College Physics 1A

for International Students

Course Instructor:

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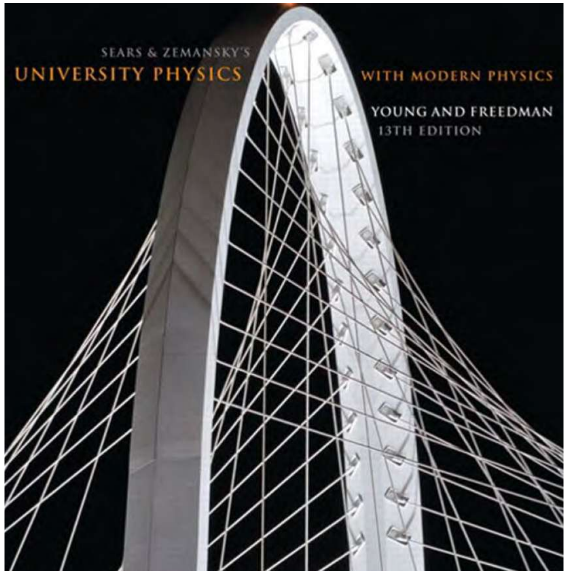
Exam Questions Breakup

- 60% - High-school level for a local student
- 70% - In class examples excluding calculus
- 80% - In class examples with calculus
- 90% - Examples and in-class exercise
- 97% - plus the homework questions
- 100% - plus some challenging problems

Tips for Survival

- Read the textbook before lectures
- Come to lectures
- Try to solve problems independently
- Proactive and interactive
- Study groups
- Don't cheat



University Physics with Modern Physics 13th Edition -Addison-Wesley (2012)

In Class…

- Maintain a quiet environment during lecture (except for questions from/to me)
- If you already learned what is being taught:
  - Work on the problem sets
  - Use your favorite chatting app on mute
- In case you are a generous person who always helps your buddies
  - Discuss during in-class exercises

Vectors and Scalars

• A ***scalar quantity*** is completely specified by a single value with an appropriate unit and has no direction.

• A ***vector quantity*** is completely described by a number and appropriate units plus a direction.

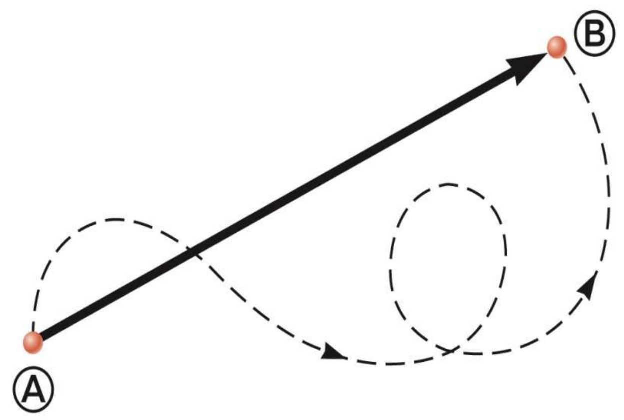
Example:

A particle travels from A to B along the path shown by the broken line.

- This is the ***distance*** traveled and is a scalar.

The ***displacement*** is the solid line from A to B

- independent of the path taken.



Vector Notation

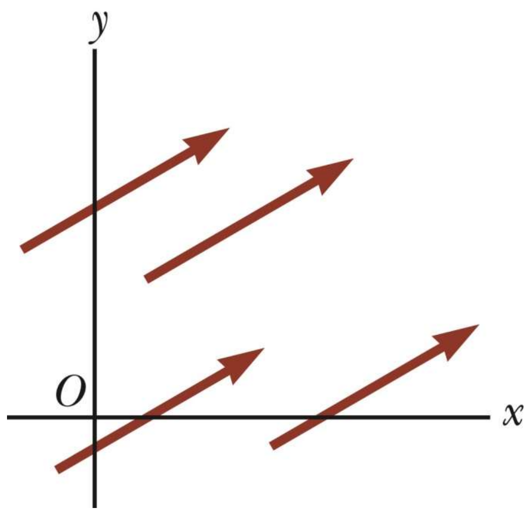
- Text uses bold with arrow to denote a vector: $\vec{\mathbf{A}}$
- Also used for printing is simple bold print: **A**
- When dealing with just the magnitude of a vector in print, an italic letter will be used: A or $|\vec{\mathbf{A}}|$

Equality of Two Vectors

Two vectors are ***equal*** if they have the same magnitude and the same direction.

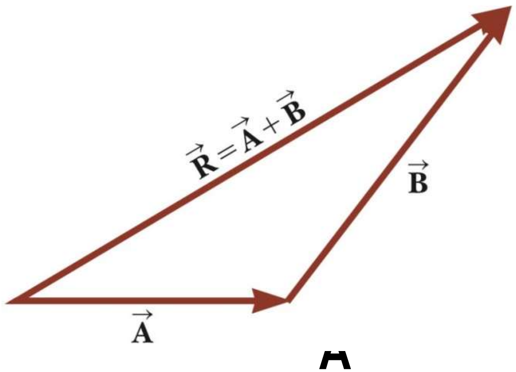
All of the vectors shown are equal.

Allows a vector to be moved to a position parallel to itself



Adding Vectors, Graphically

- Choose a scale.
- Drawing the vectors "tip-to-tail" or "head-to-tail".
- The resultant is drawn from the origin of the first vector to the end of the last vector.

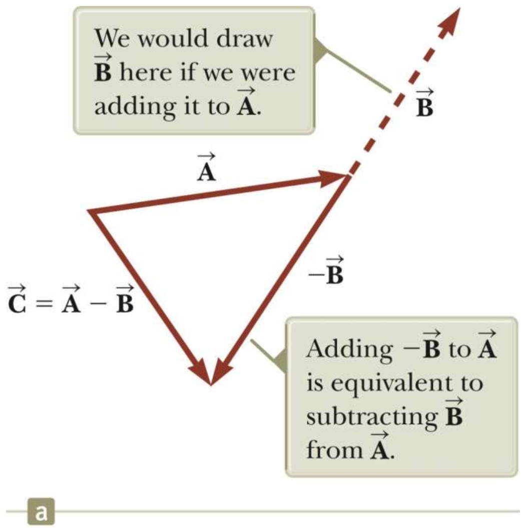


Negative Vector and Subtracting Vectors

- Negative vector: The negative of the vector will have the same magnitude, but point in the opposite direction.

Subtraction is just a special case of vector addition:

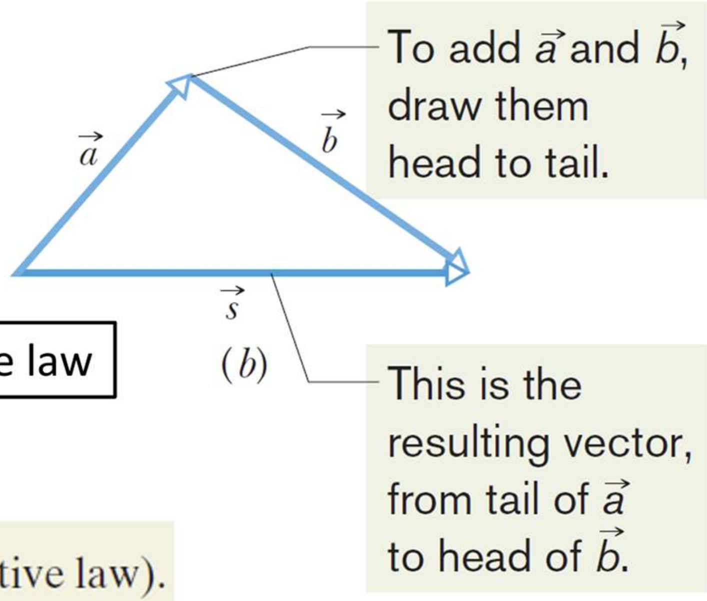
If $\vec{A} - \vec{B}$, then use $\vec{A} + \left(-\vec{B}\right)$



Vectors and their Components

- Vector ***a*** and vector ***b*** can be added geometrically to yield the <u>*resultant*</u> vector sum, ***s***

$$\vec{s} = \vec{a} + \vec{b}$$



triangle law

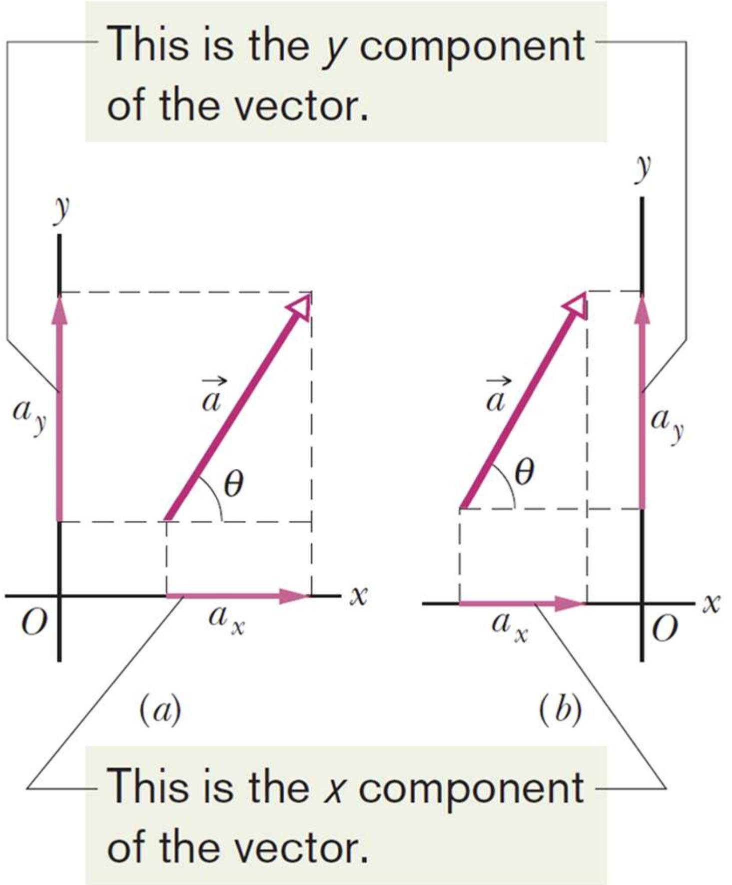
- Some rules

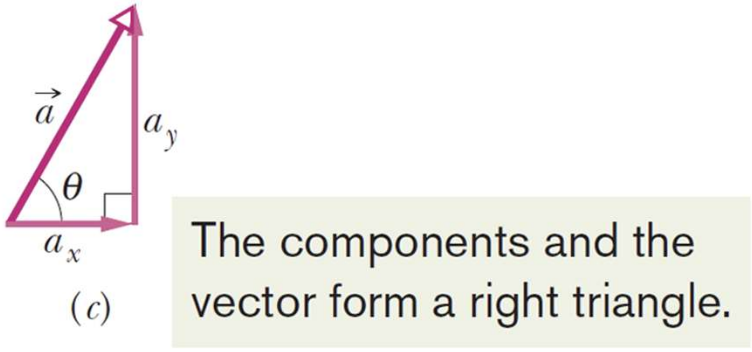
$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \qquad \text{(commutative law).}$$

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}) \qquad \text{(associative law).}$$

$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b}) \qquad \text{(vector subtraction);}$$

- The component of a vector along an axis is the projection of the vector onto that axis, the process of finding such components is called *resolution* or *decomposition*





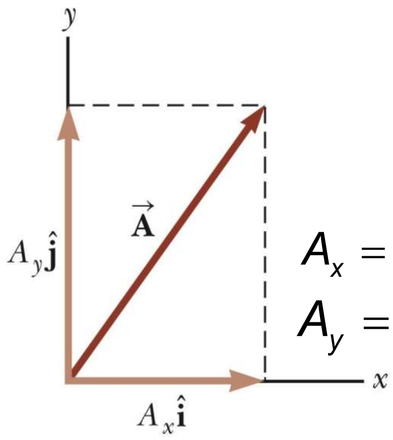
$$a_x = a\cos\theta \quad \text{and} \quad a_y = a\sin\theta.$$

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan\theta = \frac{a_y}{a_x}$$

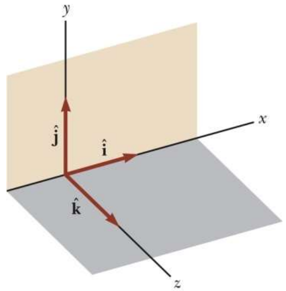
Components of a Vector

- A **component** is a projection of a vector along an axis.
- It is useful to use **rectangular components,** and express the vector in terms of unit vectors, $\hat{i}$ *and* $\hat{j}$, which represent unit vectors of x-, and y- axes, respectively.

$$\vec{A} = A_x\hat{i} + A_y\hat{j} = (A\cos\theta)\hat{i} + (A\sin\theta)\hat{j}$$

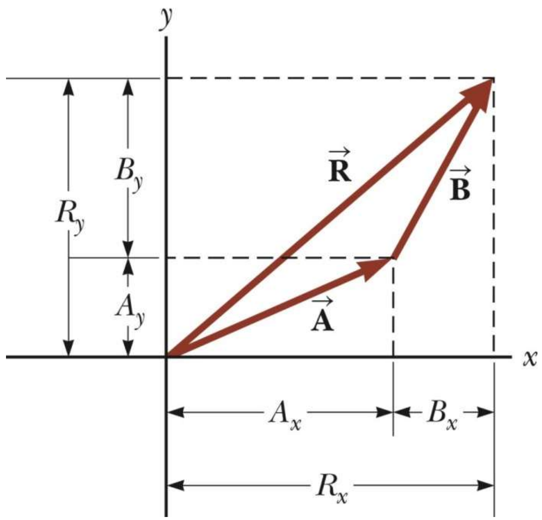


$$A_x = A\cos\theta$$
$$A_y = A\sin\theta$$



Adding Vectors with Unit Vectors

$$\begin{aligned}\vec{R} &= \vec{A} + \vec{B} \\ &= \left(A_x\hat{i} + A_y\hat{j}\right) + \left(B_x\hat{i} + B_y\hat{j}\right) \\ &= (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} \\ &= R_x\hat{i} + R_y\hat{j}\end{aligned}$$



The x component of vector $\vec{A}$ is –25.0 m and the y component is +40.0 m. (a) What is the magnitude of $\vec{A}$? (b) What is the angle between the direction of $\vec{A}$ and the positive direction of x? [47.2 m; 122°]

3.3 A vector $\vec{a}$ can be represented in the *magnitude-angle* notation (a, θ), where

$$a = \sqrt{a_x^2 + a_y^2}$$

is the magnitude and

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$$

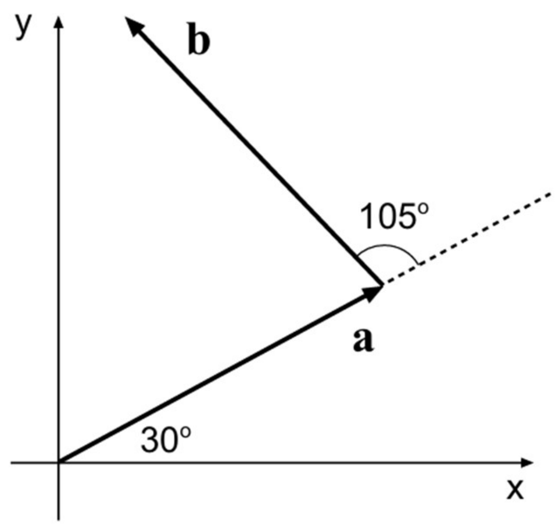
is the angle $\vec{a}$ makes with the positive x axis.

(a) Given $A_x = -25.0$ m and $A_y = 40.0$ m, $A = \sqrt{(-25.0\text{ m})^2 + (40.0\text{ m})^2} = 47.2\text{ m}$.

(b) Recalling that tan $\theta = \tan(\theta + 180°)$,

$$\tan^{-1}[(40.0\text{ m})/(-25.0\text{ m})] = -58° \text{ or } 122°.$$

The two vectors a and b in Fig below have equal magnitudes of 10.0m. Find (a) the x component and (b) the y component of their vector sum r, (c) the magnitude of r and (d) the angle r makes with the positive direction of the x axis.



(a) First, find the x and y components of the vectors $\mathbf{a}$ and $\mathbf{b}$. The vector $\mathbf{a}$ makes an angle of $30°$ with the $+x$ axis, so its components are

$$\begin{aligned} a_x &= a\cos 30° = (10.0\,\mathrm{m})\cos 30° = 8.66\,\mathrm{m} \\ a_y &= a\sin 30° = (10.0\,\mathrm{m})\sin 30° = 5.00\,\mathrm{m} \end{aligned}$$

The vector $\mathbf{b}$ makes an angle of $135°$ with the $+x$ axis ($30°$ plus $105°$ more) so its components are

$$\begin{aligned} b_x &= b\cos 135° = (10.0\,\mathrm{m})\cos 135° = -7.07\,\mathrm{m} \\ b_y &= b\sin 135° = (10.0\,\mathrm{m})\sin 135° = 7.07\,\mathrm{m} \end{aligned}$$

Then if $\mathbf{r} = \mathbf{a} + \mathbf{b}$, the x and y components of the vector $\mathbf{r}$ are:

$$\begin{aligned} r_x &= a_x + b_x = 8.66\,\mathrm{m} - 7.07\,\mathrm{m} = 1.59\,\mathrm{m} \\ r_y &= a_y + b_y = 5.00\,\mathrm{m} + 7.07\,\mathrm{m} = 12.07\,\mathrm{m} \end{aligned}$$

So the x component of the sum is $r_x = 1.59\,\mathrm{m}$, and...

(b) ...the y component of the sum is $r_y = 12.07\,\mathrm{m}$.

(c) The magnitude of the vector $\mathbf{r}$ is

$$r = \sqrt{r_x^2 + r_y^2} = \sqrt{(1.59\,\mathrm{m})^2 + (12.07\,\mathrm{m})^2} = 12.18\,\mathrm{m}$$

(d) To get the direction of the vector $\mathbf{r}$ expressed as an angle θ measured from the $+x$ axis, we note:

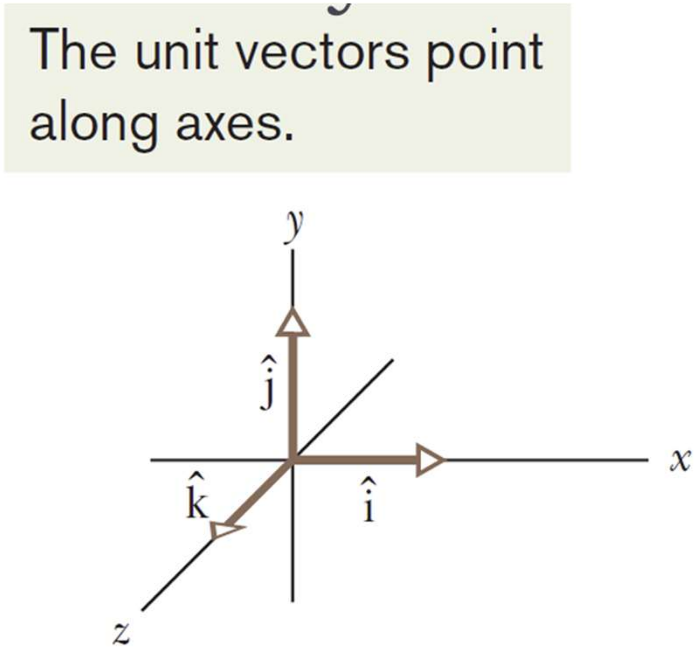
$$\tan\theta = \frac{r_y}{r_x} = 7.59$$

and then take the inverse tangent of 7.59:

$$\theta = \tan^{-1}(7.59) = 82.5^\circ$$

Since the components of $\mathbf{r}$ are both positive, the vector *does* lie in the first quadrant so that the inverse tangent operation has (this time) given the correct answer. So the direction of $\mathbf{r}$ is given by $\theta = 82.5^\circ$.

Unit Vectors, Adding Vectors by Components



- Unit vector (單位向量)
  - a vector of *unit magnitude*, pointing in a particular direction
  - unit vectors pointing in the *x*-, *y*-, and *z*-axes are usually designated by $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ respectively

- Vector $\vec{a}$ with components a_x, a_y, and a_z in the *x*-, *y*-, and *z*-directions, can be written in terms of the following vector sum: $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$

Unit Vectors, Adding Vectors by Components

- If $\vec{r} = \vec{a} \pm \vec{b}$ then
$$r_x = a_x \pm b_x$$
$$r_y = a_y \pm b_y$$
$$r_z = a_z \pm b_z.$$

e.g.
$$\vec{a} = 2\hat{\imath} + 3\hat{\jmath}$$
$$\vec{b} = \hat{\imath} - \hat{k}$$
$$\vec{a} + \vec{b} = 3\hat{\imath} + 3\hat{\jmath} - \hat{k}$$

- If k = scalar, then $k\vec{r} = kr_x\hat{\imath} + kr_y\hat{\jmath} + kr_z\hat{k}$

Example 3.5 Taking a Hike

A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

(A) Determine the components of the hiker's displacement for each day.

$$A_x = A\cos(-45.0°) = (25.0\text{ km})(0.707) = \boxed{17.7\text{ km}}$$

$$A_y = A\sin(-45.0°) = (25.0\text{ km})(-0.707) = \boxed{-17.7\text{ km}}$$

$$B_x = B\cos 60.0° = (40.0\text{ km})(0.500) = \boxed{20.0\text{ km}}$$

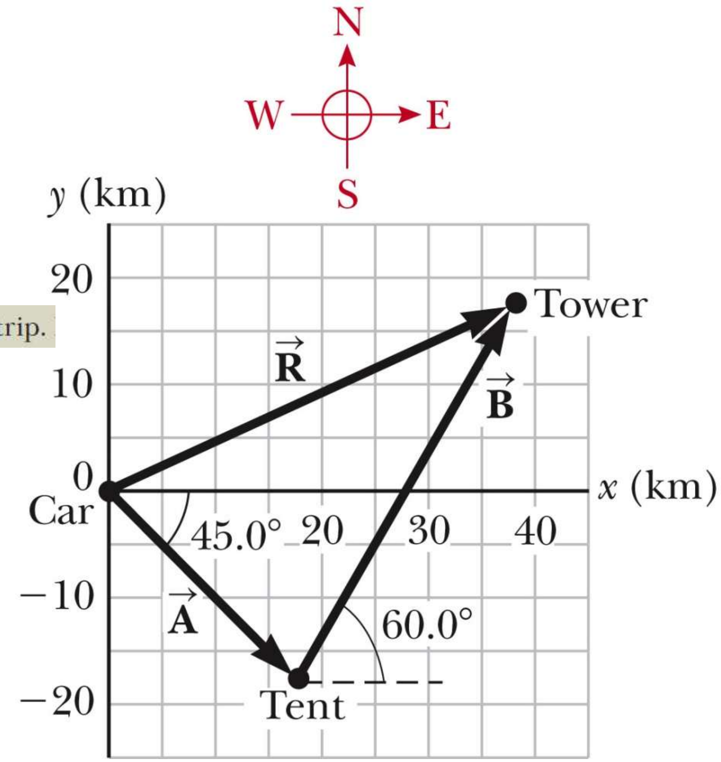
$$B_y = B\sin 60.0° = (40.0\text{ km})(0.866) = \boxed{34.6\text{ km}}$$

(B) Determine the components of the hiker's resultant displacement $\vec{\mathbf{R}}$ for the trip.

$$R_x = A_x + B_x = 17.7\text{ km} + 20.0\text{ km} = \boxed{37.7\text{ km}}$$

$$R_y = A_y + B_y = -17.7\text{ km} + 34.6\text{ km} = \boxed{17.0\text{ km}}$$

$$\vec{\mathbf{R}} = \boxed{(37.7\hat{\mathbf{i}} + 17.0\hat{\mathbf{j}})\text{ km}}$$



Sample Problem

- A rabbit runs across a field. The coordinates (metres) of the rabbit's position as functions of time t (seconds) are given by

$$x = -0.31t^2 + 7.2t + 28$$
$$y = 0.22t^2 - 9.1t + 30.$$

(a) At $t = 15$ s, what is the rabbit's position vector $\vec{r}$ in unit-vector notation and in magnitude-angle notation?

$$x = (-0.31)(15)^2 + (7.2)(15) + 28 = 66 \text{ m}$$

and $$y = (0.22)(15)^2 - (9.1)(15) + 30 = -57 \text{ m},$$

so $$\vec{r} = (66 \text{ m})\hat{\mathrm{i}} - (57 \text{ m})\hat{\mathrm{j}}, \qquad \text{(Answer)}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(66 \text{ m})^2 + (-57 \text{ m})^2}$$
$$= 87 \text{ m}, \qquad \text{(Answer)}$$

and $$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{-57 \text{ m}}{66 \text{ m}}\right) = -41°. \qquad \text{(Answer)}$$

A boat is crossing a river is traveling east at 7m/s**. The current of the river is traveling north at** 3m/s**. At what angle is the boat traveling?**

A two-dimensional vector comes in the form $\vec{r} = a\hat{i} + b\hat{j}$. The angle of the vector is found using the equation:

$$\theta = tan^{-1}\left(\frac{b}{a}\right)$$

For this problem, the velocity vector is: $\vec{v} = \left(7\hat{i} + 3\hat{j}\right)\frac{m}{s}$

The angle of the boat is:

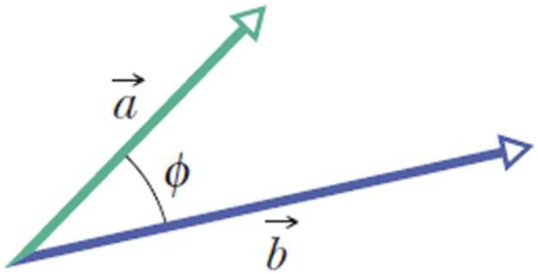
$$\theta = tan^{-1}\left(\frac{3}{7}\right) \approx 23.2^{\circ}$$

The boat is traveling 23.2° above horizontal.

Multiplying Vectors

- Scalar product (純量積) (dot product)

$$\vec{a} \cdot \vec{b} = ab \cos \phi$$



$$\vec{a} \cdot \vec{b} = (a \cos \phi)(b) = (a)(b \cos \phi)$$

- operation

$$\vec{a} \cdot \vec{b} = (a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}) \cdot (b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} + b_z\hat{\mathrm{k}})$$
$$= a_xb_x + a_yb_y + a_zb_z.$$

  - $\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = 1$
  - $\hat{\imath} \cdot \hat{\jmath} = \hat{\jmath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{k} = \hat{k} \cdot \hat{\jmath} = \hat{k} \cdot \hat{\imath} = \hat{\imath} \cdot \hat{k} = 0$

Sample Problem

What is the angle ϕ between $\vec{a} = 3.0\hat{\mathrm{i}} - 4.0\hat{\mathrm{j}}$ and $\vec{b} = -2.0\hat{\mathrm{i}} + 3.0\hat{\mathrm{k}}$?

$$\vec{a} \cdot \vec{b} = ab \cos \phi.$$

$$a = \sqrt{3.0^2 + (-4.0)^2} = 5.00,$$

$$b = \sqrt{(-2.0)^2 + 3.0^2} = 3.61.$$

$$\vec{a} \cdot \vec{b} = -6.0$$

$$\phi = \cos^{-1} \frac{-6.0}{(5.00)(3.61)} = 109° \approx 110°. \qquad \text{(Answer)}$$