Observation signal $y(t)$ from the stationary signal $x(t)$ is defined as below:

$$y(t) = x(t) + V(t)$$

Power density spectrum of noise $v(t)$ and signal $x(t)$ are provided:

$$s_{\mathrm{vv}} = \frac{N_0}{2}$$
$$s_{xx}(s) = \frac{2\alpha}{\alpha^2 - s^2}$$

Calculate the wiener estimation of the signal x at time t. (i. e. $\hat{x}(t \mid t) = ?$)

$$y = x + v$$
$$E[y] = E[x] + E[v]$$

If $v\ is\ gaussian$:

$$E[y] = E[x]$$

And we have:

$$R_y(k) = E[y(n)y(n-k)] \overset{y(t)=x(t)+v(t)}{\longrightarrow}$$
$$R_y(k) = E[x(n) + v(n))(x(n-k) + v(n-k))\big] =$$
$$E[x(n)x(n-k) + v(n)v(n-k) + v(n)x(n-k) + x(n)v(n-k)]$$
$$Since\ x \perp\!\!\!\perp v \Rightarrow V_y(n) = R_x(n) + R_v(n) + 0 + 0$$

zeros are because of the gaussian noise and $x \perp\!\!\!\perp v$, so:

$$S_y = S_x + S_v$$
$$H = \frac{S_{xy}(S)}{S_y(s)} = \frac{S_x}{S_y} = \frac{\frac{2\alpha}{\alpha^2 - S^2}}{\frac{2\alpha}{\alpha^2 - s^2} + \frac{N_0}{2}} = \frac{4\alpha}{-N_0 s^2 + N_0\alpha^2 + 4\alpha}$$

$$R_{xy} = E[x(n)y(n-k)] = E[x(n)(x(n-k) + v(n-k))] = R_x(n) + 0$$
$$S_{xy} = S_x$$

And, we have

$$\hat{S_x} = H(s)S_y \longrightarrow H(s) = \frac{\hat{S}_x}{S_y}$$

After Calculating $H(s)$, we are going to have.

$$\mathcal{L}^{-1}\{\hat{x}(s) = H(s)y(s)\}$$

Calculate this and we are done☺