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IDA* · Iterative deepening · Johnson ·

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· Prim · SMA*

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List of graph algorithms

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Edit | View history

Q

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Contents
Current events
Random article
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Contact us
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Community portal
Recent changes
Upload file

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What links here
Related changes
Special pages
Permanent link
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Edmonds' algorithm

This article is about the optimum branching algorithm. For the maximum matching algorithm, see Blossom algorithm.

In graph theory, **Edmonds' algorithm** or **Chu–Liu/Edmonds' algorithm** is an algorithm for finding a spanning arborescence of minimum weight (sometimes called an *optimum branching*). It is the directed analog of the minimum spanning tree problem. The algorithm was proposed independently first by Yoeng-Jin Chu and Tseng-Hong Liu (1965) and then by Jack Edmonds (1967).

Contents [hide]

- 1 Algorithm
 - 1.1 Description
- 2 Running time
- 3 References
- 4 External links

Algorithm [edit]

Description [edit]

The algorithm takes as input a directed graph $D=\langle V,E\rangle$ where V is the set of nodes and E is the set of directed edges, a distinguished vertex $r\in V$ called the *root*, and a real-valued weight w(e) for each edge $e\in E$. It returns a spanning arborescence A rooted at r of minimum weight, where the weight of an arborescence is defined to be the sum of its edge weights, $w(A)=\sum_{e\in A}w(e)$.

The algorithm has a recursive description. Let f(D, r, w) denote the function which returns a spanning arborescence rooted at r of minimum weight. We first remove any edge from E whose destination is r. We may also replace any set of parallel edges (edges between the same pair of vertices in the same direction) by a single edge with weight equal to the minimum of the weights of these parallel edges.

Now, for each node v other than the root, find the edge incoming to v of lowest weight (with ties broken arbitrarily). Denote the source of this edge by $\pi(v)$. If the set of edges $P = \{(\pi(v), v) \mid v \in V \setminus \{r\}\}$ does not contain any cycles, then f(D, r, w) = P.

Otherwise, P contains at least one cycle. Arbitrarily choose one of these cycles and call it C. We now define a new weighted directed graph $D' = \langle V', E' \rangle$ in which the cycle C is "contracted" into one node as follows:

The nodes of V' are the nodes of V not in C plus a new node denoted v_C .

- If (u,v) is an edge in E with $u \notin C$ and $v \in C$ (an edge coming into the cycle), then include in E' a new edge $e = (u,v_C)$, and define $w'(e) = w(u,v) w(\pi(v),v)$.
- If (u,v) is an edge in E with $u\in C$ and v
 otin C (an edge going away from the cycle), then include in E' a new edge $e=(v_C,v)$, and define w'(e)=w(u,v).
- If (u,v) is an edge in E with $u \notin C$ and $v \notin C$ (an edge unrelated to the cycle), then include in E' a new edge e = (u,v), and define w'(e) = w(u,v).

For each edge in $oldsymbol{E}'$, we remember which edge in $oldsymbol{E}$ it corresponds to.

Now find a minimum spanning arborescence A' of D' using a call to f(D', r, w'). Since A' is a spanning arborescence, each vertex has exactly one incoming edge. Let (u, v_C) be the unique incoming edge to v_C in A'. This edge corresponds to an edge $(u, v) \in E$ with $v \in C$. Remove the edge $(\pi(v), v)$ from C, breaking the cycle. Mark each remaining edge in C. For each edge in A', mark its corresponding edge in E. Now we define f(D, r, w) to be the set of marked edges, which form a minimum spanning arborescence.

Observe that f(D, r, w) is defined in terms of f(D', r, w'), with D' having strictly fewer vertices than D. Finding f(D, r, w) for a single-vertex graph is trivial (it is just D itself), so the recursive algorithm is guaranteed to terminate.

Running time [edit]

The running time of this algorithm is O(EV). A faster implementation of the algorithm due to Robert Tarjan runs in time $O(E\log V)$ for sparse graphs and $O(V^2)$ for dense graphs. This is as fast as Prim's algorithm for an undirected minimum spanning tree. In 1986, Gabow, Galil, Spencer, and Tarjan produced a faster implementation, with running time $O(E+V\log V)$.

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External links [edit]

- Edmonds's algorithm (edmonds-alg) → An implementation of Edmonds's algorithm written in C++ and licensed under the MIT License. This source is using Tarjan's implementation for the dense graph.
- NetworkX, a python library distributed under BSD, has an implementation of Edmonds' Algorithm .

Categories: Graph algorithms

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