Current and Resistance

- Electric current
- Resistance and resistors
- Ohm's law
- Temperature dependence of resistivity
- Superconductor (optional)
- Electric power

Electric Current and Resistor

Most practical applications of electricity deal with electric currents.

Electric current is the rate of net flow of charge through some region of space.

It means the amount of charge flowing through an element per unit time.

The SI unit of current is the **ampere** (A).

- 1 A = 1 C / s

The symbol for electric current is I.

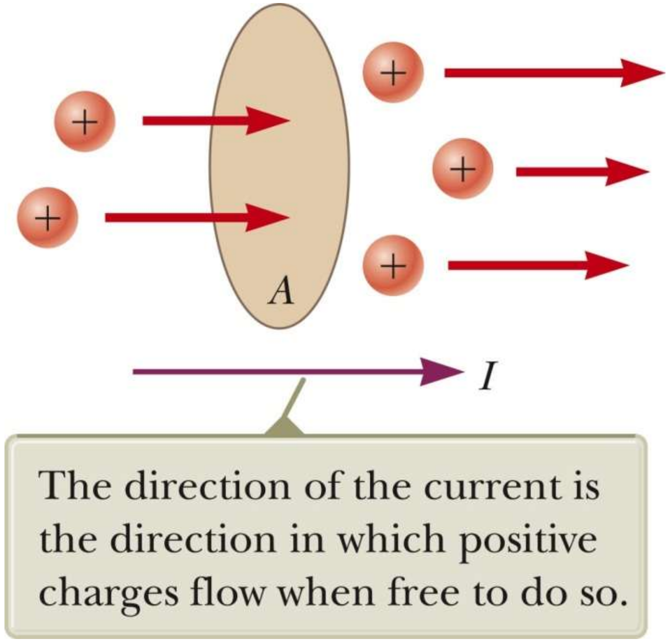
In a circuit, ***resistor*** is an element which “resists” the flow of current.

Average vs. Instantaneous Electric current

Assume charges are moving perpendicular to a surface of area *A.*

If ΔQ is the amount of charge that passes through A in time Δt, then the average current is

$$I_{avg} = \frac{\Delta Q}{\Delta t}$$



If the rate at which the charge flows varies with time, the instantaneous current, I, is defined as the differential limit of average current as $\Delta t \rightarrow 0$

$$I \equiv \frac{dQ}{dt}$$

Direction of Current

The charged particles passing through the surface could be positive, negative or both.

It is conventional to assign to the current the same direction as the flow of positive charges.

In an ordinary conductor, no positive charge can flow because only electrons are allowed to move easily. When electrons in an ordinary conductor flows, the direction of current flow is opposite to the direction of the flow of electrons.

It is common to refer to any moving charge as a *charge carrier.*
The charge carriers in ordinary conductors are electrons.

Charge Carrier Motion in a Conductor

When a potential difference is applied across the conductor, an electric field is set up in the conductor which exerts an electric force on the electrons.

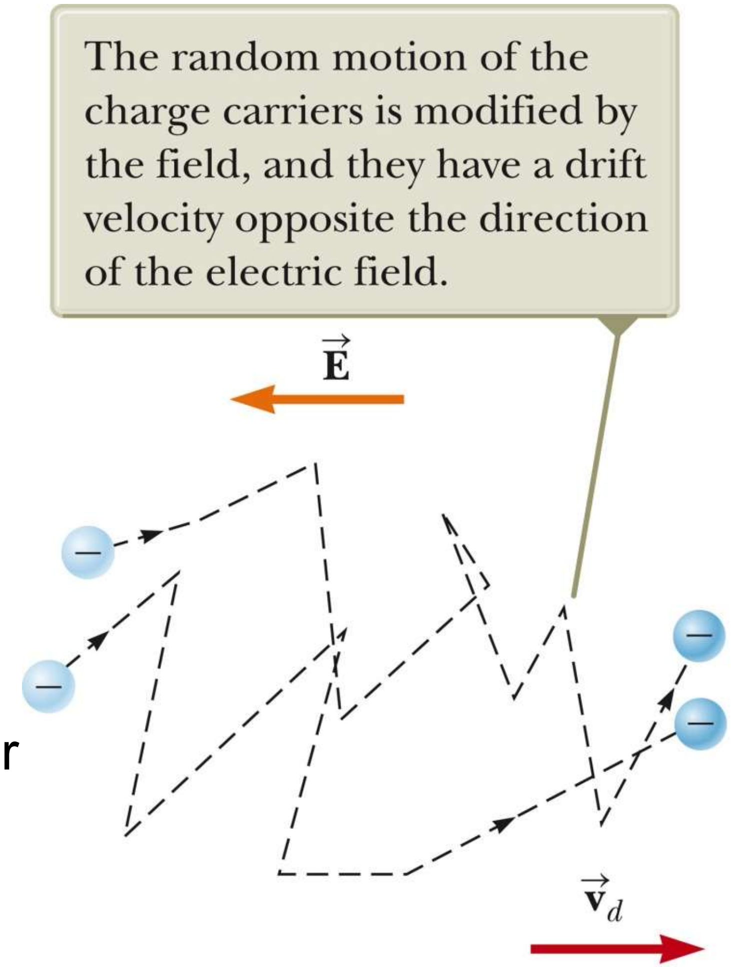
The motion of the electrons is no longer random.

The zigzag black lines represents the motion of a charge carrier in a conductor in the presence of an electric field.

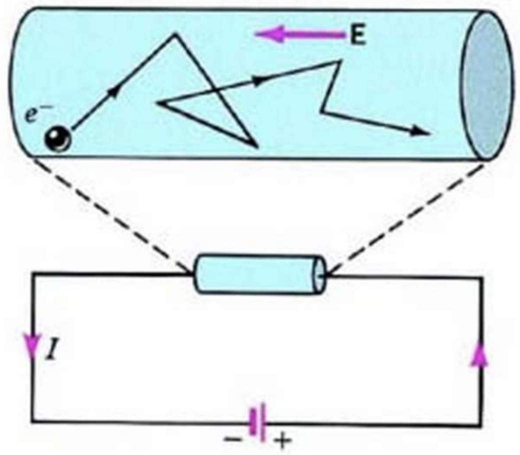
The sharp changes in direction are due to collisions.

The net motion of electrons is opposite to the direction of the electric field.

The charge carriers slowly move along the conductor with a drift velocity, $\vec{v}_d$, creating a current.

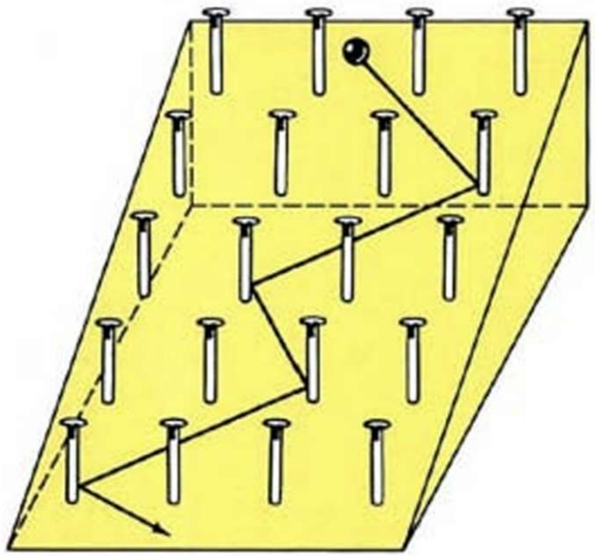


The path of a conduction electron in a current-carry wire is quite erratic, as shown in the right figure. The motion of an electron resembles that of a steel ball rolling down a nail-studded incline.



(a)

There is an analogy between a wind and an electric current. Wind: molecules speed 300m/s, wind speed 10m/s. Current: electron thermal speed 106m/s, drift speed 10^{-4}m/s.

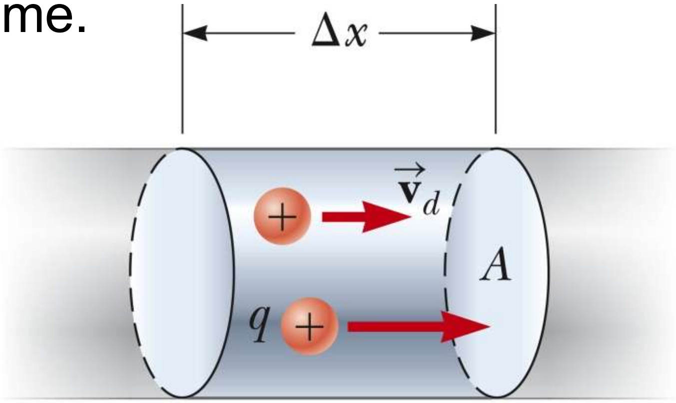

(b)

Current and Drift Speed

Charged particles move through a cylindrical conductor of cross-sectional area *A.*

$$\Delta x = v_d \Delta t$$

n is the number of mobile charge carriers per unit volume.



$N = nA\Delta x$ is the total number of charge carriers in a segment.

The total charge is the number of carriers times the charge per carrier, q

$$\Delta Q = Nq = (nA\Delta x)q$$

Assume the carriers move with a velocity v_d parallel to the axis of the cylinder such that they experience a displacement $\Delta x = v_d \Delta t$ in a time interval Δt. Then

$$\Delta Q = nqAv_\mathrm{d}\Delta t$$

v_d is an average speed called the **drift speed.** (2.23 x 10^{-4} m/s for a copper wire carrying a current of 10.0 A)

The averaged current is $\boxed{I_\mathrm{avg} = \frac{\Delta Q}{\Delta t} = nqAv_\mathrm{d}}$

Current Density

The **current density** J of a conductor is defined as the current I per unit area A.

- $J \equiv I / A = nqv_d$
- This expression is valid only if the current density is uniform and A is perpendicular to the direction of the current.

J has SI units of $\mathrm{A/m^2}$

Same as current, the current density is in the direction of the positive charge carriers.

Conductivity

A current density and an electric field are established in a conductor whenever a potential difference is maintained across the conductor.

$$J = \sigma E$$

For some materials, the current density J is directly proportional to the field E.

The constant of proportionality, σ, is called the **conductivity** of the conductor.

A copper wire carries a current of 10 A. It has a cross-section area of 0.05 cm^2. Estimate the drift velocity of the electron.

Solution:

$$\text{number density } n = \frac{N_A}{V_A} = \frac{\rho N_A}{M} = 8.5\times 10^{28} \text{ atoms/m}^3$$

$$\text{drift velocity } v_d = \frac{I}{neA} = 1.5\times 10^{-4} \text{ m/s}$$

This is the extremely small drift speed at which the electron gas as a whole moves along a wire.

Ohm's Law

Ohm's law states that for many materials, the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current.

- Mathematically, $J = \sigma E$
- Most metals obey Ohm's law
- Materials that obey Ohm's law are said to be *ohmic*
- Materials that do not obey Ohm's law are said to be *nonohmic.*

Ohm's law is not a fundamental law of nature

It is an empirical relationship valid only for certain materials.

Resistance

In an ohmic conductor, the voltage $\Delta V = V_b - V_a$ applied across the ends of the conductor is proportional to the current through the conductor.

$$\boxed{\Delta V = IR} \qquad \text{or } \frac{\Delta V}{I} = R$$

The constant of proportionality R is called the **resistance** of the conductor.

SI units of resistance are *ohms* (Ω) and $1\ \Omega = 1\ \mathrm{V} / \mathrm{A}$.

Resistance in a circuit arises due to collisions between the electrons carrying the current with the fixed atoms inside the conductor.

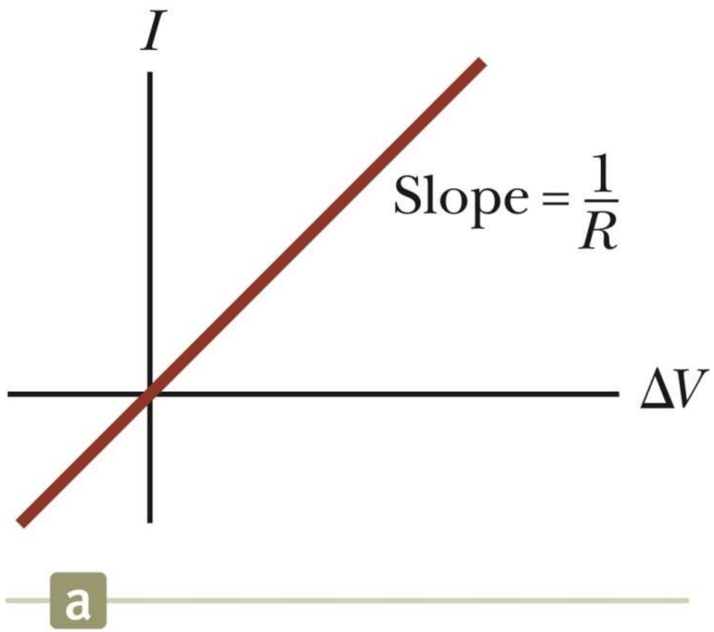
$$J = \sigma E \;\Rightarrow\; \frac{I}{A} d = \sigma E d \;\Rightarrow\; I \frac{d}{\sigma A} = V \;\Rightarrow\; IR = V$$

Ohmic Material, Graph

The resistance of an ohmic device is constant over a wide range of voltages.

The relationship between current and voltage is linear.

The slope is related to the resistance.

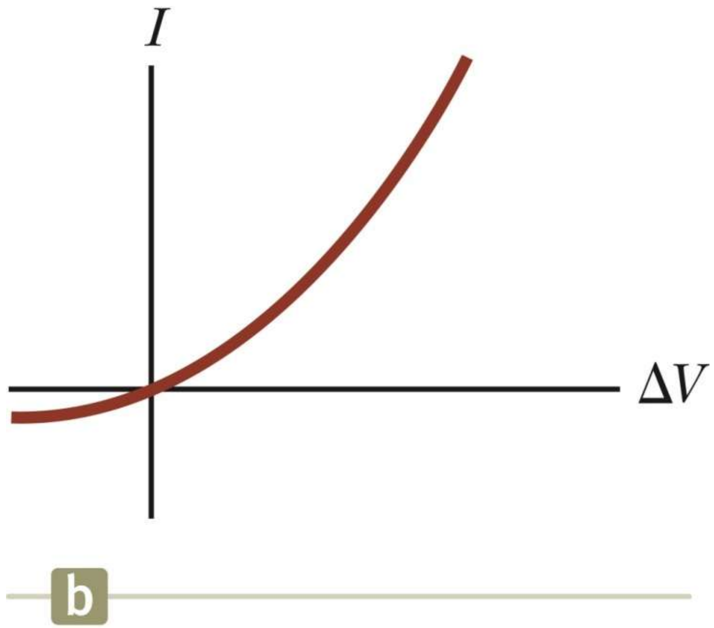


Nonohmic Material, Graph

Nonohmic materials are those whose resistance changes with voltage or current.

The current-voltage relationship is nonlinear.

A junction diode is a common example of a nonohmic device: low *R* for currents in one direction and high *R* in the reverse direction.

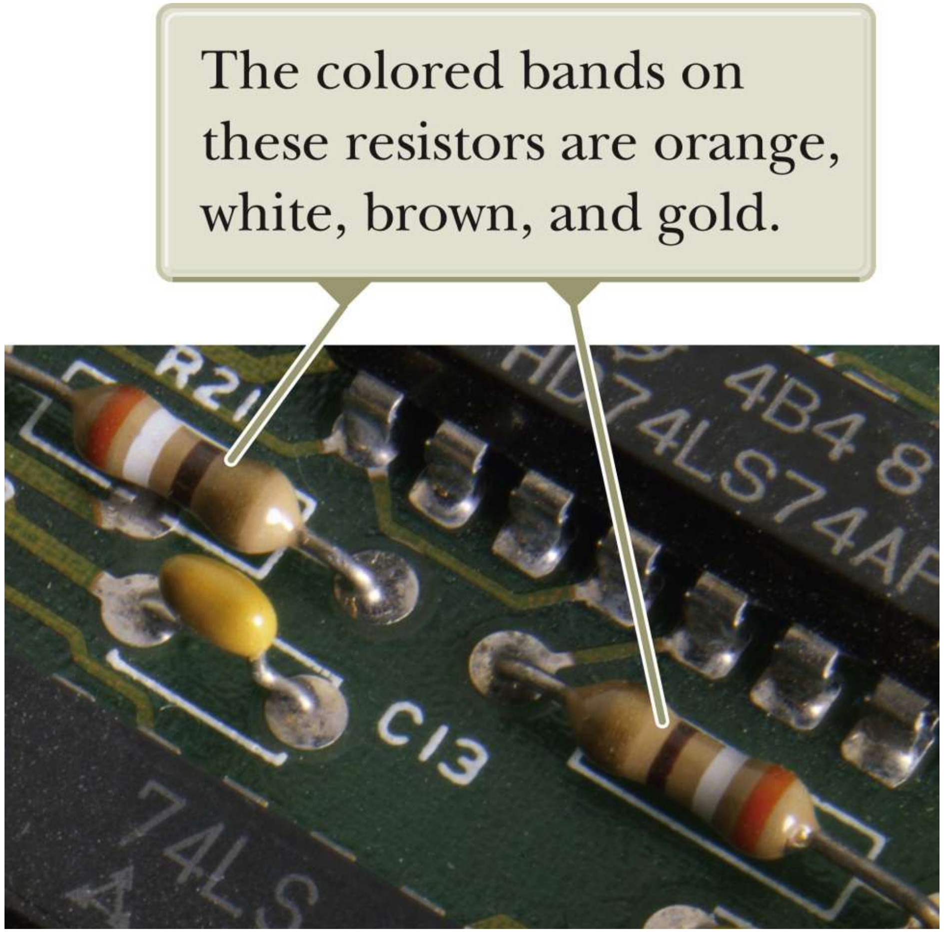


Resistors

Most electric circuits use circuit elements called **resistors** to control the current in the various parts of the circuit.

Stand-alone resistors are widely used.

- Resistors can be built into integrated circuit chips.



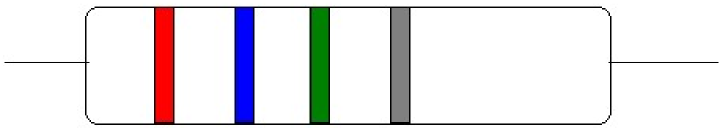
Resistor Color Codes (Optional)

Values of resistors are normally indicated by colored bands.

- The first two bands give the first two digits in the resistance value.
- The third band represents the power of ten for the multiplier band.
- The last band is the tolerance.

TABLE 27.1 *Color Coding for Resistors*

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Silver		10^{-2}	10%
Colorless			20%



Red (=2) and blue (=6) give the first two digits: 26

Green (=5) gives the power of ten in the multiplier: 10^5

The value of the resistor then is $26 \times 10^5\ \Omega$ (or $2.6\ \mathrm{M}\Omega$)

The tolerance is 10% (silver = 10%) or $2.6 \times 10^5\ \Omega$

Resistivity

The inverse of the conductivity is the **resistivity**: $\rho \equiv 1/\sigma$.

Resistivity has SI units of ohm-meters ($\Omega \cdot \mathrm{m}$)

An ideal conductor would have zero resistivity.
An ideal insulator would have infinite resistivity.

Resistance R is also related to resistivity ρ :

$$\boxed{R = \frac{l}{\sigma A} = \rho \frac{l}{A}}$$

ℓ is the length of a wire
A is the cross-section area of the wire

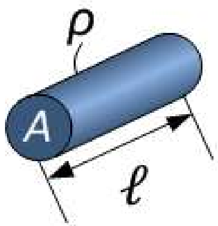


TABLE 27.2 *Resistivities and Temperature Coefficients of Resistivity for Various Materials*

Material	Resistivity[a] ($\Omega \cdot$ m)	Temperature Coefficient[b] $\alpha[(°C)^{-1}]$
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.00×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon[d]	2.3×10^{3}	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C. All elements in this table are assumed to be free of impurities.
[b] See Section 27.4.
[c] A nickel–chromium alloy commonly used in heating elements. The resistivity of Nichrome varies with composition and ranges between 1.00×10^{-6} and 1.50×10^{-6} $\Omega \cdot$ m.
[d] The resistivity of silicon is very sensitive to purity. The value can be changed by several orders of magnitude when it is doped with other atoms.

Resistance and Resistivity, Summary

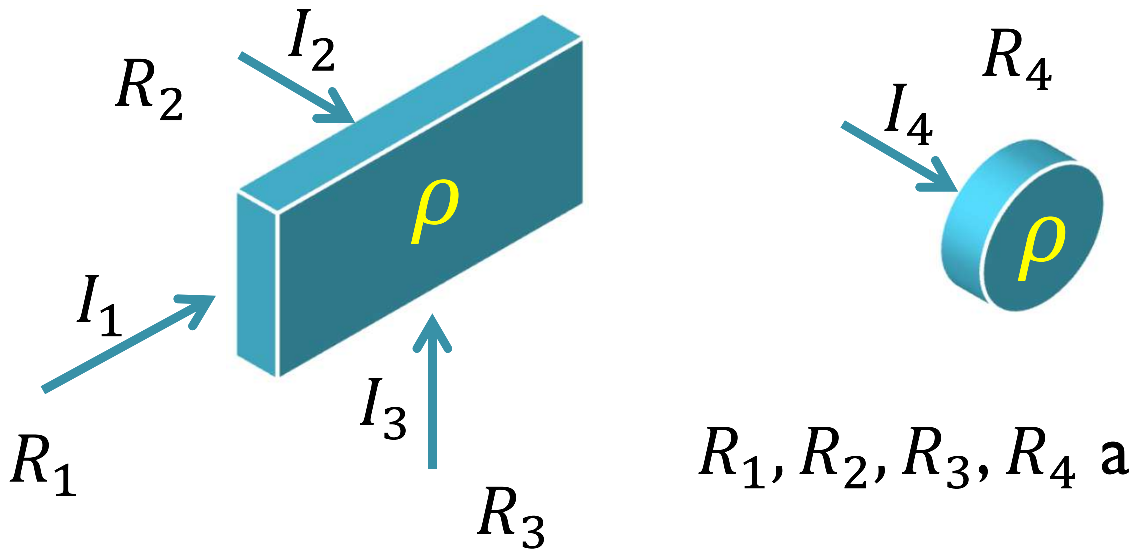
Every ohmic material has a characteristic resistivity ρ that depends on the properties of the material and on temperature.

- Resistivity ρ is a property of substances.

The resistance of a material depends on its geometry and its resistivity.

- Resistance R is a property of an object.

Resistance depends on geometry and how we apply currents.



R_1, R_2, R_3, R_4 are different

Resistance of a Cable (Optional)

Assume the polyethylene between the conductors to be concentric elements of thickness *dr.*

The resistance of the hollow cylinder of polyethylene is

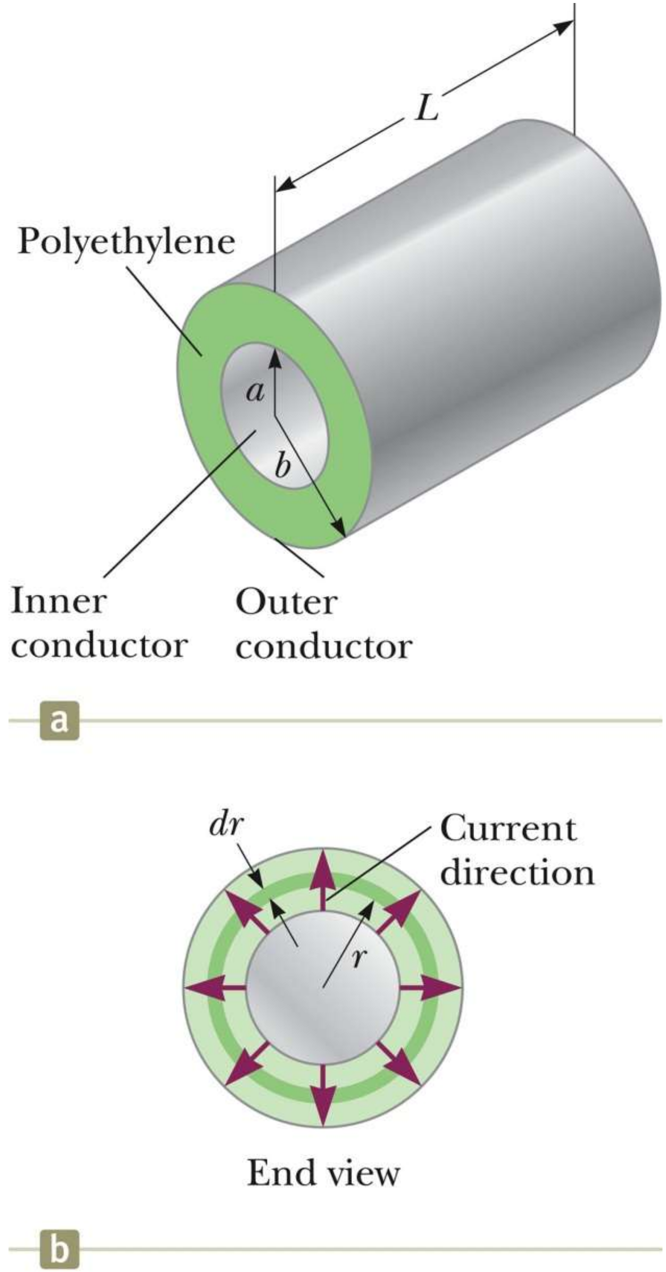
$$dR = \frac{\rho dr}{A} = \frac{\rho}{2\pi r L} dr$$

The total resistance across the entire thickness is

$$R = \int dR = \frac{\rho}{2\pi L} \int_a^b \frac{dr}{r} = \frac{\rho}{2\pi L} \ln\left(\frac{b}{a}\right)$$

This is the radial resistance of the cable.

The calculated value is fairly high, which is desirable since you want the current to flow along the cable and not radially out of it.



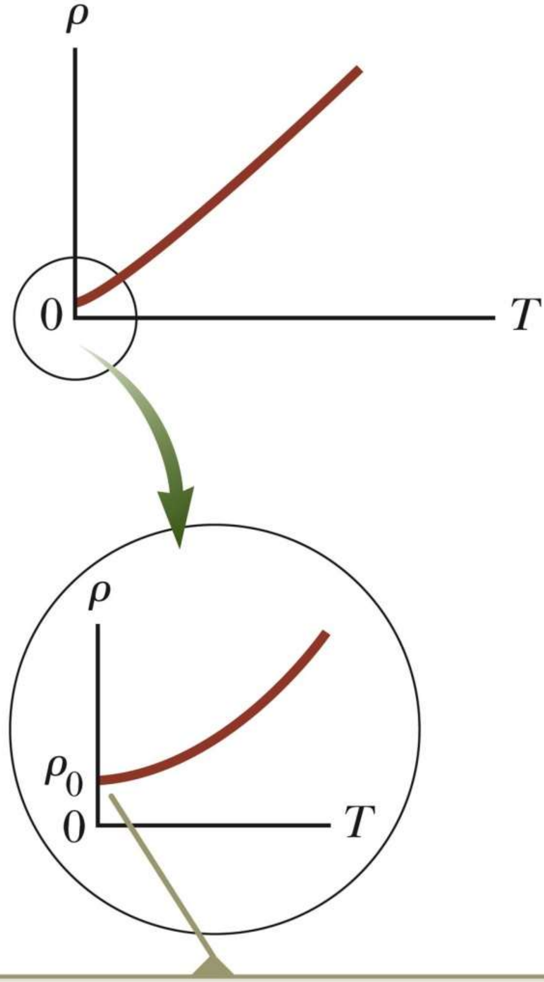
Resistivity and Temperature of normal metal

For some metals, the resistivity is nearly proportional to the temperature.

This is because the resistivity is due to collisions between electrons and atoms. Since the atoms vibrate a lot at higher temperatures, the electrons become easier to collide with the atoms which block their paths, resulting in a higher resistivity at higher temperatures.

A nonlinear region always exists at very low temperatures.

The resistivity usually reaches some finite value as the temperature approaches absolute zero.



As T approaches absolute zero, the resistivity approaches a finite value ρ_0.

Superconductors (Optional)

A class of materials and compounds whose resistances fall to virtually zero below a certain temperature, T_C.

- T_C is called the **critical temperature.**

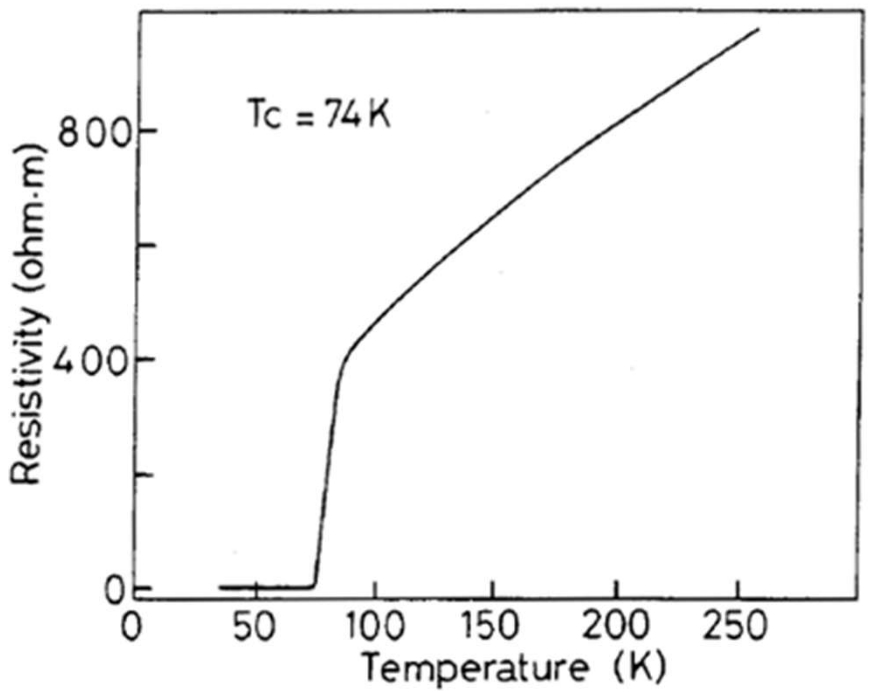
The graph is the same as a normal metal above T_C, but suddenly drops to zero at T_C.

The value of T_C is sensitive to:

- chemical composition
- pressure
- molecular structure

Once a current is set up in a superconductor, it persists without any applied voltage because of R = 0.

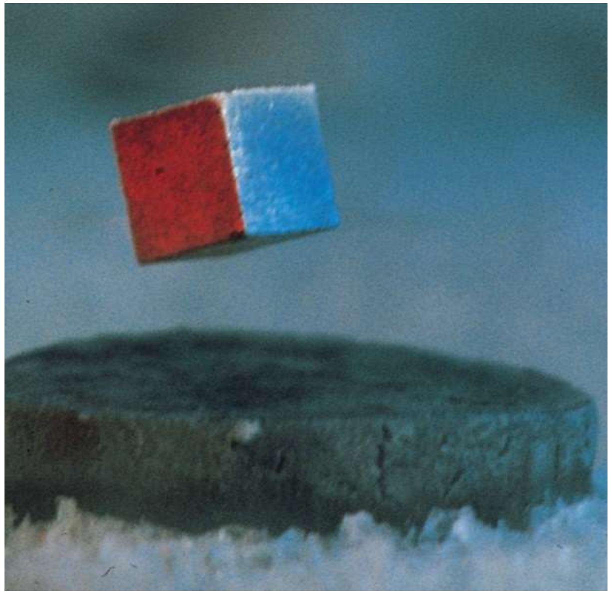
Resistivity-temperature curve of a superconductor



Superconductor Application (Optional)

Superconductor can block magnetic field and levitate magnets.

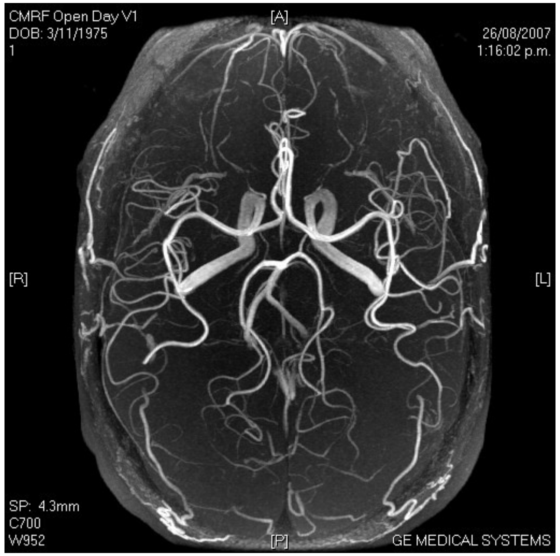
Video: Quantum Levitation



A small permanent magnet levitated above a disk of the superconductor.

Superconductor can also help producing strong magnetic fields.

A superconducting magnet can be used in Magnetic Resonance Imaging (MRI).



Example of MRI image for medical use.

Electrical Power

Assume a circuit as shown is the system.

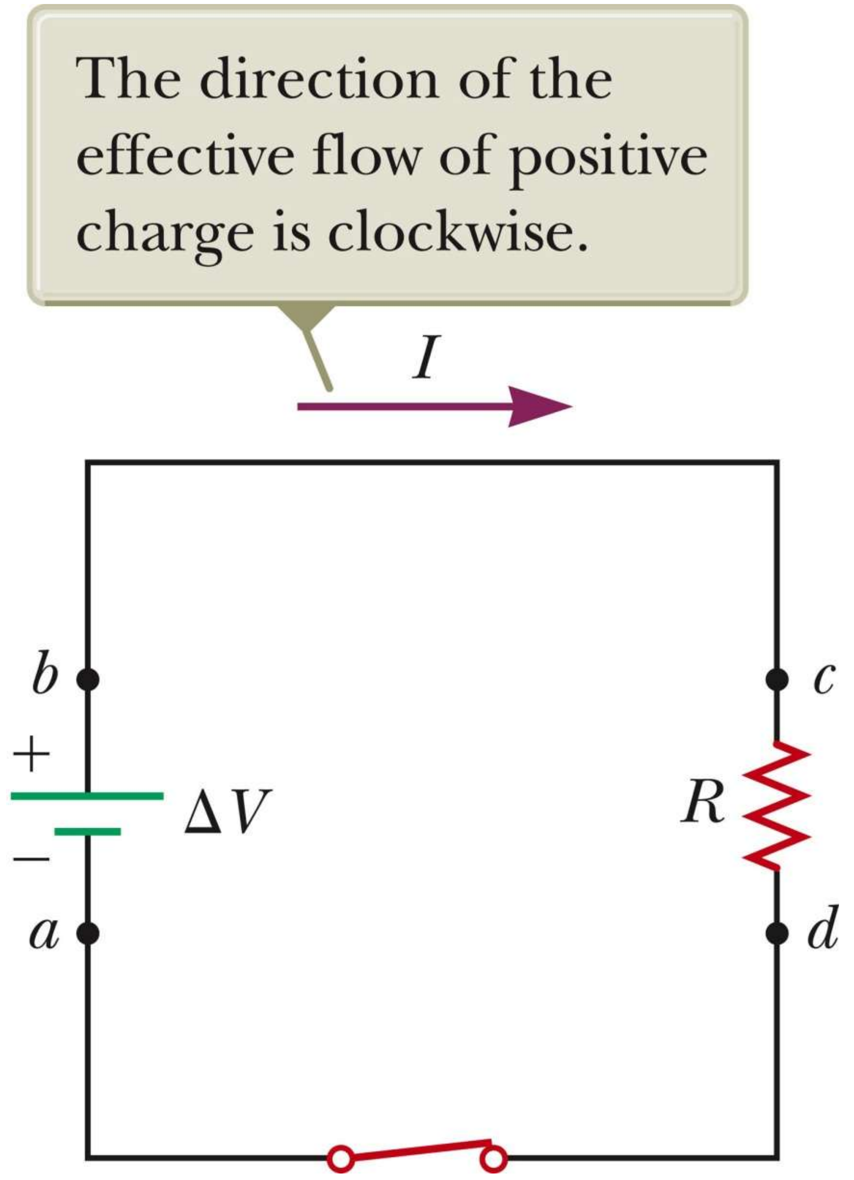
As a charge moves from *a* to *b*, the electric potential energy of the system increases by $Q\Delta V$.

This electric potential energy is transformed into internal energy in the resistor.

- Corresponds to increased vibrational motion of the atoms in the resistor

The resistor is normally in contact with the air, so its increased temperature will result in a transfer of energy by heat into the air.

The resistor also emits thermal radiation.



Electric Power, 2

After some time interval, the resistor reaches a constant temperature.

- The input of energy from the battery is balanced by the output of energy by heat and radiation.

The rate at which the system's potential energy decreases as the charge passes through the resistor is equal to the rate at which the system gains internal energy in the resistor.

The **power** is the rate at which the energy is delivered to the resistor.

The power is given by the equation $P = I\Delta V$.

$$\boxed{P = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}}$$

Units: I is in A, R is in Ω, ΔV is in V, and P is in W.