Filter type	Purpose	3x3	5x5
Mean	In the calculation, mean of the part where the filter is used is calculated.	$\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$	$\frac{1}{25}\begin{bmatrix}1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\end{bmatrix}$
Gaussian	It is used for noise removal It can be used as a pre-processing for edge detection algorithms.	$\frac{1}{16}\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix}$	$\frac{1}{256}\begin{bmatrix}1&4&6&4&1\\4&16&24&16&4\\6&24&36&24&6\\4&16&24&16&4\\1&4&6&4&1\end{bmatrix}$
Median	Takes the median number of the part where the filter is used on. It can be used for simple noise removal but makes edges blurry.	-	-
Sobel Filter in X direction	Edge detection in X direction = Gaussian * 1^{st} Derivative in X (The calculation is done in 1D)	$\begin{bmatrix}-1&0&1\\-2&0&2\\-1&0&1\end{bmatrix}$	$\begin{bmatrix}-1&-2&0&2&1\\-4&-8&0&8&4\\-6&-12&0&12&6\\-4&-8&0&8&4\\-1&-2&0&2&1\end{bmatrix}$
Sobel Filter in Y direction	Edge detection in Y direction = Gaussian + 1^{st} Derivative in Y (The calculation is done in 1D)	$\begin{bmatrix}-1&-2&-1\\0&0&0\\1&2&1\end{bmatrix}$	$\begin{bmatrix}-1&-4&-6&-4&-1\\-2&-8&-12&-8&-2\\0&0&0&0&0\\2&8&12&8&2\\1&4&6&4&1\end{bmatrix}$
	$\begin{bmatrix}1\\2\\1\end{bmatrix}\cdot\begin{bmatrix}-1&0&1\end{bmatrix}=\begin{bmatrix}1\cdot-1&1\cdot0&1\cdot1\\2\cdot-1&2\cdot0&2\cdot1\\1\cdot-1&1\cdot0&1\cdot1\end{bmatrix}=\begin{bmatrix}-1&0&1\\-2&0&2\\-1&0&1\end{bmatrix}$		
Laplacian	Isotropic (rotation-invariant) 2^{nd} Derivative Filter Detects curvatures	$\begin{bmatrix}0&-1&0\\-1&4&-1\\0&-1&0\end{bmatrix}$	$\begin{bmatrix}0&0&-1&0&0\\0&-1&-2&-1&0\\-1&-2&16&-2&-1\\0&-1&-2&-1&0\\0&0&-1&0&0\end{bmatrix}$
Prewitt Filter	Similar to Sobel filters but uniform weights in the filter.	$\begin{array}{\|c\|c\|c\|}\hline -1&0&+1\\\hline -1&0&+1\\\hline -1&0&+1\\\hline\end{array}$ G_x	$\begin{array}{\|c\|c\|c\|}\hline +1&+1&+1\\\hline 0&0&0\\\hline -1&-1&-1\\\hline\end{array}$ G_y

1. Forward difference

$$\left.\frac{dI}{dx}\right|_x \approx I[x+1] - I[x]$$

- **Uses current and next point**
- **Two-point stencil**
- Slightly biased toward future (forward)
- Simple but less accurate

Optional scaling if step size is h:

$$\frac{dI}{dx} \approx \frac{I[x+1] - I[x]}{h}$$

2. Backward difference

$$\left.\frac{dI}{dx}\right|_x \approx I[x] - I[x-1]$$

- **Uses current and previous point**
- **Two-point stencil**
- Slightly biased toward past (backward)

Optional step size h:

$$\frac{dI}{dx} \approx \frac{I[x] - I[x-1]}{h}$$

3. Central difference

$$\left.\frac{dI}{dx}\right|_x \approx \frac{I[x+1] - I[x-1]}{2}$$

- **Uses points on both sides of current point**
- **Symmetric**, more accurate
- Basis for **Sobel and derivative-of-Gaussian filters**

Optional step size h:

$$\frac{dI}{dx} \approx \frac{I[x+1] - I[x-1]}{2h}$$

✅ Typically **preferred** because it approximates the derivative at the **center pixel**.

4. 2D extension

For an image $I(x, y)$:

- Forward difference in x: $I[x+1, y] - I[x, y]$
- Forward difference in y: $I[x, y+1] - I[x, y]$

Similarly, central difference:

$$\frac{\partial I}{\partial x} \approx \frac{I[x+1, y] - I[x-1, y]}{2}, \quad \frac{\partial I}{\partial y} \approx \frac{I[x, y+1] - I[x, y-1]}{2}$$

- Central difference in 2D is the **foundation for Sobel filters**.

Second Derivative Filters

Mathematical meaning

$$\frac{\partial^2 I}{\partial x^2}, \quad \frac{\partial^2 I}{\partial y^2}$$

In 2D:

$$\nabla^2 I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$$

Discrete approximation

$$\frac{d^2 I}{dx^2} \approx I(x+1) - 2I(x) + I(x-1)$$

What it detects

- Changes of the change
- Curvature
- Zero-crossings at edges

Images for understanding filters (intuitively)

Mean Filter (Averaging)

$$F(x,y) \quad * \quad H(u,v) \quad = \quad G\,(x,y)$$

$F(x,y)$:

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$H(u,v)$: $* \frac{1}{9}$

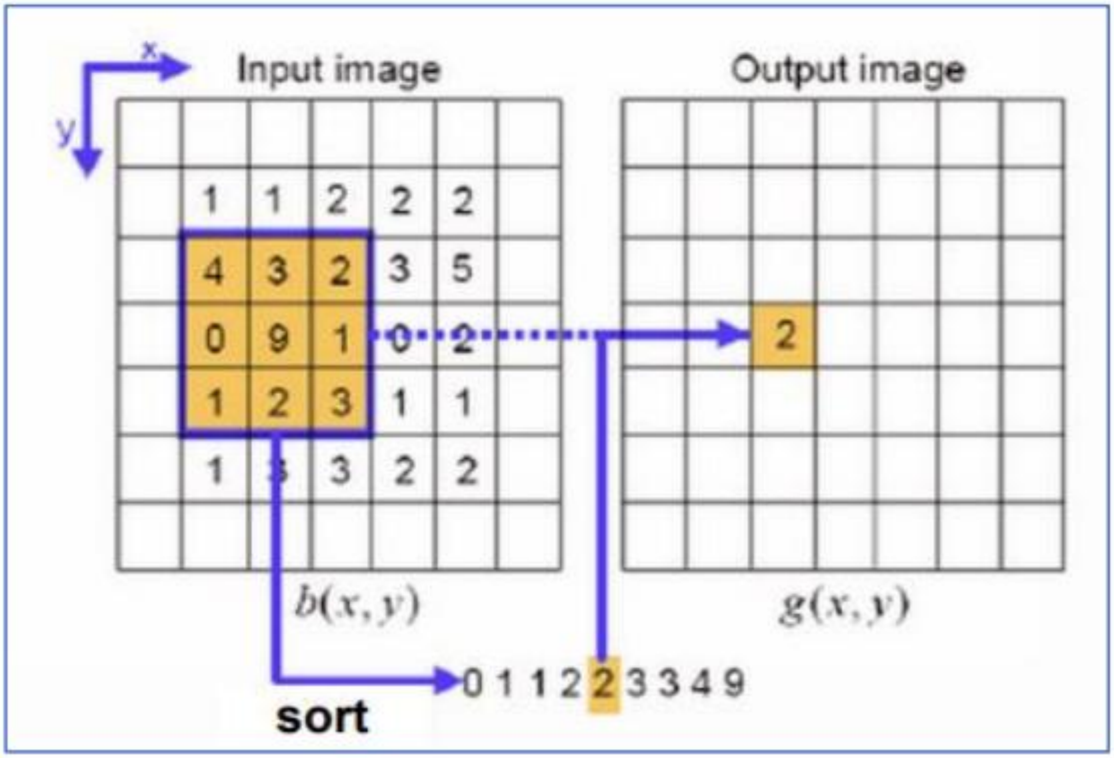
1	1	1
1	1	1
1	1	1

“box filter”

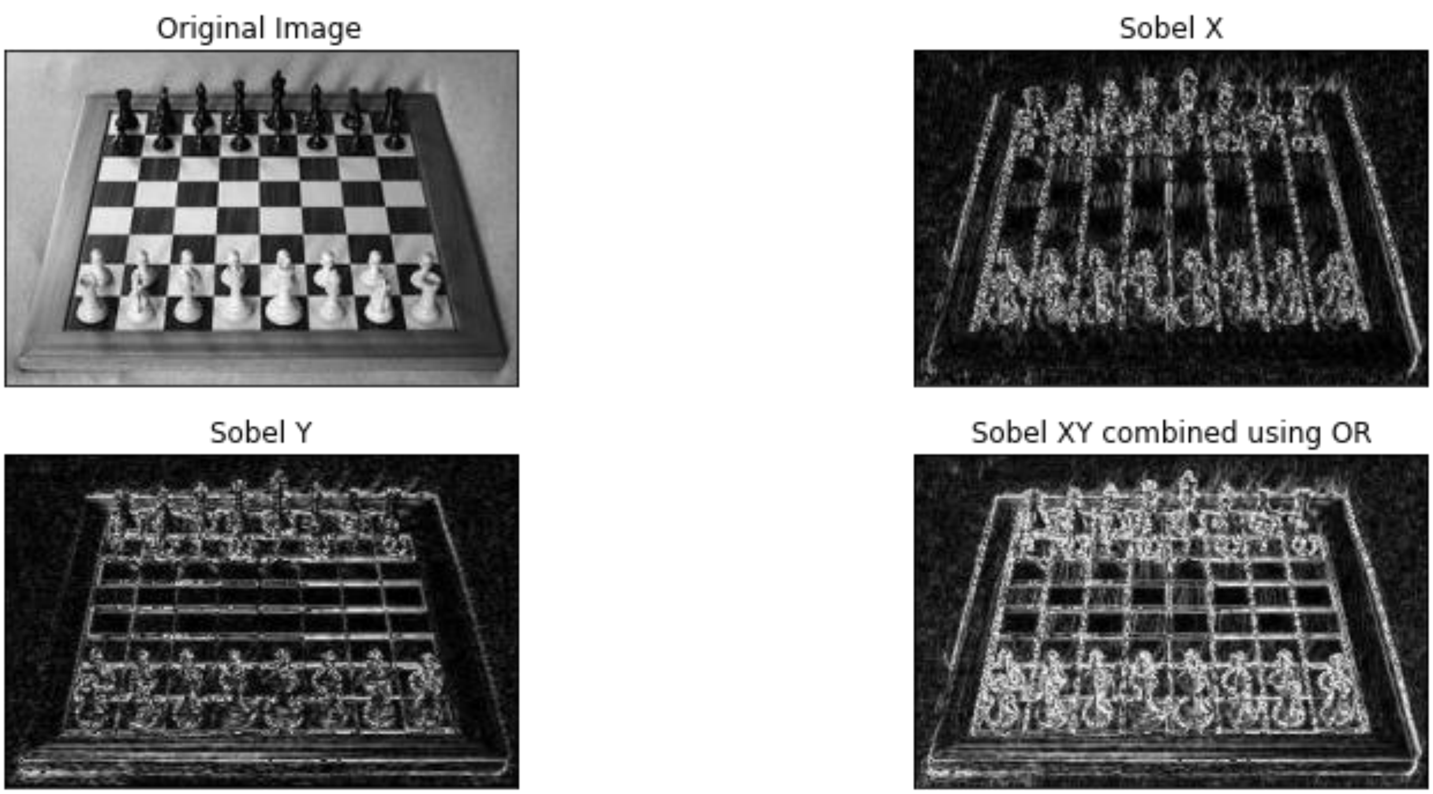
$G(x,y)$:

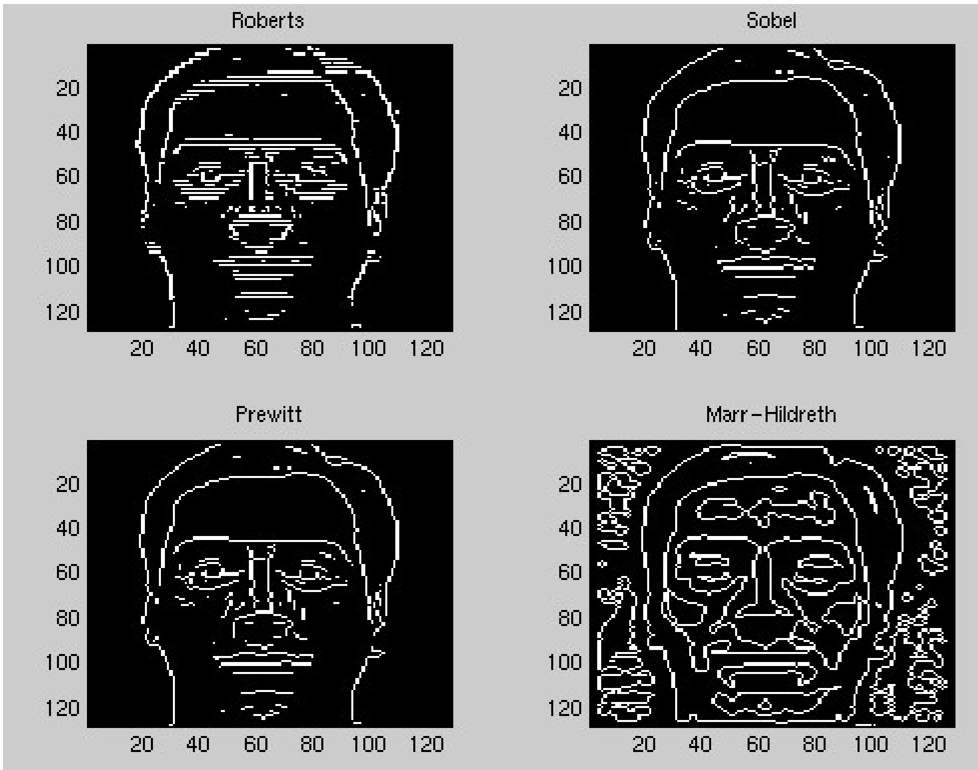
	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

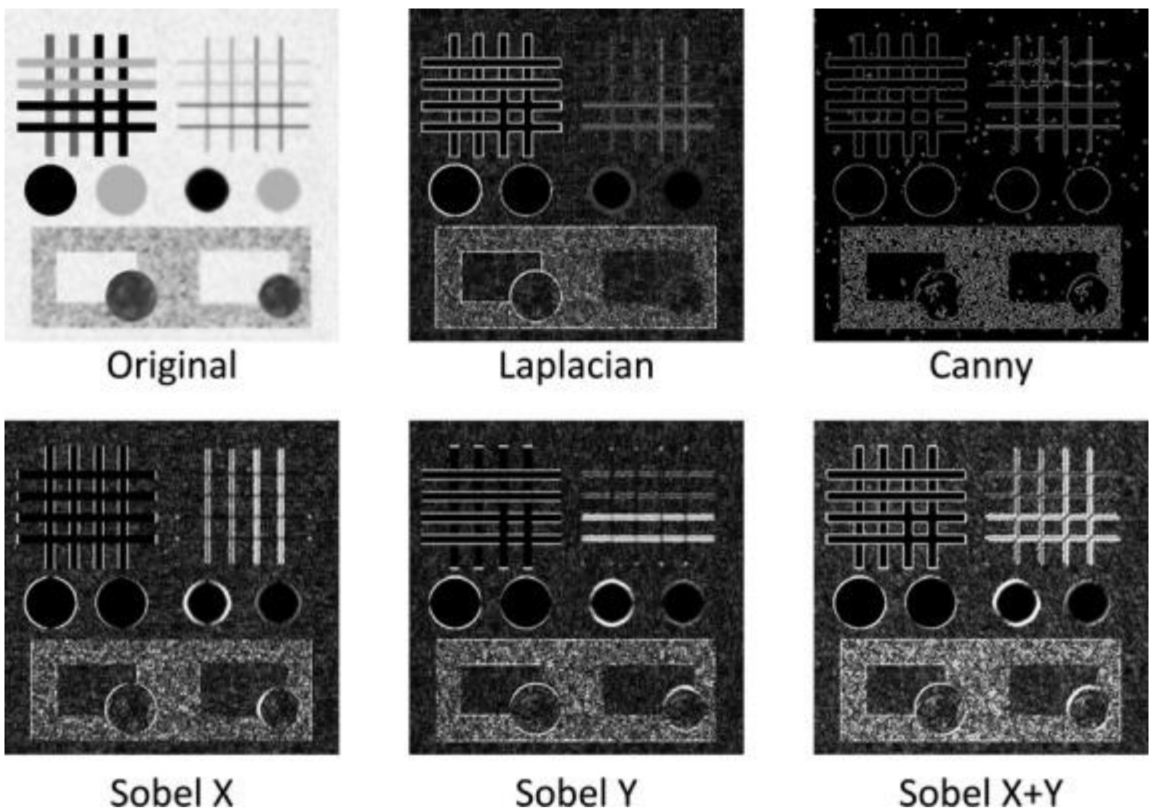
Median Filter



Sobel Filters







- Canny is the **edge detection algorithm** not a filter
- Marr-Hildreth is the **edge detection algorithm** not a filter
- Roberts is a filter (operator)