Experimental Determination of Resistive Torque Load on CVT Secondary Pulley

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Abstract

Achieving high efficiency and endurance are two of the most crucial aspects of SAE's Baja car competitions. Thus, it becomes necessary to optimize the Bruin Racing's Baja vehicle to specific race conditions. In order to utilize a mathematical model to aid in the systematic tuning of a continuously variable transmission (CVT) for race conditions, it is necessary to obtain measurements of the geometry of the CVT, as well as the internal statics and dynamics of the vehicle. This report aims to mathematically describe one of these unknown variables, specifically the resistive torque, acting on the secondary sheave by the rear-end of the drivetrain, as it may vary with speed and angle of inclination to the ground. A model of this torque load, used in conjunction with an analysis of CVT dynamics, allows Baja SAE teams to determine an optimized CVT flyweight value for competition, increasing the vehicle's efficiency and adaptability. Through simple and low-cost experimentation techniques, the resistive torque is found to be nearly constant with velocity. Furthermore, the angular dependence is shown to follow elementary analytic models to a high degree.

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Symbol	Description	Units
τ_R	Resistive torque on the secondary of the CVT as a function of velocity and angle	[in lbs]
v	Velocity of the vehicle	[MPH]
θ	Incline angle	[deg]
τ_v	Resistive torque as a function of velocity	[in lbs]
τ_θ	Resistive torque as a function of incline angle	[in lbs]
$a_l(t)$	Linear deceleration of the vehicle as a function of time	[MPH/s]
I_s	Moment of inertia of the secondary of the CVT	[lbm ft^2]
$\alpha_s(v)$	Angular deceleration of the secondary as a function of the vehicle's velocity	[rad/s^2]
$\alpha_w(v)$	Angular deceleration of the wheels of the vehicle as a function of the vehicle's velocity	[rad/s^2]
R_w	Radius of the wheels	[in]
p	Gearbox ratio	[1]

1 Introduction

The Baja vehicle's drivetrain is a complex system. Like all objects, it loses energy as it travels, meaning its efficiency is less than 100%. Though an efficiency of 100% is impossible, the ultimate goal while designing a vehicle is having the highest possible efficiency. Energy is lost in every system, therefore every system within the Baja vehicle could be analyzed and tinkered with in an effort to reduce losses and optimize performance. Earlier this year, Bruin Racing senior Brendon Anderson conducted a thorough physical analysis of the dynamics of the CVT of the Baja vehicle, defining the physical relationships present within the interactions of the CVT. Through this analysis, Anderson determined a function for the optimized flyweight mass depending on said physical relationships of the CVT. At this optimized flyweight mass, the engine will naturally run at the driver's desired RPM within the CVT's shift range. To validate and enhance this flyweight tuning method, the resistive loads acting on the CVT during operation are needed. The resistive torque, being a part of the optimization calculations, must be precisely measured. As the entire drivetrain system is extremely difficult to mathematically model, it has been decided to take an experimental approach rather than a theoretical approach. In effect, this report aims to show, from beginning to end, the thought process, experimental procedure, and results of the tests conducted on the vehicle to observe its dynamics and its relationship to the resistive torque at the secondary.

2 Theory

2.1 Separation of Variables

The function τ_R will be separated into two different functions, one dependent solely on velocity and the other dependent solely on the front to back tilt, θ.

This separation of the resistive torque is justified by all the possible resistive forces that, under any condition, may act on the drivetrain of the vehicle from the secondary pulley to the wheels. If all of these forces depend on either θ or v, but never both, then a separation of variables is just. All significant forces present that could contribute to τ_R are shown:

$$\tau_R = \tau_R(F_{aero}, F_{gear}, F_w, F_{rr}),$$

where F_{aero} is aerodynamic resistance, F_{gear} is gearbox resistance, F_w is the resistive force due to gravity, and F_{rr} is rolling resistance.

2.1.1 Aerodynamic Resistance

Due to the difficulty of modeling and predicting variations in wind during competition, the aerodynamic resistance due to gusts of wind will be considered to be negligible. This leaves the vehicle's total aerodynamic resistance to be only its drag force, which is

$$F_{aero} = kv^2.$$

As shown, the aerodynamic resistance is a function of only velocity and therefore F_{aero} is only dependent on velocity and not angle.

2.1.2 Resistive Forces in the Gearbox

The major contributor to F_{gear} is the meshing of the gears, in which energy is lost due to friction between the moving gear teeth. This friction forces produces a torque opposite the direction of motion.

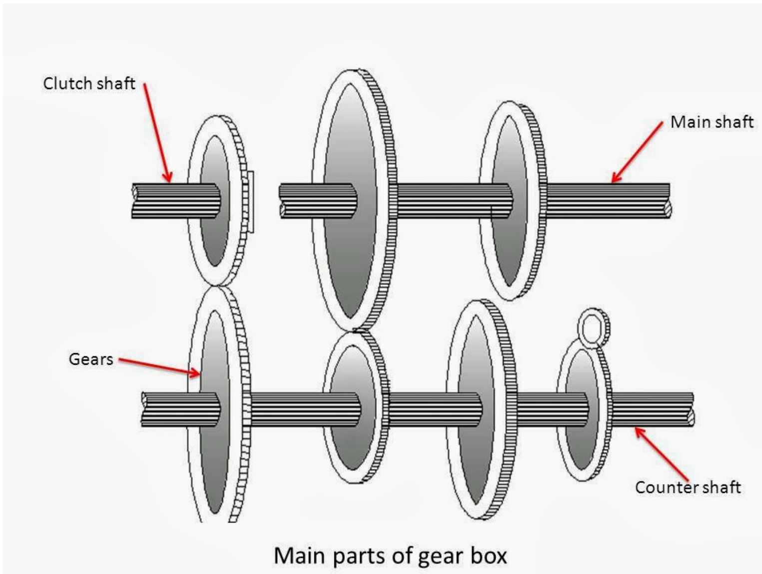

Figure 1: Gears locked rotationally.

Consider Figure 1. Each gear that is meshed with another is constrained by the shaft going through its center, which is attached to the outside parts of the car, which are fixed. This fixes the gears axially; the center-to-center distance of the gears remains constant. The meshing of the gears further locks each gear rotationally and along the shaft. As the gears are locked in all directions, they are always at a constant distance from each-other. Consider also that the friction forces between the two meshed gears depends only on the normal force between them and the coefficient of friction.

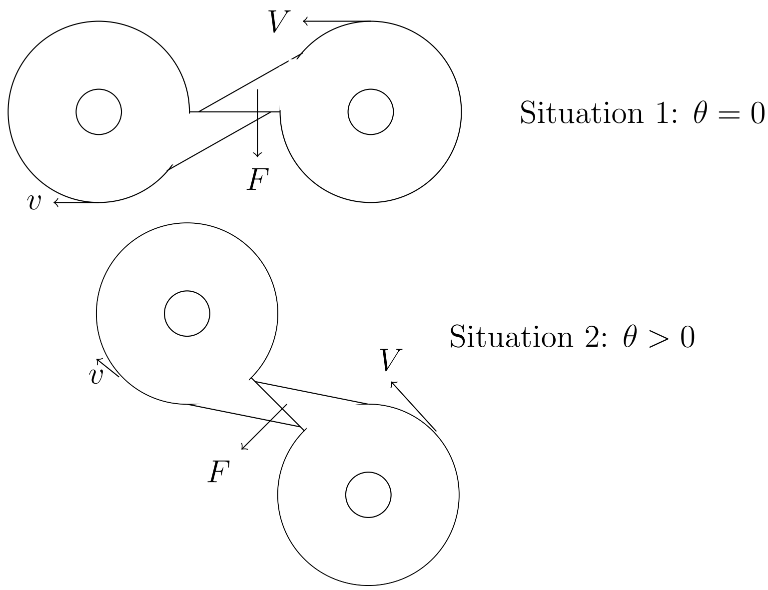

Figure 2: Angle has no effect on forces between teeth.

In Figure 2, the arrow labeled F is the force of the right gear pushing on the left gear to maintain a constant velocity. As previously explained, the position of the gears is locked by the shafts; the relative distances between gears and the normal forces between the gears do not change when the angle θ changes. Therefore, the frictional force between the two gears depends only on the force, F, with which one gear pushes on the next.

As energy is lost between gears due to a multitude of factors, each successive gear has a slightly lower energy and velocity. This is why there is a force between the two teeth at a constant velocity: the force F accounts for the loss of energy between gears to keep all gears moving at a constant velocity. The force F between the two gears depends on the difference in energy between the two gears. The greater this force F, the greater the normal force between the gears, and the greater the friction forces between the two gears.

As the resistive force at play (friction between the two gears) depends only on the energy lost between gears, F_{gear} depends on velocity. Indeed, at higher velocities energy losses become greater due to greater energy losses through heat loss. As the system heats up, the lubricant becomes thinner, reducing its minimizing effect on the friction between gears. Therefore, it is logical that as velocity increases, the resistive forces between the gears should increase.

This model is partially verified in literature through experimental means. As determined by Crook in 1961 [2], the velocity versus traction force interactions are roughly as shown in Figure 3:

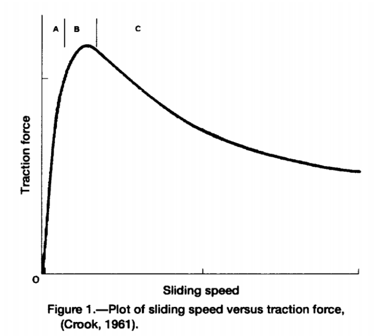

Figure 3: Plot of sliding speed versus traction force. [2]

The exact relationship between resistive forces and velocity is not paramount here, simply that such a relationship exists. The separation of variables remains justified, as F_{gear} depends only velocity but not θ.

2.1.3 Gravitational Resistance

The resistive force F_w originates from the force of gravity acting on the car. Gravity is a constant force in all conditions on the surface of the earth, including velocity.

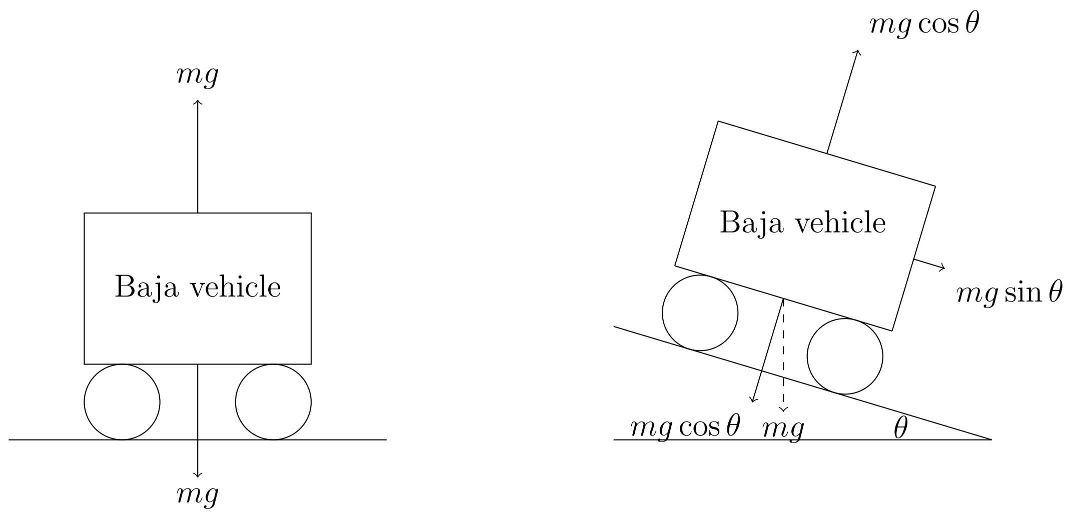

Figure 4: Gravitational forces acting on the Baja Vehicle at angle θ.

As can be seen in Figure 4, there is a net force acting backwards that varies depending on the angle θ. This is a resistive force that needs to be overcome. Therefore,

$$F_w = mg\text{sin}\theta,$$

where F_w depends only on the angle.

2.1.4 Rolling Resistance

This theory section relies heavily on literature, as the dynamics of rolling resistance are very complex and are a wide area of research and experimentation. As F_{rr} is a sort of friction force which depends on normal force, it can be written as

$$F_{rr} = C_{rr}N,$$

where C_{rr} is the coefficient of rolling resistance and N is the normal force. Here is the accepted general expression for the rolling resistance at varying velocities and no incline angle:

$$C_{rr} = 0.005 + \frac{1}{t}\left[0.01 + 0.0095\left(\frac{v}{100}\right)^2\right].$$

where t is a constant reflecting the inflation of the tire [3].

C_{rr}'s dependence on velocity is derived from the slipping movement of the tire and different velocities. Assuming that the car does not slip in this theoretical analysis, the role of velocity in the rolling resistance can be ignored. Even if in actuality there *is* significant slip, it plays a minimal role in the rolling resistance. For the typical Baja car, the largest deviation would be less than 5%, a minimal difference in a force which is already small compared to other forces at play.

Besides sliding friction, the main factor of rolling resistance is hysteresis, a process in which energy is lost due to the continuous tire deformation under the load of the vehicle. In essence, the energy required to deform the tire is greater than the energy released when the tire returns to its normal shape. Thus, any factor that affects the deformation of the tire affects the rolling resistance of the vehicle. These factors include normal force and geometric tire values.

As θ varies on an incline, the normal force perpendicular to the slope also varies; its dependency is $mg\cos\theta$. Because the tire sinkage depends directly on the weight upon it, tire sinkage should *also* vary directly with $\cos\theta$.

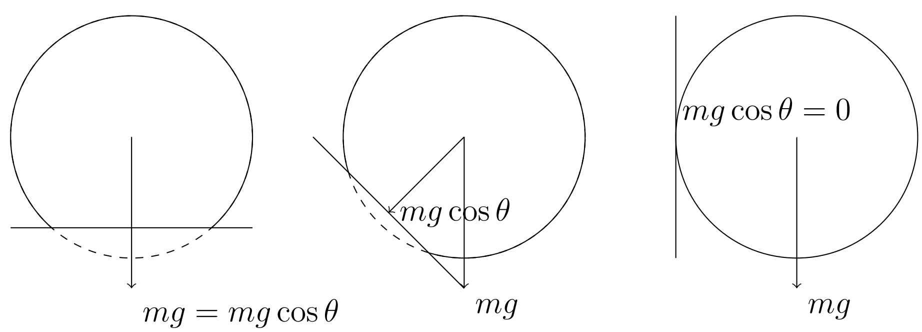

Figure 5: Tire sinkage depths at varying angle θ

This makes sense, as when the slope increases, some of the gravitational force pushing down on the wheels is turned into a backwards force that drags the car down the slope. Therefore there should be less force pushing the car into the slope as θ increases. As tire deformation is affected by the angle of the incline, and slippage of tires is ruled out, F_{rr} depends only on the angle and not on the velocity of the car.

2.2 Velocity-Dependent Resistive Torque

For this theoretical modeling of τ_v, analysis starts with the forces at the secondary of the CVT and works outwards to the forces acting on the wheels of the vehicle. For this derivation, two assumptions will be made:

1. An experimentally determined a function *v(t)* of the car rolling on flat ground has been obtained (explained in procedure).

2. There is negligible tire slippage between the ground and tires.

The torques on the secondary pulley of the CVT dependent on velocity, τ_v, can be represented in terms of its moment of inertia and its radial acceleration. Because the desired function τ_v is in terms of velocity, the angular acceleration will be represented as a function of the velocity of the Baja vehicle;

$$\tau_v = I_s \alpha_s(v). \tag{1}$$

It is difficult to analyze the deceleration of the secondary pulley of the CVT, instead it is much simpler to analyze the motion of the wheels of the vehicle and relate this to the motion of the CVT. The wheels and the secondary pulley of the CVT are connected by a series of rotating shafts and gears. As the car decelerates, both the wheels and the lips of the secondary pulley of the CVT begin to spin more slowly; their angular velocity is decreasing. Thus, the wheels and secondary must have some angular acceleration, $\alpha_w(v)$ and $\alpha_s(v)$, respectively. As the wheels and the secondary are linked by a series of spinning shafts and gears, the relationship between the two angular decelerations is related by the gearbox ratio:

$$\alpha_s(v) = p\alpha_w(v). \tag{2}$$

Using the radius of the wheel, α_w can be found in terms of linear acceleration as

$$\alpha_w(v) = \frac{a_l(v)}{R_w}. \tag{3}$$

So, in combining equations (2) and (3),

$$\alpha_s(v) = \frac{pa_l(v)}{R_w}, \tag{4}$$

which gives the angular acceleration of the secondary in terms of velocity.

The variables p and R_w are known values, so $a_l(v)$ must be found in known terms. For this, an experimentally determined velocity profile $v(t)$ is used. Because the assumption of negligible tire slippage was made, the velocity of the car, *v(t)* is also the velocity of the wheels;

$$a_l(t) = \frac{dv(t)}{dt}.$$

Therefore, substituting (4) into (1) gives

$$\tau_v = \frac{I_s p a_l(v)}{R_w}, \tag{5}$$

where $a_l(v)$ is determined from the experimentally determined velocity profile $v(t)$.

2.3 Angle-Dependent Resistive Torque

As was previously covered, there are two resistive forces that depend on the front-to-back tilt that contribute to the cumulative resistive forces. This angle-dependent torque is

$$\tau_\theta = \tau_\theta(F_w, F_{rr}). \tag{6}$$

As can be seen in Figure 4, the backwards force by gravity F_w is given by the expression

$$F_w = mg\sin(\theta). \tag{7}$$

The rolling resistance is given by

$$F_{rr} = C_{rr}N,$$

where C_{rr} is the coefficient of rolling resistance between the tires and the ground, and N is the normal force acting on the wheel. Because the normal force is a function of the angle θ as is shown in Figure 5, the equation for F_{rr} becomes

$$F_{rr} = C_{rr}mg\cos(\theta). \tag{8}$$

The coefficient of rolling resistance is typically modeled as

$$C_{rr} = \sqrt{\frac{z}{D_w}},$$

where z is the tire sinkage depth and D_w the wheel diameter. Because such equations are somewhat approximate, it will be assumed that the *proportionality* of the equation is correct, and thus

$$C_{rr} \propto \sqrt{\frac{z}{D_w}}. \tag{9}$$

The sinkage of a tire depends on the weight on it and the normal force on the wheels at any angle depends on the angle θ, leading to the assumption that tire sinkage carries the same proportionality. Then,

$$z \propto mg\cos\theta.$$

Therefore,

$$C_{rr} = \widetilde{K_{rr}}\sqrt{\frac{mg\cos\theta}{D_w}},$$

where $\widetilde{K_{rr}}$ is a constant of proportionality. Absorbing the wheel diameter into $\widetilde{K_{rr}}$ and replacing it with K_{rr} gives

$$C_{rr} = K_{rr}\sqrt{mg\cos\theta}. \tag{10}$$

Thus, due to the angular dependance of the sinkage depth and the normal force, combining equations (8) and (10) gives

$$\begin{aligned} F_{rr} &= K_{rr}(\sqrt{\text{mgcos}\theta})\text{mgcos}\theta, \\ F_{rr} &= K_{rr}(\text{mgcos}\theta)^{\frac{3}{2}}, \end{aligned} \tag{11}$$

where K_{rr} can be experimentally determined. Therefore, the angle-dependent resistive force acting on the vehicle is

$$F_\theta = K_{rr}(mg\cos\theta)^{\frac{3}{2}} + mg\sin\theta. \tag{12}$$

(12) is the force acting backwards on the car. Under the assumption that the force acts on the wheels and is translated through the drivetrain to the secondary, the proportionality is

$$T_\theta \propto K_{rr}(mg\cos\theta)^{\frac{3}{2}} + mg\sin\theta, \tag{13}$$

where the proportionality varies by a constant defined by geometry.

3 Procedure

3.1 Velocity-Dependent Experiment

3.1.1 Description

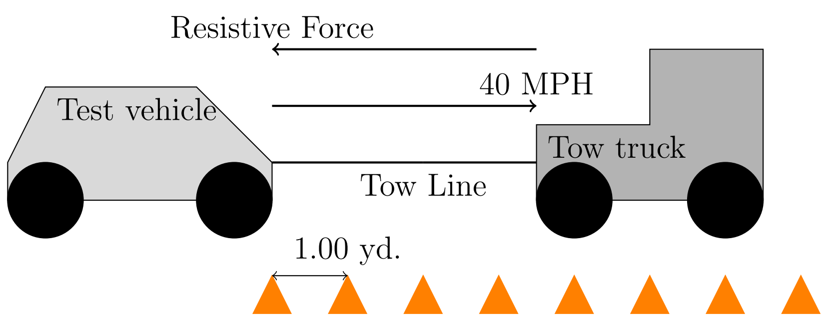

Figure 6: Schematic layout of experiment.

In order to obtain the natural deceleration of the test vehicle as a function of velocity v, and as a result of forces acting towards τ_R, the CVT belt must be disconnected from the vehicle to remove the CVT's internal resistance from affecting the data. A tow truck is used to tow the vehicle to 40 MPH before releasing the tow line such that the vehicle starts to decelerate on its own. Since the cones running alongside the test vehicle are a fixed distance apart, the time between each cone pass is inversely proportional to speed of the test vehicle.

3.1.2 Tow-Release Mechanism

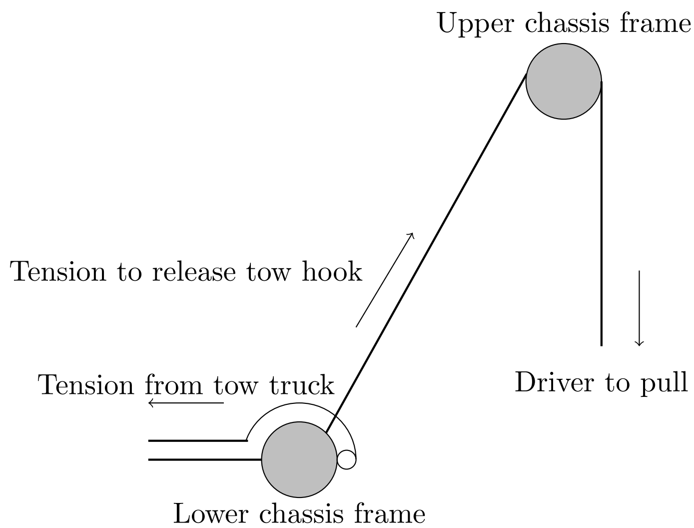

Figure 7: Cross-section of tow-release mechanism.

A chord is tied to the tow hook before attaching it to the lower chassis frame, such that by lightly pulling the other end of the chord, the hook slips off the frame and the vehicle is no longer tethered by the tow hook.

Figure 8: Hook attached to lower chassis frame.

Figure 9: Tow release system.

3.1.3 Measurements

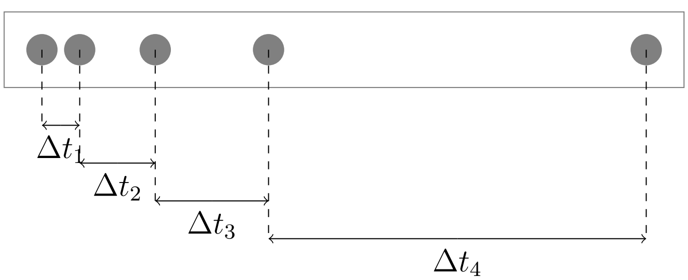

Figure 10: Visual representation of slower speed and greater time interval.

Since the cones are a fixed distance Δx apart, the time between each cone Δt is inversely proportional to the instantaneous speed of the vehicle. Thus applying the formula

$$v = \frac{dx}{dt} \approx \frac{\Delta x}{\Delta t},$$

a graph of velocity against time is able to be obtained.

Figure 11: View from back of Tow truck. Note the equally spaced cones on the right.

3.1.4 Materials

Item	**Quantity**
Vehicle	$\times$ 1
Tow Truck	$\times$ 1
Tow Line w/ release mechanism	$\times$ 1
Cone	$\times$ 100
Meter Rule	$\times$ 1
GoPro Video Recorder	$\times$ 2
GoPro Mounts	$\times$ 2
Driver	$\times$ 2
Spotter	$\times$ 1
Helmet	$\times$ 1
Secluded area w/ wide space	$\times$ 1

3.1.5 Experimentation Procedure

1. Disconnect CVT Belt.
2. Attach GoPros to view tires and cones in single line of vision.
3. Lay out cones at fixed distance apart using meter rule.
4. Line test vehicle and tow truck up, approximately 50 meters away from start of cone runway.
5. Attach Tow Line with release mechanism.
6. Start GoPros recording.
7. Driver 1 to sit in Vehicle to steer and operate brakes in emergency.
8. Driver 2 to drive Tow Truck.
9. Spotter to sit backwards in Tow Truck to communicate between both drivers.
10. Gradually accelerate Tow Truck to 45 MPH.
11. At 45 MPH (above intended range), signal to Vehicle that top speed is reached.
12. Vehicle driver to release tow line and maintain straight-line path alongside the cones.
13. Tow Truck to turn away from path of Vehicle.
14. Set of data points complete when Vehicle comes to complete stop.

3.2 Angle-Dependent Experiment

3.2.1 Description

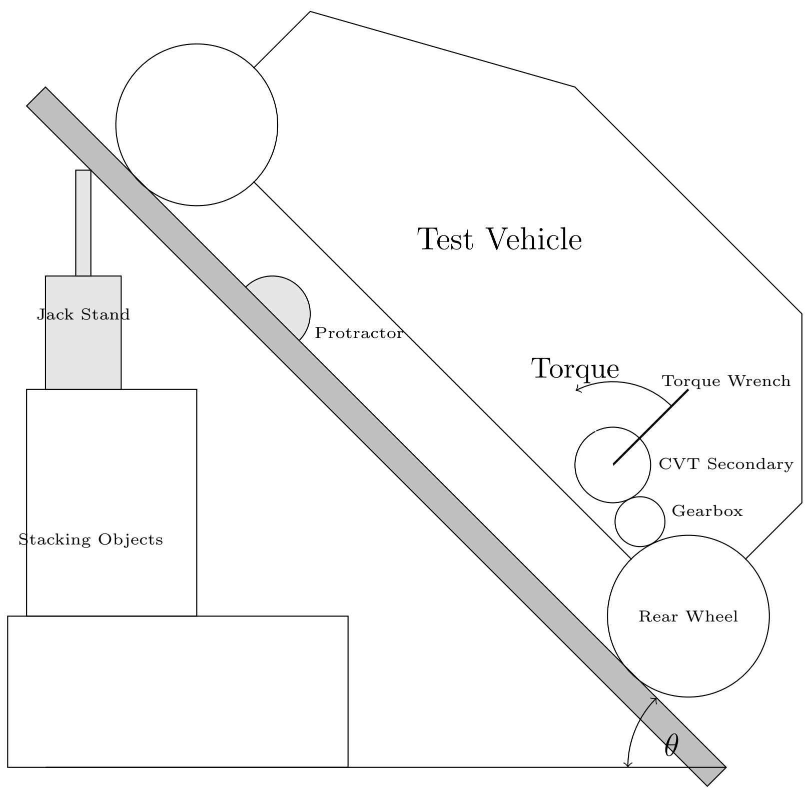

Figure 12: Schematic layout of experiment.

In order to obtain the resistive torque of the vehicle as a result of forces acting towards τ_R, a torque wrench is mounted onto the CVT's secondary sheave, and a torque will be applied until the test vehicle starts to move. The same driver who performed the velocity-dependent experiment must sit in the test vehicle for the same overall weight of the vehicle. This procedure is then repeated for various angle θ from 0° to 45°.

Figure 13: Overall view of experiment.

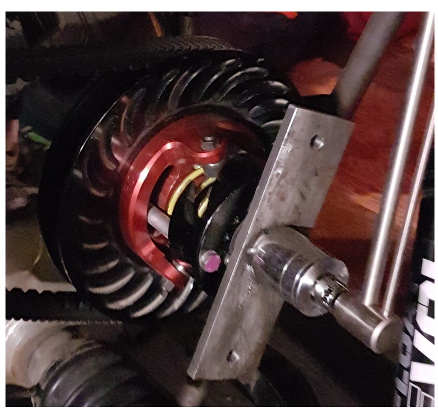

Figure 14: Custom plate to mount torque wrench onto CVT's secondary sheave.

The custom plate is locked rotationally to the sheave using the bolts on the sheave. The torque wrench can then apply torque onto the plate via the nut welded onto it, and in turn it applies a torque onto the CVT's secondary sheave.

3.2.2 Materials

Item	Quantity
Vehicle	$\times$ 1
Protractor	$\times$ 1
Torque Wrench	$\times$ 1
Custom Plate	$\times$ 1
Driver	$\times$ 1
Person 1 (apply torque)	$\times$ 1
Person 2 (read and record torque)	$\times$ 1
Ramp	$\times$ 1
Stacking Objects	Assorted
Jack Stand	$\times$ 1

3.2.3 Experimentation Procedure

1. Adjust ramp angle using jack stands and stacking objects, ensure ramp is stable.
2. Measure and record ramp angle using protractor.
3. Mount custom plate onto CVT's secondary sheave.
4. Push test vehicle up ramp, and hold in position with brakes.
5. Ensure rear wheel is fully on ramp.
6. Driver to climb into vehicle, and operate brakes.
7. Attach torque wrench to custom plate, and gradually apply torque.
8. Near torque value from previous angle, driver to release brakes.
9. Apply more torque, until tire visibly moves.
10. Record the value of torque indicated on torque wrench.

11. Repeat measurements for the same angle 3 times.

    Note: Do not let test vehicle rest for too long on the same side of tires, as prolonged state of rest will non-negligibly deform tires and affect experimental results.

12. Repeat steps for angles $0°$ to $45°$.

4 Data Analysis

4.1 Moment of Inertia

The moment of inertia about the axis of the shaft for the secondary of the CVT can be approximated by the "Mass Properties" feature in the SolidWorks 3-D model. It gives a value of $I_s = 17.34$ lbm$\cdot$ in^2, or $I_s = 0.1204$ lbm$\cdot$ ft^2.

4.2 Velocity Dependent Results

The first step is to calculate the velocity of the vehicle using the timestamps obtained through the deceleration experiment. To do so, the video footage is converted to numerical data points for analysis. The video is replayed in slow-mo frame-by-frame and the GoPro time stamp is recorded as soon as each cone is fully inside a frame.

Having acquired the time stamps of the vehicle at every cone for each trial run, the vehicle's velocity is calculated using the central difference method:

$$v_k = \frac{dx_k}{dt} = \frac{x_{k+1} - x_{k-1}}{t_{k+1} - t_{k-1}}. \tag{14}$$

(14) is applied in MATLAB to find velocity with respect to both time and distance. It is used again on the velocity data set to calculate the acceleration of the vehicle at each cone and at each time stamp. The velocity and acceleration of the vehicles with respect to distance and time are shown in Figures 15 to 17.

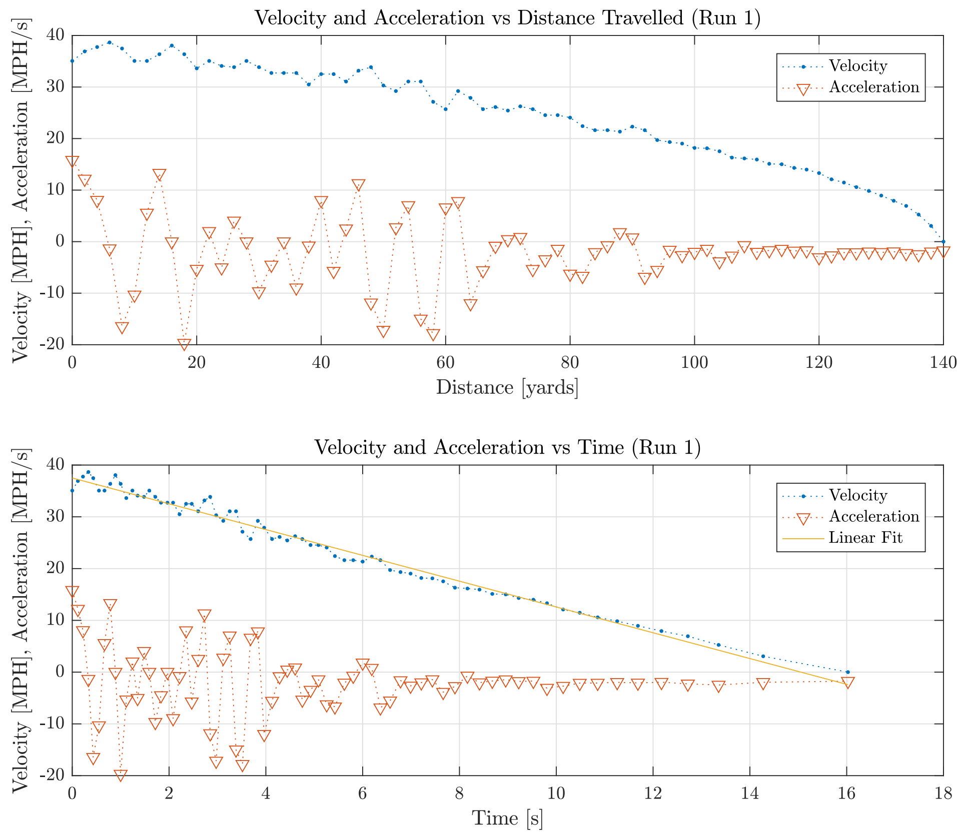

Figure 15: Calculated velocity and acceleration with respect to distance and time for Run 1.

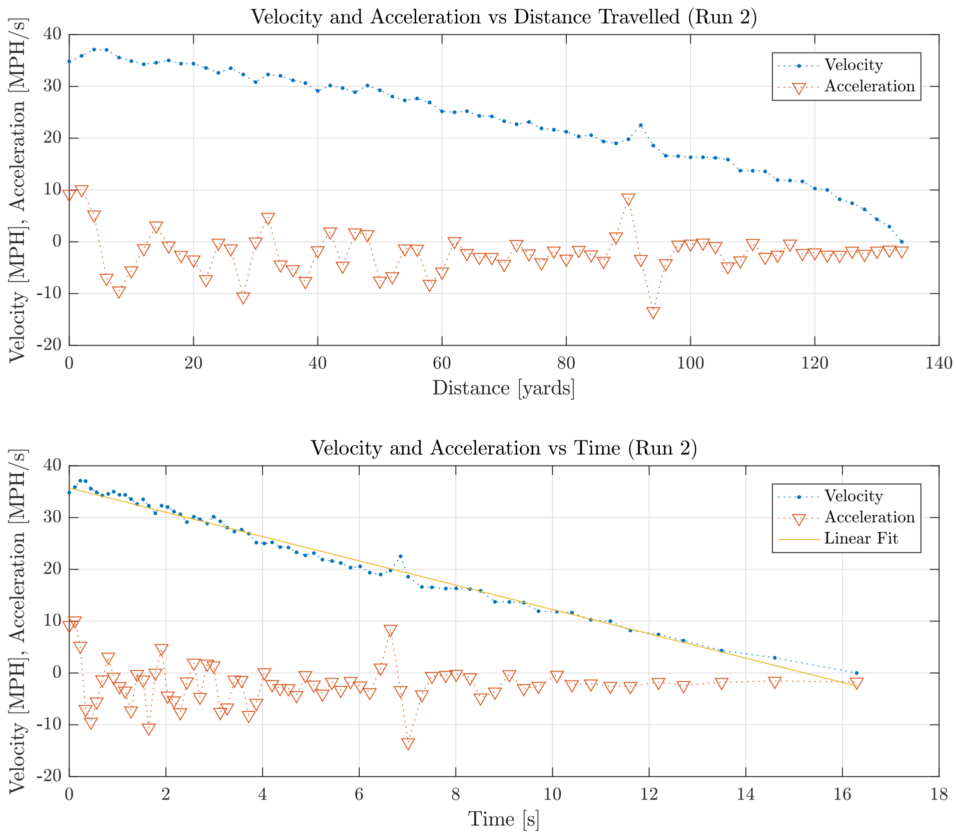

Figure 16: Calculated velocity and acceleration with respect to distance and time for Run 2.

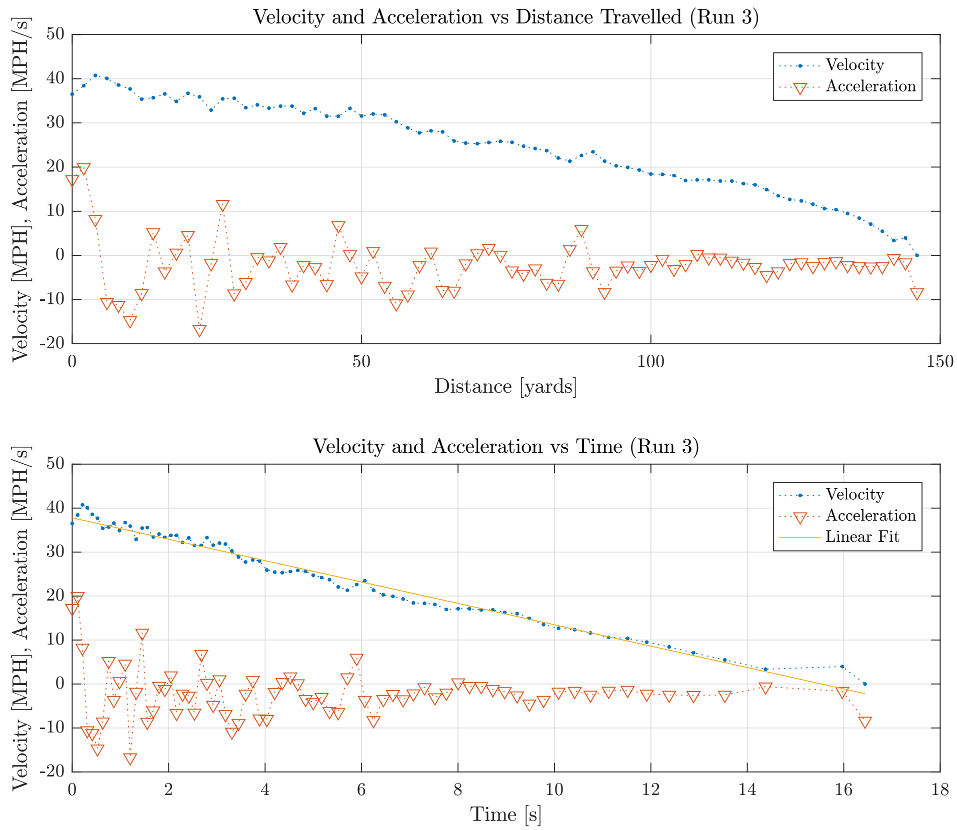

Figure 17: Calculated velocity and acceleration with respect to distance and time for Run 3.

After plotting the velocity and acceleration data, the linear regression and the correlation coefficient is calculated using Microsoft Excel's "Data Analysis" tool pack in order to determine whether a best fit line would be applicable. The correlation coefficient R for all three trials are tabulated in Table 1. The values of R are very close to one, and therefore the assumption that a linear fit for velocity is appropriate holds true. These best fit lines have been drawn in the velocity and acceleration vs. time plots of Figures 15 to 17, respectively.

Because the velocity of all three trial runs were linear, the vehicle is assumed to have experienced a constant acceleration throughout its run.

Table 1: Average accelerations taken from the slope of the linear fit lines of the velocity vs time data sets.

Run	Avg. Accel. [mph/s]	Std. Deviation [mph/s]	$\bar{a}$ [mph/s]	R
1	-2.491			0.99219
2	-2.347	0.059	-2.423	0.99401
3	-2.432			0.99001

This constant acceleration is equal to its average acceleration, or the slope of the best fit line. The average accelerations for each trial run are shown in Table 1. The mean acceleration of all three trials, $\bar{a}$, is used to find the angular acceleration of the rear wheels. Substituting $\bar{a}$ into (3) gives

$$\alpha_w = \frac{\bar{a}}{R_w} = -3.708 \text{ rad/s}^2,$$

with radius of the wheel being $R_w = 11.5$ in.

The angular acceleration of the wheel can be related to the angular acceleration of the output shaft of the CVT through the fundamental law of gearing. By referring to Figure 18, the following equations can be used to relate the angular acceleration α_s with α_w.

Because the output shaft of the secondary is locked rotationally with the first gear in the gearbox,

$$\alpha_s = \alpha_{gear_1}. \tag{15}$$

The angular acceleration of the second gear is related by a ratio of the number teeth of the first gear (N_1) to the number of teeth of the second gear (N_2) multiplied by the angular acceleration of the first gear. That is,

$$\alpha_{gear_2} = \frac{N_1}{N_2}\alpha_{gear_1}. \tag{16}$$

The angular acceleration of the third gear is that of the second gear:

$$\alpha_{gear_3} = \alpha_{gear_2} = \frac{N_1}{N_2}\alpha_{gear_1}. \tag{17}$$

Again, using the fundamental law of gearing, angular acceleration of the fourth gear is the ratio of the number of teeth of the third gear (N_3) to the

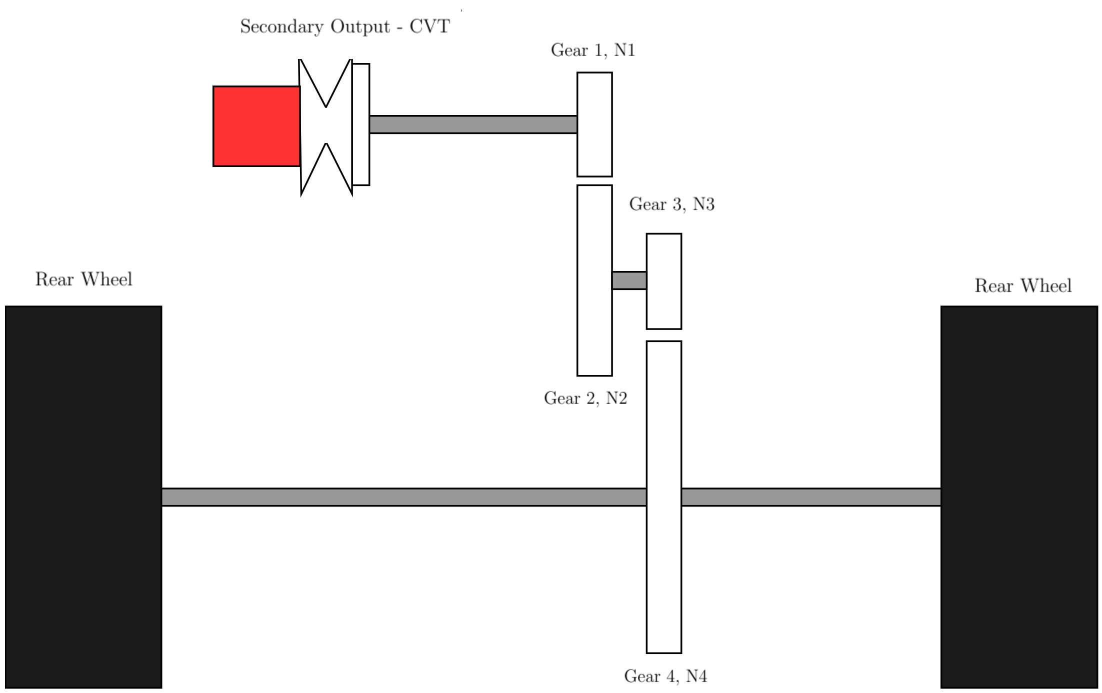

Figure 18: The diagram illustrates the route of power transfer from the secondary of the CVT to the wheels. The gears and their tooth numbers are displayed in Table 2.

number of teeth of the fourth gear (N_4) multiplied by the angular acceleration of the third gear;

$$\alpha_{gear_4} = \frac{N_3}{N_4}\alpha_{gear_3} = \frac{N_1}{N_2}\frac{N_3}{N_4}\alpha_{gear_1}. \tag{18}$$

Finally, since the rotation of the wheels are locked with the rotation of the fourth gear, the angular acceleration of the wheel can be written as

$$\alpha_w = \alpha_{gear_4} = \frac{N_1}{N_2}\frac{N_3}{N_4}\alpha_{gear_1}. \tag{19}$$

The angular acceleration of the secondary, α_s, is calculated using the known tooth counts for the gears in the gearbox which are tabulated in Table 2.

Table 2: The tooth count and angular accelerations of each item in Figure 18. As expected, the drivetrain reduces the input angular acceleration by a gear ratio.

	Secondary	Gear 1	Gear 2	Gear 3	Gear 4	Rear Wheels
N	N/A	18	41	17	65	N/A
α [rad/s^2]	-32.29	-32.29	-14.18	-14.18	-3.708	-3.708

Thus, the velocity-dependent resistive torque applied at the secondary is

$$\tau_v = I_s \alpha_s = -0.167\,\text{lb} \cdot \text{in} = -0.0139\,\text{lb} \cdot \text{ft}, \tag{20}$$

with the negative sign denoting the opposition of the direction of motion.

4.3 Angle-Dependent Results

Using the incline and torque wrench method as outlined in the experimental procedure, a data set composed of 16 different angles was acquired. Three torque measurements were taken at every angle and averaged. The range of angles for the experiment, from 4° to 35.5°, is comparable to the range of angles that the vehicle will undergo in racing conditions.

In order to encapsulate the data set as a function, a best fit curve needs to be created and plotted onto the set of data. To determine the degree of the polynomial used to fit the curve, the "Curve Fitting" function is used in MATLAB to plot and compare the goodness of fit statistics of each curve. As shown in Table 3, the polynomial with the highest R value and lowest Root Mean Square Error (RMSE) is a polynomial with degree 1. This is a linear function which is represented by

$$\tau_\theta = 9.423\theta + 22.53. \tag{21}$$

Next, the theoretical model is compared to the linear model to verify its accuracy. The force applied to the rear wheels is

$$F_\theta = K_{rr}(mg\cos\theta)^{\frac{3}{2}} + mg\sin\theta, \tag{12}$$

which is then converted to the torque τ_θ by multiplying by the radius of the wheel, R_w. The torque that is applied to the surface of the rear wheels is then transmitted through a gearbox with a ratio of 1:8.709, thus giving

$$\tau_\theta = \frac{1}{8.709} F_\theta R_w = 1.3205(K_{rr}(mg\cos\theta)^{\frac{3}{2}} + mg\sin\theta), \tag{22}$$

where τ_θ is the theoretical model of the torque applied to the output of the secondary pulley of the CVT for every incline angle θ, and mg is the weight of the car and the driver, approximately 537 lb.

First, the constant K_{rr} must be determined. This is done by solving for K_{rr} for the first value of torque and angle. This gives a value of K_{rr} to be $K_{rr} = 1.141 \cdot 10^{-4}$. Substituting the known values, (22) is plotted alongside the data as shown in Figures 19 and 20 for easier juxtaposition.

Examining Figure 19, the theoretical model appears to be approximately linear from 0° to 35.5°. This is verified by the R value for (22) shown with respect to the data set. As it is very close to 1, the theoretical model has a strong linear correlation. However, the R value for the model is not nearly as close to unity as are the given polynomial fits, and so within the ranges 0° to 35.5°, the linear fit (21) is determined to be a more accurate model.

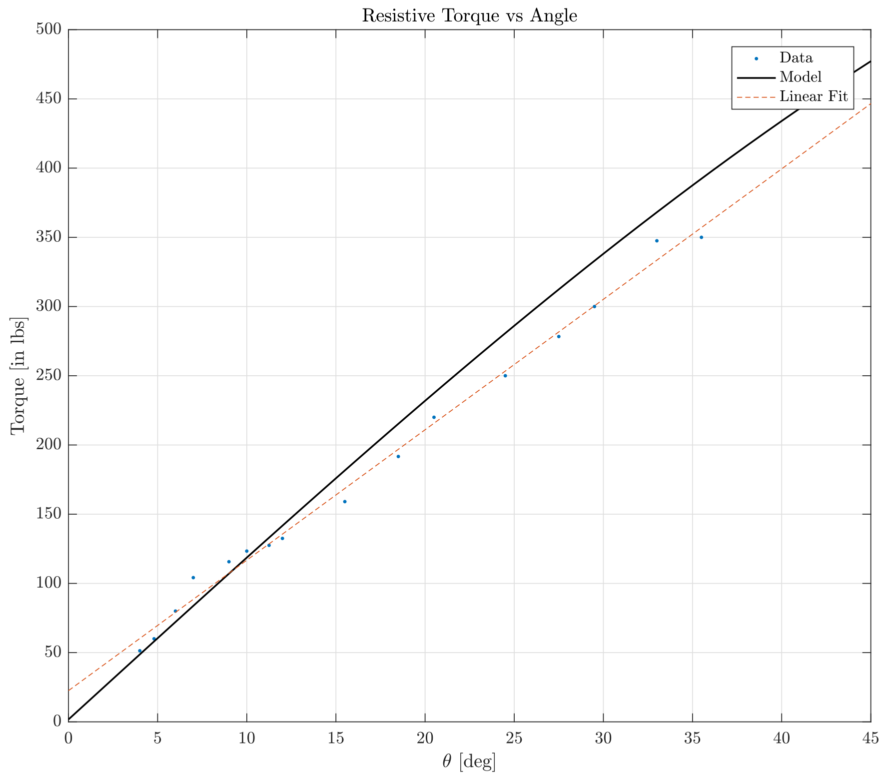

Figure 19: Applied torque versus the angle of inclination.

Table 3: Goodness-of-Fit statistical values from MATLAB.

Function	Sum Squared Error	R	Root Mean Square Error
τ_θ	1076	0.9650	8.468
Degree 1	913.7	0.9966	8.079
Degree 2	912.8	0.9964	8.379
Degree 3	884.5	0.9962	8.585

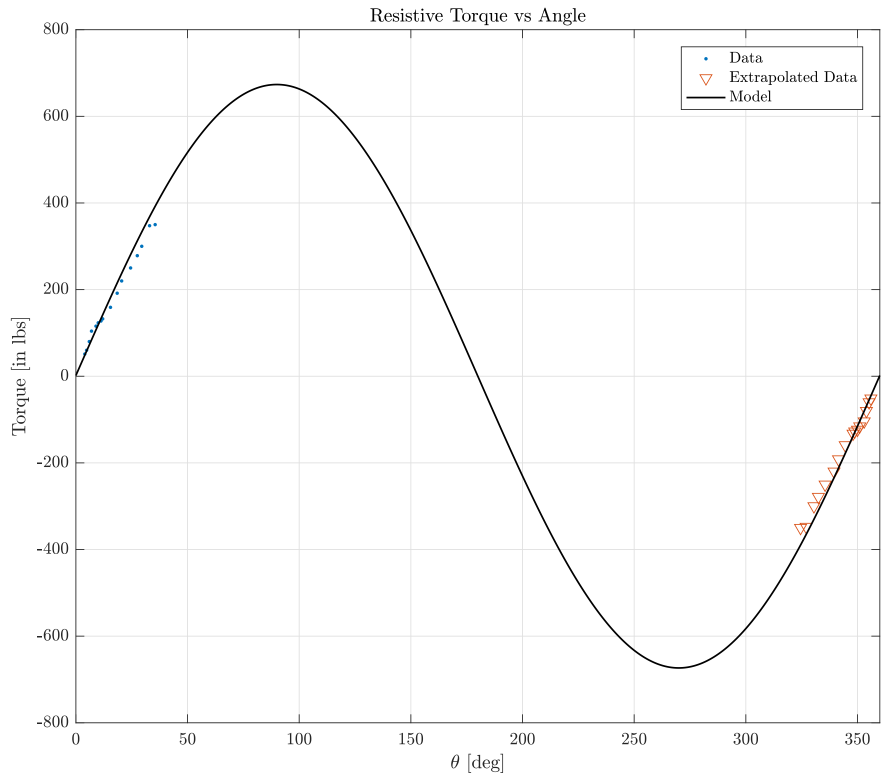

Figure 20: Applied torque versus the angle of inclination with the addition of extrapolated points.

To check the data's periodicity, the incline data is extrapolated assuming that the resistive torque of the vehicle on a decline can be approximated by taking its negative values from 0° to 90° with very minor deviations occurring from a change of the center of gravity and the direction of friction.

When plotted from $\theta = 0°$ to $\theta = 360°$, these extrapolated points create the final 30 degrees of the theoretical model, as shown in Figure 20.

Therefore, the theoretical model

$$\tau_\theta = 1.3205(K_{rr}(mg\cos\theta)^{\frac{3}{2}} + mg\sin\theta),$$

may be used to predict the amount of torque that will be applied to the vehicle for an angle θ past 35.5° or below $-35.5°$ with reasonable results.

5 Conclusion

In order to calibrate the CVT of a Baja SAE vehicle, it is crucial to have quantifiable values for the resistance. Ultimately, this resistance will be registered as a torque applied to the secondary pulley of the CVT. In the case that the functional dependence of τ_R on v is independent of that on θ, $\tau_R(v, \theta) = \tau_v(v) + \tau_\theta(\theta)$.

By measuring the natural deceleration of the vehicle after being towed at nearly 40 miles per hour, the torque applied to the secondary as a function of velocity may be calculated. From experimental findings, this torque is $\tau_v = -0.167\,\mathrm{lb} \cdot \mathrm{in}$, a nearly constant value for all velocities that the vehicle will ever race in.

The velocity-dependent torque value, although constant, is relatively small, and so the majority of the resistive torque must come from the angle-dependent function.

In order to measure the angular dependence of resistive torque, the vehicle was tested at a range of inclinations and torque was applied to the secondary output shaft when stationary. Due to the difficulty of raising and situating a 537 pound vehicle on a specific angle, the experimental data was limited to a small range of 0° to 36°. However, this range is similar to the off-road racing conditions that the vehicle will undergo during competitions.

The analytic model for τ_θ is validated by the use of statistical values and the fact that angles are periodic. For the purposes of the Baja SAE competition, a linear fit from angles ranging from 0° to roughly 35° can be used to predict the resistive torque of a vehicle with minimal error.

Having experimentally produced two independent functions that make up the resistive torque, the CVT's flyweights can be tuned for maximum performance in different racing conditions. The resistive torque of future vehicles can also be predicted with reasonable accuracy without the need to go through such rigorous experiments. However, in the future, more experimental tests should be performed to validate the findings stated in this report, so that the vehicle's CVT can be optimized even further.

References

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[2] Rebbechi, Brian, Fred B. Oswald, and Dennis P. Townsend. (1996). *Measurement of Gear Tooth Dynamic Friction*. [Online]. Available: https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19970001588.pdf.

[3] No Author. *Rolling Resistance*. [Online]. Available: http://www.engineeringtoolbox.com/rolling-friction-resistance-d_1303.html.

MATLAB Source Code

Listing 1: MATLAB data processing.

```
% Resistive Torque - Velocity Dependence

clear all; close all; clc;
set(0, 'defaulttextinterpreter', 'latex');

% Load Excel Data Document
filename = 'data.xlsx';
for i=1:3
data = xlsread(filename,i);
data = transpose(data);

% Initialize cone distance
distance = 3600*2/1760;
final = length(data);

%% Time Differences

% Create Time Array for Storage
time = zeros(1,final);                      % time stamps
time(1:final) = data(2,1:final);            % transfer from data.xlsx into easier-to-use time array


%% Velocity

velocity = zeros(1,final);
% Create Velocity Array for Storage

% Divide cone distance by ( time(k+1)-time(k-1) )
for k=1:final
    if k == 1
```

```
        % first cone exception
        velocity(k) = distance/(time(k+1)-time(k));
    elseif k == final
        % last cone exception
        velocity(k) = 0; %distance/(time(k)-time(k-1));
    else
        % Calculate average velocity using Central Distancing Method
        velocity(k) = 2*distance/(time(k+1)-time(k-1));
    end
end

%% Acceleration

% Create Acceleration Array for Storage
acceleration = zeros(1, final);

% Divide velocity by ( time(k+1)-time(k-1) )
for k=1:final
    if k == 1
        % first cone exception
        acceleration(k) = (velocity(k+1)-velocity(k))/(time(k+1)-time(k));
    elseif k == final
        % last cone exception
        acceleration(k) = (velocity(k)-velocity(k-1))/(time(k)-time(k-1));
    else
        % Calculate average acceleration using Central Distancing Method
```

```
        acceleration(k) = (velocity(k+1)-velocity(k-1))/(time(k+1)-time(k-1));
    end
end

%% Linear Regression

Cone = transpose(data(1,1:length(data)));
Time = transpose(time);
Velocity = transpose(velocity);
Acceleration = transpose(acceleration);

% Regression Values obtained through Excel Data Analysis Tool Pack
if i == 1
        b = -2.49136791;
        r = 0.992189721;
        c = 37.5;
elseif i == 2
        b = -2.34724;
        r = 0.994011;
        c = 35.72485;
elseif i == 3
        b = -2.43168;
        r = 0.990086;
        c = 37.77875;
end

y_trendline = b*time + c;

%% Plot and Tabulate:

% Plot 'v' and 'a' wrt to each cone
f1 = figure;
cone = 0:1:final-1;
subplot(2,1,1);
plot(cone.*2, velocity, ':.');
set(gca,'ticklabelinterpreter','latex');
```

```
t1 = sprintf('Velocity and Acceleration vs Distance Travelled (Run %i)', i);
title(t1);
xlabel('Distance [yards]'); ylabel('Velocity [MPH], Acceleration [MPH/s]');
hold on;
plot(cone.*2, acceleration, ':v');
legend({'Velocity', 'Acceleration'},'interpreter','latex');
hold off;
grid on;

% Plot 'v' and 'a' wrt to time
subplot(2,1,2);
plot(time, velocity, ':.');
set(gca,'ticklabelinterpreter','latex');
t2 = sprintf('Velocity and Acceleration vs Time (Run %i)', i);
title(t2)
xlabel('Time [s]'); ylabel('Velocity [MPH], Acceleration [MPH/s]');
hold on;
plot(time, acceleration, ':v');
grid on;
plot(time, y_trendline);
legend({'Velocity', 'Acceleration', 'Linear Fit'},'interpreter','latex');
hold off;

% Tabulate data
table_data = table(Cone, Time, Velocity, Acceleration);
InitialVelocity = velocity(1);
FinalVelocity = velocity(length(data));
TotalTime = time(length(data));
AverageAcceleration = (0-InitialVelocity)/TotalTime;
table_acceleration = table(InitialVelocity, FinalVelocity, TotalTime, AverageAcceleration);
```

```

% Display tables
disp(table_data);
disp(table_acceleration);
writetable(table_data, 'dataprocessed.xlsx', 'Sheet'
    , i);

% Save figures as EPS
if exist(t1, 'file') == 1
    delete(t1);
end
if exist(t2, 'file') == 1
    delete(t2);
end
tit = sprintf('Run%i', i);
%saveas(f1, tit, 'epsc');
%saveas(f2, t2, 'epsc');
end
%close all;
```

Listing 2: MATLAB data processing.

```
% Resistive Torque - Angle of Inclination Dependence
clear all; close all ; clc;
set(0, 'defaulttextinterpreter', 'latex');

% Load Excel Doc
filename = 'angle.xlsx';
angle = xlsread(filename, 'angle');
angle2 = xlsread(filename, 'angle2');

% Number of Data Points
points = size(angle,1);

% Separate columns and store into separate arrays
degrees = zeros(1,points);
torque = zeros(1, points);
degrees_extrapolated = zeros(1,points);
```

```
torque_extrapolated = zeros(1, points);

for i=1:points
   degrees(i) = angle(i,1);
   torque(i) = angle(i,2);
   degrees_extrapolated(i) = angle2(i,1);
   torque_extrapolated(i) = angle2(i,2);
end

% Polynomial Fit
final = 360;
x = linspace(0,final,final);
polyfit1 = 9.423*x + 22.53;

% Model
k_rr = 1.141e-4;                % input('k_rr = : ');       % k_rr is experimentally determined using "Curve Fitting"
weight = 510;%347+180*.9;                % 400 lbs for car; 180 lbs for sej
gearconstant = (65*41)/(17*18);
r_wheel = 11.5;
C = r_wheel/gearconstant;
model = C*(weight*sind(x)+ k_rr*(weight*cosd(x)).^1.5);

% Plot
g1= figure;
plot(degrees,torque,'.');
hold on;
set(gca,'ticklabelinterpreter','latex');
title('Resistive Torque vs Angle');
ylabel('Torque [in lbs]'); xlabel('$\theta$ [deg]');
hold on;
plot(x(1:46), model(1:46), '-k', 'LineWidth', 1);
plot(x(1:46), polyfit1(1:46), '--');
legend({'Data', 'Model', 'Linear Fit'},'interpreter','latex');
```

```
xlim([0 45]);
grid on;
hold off;
% name = 'TorquevsAngle.pdf';
% saveas(gca,name);
% system(['pdfcrop ',name,' ',name]);

g2= figure;
plot(degrees,torque,'.');
hold on;
plot(degrees_extrapolated,torque_extrapolated, 'v');
set(gca,'ticklabelinterpreter','latex');
title('Resistive Torque vs Angle');
ylabel('Torque [in lbs]'); xlabel('$\theta$ [deg]');
hold on;
plot(x, model, '-k', 'LineWidth', 1);
legend({'Data', 'Extrapolated Data', 'Model'},'
    interpreter','latex');
xlim([0 360]);
hold off;
grid on;
% name = 'Extrapolated.pdf';
% saveas(gca,name);
% system(['pdfcrop ',name,' ',name]);
```