Signals and Systems MATLAB HW3

Deadline: 2022/5/03 23:59

Digital Filters

The objective of this section is to learn several MATLAB commands about the digital filter design and its applications.

1. Background

A causal, non-ideal lowpass filter is designed with frequency response $H(e^{jw})$, and its difference equation is specified as

$$a_1 y[n] = \sum_{k=1}^{N+1} b_k x[n-k+1] - \sum_{k=2}^{M+1} a_k y[n-k+1]\,,$$

where $x[n]$ and $y[n]$ denote the input signal and the output signal, respectively.

- To obtain the frequency response of the filter, we may use the MATLAB command **freqz**:

  $$[H, w] = \mathbf{freqz}(b, a, K);$$

  This returns the K-point complex frequency response vector H and the K-point frequency vector w in radians/sample of the filter:

  $$H(e^{jw}) = \frac{b_1 + b_2 e^{-jw} + \cdots + b_{N+1} e^{-jNw}}{a_1 + a_2 e^{-jw} + \cdots + a_{M+1} e^{-jMw}},$$

  given the numerator and denominator coefficients in vectors b and a, where

  $$a = [a_1, a_2, \cdots, a_{M+1}]$$
  $$b = [b_1, b_2, \cdots, b_{N+1}].$$

- The MATLAB command **butter** designs an IIR filter with a Butterworth response of order L:

  $$[b, a] = \mathbf{butter}(L, f_c);$$

  This returns the transfer function with numerator b and denominator a coefficient vectors (of length L+1) of an L^{th}-order lowpass digital Butterworth filter with normalized cutoff frequency f_c. The cutoff frequency f_c of the filter is normalized so that it lies in the interval [0, 1], with 1 corresponding to $w = \pi$. Moreover, you can design a lowpass, highpass, bandpass, or bandstop Butterworth filter by specifying the value of ftype:

$$[b,a] = \textbf{butter}(L, f_c, \text{ftype});$$

- Given an input signal vector x, and filter it by a lowpass filter with numerator b and denominator a. To obtain the output signal y, we may use the MATLAB command **filter**:

$$y = \textbf{filter}(b,a,x);$$

2. Questions

Part I

A discrete-time signal is written as

$$x[n] = \cos(2\pi(n-1)T_s), \qquad n = 1, 2, \cdots, 100.$$

T_s denotes the sampling interval, and the sampling frequency is $f_s = 1/T_s = 20$ Hz.

Please write a MATLAB script (saved as **mybutter1.m**) to implement the following problems.

(a) (5%) Use the MATLAB function **plot** to plot $x[n]$ vs n.

(b) (15%) Obtain a Butterworth lowpass digital filter $H(e^{jw})$ by using the MATLAB function **butter** with the following specifications:

Filter order: $L = 3$

Cutoff frequency: $f_c = 0.05$

Please write down the transfer function $H(e^{jw})$ of the filter in your report, and use the MATLAB function **plot** to plot the magnitude response (in dB) vs w (in interval $[0, \pi]$) and the phase response (in degree) vs w (in interval $[0, \pi]$) of this filter $H(e^{jw})$. In addition, use the MATLAB function **plot** to plot the output signal $y[n]$ vs n when inputting $x[n]$ into the filter $H(e^{jw})$. There will be 3 figures in total in this problem.

(c) (15%) Please repeat Problem (b) with $L = 7$, $f_c = 0.05$ and $f_s = 20$ Hz.

(d) (15%) Please repeat Problem (b) with $L = 3$, $f_c = 0.5$ and $f_s = 20$ Hz.

(e) (10%) What is the effect of increasing L? What about increasing f_c? Please give some explanation in your report.

Note: It would be better to show the 9 figures from Part I (b)(c)(d) in 9 sub-figures, which are integrated into one figure. For example, you can use the MATLAB function **subplot** in your **mybutter1.m** file.

Part II

An input signal is written as

$$x[n] = \cos(2\pi(n-1)T_s) + 2\cos(2\pi f_1(n-1)T_s), \qquad n = 1, 2, \cdots, M,$$

where $T_s = 0.002$, $f_1 = 100$ and $M = 1000$.

Please write a MATLAB script (saved as **mybutter2.m**) to implement the following problems.

(a) (10%) Use the MATLAB function **plot** to plot $x[n]$ vs n.

(b) (15%) Obtain a 16-order Butterworth lowpass digital filter by using the MATLAB function **butter** such that the output

$$y[n] \approx \cos(2\pi(n-1)T_s), \qquad n = 1, 2, \cdots, M$$

when inputting $x[n]$ into the filter.

Please write down the transfer function $H(e^{jw})$ of this filter and the cutoff frequency in your report, and use the MATLAB function **plot** to plot the output signal $y[n]$ vs n.

(c) (15%) Obtain a 16-order Butterworth bandpass digital filter by using the MATLAB function **butter** such that the output

$$y[n] \approx 2\cos(2\pi f_1(n-1)T_s), \qquad n = 1, 2, \cdots, M$$

when inputting $x[n]$ into the filter.

Please write down the transfer function $H(e^{jw})$ of this filter and the bandpass frequency in your report, and use the MATLAB function **plot** to plot the output signal $y[n]$ vs n.

Note: It would be better to show the 3 figures from Part II (a)(b)(c) in 3 sub-figures, which are integrated into one figure. For example, you can use the MATLAB function **subplot** in your **mybutter2.m** file.

3. NTU COOL Submission

- Please upload a compressed file (.zip), which includes your **m-files** (saved as **mybutter1.m** and **mybutter2.m**) and a **word file** (saved as **report.doc**). Please show the figures mentioned above in the word file (report.doc) and answer the questions.
- The compressed file should be named as **ID_MATLAB3.zip**.
  (e.g., B09901xxx_MATLAB3.zip)