A Treatise of Climate-Impacted Credit Risk Models Using Distortion Functions: The Climate-Calibrated Proportional Hazards Distortion Model

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Abstract

This study delves into the utilization of distortion functions to distort the credit quality driver within the Vasicek single-threshold default model, enabling the capture of physical risks stemming from climate-impacted events. Additionally, it scrutinizes how distortion impacts credit transition probabilities in a multi-threshold context. The primary focus centers on the Proportional Hazards Distortion, which increases the probabilities of downgrade and default as its parameter a decreases. This parameter a is calibrated using a climate variable, the change in sea surface temperature (ΔSST), under the assumption of a linear relationship. With increasing ΔSST, indicative of deteriorating economic conditions due to climate-impacted events, the distortion parameter amplifies the likelihood of credit rating downgrades. As ΔSST approaches a threshold of $2°C$, the non-linear increase of downgrade probabilities results in a significant devaluation of the corresponding loan portfolios. Subsequently, escalating default probabilities due to adverse economic conditions driven by the rising climate parameter lead to the expansion of portfolio risk metrics, particularly, economic capital. Notably, despite the challenge posed by the limited availability of precise climate data, this model establishes a rudimentary framework for integrating climate risk into a well-established credit risk model.

Acknowledgments

I want to take a moment to express my gratitude to those who offered invaluable contributions to my master's project. Dr. Devan Becker, our conversations about machine learning and scrutinizing model assumptions added immense depth to my ability to not only build models, but build insight models and validate them. Dr. Adam Metlzer, your open-door policy and the welcoming atmosphere of you created provided me with a space to bounce ideas off you and gain insights into market dynamics; it made a world of difference. Dr. McCluskey, your generous help in designing the calibration was a turning point in the project. Your expertise helped make this calibration simple and intuitive. Paul Sweeting, your extensive knowledge on copulas and your willingness to share it with me was incredibly generous. It brought a unique depth to my research that wouldn't have been possible without you. To, Dr. Mark Reesor, my supervisor - your unwavering patience and dedication to my academic growth have been nothing short of extraordinary. I can't thank you enough for pushing me out of my comfort zone and allowing me to essentially set up camp in your office. The investment you made in me and the consequent dividends made a significant impact in my academic, professional and personal life. Thank you all for being such crucial parts of this journey for me.

Contents

1 Introduction

Since the implementation of the Basel 1 Accord in 1998, financial institutions have devoted significant research to portfolio credit risk models, which are vital for estimating potential losses and determining the economic capital required to support risk-taking activities. Initially, under Basel 1, economic capital allocation was based on predetermined asset values with specific weights assigned to different asset classes, regardless of a borrower's financial standing. However, the 2008 financial crisis highlighted the limitations of this framework, leading to the introduction of Basel II, with standardized and internal ratings-based approaches (IRB). Vasicek's Merton-type asymptotic single factor model is widely used in the industry, and is a standard framework in the Basel IRB approach. Though, it's base form lacks sufficient structure to capture the impact of climate-related events on portfolio risk measures. Consequently, the evolving challenges posed by climate change present credit risk modeling with a complex task - capturing systemic risk due to climate-impacted events.

Climate change introduces new and unpredictable risks associated with physical risks in the form of hurricanes, rising sea levels, drastic weather shifts, and wildfires. Additionally, climate-related risks encompass transition risks, arising from climate regulations and mandates that affect the creditworthiness of companies reliant on fossil fuels or exposed to climate-impacted sectors. As a result, financial institutions are in the process of augmenting their credit risk models to account for potential depreciation of assets associated with climate risks. By incorporating these climate-related factors into their models, financial institutions can gain a more comprehensive understanding of the risks they face and make informed assessments to manage any consequent exposures. However, this poses the question,"Do financial institutions develop new models or augment existing models to capture this new systemic factor?" This paper seeks to explore this question by exploring how distortion functions and their parameters impact credit transition matrices and portfolio risk measures and the viability of the model in practice.

2 Literature Review

The scientific understanding and recognition of climate change as a significant global issue has been developing over several decades, with the scientific discussion on climate change emerging in the 1960s and 1970s as researchers began studying the potential impact of human activities on the Earth's climate system. Thus, climate change itself is not a new concept. Yet the implementation of climate regulations was still decades away due to the lack of scientific consensus. Achieving scientific consensus took time, as all the necessary steps to create firm regulations that struck an equilibrium of financial stability and environmental sustainability involved a rigorous process. The two key steps in this effort were; conducting extensive research to gain an exhaustive understanding of climate change due to its complex and dynamic nature. This resulted in climate change being viewed as a new type of systemic risk. The second step was engaging in broad-spectrum stakeholder consultations to frame possible solutions in such a way that they were equitable, both politically and economically. Though the strides were short for these steps, they ultimately lead to the development of robust frameworks and formed diverse perspectives of climate-impacted financial risks and how to account for them.

The Bank of Canada and the Office of the Superintendent of Financial Institutions [12] published a report detailing a climate scenario analysis pilot with six Canadian financial institutions. The report outlined key steps it took to lay a foundation for scenario analysis and consequent climate regulations. These steps were;

- enhancing the regulator's and financial institutions' analytical capacity to handle climate transition scenarios
- improving assessment and disclosure of climate-related risks
- understanding the financial sector's exposure to climate transition risk

Scenario analysis proved valuable for identifying risks in uncertain climate transition environments and the endeavor aided global development of climate transition scenarios aligned with targets specific to a scenario. The analysis also highlighted the need for sectoral restructuring of portfolios to capture specific climate related macroeconomic impacts. Consequently, the pilot established a foundation for understanding and assessing climate-related risks in the Canadian economy and financial system. However, the need for the development of standardized methodologies to form a rigorous framework for capturing the impact of climate-related risks was specified as an area of significance for further development.

Garnier et al.[7] addresses this need and the approach presented is primarily aimed at the financial community and focuses on assessing the financial risks related to climate change. Specifically, it addresses the estimation of climate risk within a loan portfolio with the added consideration of transition risk arising from uncertain political and legal actions taken to mitigate climate impact. To address these risks, the paper introduces the Climate Extended Risk Model (CERM), but allows for multiple systemic risk factors, providing a methodology to calculate incremental credit losses in a loan portfolio by incorporating both physical and transition risks. This approach allows one to examine which systemic factors are most significant or which scenario path is likely to cause an exceeding of an institutions' loss threshold. The implications of this research are significant for financial institutions, as it enables them to better understand and manage climate-related risks within their loan portfolios.

However, there are several drawbacks with the CERM. Firstly, it relies on the availability and quality of climate-related data. A lack of such could affect the accuracy and reliability of the model's outputs. Risk Live Europe ESG & Climate Risk Europe, is an event where senior risk management and investment strategy decision-makers such as asset managers, hedge funds, life insurance firms, pension funds, and investment banks in the UK and Europe, meet to discuss insights in risk management. At Risk Live 2023, there was widespread discontent expressed about the lackluster access to transparent data for climate risk model validation; this highlights modern limitations of CERM for the efficient assessment of climate change in credit risk. Discontent was also expressed at the absence of a standardized framework, similar to those in the IRB, to guide risk professionals in setting reasonable thresholds. Moreover, climate change can have systemic monolithic implications, affecting multiple sectors and stakeholders simultaneously. Thus, credit risk models should also account for the risks, born from interconnectedness and cascading effects of climate-impacted events. This may pose a challenge for the CERM or similar models as the inclusion of such scenarios adds to the complexity of the model. Thus, implementing these methods may be costly and time consuming for financial institutions as impeding climate regulations loom. In that regard, another method of capturing climate related risks is explored, but with a relatively minimal, but significant, modification to an existing model.

Drmac[5] takes an alternative approach; he augments an existing model framework by using a multi-factor setting. In this work, Vasicek's Asymptotic Single Factor model is transformed by assuming the systemic factor is a function of a finite amount of macro-variables. This application is advantageous because it allows the conditional default probability to be decomposed into point-in-time and through-the-cycle default probabilities. Moreover, the multi-factor assumption allows for the introduction for climate-related macro-variables as systemic risk. Drmac then showed that this model is simply a special case of applying the Gaussian distortion to Yang's default probability.

Zeldenrijk[16] took a more direct approach, distorting the systemic factor of Vasicek's model to re-weight likelihood of the defaults and then calibrating the distortion function parameters with climate data. The two distortions he used were the Gaussian and the Beta Distortion. Through this work, he showed how, by using different climate metrics to calibrate the distortion parameters, one can capture the impact of climate events on portfolio risk measures without significant modifications to the existing model.

Building on the latter, this work follows suit, using distortion functions to explore the impact of climate-impacted events on portfolio risk measures. However, this work takes a novel approach by distorting the distribution of the default driver and examining its impact portfolio risk measures. Likewise, the effect of distorting the distribution of the default driver on credit transition matrices is also explored. The focus of this work will the physical risks induced by climate risk, as opposed to transition risks which Garnier et al.[7], Drmac[5], Zeldenrijk[16] previously explored.

3 Methodology

3.1 Vasicek's Asymptotic Single Risk Factor Model

Since there are numerous works delineating the derivation and properties of the Vasicek model, some included in the references, a light description of the model and its main properties is offered.

The Vasieck model is a Merton-type model in the sense that it models credit defaults as an asset value problem. When a borrower's asset value cross some predetermined threshold, a default is triggered. More formally, let Y_i be the i^{th} borrower's asset value. Vasieck[14] proposed that the i^{th} borrower's asset value can be expressed as:

$$Y_i = \sqrt{\rho}Z + \sqrt{1-\rho}\epsilon_i, \tag{1}$$

where

- $Z \sim$ N(0,1) is a systemic factor to which all borrowers in the portfolio are affected by
- $\epsilon_i \sim$ N(0,1) is an idiosyncratic factor, unique to each borrower.
- ρ (assumed to be homogeneous for simplicity) is the relative weight of the systemic factor compared to the idiosyncratic factor in (1).
- Z and ϵ_i are i.i.d.

Since Z and ϵ_i are $N(0,1)$, it easily follows that:

$$\mathbb{E}[Y_i] = 0,\, Var(Y_i) = 1,$$

and, $Y_i \sim N(0,1)$ as well. This property makes finding the unconditional default probability relatively easy in the below paragraph. Moreover, the correlation between two distinct asset values, is:

$$Corr(Y_i, Y_j) = \rho.$$

A default event occurs when a borrower's asset value falls below a certain threshold and this can be written as $Y_i \leq b$, where b is the threshold. This is indicated by a default indicator as defined by:

$$\mathbb{I}_{D_i} = \begin{cases} 1 & \text{if } Y_i \leq b \\ 0 & \text{if } Y_i > b \end{cases} \tag{2}$$

where D_i is the event that borrower i defaults. Thus, since $Y_i \sim N(0,1)$, the unconditional probability of default, $P(D_i)$ is:

$$P(D_i) = P(\mathbb{I}_{D_i} = 1) = P(Y_i \leq b) = \Phi(b) \implies b = \Phi^{-1}(P(D_i))$$

where Φ and Φ^{-1} are the standard normal cumulative distribution function and it's inverse, respectively. Finally, the conditional default, or the default given some realization of the random systemic factor Z is:

$$\begin{aligned} P(Y_i \leq b | Z = z) &= P(\sqrt{\rho} Z + \sqrt{1-\rho}\epsilon_i \leq b | Z = z) \\ &= P(\sqrt{\rho} z + \sqrt{1-\rho}\epsilon_i \leq b) \\ &= \Phi\left(\frac{b - \sqrt{\rho} z}{\sqrt{1-\rho}}\right). \end{aligned} \tag{3}$$

The correlation between default indicators can also be derived using (3). For simplicity, it is assumed that default probability is homogeneous amongst borrower, i.e. $P(Y_i \leq b) = P(D_i) = P(D_j)$. Then, the correlation between default indicators is expressed as:

$$Corr(\mathbb{I}_{D_i}, \mathbb{I}_{D_j}) = \frac{\int_{\infty}^{-\infty} \Phi(\frac{b-\sqrt{\rho}z}{\sqrt{1-\rho}})^2 \phi(z) dz - P(Y_i \leq b)^2}{P(Y_i \leq b))(1 - P(Y_i \leq b))} = \frac{\int_{\infty}^{-\infty} \Phi(\frac{b-\sqrt{\rho}z}{\sqrt{1-\rho}})^2 \phi(z) dz - \Phi(b)^2}{\Phi(b)(1 - \Phi(b))}. \tag{4}$$

From (4), it is observed that the correlation depends on the choice of default threshold and the relative weight of the systemic factor ρ. In the following section, how distorting the systemic factor changes this relationship is observed.

3.2 A Brief Overview of Distortion Functions and Their Existing Applications to Credit Risk Models

3.2.1 Distortion Functions

According to Reesor et al.[13], a distortion function g, is a function such that:

- g:[0,1] $\rightarrow$ [0,1]
- g is non-decreasing on the interval [0,1] and $g(0) = 0$ and $g(1) = 1$

Essentially, a distortion function can be conceptualized as a cumulative distribution function (CDF) with support on [0,1]. Let X be a random variable with CDF F_X, then a distorted distribution is defined by: (5)

$$F^g(x) = g(F_X(x)). \tag{5}$$

It is noteworthy to mention that the random variable itself is not being distorted, but its distribution, with the distortion function re-weighting the probability of outcomes. Additionally, g is assumed to be differentiable.

3.2.2 Distorting The Distribution of The Systemic Factor in Vasicek's ASRF Model

Distorting the distribution of the systemic factor:

$$\Phi(z) \rightarrow \Phi^g(z) = g(\Phi(z)),$$

where $\Phi^g(z)$ represents the distorted cumulative distribution function of the systemic factor Z. Now, Z is no longer normal and the distribution of Z depends on the distortion function as well as its parameters. So, it follows that:

$$\mathbb{E}^g[Y_i] = \sqrt{\rho}\mathbb{E}^g[Z] \text{ and } Var^g(Y_i) = 1 + \rho(Var^g(Z) - 1).$$

Observe, that both moments are now dependent on how Z is distorted. The same also applies for the correlation between default drivers. The correlation between default drivers is written as:

$$Corr^g(Y_i, Y_j) = \frac{\rho Var^g(Z)}{1 + \rho(Var^g(Z) - 1)},$$

and, following a similar derivation for the correlation between default indicators in the undistorted case, the correlation between default indicators in the distorted case is expressed as:

$$Corr^g(\mathbb{I}_{D_i}, \mathbb{I}_{D_j}) = \frac{\int_{\infty}^{-\infty} \Phi(\frac{b-\sqrt{\rho}z}{\sqrt{1-\rho}})^2 g'(\Phi(z))\phi(z)dz - g(\Phi(b))^2}{g(\Phi(b))(1 - g(\Phi(b)))}.$$

In both cases, the correlation between default drivers and default indicators depend on ρ, b, g and the parameters of the distortion function. Zeldenrijk[16] vividly illustrates different distortion functions and parameters with visualizations that depict the impact on a portfolio's risk measures.

3.3 Brief Primer on Joint Distributions and Copulas

In the next section of this paper, the effects of distorting the default driver distribution are explored. However, an interesting implication of this is the impact on the marginals CDF's in joint distributions. Consequently, this section offers a light primer on joint densities and their dependence structure, copulas.

A joint distribution captures the behavior of multiple random variables simultaneously, providing a comprehensive view of their combined probabilities. It is characterized by a joint distribution function that offers insights into the dependence structure among the variables. The correlation matrix, a crucial parameter of joint distributions, determines the shape and orientation of the joint density function's surface. A strong positive correlation results in variables changing together, while a strong negative correlation indicates an inverse relationship. The magnitude and direction of the correlation reflects the degree of elongation and probability concentration in the joint density function, shedding light on the collective behavior of the variables within the system. This is illustrated in Figure 1 using the Gaussian copula and standard normal marginals. It can also be observed that when there is no correlation between the variables, this results in circular contours, indicating no linear dependence. In general, modelling marginals and their dependence structures is complicated, but the following theorem provides a robust framework to address the challenge.

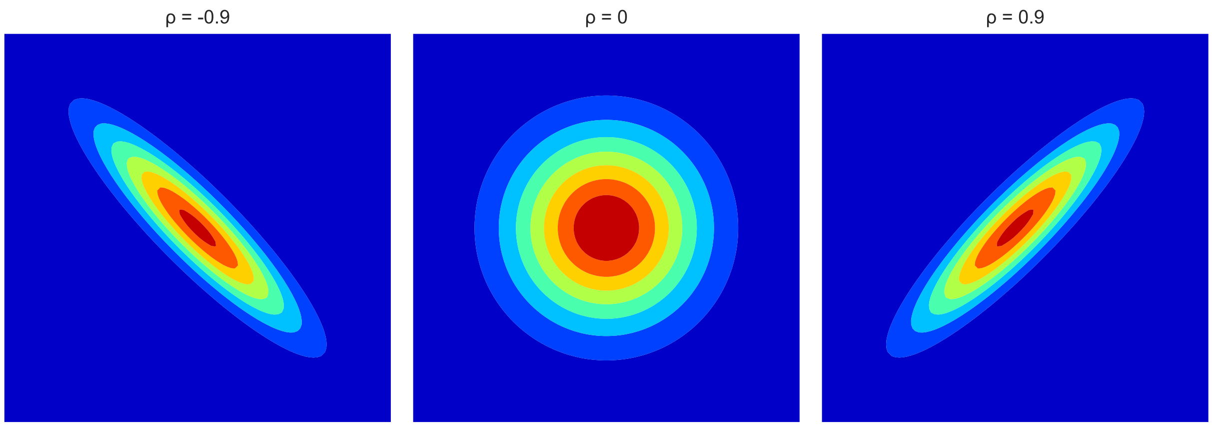


Figure 1: Bivariate Standard Normal Density Contour Plots for Different ρ using Gaussian Copula

Sklar's theorem simplifies the modeling of joint probability measures by decomposing the joint distribution into marginal distributions and a copula function that captures the dependence structure. This allows for independent modeling of the marginal distributions and flexible modeling of the dependence using copulas. By separating the modeling of the marginal distributions from the dependence structure, Sklar's theorem provides a more manageable approach and enables capturing complex dependence patterns. Overall, Sklar's theorem is a powerful tool that streamlines the modeling process and enhances the understanding of the joint behavior of multiple variables. Formally, it is stated as follows:

Theorem 1 (Sklar's Theroem)**.** Let F be a multivariate distribution function for a random vector $\mathbf{X} \in \mathbb{R}^d$. Then:

$$\begin{aligned} F(\mathbf{X}) &= F(X_1, X_2, ..., X_d) \\ &= C(F_1(X_1), F_2(X_2), ..., F_d(X_d)), \end{aligned}$$

where F_i is the marginal CDF for X_i and C is a copula that describes the dependence structure between each X_i.

Thus, a copula is any CDF with uniform marginals and any joint distribution can be expressed as a copula. In this work, the Gaussian copula is utilized due to its modelling and visual simplicity. This copula has the following form:

$$C(U_1, U_2) = \Phi_2(\Phi^{-1}(U_1), \Phi^{-1}(U_2)),$$

and $U_i = F_i(X)$, where Φ_2 is the bivariate normal CDF and has correlation parameter, ρ_{Corr}.

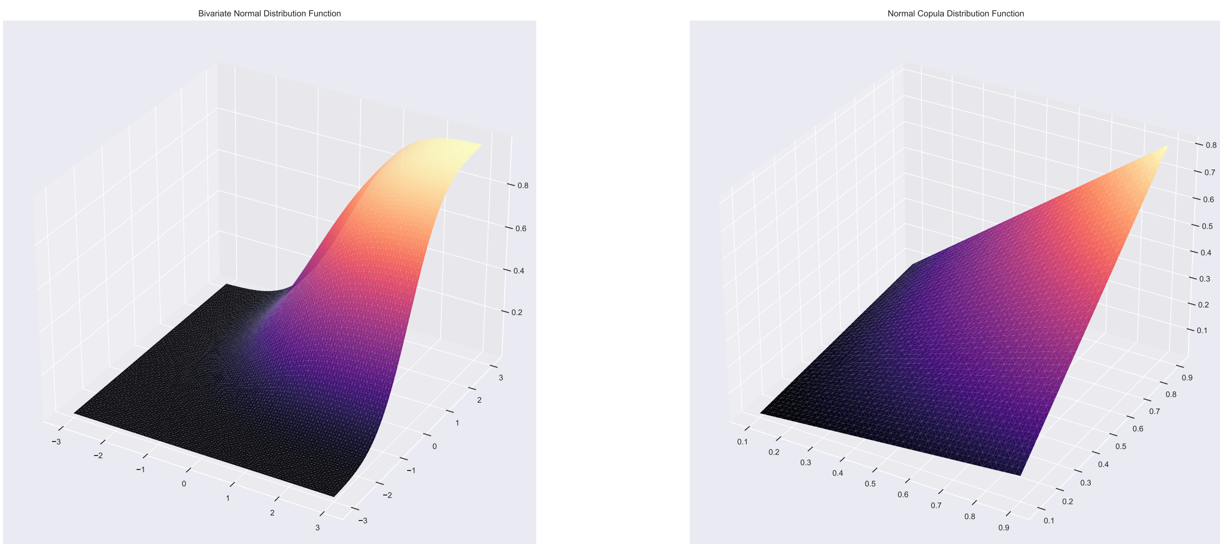


Figure 2: Bivariate Standard Normal Distribution and Gaussian Copula Distribution

Figure 2 visualizes the distinct differences between the bivariate normal CDF and the normal copula. That is, that the Copula's output is the probability that the marginal CDF's take on a particular pair of values. This property of the Copula will become a critical foundation for the forthcoming analysis of joint defaults.

The joint density can be easily derived by taking the mixed partial derivative of the joint distribution. This is written formally as:

$$f(\mathbf{X}) = \prod_{i=1}^{d} f_i(X_i) c(F_1(X_1), F_2(X_2), ..., F_d(X_d)) \tag{6}$$

where f_i is the marginal density of X_i and $c = \frac{\partial^d C(F_1(X_1), F_2(X_2), ..., F_d(X_d))}{\prod_{i=1}^{d} \partial X_i}$ is the copula density.

3.4 An Alternative Approach: Distorting the Distribution of The Default Driver

For this work, the distribution of the default driver is distorted and similar to the approach by Zeldenrijk[16], the characteristics of the distorted distribution also depend on ρ, g and the parameters of the distortion function. However, Zeldenrijk's distortion assumes that only the systemic factor is impacted. By distorting the default driver, it is assumed that both systemic and idiosyncratic factors are impacted. Refer to Appendix C for more comparative analysis and discussion on the two methods.

Using the same base assumptions from Vasicek's ASRF, i.e. Z, ϵ_i and Y_i are $\sim N(0, 1)$. In the same fashion that the systemic factor was distorted, the distorted default driver now has cumulative distribution function:

$$\Phi(y) \to \Phi^g(y) = g(\Phi(y)), \tag{7}$$

and distorted probability density function:

$$\phi^g(y) = \frac{d(\Phi^g(y))}{dy} = g'(\Phi(y))\phi(y), \tag{8}$$

where g is some differentiable distortion function. Prior, the distortion only impacted the systemic factor, however, by distorting the default driver, both systemic and idiosyncratic factors are simultaneously impacted. The mean of the distorted default driver now becomes:

$$\begin{aligned}\mathbb{E}^g[Y] &= \int y\phi^g(y)dy \\ &= \int yg'(\Phi(y))\phi(y)dy. \end{aligned} \tag{9}$$

Consequently, the distorted distribution has variance:

$$\text{Var}^g(Y) = \mathbb{E}^g[(Y - \mathbb{E}^g[Y])^2] = \int (y - \mathbb{E}^g[Y])^2 g'(\Phi(y))\phi(y)dy. \tag{10}$$

A more convenient form for the variance of the distorted distribution is:

$$\text{Var}^g(Y) = \mathbb{E}^g[Y^2] - \mathbb{E}^g[Y]^2. \tag{11}$$

The motivation for (9) and (10) is that it allows one to observe explicitly how the distortion impacts these moments. From the aforementioned equations, it can be observed exactly how the distortion re-weights the probabilities in its distribution; this is achieved by the factor $g'(\Phi(y))$ in (9) and 10. This property of distortion is important in regards to the goal of this paper and will be used in a later section to introduce climate risk.

The distortion also has interesting implications for the correlation between default drivers. Given two default drivers Y_i and Y_j, they have correlation:

$$\begin{aligned}Corr^g(Y_i, Y_j) &= \frac{Cov^g(Y_i, Y_j)}{\sqrt{\text{Var}^g(Y_i)\text{Var}^g(Y_j)}} \\ &= \frac{\mathbb{E}^g(Y_iY_j) - \mathbb{E}^g(Y_i)\mathbb{E}^g(Y_j)}{\sqrt{\text{Var}^g(Y_i)\text{Var}^g(Y_j)}}. \end{aligned} \tag{12}$$

To evaluate (12), $\mathbb{E}(Y_iY_j)$ is required. This expression has the following representation,

$$\mathbb{E}^g(Y_iY_j) = \int\int y_iy_jf^g(y_i, y_j)dy_idy_j, \tag{13}$$

where $f^g(y_i, y_j)$ is the distorted joint probability density function. Using Sklar's theorem, the joint probability density function is

$$
\begin{aligned}
f_g(y_i, y_j) &= \frac{\partial^2(F_g(y_i, y_j))}{\partial y_i \partial y_j} \\
&= \phi^g(y_i)\phi^g(y_j)c[g(\Phi(y_i)), g(\Phi(y_j))]
\end{aligned} \tag{14}
$$

where $c(g(\Phi(y_i)), g(\Phi(y_j))) = \frac{\partial^2(C(g(\Phi(y_i)), g(\Phi(y_j)))}{\partial y_i \partial y_j}$ is the copula density function. Using (13), (12) can now be computed and the correlation between the distorted default drivers is now:

$$
Corr^g(Y_i, Y_j) = \frac{\int\int y_i y_j f_g(y_i, y_j) dy_i dy_j - \mathbb{E}^g(Y_i)\mathbb{E}^g(Y_j)}{\sqrt{\text{Var}^g(Y_i)\text{Var}^g(Y_j)}}. \tag{15}
$$

Similarly, assuming a homogeneous unconditional default probability, the correlation between default indicators - which are Bernoulli random variables - is formally defined as:

$$
\begin{aligned}
Corr^g(\mathbb{I}_{D_i}, \mathbb{I}_{D_j}) &= \frac{P^g(D_i \cap D_j) - P^g(D_i)P^g(D_j)}{\sqrt{Var(\mathbb{I}_{D_i})Var(\mathbb{I}_{D_j})}} \\
&= \frac{P^g(\mathbb{I}_{D_i} \cap \mathbb{I}_{D_j}) - g(\Phi(b))^2}{g(\Phi(b))(1 - g(\Phi(b)))}.
\end{aligned} \tag{16}
$$

Again, using Sklar's Theorem, the joint unconditional default probability: $P^g(\mathbb{I}_{D_i} \cap \mathbb{I}_{D_j})$ can be written as,

$$
\begin{aligned}
P^g(D_i \cap D_j) &= P^g(Y_i \leq b, Y_j \leq b) \\
&= C(\Phi_g(b), \Phi_g(b)) \\
&= C(g(\Phi(b)), g(\Phi(b))).
\end{aligned} \tag{17}
$$

Now, using (17), (16) can be written as:

$$
Corr^g(\mathbb{I}_{D_i}, \mathbb{I}_{D_j}) = \frac{C(g(\Phi(b)), g(\Phi(b)) - g(\Phi(b))^2}{g(\Phi(b))(1 - g(\Phi(b)))}. \tag{18}
$$

Finally, (18) and the other distorted properties are,

$$\mathbb{E}^g[Y] = \int yg'(\Phi(y))\phi(y)dy, \tag{19}$$

$$\text{Var}^g(Y) = \int (y - \mathbb{E}^g[Y])^2 g'(\Phi(y))\phi(y)dy, \tag{20}$$

$$f_g(y_i, y_j) = \phi_i^g(y_i)\phi_j^g(y_j)\phi_2[g(\Phi(y_i)), g(\Phi(y_j))], \tag{21}$$

$$Corr^g(Y_i, Y_j) = \frac{\int\int y_i y_j f_g(y_i, y_j)dy_i dy_j - \mathbb{E}^g(Y_i)\mathbb{E}^g(Y_j)}{\sqrt{\text{Var}^g(Y_i)\text{Var}^g(Y_j)}}, \text{ and} \tag{22}$$

$$Corr^g(\mathbb{I}_{D_i}, \mathbb{I}_{D_j}) = \frac{\Phi_2[\Phi^{-1}(g(\Phi(b))), \Phi^{-1}(g(\Phi(b)))] - \Phi^g(b)^2}{\Phi^g(b)(1 - \Phi^g(b))}. \tag{23}$$

where ϕ_2 is the bivariate standard normal density function. Now, with this suite of equations, the impact of the distortion on the surfaces of joint measures can be visualized, particularly the joint density function and the correlation between default events. Though there are numerous distortion functions, for this analysis, the Proportional Hazards Distortion is used.

3.5 Proportional Hazards Distortion

Let g be the Proportional Hazards distortion function $g(u) = u^a$ with distortion parameter a. Applying this to the distribution of Y, the following distorted distribution is obtained:

$$\Phi^g(y) = \Phi(y)^a \tag{24}$$

where the distorted PDF is $\phi_g(y) = a\Phi(y)^{a-1}\phi(y)$. Thus, using (24), the suite of equations now become:

$$\mathbb{E}^g[Y] = a\int y\Phi(y)^{a-1}\phi(y)dy, \tag{25}$$

$$\text{Var}^g(Y) = a\int (y - \mathbb{E}^g[Y])^2\Phi(y)^{a-1}\phi(y)dy, \tag{26}$$

$$f_g(y_i, y_j) = a^2\phi(y_j)\phi(y_i)[\Phi(y_i)\Phi(y_j)]^{a-1}\phi_2[\Phi^{-1}(\Phi(b)^a), \Phi^{-1}(\Phi(b)^a)], \tag{27}$$

$$Corr^g(Y_i, Y_j) = \frac{\int\int y_i y_j f_g(y_i, y_j) - \mathbb{E}^g(Y_i)\mathbb{E}^g(Y_j)}{\sqrt{\text{Var}^g(Y_i)\text{Var}^g(Y_j)}}, \text{ and} \tag{28}$$

$$Corr^g(\mathbb{I}_{D_i}, \mathbb{I}_{D_j}) = \frac{\Phi_2[\Phi^{-1}(\Phi(b)^a), \Phi^{-1}(\Phi(b)^a)] - \Phi(b)^{2a}}{\Phi(b)^a(1 - \Phi(b)^a)}. \tag{29}$$

Based on the form of (24) , when $a = 1$, the proportional hazards distortion returns the original distribution. This case will be referred to as the base case.

3.5.1 Non-linear Behavior of Distorted 1^{st} and 2^{nd} Moments

From (25) and (26), the distortion parameter has a significant impact on the mean and variance of the distorted distribution. In the following figure, these changes are visualized, revealing the crucial role the distortion parameter plays in inducing nonlinear behavior of the first moment (mean) and second moment (variance), as $a \to 0^+$. From an examination of Figure 3, it is observed that the distorted mean decreases quickly while the distorted variance grows as a shrinks.

Also, from Figure 3, the steepness of each plot has interesting behavior when $a \leq 1$; particularly interesting is the magnitude of steepness in both graph's explode. This property will be expanded upon further in later sections, but it is sufficient to note that they will have a significant impact on portfolio risk measures. As the mean plunges toward $-\infty$,

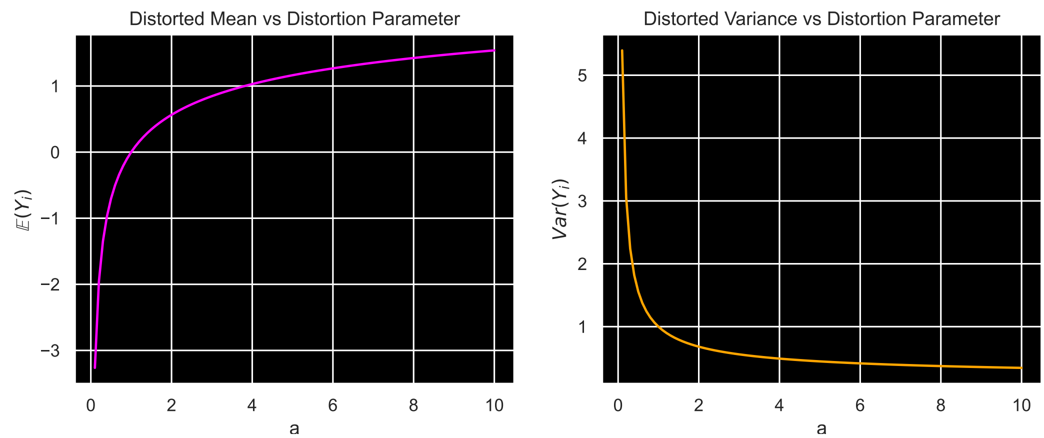


Figure 3: Distorted Mean and Variance Plots for Different a

the weight in the left tail of the distribution increases significantly and the consequence is rapidly increasing probability of default. Similarly, as the variance explodes toward ∞, this implies a greater likelihood of values far away from the mean - extreme values - occurring. This fact coupled with an already fattened tail further increases the probability of default. This is property is not surprising as Drmac[5] briefly explored this property.

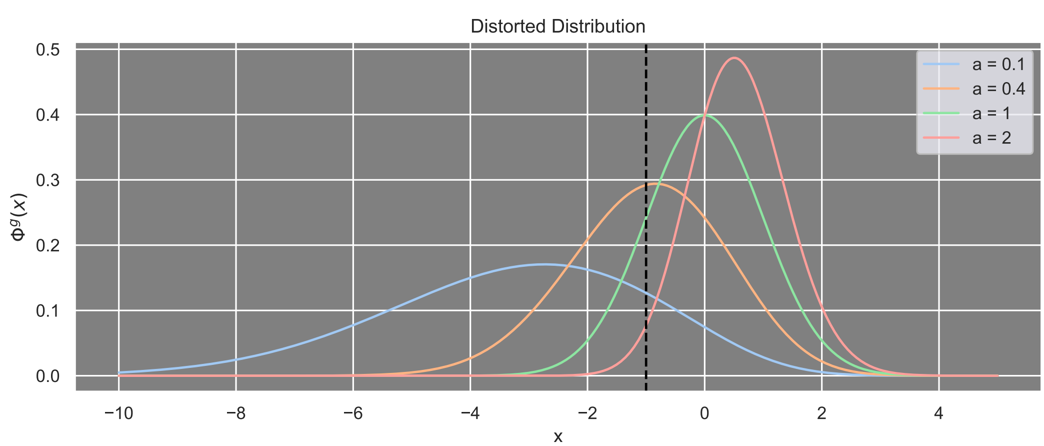


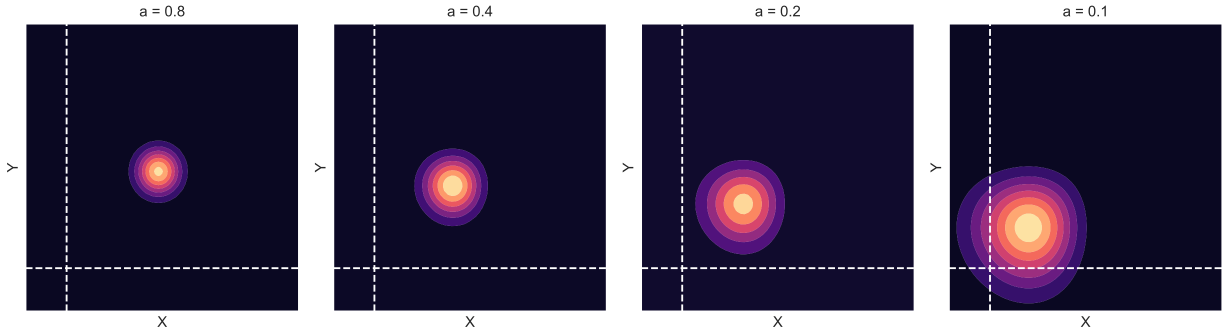
Figure 4: Distorted Density Function for Different a

Thus, with respect to credit risk models, this rapid change in steepness for the first two moments of the distorted distribution attributes to an increase in default probability. To further substantiate this claim, Figure 4 is examined. The vertical dashed line represents the default threshold. Observe, that for different values a, the area under each graph to the left of the default threshold changes; this area represents the probability of default. Thus, the figure depicts that as a decreases, the default probability increases. It is noteworthy to mention that this figure only represents the typical 'default/no default' or single threshold setting. In later sections, it is shown how distortion extends to the more general multi-threshold model for credit states.

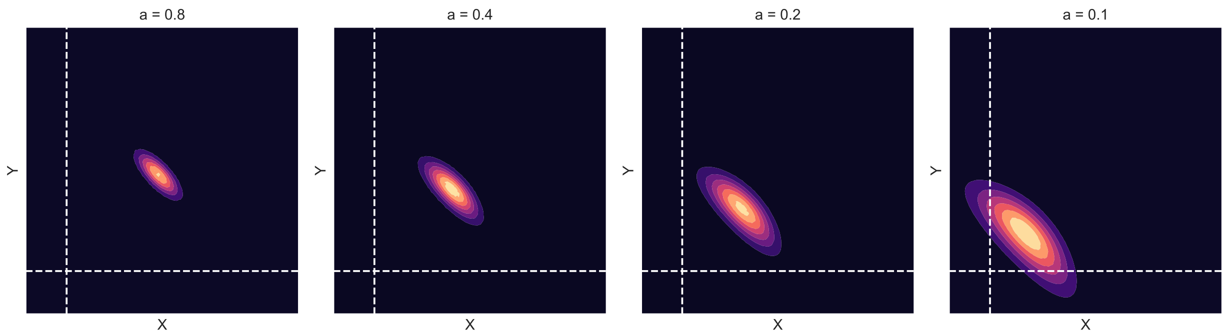
3.5.2 Distorted Joint Measures Consequences and Defaults

From the previous section, it has been established that decreasing values of the distortion parameter a, particularly when $a \leq 1$, causes an inflation of the left tail, consequently increasing the risk of default as depicted in Figure 4. These same conditions are imposed and the impact on the the distorted bivariate density function (27) is visualized via the following contour plots.

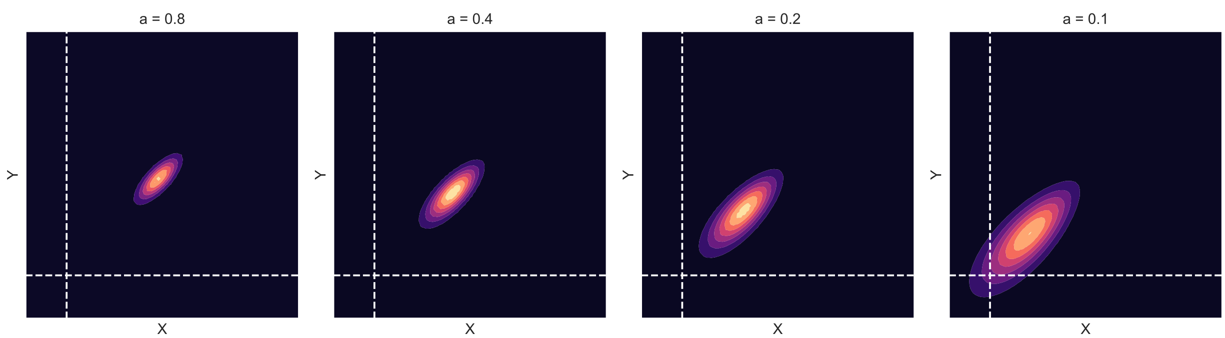
A homogeneous default threshold amongst borrowers is assumed and is represented by the white dashed lines in Figure 5. The area bounded by the vertical dashed line is the default region for the i^{th} borrower and the area bounded by the horizontal dashed line is the default region for the j^{th} borrower. Thus, the area bounded by these thresholds in the bottom-left corner of the plot is the joint-default region. It is observed in 5(b) the contours are elliptical and elongated along a negative diagonal. Due to this shape, decreasing values of a, density in the joint default region increases and consequently, default probabilities are increased. In 5(c), by a similar argument, the contours are elliptical and elongated along a negative diagonal and mainly, density in the individual default regions increase. However, in 5(a) the distorted joint density contours are significantly larger with noticeable increases in individual and joint default regions. As a result of this characteristic, this work observes the case when $\rho_{Corr} = 0$ for decreasing a.



(a) Distorted Joint Density with $\rho_{Corr} = 0$



(b) Distorted Joint Density with $\rho_{Corr} = -0.9$



(c) Distorted Joint Density with $\rho_{Corr} = 0.9$

Figure 5

3.6 Credit Transition Matrices and Distortion

Credit transition matrices are valuable tools in credit risk analysis; they provide insights into the movement of borrowers across different credit states over time. These matrices capture the probabilities of transition between various credit states, which are generally defined by credit rating categories - the default category is always the lowest category. They play a crucial role in assessing credit portfolio quality, estimating default probabilities, and modelling credit migration patterns. By analyzing historical credit data or using expert judgment, these matrices are constructed to quantify the dynamics of credit migrations within a portfolio.

To calculate a credit transition matrix, historical data on borrowers' credit ratings or credit events is utilized and this data includes information on the initial credit states of borrowers and their subsequent credit states over a specific period. The credit states often represent different rating categories, such as investment grade, speculative grade, or default. The calculation process involves examining the transitions between these states and determining the frequencies or counts of these transitions. These counts are then normalized to obtain transition probabilities. By dividing the count of transitions from a specific credit state to each possible destination state by the total count of transitions from that particular state, the probabilities of transition are derived. TDistorting Zology enables credit risk professionals to monitor portfolio performance, identify credit concentration risks, and evaluate the effectiveness of credit risk models. However, the methodology is also complex and beyond the scope of this work. Below, the following table contains fictitious transition probabilities derived from transition ratings in Gupton[8].

from/to:	**A**	**B**	**C**	**Default**
A	90%	6%	3%	1%
B	3%	85%	8%	4%
C	1%	11%	73%	15%
Default	0	0	0	100%

Table 1: Credit Transition Matrix with 4 ratings categories

In Table 1 a simple transition matrix with ratings A, B, C and D with single-period transition probabilities is shown. These categories begin with A being the highest credit rating (lowest default probability) and D (Default) being the lowest rating. Once a borrower has defaulted there is no recovery. The first column indicates the rating at the start of the period, $t = 0$, while the other columns indicate the probability of transitioning to that rating at the end of the period, $t = 1$.

Distortion can also be used to alter these transition probabilities. Since the credit transition probabilities across each row represent a conditional distribution function, a distortion function can be applied to each row's cumulative distribution function in the same fashion as in prior sections. For example, consider the second row of Table 1. The probabilities in each column of row 2 represent the credit transition likelihood at $t = 1$ given that at $t = 0$, the credit rating was A. Correspondingly, column 2 of row 2 indicates there is a 90% probability a borrower will remain at credit rating A at $t = 1$ if they had a credit rating of A at $t = 0$. Transition probabilities are estimated using intensive calibration and modelling process and thus, a deeper explanation behind the factors that drive them are beyond the scope of this work. Nevertheless, to define each row's CDF, let the set of credit ratings $\{D, C, B, A\}$ be represented by the set of values $\{1, 2, 3, 4\}$ and let a random variable X_t denote the time t credit rating with $X_t \in \{1, 2, 3, 4\}$. In general, using the latter, the CDF for a row can be written as $P(X_t \leq x_t | X_{t-1} = x_{t-1})$, where X_t represents the credit state at some period t. Similarly, the probability function of a row can be expressed as $P(X_t = x_t | X_{t-1} = x_{t-1})$. For example, the CDF for row 2 is represented as follows:

$$P(X_1 \leq x_1 | X_0 = 4) = \begin{cases} 0 & \text{if } x_1 < 1 \\ 0.01 & \text{if } 1 \leq x_1 < 2 \\ 0.04 & \text{if } 2 \leq x_1 < 3 \\ 0.1 & \text{if } 3 \leq x_1 < 4 \\ 1 & \text{if } x_1 \geq 4 \end{cases} \tag{30}$$

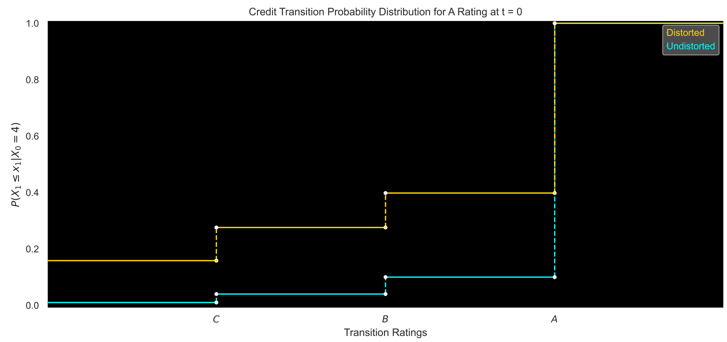
With (30), the same proportional hazards distortion procedure used in the previous sections is followed and setting the distortion parameter to be $a = 0.4$:

$$P^g(X_1 \leq x_1 | X_0 = 4) = \begin{cases} 0 & \text{if } x_1 < 1 \\ 0.01^a & \text{if } 1 \leq x_1 < 2 \\ 0.04^a & \text{if } 2 \leq x_1 < 3 \\ 0.1^a & \text{if } 3 \leq x_1 < 4 \\ 1^a & \text{if } x_1 \geq 4 \end{cases} = \begin{cases} 0 & \text{if } x_1 < 1 \\ 0.16 & \text{if } 1 \leq x_1 < 2 \\ 0.28 & \text{if } 2 \leq x_1 < 3 \\ 0.39 & \text{if } 3 \leq x_1 < 4 \\ 1 & \text{if if } x_1 \geq 4 \end{cases} \tag{31}$$

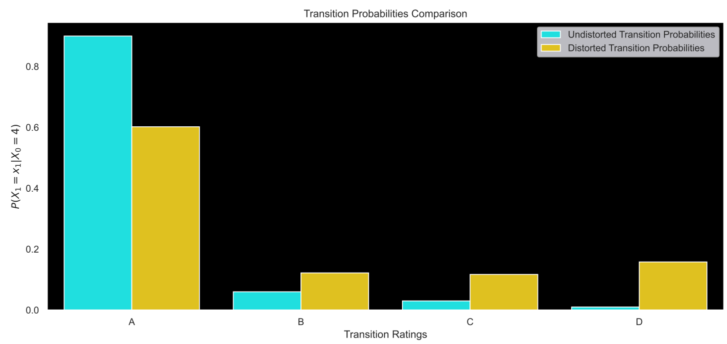
The impact distortion has on (31) is shown in Figure 6 and this visual illustrates how distortion also increases downgrade probabilities in the credit transition matrix. By distorting the first row of the transition matrix and re-weighting the credit migration probabilities, the likelihood of a borrower in state A in $t = 0$, remaining in state A in $t = 1$ is lowered. However, the probability siphoned in the latter is simultaneously redistributed to the probability of the same borrower being downgraded. Thus, as shown in Section 3.6, under distortion, as a decreases, downgrade and default likelihood increases. This redistribution of transition probabilities for row 2 is also shown in Table 2 and visualized in Figure 6(b).

	A	**B**	**C**	**D**
$P(X_1 = x_1 \vert X_0 = 4)$	90%	6%	3%	1%
$P^g(X_1 = x_1 \vert X_0 = 4)$	61%	11%	12%	16%

Table 2: Comparing Undistorted/Distorted Transition Probabilities for Row 2 of Table 1



(a) Distorted($a = 0.4$) and Undistorted Transition Probability CDF



(b) Distorted($a = 0.4$) and Undistorted Transition Probability PDF

Figure 6

It is noteworthy to mention that the 'default/no default' scenario is a special case of credit migration. It considers only two states: no-default and default. Thus, at $t = 0$, the only sensible option is for the borrower to be solvent. An interesting impact is how this reallocation of weight evolves for progressively smaller values of a.

Overall, the distortion in the credit transition matrix reflects changes in the credit environment due to climate. These changes have important implications for credit risk management, as they highlight shifts in the distribution of credit transition states and potential impacts on the creditworthiness of entities.

In the preceding section, the previously mentioned characteristic of the proportional hazards distortion (increasing default probability as a decreases) is employed to demonstrate how climate-impacted risk and their effect on credit transition matrices and portfolio risk measures.

3.7 Impact of Distortion on Portfolio Risk Measures and Credit Transition Matrices

During the forthcoming scenario analysis, it is assumed that the group of borrowers in the simulations are based in The Bahamas and because of this, it is assumed a has some relationship to the climate variable change in sea surface temperature (ΔSST) and ΔSST is the only climate hazard that affects credit ratings. The rationale for using ΔSST (obtained from SST) is derived from Finlayson[6].

Sea surface temperature (SST) increases can influence increases in convective activity in the ocean, adding to a hurricane's fuel source and consequently, increasing their intensity. The latter coupled with the fact that The Bahamas is most likely to be impacted by hurricanes than other countries in the region is why SST was chosen for this scenario analysis. Thus, as SST increases, a should decrease and as a result, induce deteriorating economic conditions and ultimately, increase the likelihood of downgrades due to climate-impacted events. It is important to note that this is a strong assumption and is made for simplicity as Finlayson also explains that SST is only one of many factors that may contribute to a climate-impacted event. Other factors such as vertical wind-shear, absolute humidity and the equatorial phase of the Pacific are other significant drivers as well and they all interact to influence cyclone activity in the Atlantic.

In 2019, Hurricane Dorian revealed the broad-spectrum impact of intense storms on the Bahamian economy. The Inter-American Development Bank [2] estimated a \$3.4 billion impact on The Bahamian economy due to Hurricane Dorian - 72% in direct physical damages, 21% in business losses and 7% in additional costs with the private sector absorbing almost 90 % of total losses. Of these damages, only 38% were insured and this primarily reflects private home insurance. Similarly, third quarter real GDP reflect a 10.9% contraction from the previous quarter directly attributed to Hurricane Dorian. These harsh economic conditions may impact the risk profiles of borrowers exposed to the economy. This logic lays the foundation for the following simulations.

3.7.1 Calibrating The Distortion Parameter with Climate Variables

To exploit the property of the Proportional hazards distortion — increasing credit downgrade likelihood and defaults for decreasing a — the distortion parameter is calibrated to the climate variable. This calibration allows the risk due to climate-impacted events to affect credit ratings through the distortion parameter. Though ΔSST is the only climate variable used in this model, it is important to acknowledge that climate impacts are the outcome of numerous interacting factors. Nevertheless, to simplify the analysis, a linear relationship between the distortion parameter and ΔSST changes is assumed. Also, the following boundary conditions are imposed:

For some function $f(\Delta SST)$,

$$f(0) = 1 \text{ and } f(2) = 0. \tag{32}$$

These conditions follow from the fact that when the there is no change in sea surface temperature ($\Delta SST = 0$), the model should output the base case i.e $a = 1$. The second conditions is derived from the Intergovernmental Panel on Climate Change (IPCC)[11]. The report highlighted the importance of limiting global warming to below 1.5°C or 2°C above pre-industrial levels as crossing these thresholds could lead to more severe and irreversible climate shifts and climate-impacted events. As a result, 2°C is choosen as the threshold for ΔSST. Using conditions (32) and the assumption of linearity between the distortion parameter a and ΔSST, the following relationship is obtained,

$$a(\Delta SST) = -0.5\Delta SST + 1 \tag{33}$$

where $\Delta SST \in [0, 2)$ and $a(\Delta SST) \in (0, 1]$.

3.8 Climate Scenario Simulations

3.8.1 Credit Transition Matrices and Portfolio Value

For Figure 7, the transition matrix shown in Table 1 is used as the baseline scenario. In each of the succeeding tables, (33) provides the relevant distortion parameter according to the ΔSST. The climate-calibrated distortion parameter in each scenario simulates worsening climate conditions, and all borrowers represented by the transition matrix are exposed to this risk.

	A	B	C	D
A	90%	6%	3%	1%
B	3%	85%	8%	4%
C	1%	11%	73%	15%
D	0	0	0	1

(a) Baseline:
$\Delta SST = 0^{\circ} C \rightarrow a = 1$

	A	B	C	D
A	82%	9%	6%	3%
B	2%	77%	11%	10%
C	1%	8%	67%	24%
D	0	0	0	1

(b) Mild:
$\Delta SST = 0.5^{\circ} C \rightarrow a = 0.75$

	A	B	C	D
A	68%	12%	10%	10%
B	2%	64%	15%	19%
C	1%	5%	55%	39%
D	0	0	0	1

(c) Severe:
$\Delta SST = 1^{\circ} C \rightarrow a = 0.5$

	A	B	C	D
A	44%	11%	13%	32%
B	1%	14%	40%	45%
C	0%	3%	35%	62%
D	0	0	0	1

(d) Catastrophe:
$\Delta SST = 1.5^{\circ} C \rightarrow a = 0.25$

Figure 7: Credit Transition Matrix under Various Climate Scenarios

Since the transition probabilities are impacted by the distortion, it follows the value of the loan portfolio is impacted as well. To illustrate this, it is assumed that all borrowers in each distinct credit state have equally valued loans. Then, the credit ratings have the following values in a generic unit of currency ($): A rating - $1, B rating - $0.75, C rating - $0.25 and Default ($D$ Rating) are worthless. The portfolio is initialized assuming 1000 borrowers inside the following credit ratings: A rating - 900, B rating - 75 and C rating - 25 (defaulted loans can not be included in set of borrowers at the start of the period). Using this portfolio composition, Table 3 depicts the impact distortion has on the value of a portfolio in a single period step while Figure 8 displays the impact over 12 periods. Figure 8 assumes the baseline transition matrix for each scenario is constant over all periods and this is why the portfolio value consistently declines over the period. The assumption is not practical, but allows for simplicity in visualizing the climate impact on portfolio value over multiple periods.

	$t = 0$	$t = 1$			
	Initial	**Baseline**	**Mild**	**Severe**	**Catastrophe**
A Rating	\$900.00	\$812.00	\$742.00	\$616.00	\$395.00
B Rating	\$56.25	\$90.75	\$105.50	\$115.50	\$101.25
C Rating	\$5.00	\$12.75	\$19.50	\$28.75	\$34.50
Total Portfolio	**\$961.25**	**\$915.50**	**\$866.50**	**\$760.25**	**\$530.75**

Table 3: Expected Portfolio Value for Each Climate Scenario

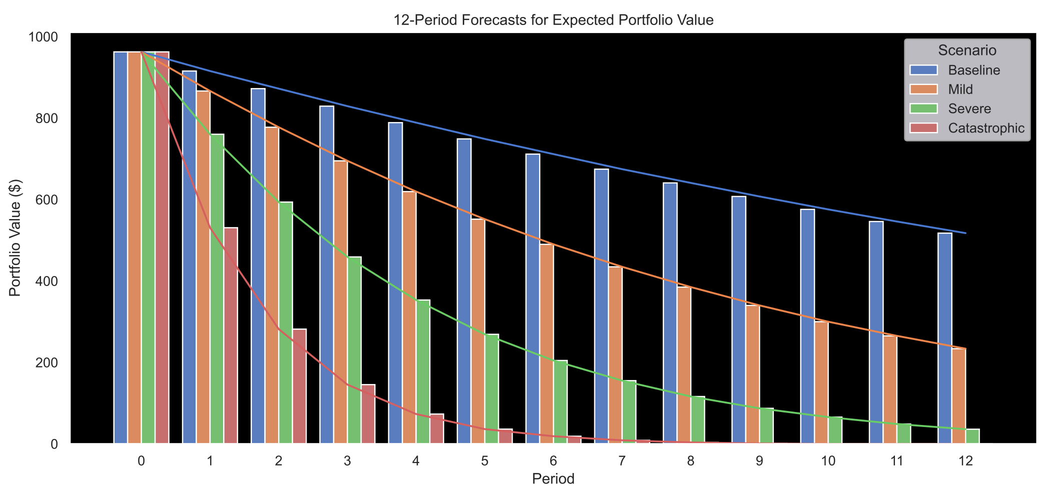


Figure 8

3.8.2 Portfolio Risk Measures

In this section, the loss distribution for A Rated borrowers is illustrated for each climate scenario in Figure (9). Similarly, using borrowers from each Rating group, the loss (due to default) distribution and it's corresponding portfolio risk measures are simulated with Monte Carlo simulations (10,000 trials). The methodology and code behind generating these measures are shown in Appendix A. We assume the following for simplicity:

- All portfolios comprise of 1,000 borrowers.
- Exposure-at Default (EAD) = Loss-Given-Default (LGD) = 1
- From the baseline transition matrix, each borrowers chance of transitioning to default is derived from the Column D in (Figure 7(a)) and is homogeneous amongst all borrowers in each group. The tolerance for tail risks is 99.9%.
- ΔSST and the corresponding distortion parameters for each scenario are the same as those used in the previous section.

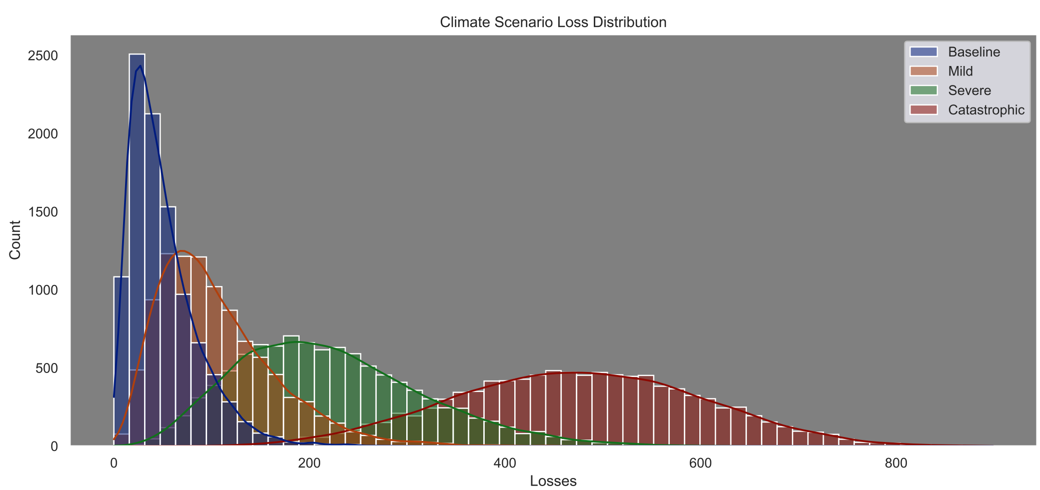


Figure 9: Loss Distribution for A Rated Borrowers

	Baseline	Mild	Severe	Catastrophe
VaR$_{99.9\%}$	\$83.00	\$184.00	\$376.00	\$702.00
CVaR$_{99.9\%}$	\$93.27	\$203.00	\$401.50	\$723.99
Expected Loss	\$10.17	\$31.89	\$98.96	\$315.46
Economic Capital	\$74.84	\$143.97	\$269.51	\$384.39

(a) *A* Rated Borrowers

	Baseline	Mild	Severe	Catastrophe
VaR$_{99.9\%}$	\$209.00	\$379.00	\$562.00	\$810.00
CVaR$_{99.9\%}$	\$233.60	\$418.40	\$584.10	\$829.91
Expected Loss	\$40.11	\$89.30	\$199.21	\$447.35
Economic Capital	\$168.89	\$274.22	\$362.79	\$362.65

(b) *B* Rated Borrowers

	Baseline	Mild	Severe	Catastrophe
VaR$_{99.9\%}$	\$487.00	\$625.00	\$775.00	\$908.00
CVaR$_{99.9\%}$	\$535.70	\$654.50	\$799.75	\$924.60
Expected Loss	\$150.62	\$241.16	\$386.95	\$622.68
Economic Capital	\$336.37	\$383.83	\$388.04	\$285.31

(c) *C* Rated Borrowers

Figure 10: Portfolio Level Risk Measures for each Borrower Group

4 Discussion

Although in the simulations, results for all scenarios were included, for the breadth of this discussion, the catastrophe scenario will the prime focus. This is because, from a industry perspective, though all likelihoods are important, contingencies mainly hedged against risk born the from worst possible scenario.

Figure 7 illustrates the impact that climate impacted risk can have on a credit transition matrix via distortion functions. It is observed that as ΔSST increases, there is proportional decrease in the distortion parameter, a. The latter is a direct consequence of the manner in which the climate variable is calibrated to a. As a result of the deteriorating economic conditions, simulated by the unique calibration of ΔSST to a, the likelihood of borrowers remaining in a higher credit state decreases. Subsequently, because the total probability must sum to one, the deducted probability is then, redistributed to the probability of a borrower/asset transitioning to a lower credit state. Though, the probability redistribution is non-linear and becomes more extreme as a becomes increasingly small. This is most observable in the catastrophe scenario, Figure 7(d). As the parameter a becomes very small, the distortion from proportional hazards appears to allocate progressively smaller probabilities to transitioning to a lower intermediate credit state, but increasingly more to the likelihood of defaulting. This is inferred from the observation that probability of defaulting accounts for 32-62% of the total probability regardless of the initial credit rating in the catastrophe scenario. As mentioned prior, the proportional hazards distortion increased default probability for decreasing a, in the single-threshold scenario. These results now show that the distortion also impacts preceding credit thresholds, though in an exponential way.

Moreover, Table 3, also illustrates climate-adjusted risk impacts on the unrealized value of a portfolio of borrowers. As with the transition matrices, the value of the portfolio depreciates quickly as the climate scenario declines. Most notably, in the catastrophe scenario, the total portfolio expected value sinks by 42%, relative to baseline. This shows that, under this framework, financial institutions are likely to suffer large portfolio devaluations as a result of the bleak economic environment induced by climate-impacted events. The same rapid devaluation is observed in Figure 8. The catastrophe scenario is the only case where the loan portfolio becomes virtually worthless and does so after only 7 months.

Figure 9 depicts how climate-impacted risks impacts the loss distribution for A rated borrowers. As mentioned in previous sections, as a decreases, representing deteriorating economic conditions, the default probability increases. This can be observed visually Figure 9 as the mean increases as the climate scenario worsens. This represents the expected loss of the portfolio. The right tail of the distribution also fattens as the climate scenario worsens and increases the portfolio VAR estimate. The consequence of these two statistics increasing is a corresponding increase in economic capital requirements, since by definition, it is the difference between the VAR and expected loss. The expansion of these portfolio risk measures and others are quantified in for each borrower group in Figure 10. This figure highlights analogous impacts of the climate-calibrated distortion on portfolio risk measures in the single threshold case. For borrowers exposed the worsening economic conditions in each scenario, their risk profile also deteriorates in a proportionate way. During the catastrophe scenario, the VaR and CVaR risk grew significantly, by an average factor of 1.9, from their baseline levels. Likewise, the Expected Loss experience the most significant increase by a factor of approximately 4.1, from the baseline scenario while Economic Capital exhibited interesting behavior for C Rated borrowers. For each borrower group, except C Rated borrowers, economic capital expanded modestly by an average factor of 3.6 from baseline. However, economic capital for C Rated borrowers appears to increase marginally, then decrease in the catastrophic case. These results show that, just as with transition matrices, the climate-calibrated distortion induces the expansion of portfolio level risk measures through worsening climate conditions. This significantly affects financial institutions as they are mandated to allocate sufficient capital to support their financial activities and the consequent widening in Economic Capital allocations increases the expense and risk associated with lending activities for exposed financial institutions.

4.1 Model Caveats and Improvements to Robustness

A strength of this model is its simplicity; yet this may also be a critical caveat. Climate change is a very complex phenomena, as the weather is a highly complex dynamical system whose variables have extremely non-linear and chaotic interactions. Coupled with the equally complex and dynamic socioeconomic and financial systems, a more sophisticated calibration process is required. This process would involve the modelling of how climate variables, relevant to a group of borrowers, interact with each other to produce a climate-impacted event, then calibrating this output with the distortion parameter. Surely, this alone is worthy of an entirely separate and challenging undertaking. Though, the consequences of the latter may be a more robust climate-calibrated model, the subsequent obstacle of model validation presents a challenge. Since climate change impacts are an ongoing and relatively recent development, obtaining historical and transparent data to thoroughly validate model assumptions is a prominent issue. A consequence of the data availability issue is that it undermines the viability of a data-driven approach to effectively calibrate the define a relationship with the desired climate variables and the distortion parameter.

A possible way to circumvent this restriction would be to utilize expert judgement to make informed guesses about what an appropriate climate parameter would be. This is expert judgement also extends to defining the relationship between the climate variable and default probabilities as a linear relationship is not the only choice - nor the most robust either. For example, let the following equation be the result of several expert judgements on the relation:

$$a(\Delta SST) = \theta e^{-\gamma(\Delta SST - \mu)}, \tag{34}$$

where γ is the sector correlation with the climate variable, μ is chosen such that when $\Delta SST = 0$, there is no change in the macroeconomic status-quo i.e. $a(0) = 1$ and θ is calibrated as a result of expert opinion. Consider if an expert on the climate-impacted risk was consulted and provided an opinion on the impact of climate risk on a sector. They expect that when $\Delta SST = 0.5^{\circ}C$ the probability of default in group A will increase by a factor of 1.3 and when $\Delta SST = 1^{\circ}C$, it increases by a factor of 2. Then, specific values of γ and θ can be found. Thus, consulting a large number of experts would generate a distribution of parameters from which bootstrapped estimates can be computed. These bootstrap estimates would allow for the appropriate choice of distortion parameter given change in the climate variable and the model to capture climate-impacted risk according to a summary of expert opinion according to reasonable assumptions. A similar process may also be applied to choosing an appropriate distortion function, but a discussion is beyond the scope of this work.

Nevertheless, the climate-calibrated proportional hazards parameter in Vasicek's ASF model provides a rudimentary and explainable framework that offers a basic structure to account for climate-impacted risk in credit risk models using distortion. Naturally, immediate extensions of this would be assuming non-homogeneous default probabilities amongst borrowers in the same group as it is a more expansive assumption. Though, a particularly interesting addition of the work presented in this paper would be to use different copulas.

It is well known that the Gaussian copula has negligible tail dependency. However, a well known solution to this is to simply use a Laplace or, most known, the Student-t copula. In this regard, a study of how distortion impacts joint defaults with different copulas may be a worthwhile and fruitful venture.

5 Conclusion

This work endeavored to provide an alternative way to account for climate risk in credit risk models without developing new frameworks, but by augmenting an existing standardized frameworks, Vasicek's ASRF model. This was done by using distortion functions to re-weight the likelihood of events in a way that was biased towards towards changes in climate. The result of this endeavor was calibrating the distortion parameter in the Proportional Hazards distortion, a to the climate variable ΔSST, such that worsening climate environments and their consequent effect on the economic environment, induced higher risks of defaults in transition matrices and significant economic capital allocations for portfolios. The strength of the model is its simplicity, as it may provide a framework for practitioners to capture the risk associated with climate change across different industries and regions. Each exposed sector may possess a unique climate-calibrated distortion parameter. In the simulations of this work, borrowers were assumed to have significant exposure to The Bahamian economy, thus the unique choice of a being calibrated to ΔSST. However, consider if a creditor had exposure to borrowers in California; then it would be conducive to choose climate variables that would impact the likelihood of wildfires. Through the strong assumption of linearity, the resultant calibration would not only be simple, but explainable. Each unit change of the climate variable would have a proportionate change in the distortion parameter. Even though the model's output intensifies significantly for subsequently smaller values of a, there still exists a quantifiable relationship. Moreover, a need not only represent risk due to physical events, but transition risk as well. An example of this would be using Carbon Price as a calibration variable; as Carbon Prices increases, some business exposed to the fossil fuel sector may find it progressively challenging to remain profitable. This may also alter their risk profiles and increase their default likelihood due to higher climate-related taxes.

With climate change becoming a realized risk, financial regulators are actively developing mandates that will soon require financial institutions to account for this risk. Thus, it is an imperative those institutions develop the technical capacity to meet the requirements of those mandates. Yet, the model development pipeline is long and costly. Notwithstanding the shortfalls of the model delineated in this paper, at minimum it provides proof of concept or a simple challenger model to compare with a more sophisticated production line risk model. Overall, this work offers an alternative way to capture and account for the risks associated with climate-impacted events that may prove to have a shorter pipeline and a smaller purse.

References

[1] Technical report.

[2] Inter-American Development Bank and UNECLAC. Assessment of the Effects and Impacts of Hurricane in The Bahamas. Technical report, Inter-American Development Bank, 2020.

[3] F. Benth, G. Di Nunno, and D. Schroers. Copula Measures and Sklar's Theorem in Arbitrary Dimensions. *Scandanavian Journal of Statistics*, 49(3):917–1419, 2020.

[4] D. Bolder. *Credit-Risk Modelling: Theoretical Foundations, Diagnostic Tools, Practical Examples, and Numerical Recipes in Python*. Springer, 2018.

[5] M. Drmac. Incorporating Climate Transition Risk into Credit Risk Assessments via Distortion. Graduate Research Project, Department of Mathematics, Wilfrid Laurier University, 2023.

[6] M. Finlayson. 2023 Predictions for the Atlantic Hurricane Season: Concerns for The Bahamas, 2023.

[7] J. Garnier, J. Gaudemet, and A. Gruz. The Climate Extended Risk Model (CERM). *arXiv preprint arXiv:2103.03275*, 2022.

[8] G. Gupton, C. Finger, RiskMetrics Group, and M. Bathia. Introduction to Credit Metrics. Technical report, J.P Morgan, 2007.

[9] D. Li. On Default Correlation: A Copula Function Approach. *RiskMetrics Group*, 2000.

[10] Y. Malevergne and D. Sornette. Testing the Gaussian Copula Hypothesis for Financial Assets Dependences. *Quantitative Finance*, 3(4):231–250, 2003.

[11] V. Masson-Delmotte, P. Zhai, H. O. Pörtner, D. Roberts, J. Skea, P.R. Shukla, A. Pirani, W. Moufouma-Okia, C. Péan, R. Pidcock, S. Connors, J. B. R. Matthews, Y. Chen, X. Zhou, M. I. Gomis, E. Lonnoy, T. Maycock, M. Tignor, and T. Waterfield (eds.).

Global Warming of 1.5°C. An IPCC Special Report on the impacts of Global Warming of 1.5°C above pre-industrial levels. Technical report, Intergovernmental Panel of Climate Change, 2022.

[12] Bank of Canada and Officie of Superintendent of Financial Insititutions. Using Scenario Analysis to Assess Climate Transition Risk. Technical report, Bank of Canada, 2022.

[13] M. Reesor and McLeish D.L. Risk, Entropy and The Transformation of Distributions. *North American Actuarial Journal*, 7(3):128–144, 2003.

[14] O. Vasicek. Probability of Loss on Loan Portfolio. *John Wiley Sons, Inc. eBooks*, Nov 2015.

[15] S. Watts. The Gaussian Copula and the Financial Crisis: A Recipe for Disaster or Cooking the Books? *University of Oxford*, 2016.

[16] A. Zeldenrijk. Credit Risk Distortion Models, Properties and their Application. Undergraduate Research Project, Department of Mathematics, Wilfrid Laurier University, 2023.

A Simulating Proportional Hazards Distorted Default Drivers

1. Simulate one Z and N ϵ_i's, where Z is the systemic factor representing macro-economic conditions, ϵ_i is the idiosyncratic factor representing the financial standing of the i^{th} borrower and N is the amount of borrowers.

2. Select a value of ρ then calculate y_i for each borrower. This is the process to simulate undistorted default drivers.

3. Calculate $\Phi(y_i)$, where Φ is the standard normal CDF. By the Probability Integral Transform, $\Phi(y_i) = u$.

4. To simulate the distorted default driver, y_i^g, using the method of inversion, $y_i^g = \Phi^{-1}(u^{1/a})$.

5. Using the default threshold from the undistorted Vasicek model, sum the losses.

6. Repeat the following process 10,000 times. This generates a loss distribution and estimate the risk measures.

This process is depicted in Python Code C.2 in the *loss_drivers* function.

B Portfolio Risk Measure Confidence Intervals

These figures are estimated by performing 50 repeated valuations of the *risk_measures* function shown in Python Code C.2, obtaining 50 sets of $\text{VAR}_{99.9\%}$, $\text{CVAR}_{99.9\%}$, Expected Loss, and Economic Capital. From these results, the mean and sample standard deviation for each risk measure is computed and used to calculate the corresponding 95% confidence interval. This process is depicted in Python Code C.2 in the *risk_measures_CI* function.

Climate Scenario	VAR		CVAR		EL		EC	
	Mean ($)	SE ($)	Mean ($)	SE ($)	Mean ($)	SE ($)	Mean ($)	SE ($)
Baseline	81.16	1.21	95.29	1.64	10.02	0.03	71.14	1.20
Mild	181.44	2.65	204.26	3.46	31.66	0.07	149.78	2.64
Severe	376.96	2.81	407.82	4.15	100.16	0.18	276.80	2.76
Catastrophe	703.20	2.86	730.94	3.77	316.34	0.32	386.86	2.85

(a) A Rated Borrowers

Climate Scenario	VAR		CVAR		EL		EC	
	Mean ($)	SE ($)	Mean ($)	SE ($)	Mean ($)	SE ($)	Mean ($)	SE ($)
Baseline	212.38	2.24	235.87	3.00	40.06	0.09	172.32	2.22
Mild	353.46	2.94	386.39	3.81	89.43	0.15	264.03	2.93
Severe	562.56	3.19	590.65	3.44	199.95	0.29	362.61	3.16
Catastrophe	815.52	2.42	835.73	2.87	447.10	0.34	368.42	2.36

(b) B Rated Borrowers

Climate Scenario	VAR		CVAR		EL		EC	
	Mean ($)	SE ($)	Mean ($)	SE ($)	Mean ($)	SE ($)	Mean ($)	SE ($)
Baseline	479.80	4.17	510.65	4.98	150.26	0.20	329.54	4.13
Mild	618.46	3.44	647.12	3.88	240.99	0.28	377.47	3.39
Severe	772.44	2.55	794.46	2.78	387.34	0.37	385.10	2.57
Catastrophe	915.40	1.33	926.58	1.54	622.49	0.32	292.91	1.34

(c) C Rated Borrowers

Figure 11: Portfolio Risk Measure Estimate Standard Errors

C Distorting the Default Driver and Systemic Factor: Comparison Results

This work makes an argument for the distorting the default driver Y_i, as opposed to distorting the systemic factor Z, as in Zeldenrijk's work. However, Zeldenrijk takes several additional steps in his work in regards to fixing the default probability and calibrating the distorted correlation. This work forgoes these additional steps for simplicity and distorts the systemic factor using the same process from section 3.5. Nonetheless, the rationale for this alternative approach is that it assumes that climate-impacted events affect both systemic and idiosyncratic. Due to this, the model is expected to produce less conservative estimates. This expectation is shown to be true from the results shown in Figure 12. This figure depicts that distorting the default driver consistently produces more pessimistic estimates, particularly with Expected Losses, as it is noticeably larger in each of the climate scenarios.

Risk Measure	Distorting Y_i	Distorting Z
VaR$_{99.9\%}$	\$83.00	\$81.00
CVaR$_{99.9\%}$	\$93.27	\$91.80
Expected Loss	\$10.17	\$9.87
Economic Capital	\$74.84	\$71.13

(a) Baseline

Risk Measure	Distorting Y_i	Distorting Z
VaR$_{99.9\%}$	\$184.00	\$121.00
CVaR$_{99.9\%}$	\$203.00	\$147.60
Expected Loss	\$31.89	\$13.12
Economic Capital	\$143.97	\$107.88

(b) Mild

Risk Measure	Distorting Y_i	Distorting Z
VaR$_{99.9\%}$	\$376.00	\$213.00
CVaR$_{99.9\%}$	\$401.50	\$237.20
Expected Loss	\$98.96	\$20.50
Economic Capital	\$269.51	\$192.50

(c) Severe

Risk Measure	Distorting Y_i	Distorting Z
VaR$_{99.9\%}$	\$702.00	\$456.00
CVaR$_{99.9\%}$	\$723.99	\$506.70
Expected Loss	\$315.46	\$46.56
Economic Capital	\$384.39	\$409.44

(d) Catastrophe

Figure 12: Comparison of Risk Measures for A Rated Borrowers

A Python Code

B Importing neccessary modules

[2]:
```
#Visualization Libraries
import seaborn as sb
from mpl_toolkits.mplot3d import Axes3D
from matplotlib import pyplot as plt
import matplotlib as mpl
sb.set()

#Statistical and Numerical Libraries
import copulas as cop
from scipy.integrate import quad
from scipy.integrate import dblquad
import itertools as it
from itertools import permutations
import numpy as np
import pandas as pd
import scipy.stats as st
from scipy.stats import multivariate_normal as mvn




#set parameters for plots and removing scientific notation
np.set_printoptions(suppress=True)
mpl.rcParams['figure.dpi'] = 300
```

C Primary and Auxillary Functions

C.1 Functions used for Proportional Hazards Distortion

[129]:
```
def ph_cdf(x,a):
    return st.norm.cdf(x)**a #returns prop hazard distorted cdf


def ph_pdf(x,a):
    return a*(st.norm.cdf(x)**(a-1))*st.norm.pdf(x)  #returns prop hazard␣
↪distorted pdf

def distorted_joint_pdf(x,y,a,P):

    results = [( x_val, y_val, #first two elements are just the x and y␣
↪coordinates. This allows me to extract the pair of combinations as two list
            ph_pdf(x_val,a) * ph_pdf(y_val,a) *
```

```
                dist.pdf([st.norm.ppf(ph_cdf(x_val,a)), st.norm.
↪ppf(ph_cdf(y_val,a))]) #copula density
                )
                for x_val in x #iterates through first list
                for y_val in y #iterates through second list
                ]
    return results #returns distorted joint density value

def distorted_joint_cdf(x,y,a,P):

    results = [(x_val, y_val, dist.cdf([st.norm.ppf(ph_cdf(x_val, a)), st.norm.
↪ppf(ph_cdf(y_val, a))]))
               for x_val in x
               for y_val in y
               ]
    return results #returns distorted joint density value

def ph_mean(a):
    return quad(lambda y: y*ph_pdf(y,a),-30,30)[0] #calculates mean as a␣
↪function of distortion parameter

def ph_var(a):
    e = ph_mean(a)
    return quad(lambda y: ((y - e)**2)*ph_pdf(y,a),-30,30)[0] #calculates mean␣
↪as a function of distortion parameter

def ph_j_pdf(x,y,a,P): #this function is only for integration purposes
    return ( ph_pdf(x_val,a) * ph_pdf(y_val,a) *
            dist.pdf([st.norm.ppf(ph_cdf(x_val,a)),st.norm.ppf(ph_cdf(y_val,a))])
           )


#unused functions
def default_corr_homogenous_thresh(array,a):
    results = [(dist.cdf([st.norm.ppf(ph_cdf(x, a)), st.norm.ppf(ph_cdf(x,␣
↪a))])) - ph_cdf(x,a)**2 for x in array]
    return results

def driver_corr(a,b,P):

    dist = mvn(mean = np.array([0,0]), #setting mean for bivariate normal␣
↪distribution
            cov = np.array( [
                [1, P], #defining covariance in terms of variance and correlation

                [P, 1]
```

```python
            ] ) #setting covariance matrix for bivariate distribution
            )
    #defining distorted joint density
    func = lambda x,y :x * y * ph_pdf(x,1) * ph_pdf(y,1) * dist.pdf([st.norm.
 ↪ppf(ph_cdf(x,1)),st.norm.ppf(ph_cdf(y,1))])

    integral = dblquad(func,-2,2,-2,2)

    exp = ph_mean(a)*ph_mean(b)

    quot = np.sqrt(ph_var(a)*ph_var(b))

    results = (integral[0] - exp)/quot

    return results

def default_corr(x,y,a,P):

    dist = mvn(mean = np.array([0,0]), #setting mean for bivariate normal␣
 ↪distribution
            cov = np.array( [
                [1, P], #defining covariance in terms of variance and correlation
                [P, 1]
            ])
              )

    return (distorted_joint_cdf(x,y,a,P) - PD)/(PD-PD**2)
```

C.2 Functions used for Simulating Distorted Portfolio Risk Measures

[176]:
```python
#functions used for portfolio risk measures
def loss_drivers(a, PD, borrowers, rho): #vectorized as much as possible to␣
 ↪improve performance

    #generating default driver from vasicek model
    Z=np.random.normal() #systemic factor
    e_i=np.random.normal(size=(1,borrowers)) #idiosyncratic factor
    Y_i=np.sqrt(rho)*Z+np.sqrt(1-rho)*e_i #default driver

    #generating distorted default drivers
    u = st.norm.cdf(Y_i)
    Credit_quality = st.norm.ppf(u ** (1/a))

    #calculating default events
    Threshold = st.norm.ppf(PD)
    Defaults = np.where(Credit_quality < Threshold, 1, 0)
```

```
    individual_losses = np.sum(Defaults)  # Sums losses for each simulation

    return individual_losses

#this is for a brief comparison with the previous work
def loss_drivers_Arno(a, PD, borrowers, rho): #vectorized as much as possible to␣
↪improve performance

    #generating default driver from vasicek model
    Z = np.random.normal() #systemic factor

    #generating distorted systemic factor
    u = st.norm.cdf(Z)
    Z_Dist = st.norm.ppf(u ** (1/a))

    e_i = np.random.normal(size=(1,borrowers)) #idiosyncratic factor
    Y_i = np.sqrt(rho) * Z_Dist + np.sqrt(1-rho) * e_i #default driver


    #calculating default events
    Threshold = st.norm.ppf(PD)
    Defaults = np.where(Y_i < Threshold, 1, 0)

    individual_losses = np.sum(Defaults)  # Sums losses for each simulation

    return individual_losses


#estimates risk meausres based on distorted distribution
def risk_measures(loss_function, a, PD, borrowers, rho, tol):

    # #house keeping
    num_sim = 10000
    indx = int(tol*num_sim) #converts float to integer because indexes are␣
↪prejudice against against floats

    individual_losses = np.array([loss_function(a, PD, borrowers, rho) for i in␣
↪range(10000)]) #generates loss distribution

    #calculations
    var = sorted(individual_losses)[indx] #estimates var from simulation
    es = np.mean(individual_losses[individual_losses >= var]) #estimates␣
↪expected shortfall
    el = np.mean(individual_losses) #expected loss
    ec = var - el #economic capital requirement
```

```python
    return var, es, el, ec

# #monte carlo simulator [deprecated]
# def MC(a, N, PD):
#     return np.array([loss_drivers(a, PD) for i in range(10000)])

#computes confidnce interval for each risk measure using repeated valuations
def risk_measures_CI(loss_function, a, PD, borrowers, rho, tol, num_iterations):

    results = [[risk_measures(loss_function, a, PD, borrowers, rho, tol) for _ 
 ↪in range(num_iterations)] for a in Cal_Param]

    # Calculate statistics across iterations for each risk measure
    var_values = np.array([[result[0] for result in parameter_results] for 
 ↪parameter_results in results])
    es_values = np.array([[result[1] for result in parameter_results] for 
 ↪parameter_results in results])
    el_values = np.array([[result[2] for result in parameter_results] for 
 ↪parameter_results in results])
    ec_values = np.array([[result[3] for result in parameter_results] for 
 ↪parameter_results in results])

    var_mean = np.mean(var_values, axis=0)
    var_std = np.std(var_values, axis=0)

    es_mean = np.mean(es_values, axis=0)
    es_std = np.std(es_values, axis=0)

    el_mean = np.mean(el_values, axis=0)
    el_std = np.std(el_values, axis=0)

    ec_mean = np.mean(ec_values, axis=0)
    ec_std = np.std(ec_values, axis=0)

    # Ensure all arrays have the same length
    length = len(Cal_Param)

    # Calculate confidence intervals using the normal distribution
    confidence_level = 0.95
    confidence_multiplier = st.norm.ppf((1 + confidence_level) / 2)

    var_confidence_interval = confidence_multiplier * (var_std / np.
 ↪sqrt(num_iterations))
    es_confidence_interval = confidence_multiplier * (es_std / np.
 ↪sqrt(num_iterations))
```

```
    el_confidence_interval = confidence_multiplier * (el_std / np.
↪sqrt(num_iterations))
    ec_confidence_interval = confidence_multiplier * (ec_std / np.
↪sqrt(num_iterations))

    # # Print or use the results
    # for i, a in enumerate(Cal_Param):
    #     print(f"Climate Parameter: {a}")
    #     print(f"VAR: {var_mean[i]:.4f} +/- {var_confidence_interval[i]:.4f}")
    #     print(f"ES: {es_mean[i]:.4f} +/- {es_confidence_interval[i]:.4f}")
    #     print(f"EL: {el_mean[i]:.4f} +/- {el_confidence_interval[i]:.4f}")
    #     print(f"EC: {ec_mean[i]:.4f} +/- {ec_confidence_interval[i]:.4f}")
    #     print("=" * 20)

    data = {
    'Climate Parameter': [f"{temp}°C" for temp in Diff_SST],
    'VAR Mean': var_mean,
    'VAR SE': [f"{ci:.2f}" for ci in var_confidence_interval],
    'CVAR Mean': es_mean,
    'CVAR SE': [f"{ci:.2f}" for ci in es_confidence_interval],
    'EL Mean': el_mean,
    'EL SE': [f"{ci:.2f}" for ci in el_confidence_interval],
    'EC Mean': ec_mean,
    'EC SE': [f"{ci:.2f}" for ci in ec_confidence_interval]
    }
    results_df = pd.DataFrame(data)

    return(results_df)


def loss_drivers_Arno(a, PD, borrowers, rho): #vectorized as much as possible to␣
↪improve performance

    #generating default driver from vasicek model
    Z = np.random.normal() #systemic factor

    #generating distorted systemic factor
    u = st.norm.cdf(Z)
    Z_Dist = st.norm.ppf(u ** (1/a))

    e_i = np.random.normal(size=(1,borrowers)) #idiosyncratic factor
    Y_i = np.sqrt(rho) * Z_Dist + np.sqrt(1-rho) * e_i #default driver


    #calculating default events
    Threshold = st.norm.ppf(PD)
    Defaults = np.where(Y_i < Threshold, 1, 0)
```

```
    individual_losses = np.sum(Defaults)  # Sums losses for each simulation

    return individual_losses
```

C.3 Functions used for Distorted Credit Transition Matrices

```
[ ]: #this function takes a credit transition matrix as input and returns the␣
      ↪distorted transition probabilities
     def CTM_Distortion(CTM,a):
         cdf = np.cumsum(np.fliplr(CTM), axis=1) #converts row transition␣
      ↪probabilities to cdf

         dist_cdf = [np.array(cdf[i])**a for i in range(len(cdf))] #distorts␣
      ↪distribution

         res = np.diff(dist_cdf, prepend = 0) #this undose what  np.cumsum does

         return np.fliplr(res) #reversing order so it reads as is in credit matrix


     #returns amount of borrowers in each credit state given each initial credit␣
      ↪rating
     def borrower_count(CTM,int_state):

         borrowers = np.round(CTM * int_state[:, np.newaxis],0) #calculates possible␣
      ↪borrowers in next state based on initial state

         return borrowers

     # def borrower_count(CTM,u):
     #     cdf = np.cumsum(np.fliplr(CTM), axis=1) #creates cdf of credit transition␣
      ↪row
     #     borrowers = [] #initializing list to store borrowers

     #     #looping through roughs of CTM Distribution
     #     for i in cdf:

     #         B_Condition = np.logical_and(i[1] < u, u <= i[2]) #np.where can only␣
      ↪handle multiple boleans using np.logical_
     #         C_Condition = np.logical_and(i[0] < u, u <= i[1])

     #         A_Borrowers = np.sum(np.where(u > i[2],1,0)) #sums all borrowers where␣
      ↪the condition is meet
     #         B_Borrowers = np.sum(np.where(B_Condition,1,0))
     #         C_Borrowers = np.sum(np.where(C_Condition,1,0))
```

```
#         D_Borrowers = 1000 - np.sum([A_Borrowers,B_Borrowers,C_Borrowers])

#         borrowers.append(np.array([A_Borrowers, B_Borrowers, C_Borrowers,␣
↪D_Borrowers])) #adds list of borrrowers to list

#     return np.array(borrowers) #returns borrower count for each credit state

#this function calculates the expected value of each intial
# def portfolio_value(CTM,int_state,debt_value):

#     Debtors = borrower_count(CTM,int_state) #generates amount of debtors in␣
↪each possible credit state given the initial credit state
#     Category_Values = Debtors*debt_value #expected portfolio value for each␣
↪category
#     Category_Totals = np.sum(Category_Values, axis=0, keepdims=True) #expected␣
↪value for each initial credit state
#     Final = np.sum(Category_Totals, keepdims = True) #total portfolio value

#     return np.append(Category_Totals, Final, axis=1) #returns array with total␣
↪appended to tail

def portfolio_value_2(CTM,int_state,debt_value):

    Debtors = borrower_count(CTM,int_state) #generates amount of debtors in each␣
↪possible credit state given the initial credit state
    # Category_Totals = np.sum(Debtors, axis=0, keepdims=True) #expected number␣
↪of borrowers in each credit state at t = 1
    Category_Val = Debtors.sum(axis=0)*debt_value #expected value of each loan␣
↪group
    # Total = np.sum(Category_Val, axis = 1, keepdims=True) #total expected␣
↪portfolio value

    return np.reshape(np.append(Category_Val, Category_Val.sum()),(5,1))

#10 period ahead expected value
def forecast(CTM, period,int_state,debt_value):
    exp_val = []
    exp_val.append((int_state*debt_value).sum()) #adding intial value to list
    for i in range(period):
        Debtors = borrower_count(CTM,int_state) #generates amount of debtors in␣
↪each possible credit state given the initial credit state
        Category_Val = Debtors.sum(axis=0)*debt_value #expected value of each␣
↪loan group
        int_state = Debtors.sum(axis=0) #updating initial state of borrowers
        exp_val.append(Category_Val.sum(axis=0)) #storing expected portfolio␣
↪value to list
```

```
        # print(i)
    return exp_val
```

C.4 Auxillary Functions

[6]:
```
#defining relationship between climate varaible and distortion parameter
def distortion_climate_cal(SST):
    valid_mask = (SST != 2) & (SST >= 0) #checks where conditions are meet
    distortions = np.where(valid_mask, -0.5 * SST + 1, np.nan)
    return distortions

#returns credit transition matrix in a formatted dataframe.
def tidy(x):
    #storing values to dataframe and naming rows and columns
    ratings = ['A', 'B', 'C', 'D']
    frame = pd.DataFrame(x, columns=ratings, index=ratings)

    # Apply the function to each entry in the DataFrame (rounds to 2 significant␣
 ↪figures
    frame = frame.applymap(decimal_to_percent)

    return frame

#formatting
def decimal_to_percent(decimal):
    return f"{decimal :.2f}"
```

D Visualizing Impacts of Proportional Hazards Distortion

[127]:
```
#setting parameters of gaussian copula

a =  3 #distortion parameter
P = 0.1 #correlation between distributions
mu = 0

dist = mvn(mean = np.array([0,0]), #setting mean for bivariate normal␣
 ↪distribution
            cov = np.array( [
                [1, P], #defining covariance in terms of variance and correlation

                [P, 1]

            ] ) #setting covariance matrix for bivariate distribution
            )
```

[132]:

```python
# creating joint density contours for 0 correlation and decreasing values of a <␣
↪1

sb.set(rc = {'figure.figsize':(14,4)}) #setting figure size

a = [0.8, 0.4, 0.2, 0.1] #list of distortion parameters

cons = 3.5 #constant to multiply standard deviations by. For some plots, I␣
↪needed to scale the graph up to get complete visual

sig = 2

P=0.9 #correlation parameter for bivariate normal distribution


#creating figure object for plots
fig = plt.figure()

#looping through each value in list and taking its index
for idx, val in enumerate(a):

    dist = mvn(mean=np.array([0, 0]), cov=np.array([[1, P], [P, 1]]))␣
↪#initializing bivariate standard normal with parameters

    m = np.linspace(-cons*sig, cons*sig, num=75) #generating list of evenly␣
↪spaced grid ploints to plot over
    n = np.linspace(-cons*sig, cons*sig, num=75)

    #calculating distorted values
    X,Y,Z = zip(*distorted_joint_pdf(m,n,val,P)) #unpacking list of tuples into␣
↪individual lists for plotting

    #plotting the contours
    ax = fig.add_subplot(1, len(a), idx+1) #adding 1 row of subplots with␣
↪columns equal to the length of a
    ax.tricontourf(X, Y , Z , cmap='magma')

    ax.axhline(np.percentile(m,15), linestyle = 'dashed', color = 'white')
    ax.axvline(np.percentile(m,15), linestyle = 'dashed', color = 'white')

    ax.set_xlabel("X") #lableling axes
    ax.set_ylabel("Y")
    ax.axes.xaxis.set_ticks([]) #removing axis ticks - its less clutter
    ax.axes.yaxis.set_ticks([])
    temp = 'a' #'\u03C1'
    ax.set_title(f'{temp} = {val}') #formatting title of each subplot
```

```python
plt.tight_layout() #making subplots layout less tight
plt.savefig("Distorted Joint Density - Varied a Pos Correlation.pdf") #saving plot
plt.show()
```

output_13_0.png

[ ]:

```python
# #surface plot for a = 0.15 - you can't really see anything insightful from this so it wasn't used.

# fig = plt.figure();
# a = [0.8, 0.4, 0.2, 0.1]
# cons = 3.5
# sig = 2
# P=0
# dist = mvn(mean=np.array([0, 0]), cov=np.array([[1, P], [P, 1]]))

# m = np.linspace(-cons*sig, cons*sig, num=150)
# n = np.linspace(-cons*sig, cons*sig, num=150)

# X,Y,Z = zip(*distorted_joint_pdf(m,n,0.15,P)) #a = 0.15

# ax = fig.add_subplot(projection = '3d')
# ax.plot_trisurf(X, Y , Z , cmap=plt.cm.Spectral, linewidth=0, antialiased=False)
# ax.set_xlabel("X")
# ax.set_ylabel("Y")
# ax.axes.xaxis.set_ticks([])
# ax.axes.yaxis.set_ticks([])
# temp = 'a' #'\u03C1'
# ax.set_title('Distorted Joint Density ')

# # plt.tight_layout()
# # # plt.savefig("Distorted Joint Density - Varied a.pdf") #saving plot
# plt.show()
```

[ ]:

```python
#plotting copula changes
sb.set(rc = {'figure.figsize':(14,4)})

a = [0.8, 0.4, 0.2, 0.1]
```

```
cons = 3.5
sig = 2
P=0
fig = plt.figure()

for idx, val in enumerate(a):

    dist = mvn(mean=np.array([0, 0]), cov=np.array([[1, P], [P, 1]]))

    m = np.linspace(-cons*sig, cons*sig, num=75)
    n = np.linspace(-cons*sig, cons*sig, num=75)

    X,Y,Z = zip(*distorted_joint_cdf(m,n,val,P))
    cdf_x, cdf_y = st.norm.cdf(X), st.norm.cdf(Y)

    ax = fig.add_subplot(1, len(a), idx+1, projection = '3d')

    #the extra arguments allow the surface plots to show in full resolution when␣
 ↪added to the latex document
    ax.plot_trisurf(cdf_x, cdf_y, Z , cmap='magma', linewidth=0,␣
 ↪antialiased=False)
    ax.set_xlabel("$\Phi(X)$")
    ax.set_ylabel("$\Phi(Y)$")
    ax.axes.xaxis.set_ticks([])
    ax.axes.yaxis.set_ticks([])
    ax.axes.zaxis.set_ticks([])
    temp = 'a' #'\u03C1'
    ax.set_title(f'{temp} = {val}')

plt.tight_layout()
# plt.savefig("Distorted Copula - Varied a.pdf") #saving plot
plt.show()
```

[ ]:
```
# creating joint distribution contours for 0 correlation and decreasing values␣
 ↪of a < 1 - UNUSED

sb.set(rc = {'figure.figsize':(14,4)})

a = [0.8, 0.4, 0.2, 0.1]
cons = 3.5
sig = 2
P=0
fig = plt.figure()

for idx, val in enumerate(a):

    dist = mvn(mean=np.array([0, 0]), cov=np.array([[1, P], [P, 1]]))
```

```
    m = np.linspace(-cons*sig, cons*sig, num=75)
    n = np.linspace(-cons*sig, cons*sig, num=75)

    X1,Y1,Z1 = zip(*distorted_joint_cdf(m,n,val,P))

    ax = fig.add_subplot(1, len(a), idx+1, projection = '3d')
    # ax.plot_trisurf(X1, Y1 , Z1 , cmap='magma')
    ax.plot_trisurf(X1, Y1, Z1, cmap='magma', linewidth=0, antialiased=False)
    ax.set_xlabel("X")
    ax.set_ylabel("Y")
    ax.axes.xaxis.set_ticks([])
    ax.axes.yaxis.set_ticks([])
    ax.axes.zaxis.set_ticks([])
    temp = 'a' #'\u03C1'
    ax.set_title(f'{temp} = {val}')

plt.tight_layout()
# plt.savefig("Distorted Joint Distribution - Varied a.pdf") #saving plot
plt.show()
```

[ ]:
```
# creating joint density contours for various correlation values - UNUSED

sb.set(rc = {'figure.figsize':(11,4)})

P = [-0.9,0,0.9]
cons = 3.5
sig = 2
a = 0.1
fig = plt.figure()

for idx, val in enumerate(P):

    dist = mvn(mean=np.array([0, 0]), cov=np.array([[1, val], [val, 1]]))

    m = np.linspace(-cons*sig, cons*sig, num=75)
    n = np.linspace(-cons*sig, cons*sig, num=75)

    X,Y,Z = zip(*distorted_joint_pdf(m,n,a,P))

    ax = fig.add_subplot(1, len(P), idx+1)
    ax.tricontourf(X, Y , Z , cmap='magma')
    ax.set_xlabel("X")
    ax.set_ylabel("Y")
    ax.axes.xaxis.set_ticks([])
    ax.axes.yaxis.set_ticks([])
    temp = '\u03C1'
```

```
    ax.set_title(f'{temp} = {val}')

plt.tight_layout()
# plt.savefig("Distorted Joint Density - Varied correlation.pdf") #saving plot
plt.show()
```

[ ]:
```
#lazy coding disclaimer

fig, axes = plt.subplots(1, 2)
temp = np.linspace(10,0.1,num=100)

lis1 = [ph_mean(x) for x in temp]
lis2 = [ph_var(x) for x in temp]

sb.lineplot(y = lis1, x = temp, ax=axes[0], color = 'magenta').set(title =␣
↪'Distorted Mean vs Distortion Parameter',
                                                                    xlabel = 'a',␣
↪ylabel = '$\mathbb{E}(Y_i)$');
axes[0].set_facecolor("grey") #setting axis colour

sb.lineplot(y = lis2, x = temp, ax=axes[1], color = 'orange').set(title =␣
↪'Distorted Variance vs Distortion Parameter',
                                                                    xlabel = 'a',␣
↪ylabel = '$Var(Y_i)$');
axes[1].set_facecolor("grey")

# plt.savefig("Distorted Statistics - Varied a.pdf")
```

[ ]:
```
fig, ax = plt.subplots()

sb.set_palette("pastel")
ax.set_facecolor('grey');

temp1 = np.linspace(-10,5,num=400) #evenly spalce points for ploting

for idx,x in enumerate([0.1,0.4, 1, 2]):
    sb.lineplot(y = ph_pdf(temp1,x), x = temp1, label = f'a = {x}' );

plt.ylabel("$\Phi^{g}(x)$");
plt.xlabel("x");
plt.axvline( x = -1, color = 'black', linestyle='dashed');
plt.title("Distorted Distribution")
# plt.savefig("Distorted 2D CDF - Varied a.pdf")
plt.show();
```

[ ]:
```
# # this shows that the correlation formula works assuming that the threshold is␣
↪U(0,1) - unused as well
```

```
# xaxis = np.linspace(0,1,num=100)

# for a in [0.1,0.4,0.8,2,5]:
#     gg = [( dist.cdf([st.norm.ppf(ph_cdf(x, a)), st.norm.ppf(ph_cdf(x,␣
↪a))])-st.norm.cdf(x)**2)/st.norm.cdf(x)*(1-st.norm.cdf(x)) for x in xaxis]
#     sb.lineplot(y = gg, x = xaxis, label = f'a = {a}');

# plt.xlabel('b');
# plt.ylabel('Default Correlation');
```

[ ]:
```
#plotting cumulative distribution for row of credit transition matrix

data = np.arange(1, 6)
y = np.array(np.cumsum([0.01, 0.03, 0.06, 0.9]))  # undistorted distribution
y_dist = np.array(np.cumsum([0.01, 0.03, 0.06, 0.9])) ** 0.4  # distorted␣
↪distribution

# defining figure and axis characteristics
fig, ax = plt.subplots()
ax.set_facecolor('black')

# undistorted cdf
ax.hlines(y=y, xmin=data[:-1], xmax=data[1:], color='aqua', zorder=4) #adding␣
↪horizontal lines
ax.vlines(x=data[1:-1], ymin=y[:-1], ymax=y[1:], color='aqua',␣
↪linestyle='dashed', zorder=4) #adding vertical lines
ax.scatter(data[1:-1], y[1:], color='white', s=12, zorder=4) #adding closed␣
↪interval dot
ax.scatter(data[1:-1], y[:-1], color='white', s=12, zorder=4, edgecolor='white')␣
↪#adding open interval dot

# distorted cdf
ax.hlines(y=y_dist, xmin=data[:-1], xmax=data[1:], color='gold', zorder=4)
ax.vlines(x=data[1:-1], ymin=y_dist[:-1], ymax=y_dist[1:], color='gold',␣
↪linestyle='dashed', zorder=4)
ax.scatter(data[1:-1], y_dist[1:], color='white', s=12, zorder=4)
ax.scatter(data[1:-1], y_dist[:-1], color='gold', s=12, zorder=4,␣
↪edgecolor='white')

# naming horizontal axis ticks to threshold values
ax.set_xticklabels(['', '', '$C$', '', '$B$', '', '$A$'])

# graph housekeeping
ax.grid(False)
ax.set_xlim(data[0], data[-1])
ax.set_ylim([-0.01, 1.01])
```

```
# legend formatting
legend_labels = ['Distorted','Undistorted']
legend_colors = ['gold','aqua']
ax.legend(legend_labels, facecolor='grey', framealpha=0.6, edgecolor='white',
          loc='upper right', title='', title_fontsize='12',␣
 ↪labelcolor=legend_colors,
          handlelength=0, handletextpad=0)

# figure housekeeping
plt.ylabel('$P(X_{1} \leq x_{1} |X_{0}=4)$')
plt.xlabel('Transition Ratings')
plt.title('Credit Transition Probability Distribution for A Rating at t = 0')

# plt.savefig("CTM CDF Example.pdf")

plt.show()
```

[ ]:
```
# density plot
sb.set(rc = {'figure.figsize':(14,6)})

y = np.array(np.cumsum([0.01, 0.03, 0.06, 0.9]))  # undistorted distribution
y_dist = np.array(np.cumsum([0.01, 0.03, 0.06, 0.9])) ** 0.4  # distorted␣
 ↪distribution
y_pdf = np.diff(y, prepend=0)
y_dist_pdf = np.diff(y_dist, prepend=0)

# Create a dictionary containing the data
data = {
    'Undistorted Transition Probabilities': y_pdf,
    'Distorted Transition Probabilities': y_dist_pdf
}

# Create the DataFrame
CTM_Density = pd.DataFrame(data, index=['D','C','B','A']).transpose().round(3)

# Reset index to convert the columns into a regular column and create a␣
 ↪'Transition Type' column
CTM_Density = CTM_Density.reset_index().rename(columns={'index': 'Transition␣
 ↪Type'})

# Melt the DataFrame to convert the columns into rows
melted_df = CTM_Density.melt(id_vars='Transition Type', var_name='Category',␣
 ↪value_name='Density')

# Sort the DataFrame by 'Category' and 'Density' in descending order
```

```
melted_df.sort_values(by=['Category', 'Density'], ascending=[True, False],␣
↪inplace=True)

# visualizing bar plots and formatting plot
sb.barplot(data=melted_df, x='Category', y='Density', hue='Transition Type',␣
↪errorbar=None, palette=['aqua','gold'])
plt.grid(False)
plt.legend(title='')
plt.gca().set_facecolor('black')
plt.xlabel('Transition Ratings')
plt.ylabel('$P(X_{1} = x_{1} |X_{0}=4)$')
plt.title('Transition Probabilities Comparison')

# plt.savefig("CTM PDF Example.pdf")

plt.show()
```

[ ]:
```
#loss distribution for each climate scenario
Loss_Dist_Per_Param = pd.DataFrame(columns = scenario_names)
PD_Generic = 0.05 #generic default probability

for column,a in zip(Loss_Dist_Per_Param,Cal_Param):

    Loss_Dist_Per_Param[column] = np.array([loss_drivers(a, PD_Generic,␣
↪borrowers, rho) for i in range(10000)])
```

[ ]:
```

```

E Simulation: Transition Matrices and Portfolio Losses under Distortion

E.1 Transition Matrices and Portfolio Value

[ ]:
```
#defining transition matrix with baseline transition probabilities i.e 0 degree␣
↪increase
CTM = [[0.9, 0.06, 0.03, 0.01], [0.03, 0.85, 0.08, 0.04],[0.01, 0.11, 0.73, 0.
↪15],[0,0,0,1]]

#defining array with value of asset for each credit rating
debt_value = np.array([1, 0.75, 0.25, 0])

int_state = np.array([900, 75, 25, 0])
#portfolio value for baseline scenario
Port_Val_Baseline = portfolio_value_2(CTM,int_state,debt_value)
```

```
[ ]: #mild scenario, 0.5 degree increase
     sst_mild = 0.5 #climate variable
     mild_param = distortion_climate_cal(sst_mild) #climate calibrated distortion␣
      ↪param
     CTM_Mild = CTM_Distortion(CTM, mild_param) #distorting transition matrix
     Port_Val_Mild = portfolio_value_2(CTM_Mild,int_state,debt_value) #portfolio␣
      ↪value under scenario
     tidy(CTM_Mild) #formatting matrix
```

```
[ ]: #severe scenario, 1 degree increase
     sst_severe = 1 #climate variable
     severe_param = distortion_climate_cal(sst_severe) #climate calibrated distortion␣
      ↪param
     CTM_Severe = CTM_Distortion(CTM, severe_param) #distorting transition matrix
     Port_Val_Severe = portfolio_value_2(CTM_Severe,int_state,debt_value) #portfolio␣
      ↪value under scenario
     tidy(CTM_Severe) #formatting matrix
```

```
[ ]: #severe scenario, 1 degree increase
     sst_catas = 1.5 #climate variable
     catas_param = distortion_climate_cal(sst_catas) #climate calibrated distortion␣
      ↪param
     CTM_Catas = CTM_Distortion(CTM, catas_param) #distorting transition matrix
     Port_Val_Catas = portfolio_value_2(CTM_Catas,int_state,debt_value) #portfolio␣
      ↪value under scenario
     tidy(CTM_Catas) #formatting matrix
```

```
[392]: #placing the porfolio values into table format

       Scenarios = ['Baseline', 'Mild','Severe', 'Catastrophic']
       group_names = ['A Rating', 'B Rating', 'C Rating', 'Defaulted', 'Total␣
        ↪Portfolio']
       Port_Val_Scenarios = pd.DataFrame(
           np.concatenate([Port_Val_Baseline, Port_Val_Mild, Port_Val_Severe,␣
        ↪Port_Val_Catas] #concatenating values column wise
                          ,axis=1)
           , columns = Scenarios #naming columns
           , index = group_names
       ).round(2) #rounding entries to two decimal places

       Port_Val_Scenarios
```

[392]:

	Baseline	Mild	Severe	Catastrophic
A Rating	812.00	742.0	616.00	395.00
B Rating	90.75	105.0	115.50	101.25
C Rating	12.75	19.5	28.75	34.50
Defaulted	0.00	0.0	0.00	0.00

```
Total Portfolio    915.50  866.5  760.25        530.75
```

[ ]:
```
#conerting dataframe to base latex code
print(Port_Val_Scenarios.to_latex(bold_rows=True, escape=False))
```

E.1.1 12-period forecasts for Expected Portfolio Value

[ ]:
```
sb.set(rc = {'figure.figsize':(14,6)})

#storing lists of expected values for each scenario in tuples. The latter makes␣
↪displaying as a dataframe easier
period = 12

f_vals = (forecast(CTM,period,int_state, debt_value),
 forecast(CTM_Mild,period,int_state, debt_value),
 forecast(CTM_Severe,period,int_state, debt_value),
 forecast(CTM_Catas,period,int_state, debt_value)
)

#adding to dataframe and adding a column for the scenario names
forecasts = pd.DataFrame(f_vals, index = scenario_names).reset_index().
↪rename(columns={'index': 'Scenario'})

#melting data for visualizations purposes
forecasts_melted = forecasts.melt(id_vars = 'Scenario', var_name="Period",␣
↪value_name="ExpectedValue")

#visualizing forecasted values
color = 'muted'
sb.barplot(data = forecasts_melted, x = 'Period', y = 'ExpectedValue', hue =␣
↪'Scenario', palette= color);
sb.lineplot(data = forecasts_melted, x = 'Period', y = 'ExpectedValue', hue =␣
↪'Scenario', palette= color, legend=False)

#making the visualization sexy
plt.gca().set_facecolor('black')
plt.ylabel('Portfolio Value ($)')
plt.title('12-Period Forecasts for Expected Portfolio Value')
plt.grid(False)

# plt.savefig("forecast.pdf")
```

E.2 Portfolio Risk Measures

[185]:
```
#variables assigned in this block are used throughout this section and are won't␣
↪change
scenario_names = ['Baseline', 'Mild', 'Severe', 'Catastrophic']
riskmeasure_names = ['$\textbf{VaR}_{99.9\%}$', '$\textbf{CVaR}_{99.
↪9\%}$','Expected Loss', 'Economic Capital']
Diff_SST = np.array([0, 0.5, 1, 1.5]) #sea surface temperature changes
Cal_Param = distortion_climate_cal(Diff_SST) #climate parameter
tol = 0.999 #toleratnce for risk measures
num_iterations = 50 #repeated valuations for CI
borrowers = 1000 #assuming same sized portfolio for each borrower group
rho = 0.1 #weight of systemic factor in vasicek model
```

[177]:
```
PD_A = 0.01 #default probability for a rated borrowers - assumed to be the same␣
↪for all borrowers

#initializing dataframe
Risk_Measures = pd.DataFrame(columns = scenario_names, index = riskmeasure_names)

#adding risk measures to their appropriate place in dataframe
for column,a in zip(Risk_Measures,Cal_Param): #a column for each scenario is␣
↪created,
                                              #then the appropriate risk␣
↪measures are calculated and added to that column

    Risk_Measures[column] = risk_measures(loss_drivers, a, PD_A, borrowers, rho,␣
↪tol) #risk measures being calculated based on parameter inputs

Risk_Measures.round(2) #rounding figures to 2 decimal places
```

[177]:

	Baseline	Mild	Severe	Catastrophic
$\textbf{VaR}_{99.9\%}$	85.00	176.00	370.00	701.00
$\textbf{CVaR}_{99.9\%}$	97.70	211.70	409.60	736.40
Expected Loss	10.16	32.03	100.49	316.61
Economic Capital	74.84	143.97	269.51	384.39

[ ]:
```
# results_A = [[risk_measures(a, PD_A, borrowers, rho, tol) for a in Cal_Param]␣
↪for _ in range(num_iterations)]
risk_measures_CI(a, PD_A, borrowers, rho, tol, num_iterations)
```

[183]:
```
#risk measures by distorting the systemic factor
Risk_Measures = pd.DataFrame(columns = scenario_names, index = riskmeasure_names)

#adding risk measures to their appropriate place in dataframe
for column,a in zip(Risk_Measures,Cal_Param): #a column for each scenario is␣
↪created,
```

```
                                                      #then the appropriate risk␣
 ↪measures are calculated and added to that column

    Risk_Measures[column] = risk_measures(loss_drivers_Arno, a, PD_A, borrowers,␣
 ↪rho, tol) #risk measures being calculated based on parameter inputs

Risk_Measures.round(2) #rounding figures to 2 decimal places
```

```
                         Baseline    Mild  Severe  Catastrophic
$\textbf{VaR}_{99.9\%}$     77.00  108.00   185.0        460.00
$\textbf{CVaR}_{99.9\%}$    90.27  139.40   210.8        499.10
Expected Loss                9.86   13.27    19.9         47.62
Economic Capital            67.14   94.73   165.1        412.38
```

[178]:
```
PD_B = 0.04 #default probability for B rated borrowers

#initializing dataframe
Risk_Measures = pd.DataFrame(columns = scenario_names, index = riskmeasure_names)

#adding risk measures to their appropriate place in dataframe
for column,a in zip(Risk_Measures,Cal_Param): #a column for each scenario is␣
 ↪created,
                                                      #then the appropriate risk␣
 ↪measures are calculated and added to that column

    Risk_Measures[column] = risk_measures(loss_drivers, a, PD_B, borrowers, rho,␣
 ↪tol) #risk measures being calculated based on parameter inputs

Risk_Measures.round(2) #rounding figures to 2 decimal places
```

[178]:
```
                         Baseline    Mild  Severe  Catastrophic
$\textbf{VaR}_{99.9\%}$    213.00  364.00  564.00        830.00
$\textbf{CVaR}_{99.9\%}$   232.50  392.40  607.00        846.60
Expected Loss               40.03   89.78  200.08        448.35
Economic Capital           172.97  274.22  363.92        381.65
```

[142]:
```
# results_B = [[risk_measures(a, PD_B, borrowers, rho, tol) for a in Cal_Param]␣
 ↪for _ in range(num_iterations)]
risk_measures_CI(a, PD_B, borrowers, rho, tol, num_iterations)
```

[142]:
```
  Climate Parameter  VAR Mean     VAR CI   CVAR Mean    CVAR CI     EL Mean  \
0             0.0°C    212.38  +/-2.2394  235.869315  +/-2.9994   40.058932
1             0.5°C    353.46  +/-2.9387  386.390909  +/-3.8128   89.427624
2             1.0°C    562.56  +/-3.1938  590.652000  +/-3.4385  199.954606
3             1.5°C    815.52  +/-2.4172  835.734394  +/-2.8697  447.101140

       EL CI     EC Mean      EC CI
```

```
0  +/-0.0911  172.321068  +/-2.2158
1  +/-0.1453  264.032376  +/-2.9327
2  +/-0.2852  362.605394  +/-3.1622
3  +/-0.3367  368.418860  +/-2.3566
```

[179]:
```python
PD_C = 0.15 #default probability for C rated borrowers

#initializing dataframe
Risk_Measures = pd.DataFrame(columns = scenario_names, index = riskmeasure_names)

#adding risk measures to their appropriate place in dataframe
for column,a in zip(Risk_Measures,Cal_Param): #a column for each scenario is␣
↪created,
                                              #then the appropriate risk␣
↪measures are calculated and added to that column

    Risk_Measures[column] = risk_measures(loss_drivers, a, PD_C, borrowers, rho,␣
↪tol) #risk measures being calculated based on parameter inputs

Risk_Measures.round(2) #rounding figures to 2 decimal places
```

[179]:

	Baseline	Mild	Severe	Catastrophic
$\textbf{VaR}_{99.9\%}$	476.00	602.00	770.00	911.00
$\textbf{CVaR}_{99.9\%}$	511.30	637.30	784.60	919.82
Expected Loss	148.72	239.34	387.91	622.15
Economic Capital	327.28	362.66	382.09	288.85

[143]:
```python
# results_C = [[risk_measures(a, PD_C, borrowers, rho, tol) for a in Cal_Param]␣
↪for _ in range(num_iterations)]
risk_measures_CI(a, PD_C, borrowers, rho, tol, num_iterations)
```

[143]:
```
  Climate Parameter  VAR Mean     VAR CI   CVAR Mean    CVAR CI     EL Mean  \
0             0.0°C    479.80  +/-4.1710  510.647394  +/-4.9818  150.259300
1             0.5°C    618.46  +/-3.4371  647.115636  +/-3.8767  240.986634
2             1.0°C    772.44  +/-2.5532  794.456000  +/-2.7807  387.339610
3             1.5°C    915.40  +/-1.3293  926.582853  +/-1.5385  622.486920

       EL CI     EC Mean      EC CI
0  +/-0.2005  329.540700  +/-4.1320
1  +/-0.2828  377.473366  +/-3.3915
2  +/-0.3720  385.100390  +/-2.5657
3  +/-0.3191  292.913080  +/-1.3406
```