{y}(n)$; the results should be 100 -element row vectors s and y.

(c) Use filter to produce sample realizations v and vprime from y. Then add s and v to form z. [Problem 2.5 introduces filter.] Plot s and z on the same graph. On a separate graph, plot vprime.

(d) Estimate $R_{v'}(k)$ and $R_{vv'}(k)$ from vprime and z. [See Problem 2.4.] Store the estimates in vectors Rvprime and Rvvprime, respectively.

(e) From Rvprime and Rvvprime, set up the Wiener-Hopf equation for a 4-tap ($N = 4$) FIR Wiener smoothing filter. Solve the Wiener-Hopf equation to obtain the filter coefficients h. [The function inv will compute a matrix inverse.]

(f) Use filter to apply the Wiener filter to vprime and call the output of the filter vhat. Then estimate the signal shat from z and vhat. Plot s and shat on the same graph to see the effect of the noise cancellation system.

(g) Use ensemble averaging to estimate $\sigma_v^2 = R_v(0)$. Call the estimate varv. [See Problem 2.2 for computing ensemble averages.] Then compute the mean square error,

$$\mathrm{E}[(v(n) - \hat{v}(n))^2] = \sigma_v^2 - h^{\mathrm{T}} r_{vv'}$$

Note that the MSE may be negative since σ_v^2, $R_{v'}(k)$, and $R_{vv'}(k)$ are all estimated quantities. Also compute the experimental MSE for the sample realization

$$\text{experimental MSE } = (\text{v-vhat}) * (\text{v-vhat})'/100$$

Depending upon the values in the sample realization, the experimental MSE may be larger or smaller than the theoretical MSE. However, the two quantities should be somewhat similar.

(h) Repeat parts (e)-(g) for $N = 6$ and $N = 8$.

Problem 2. Observation signal $y(t)$ from the stationary signal $x(t)$ is defined as below:

$$y(t) = x(t) + V(t)$$

Power density spectrum of noise $v(t)$ and signal $x(t)$ are provided:

$$S_{\mathrm{vv}} = \frac{N_0}{2}$$

$$S_{xx}(s) = \frac{2\alpha}{\alpha^2 - s^2}$$

Calculate the wiener estimation of the signal x at time t. (i. e. $\hat{x}(t \mid t) = ?$)