Electricity

Contents for Electricity

Topics being covered in Chapter 23 – Electric fields

- Properties of electric charges
- Charging objects by induction
- Coulomb's law
- The electric field
- Electric field of a continuous charge distribution
- Electric field lines
- Motion of a charged particle in a uniform electric field

Electric Charges

There are two kinds of electric charges

- called positive charges and negative charges

  e.g.,

  - Electrons carry negative charges
  - Protons carry positive charges.

Charges of the same sign repel one another and charges with opposite signs attract one another. (Like charges repel; Unlike charges attract)

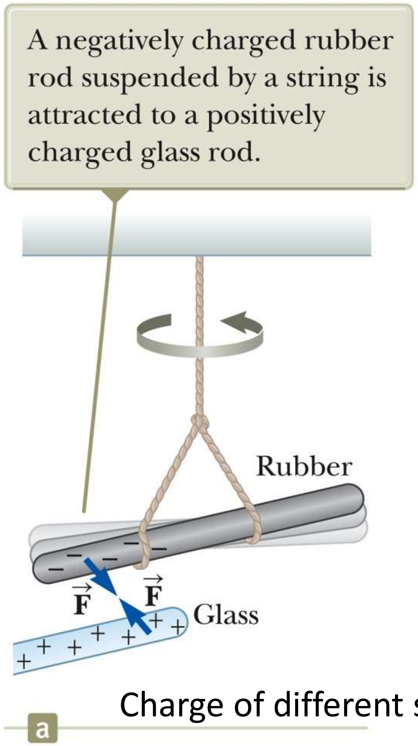
Electromagnetic Forces

•The concept of force links the study of electromagnetism to previous study.

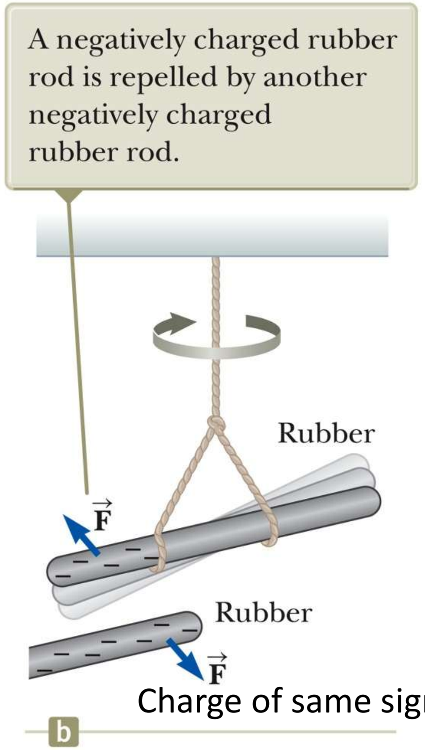
•The electromagnetic force between charged particles is one of the fundamental forces of nature.

[Video: Electric Charge and Electric Fields]

Electric Charges and forces



Charge of different signs



Charge of same sign

- The rubber rod is negatively charged and the glass rod positively charged. The two rods will **attract**.

- The rubber rod is negatively charged. The second rubber rod is also negatively charged. The two rods will **repel**.

Examples of Field Forces

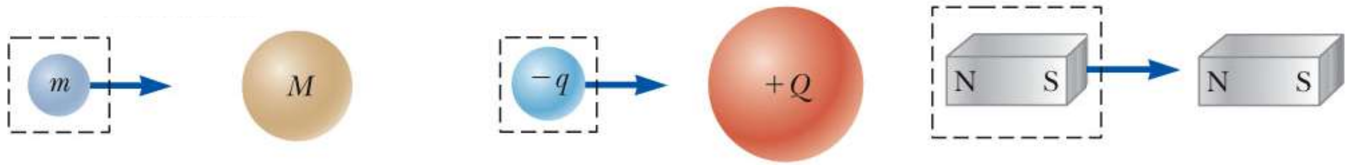
- *Field forces (non-contact forces) act through empty space*

Electromagnetic Forces

Gravitational force

Electric force

Magnetic force



Between two masses

Between two electric charges

Between two magnets

There exists interaction among electric charges and magnets too.

Elementary Particles

TABLE 23.1 *Charge and Mass of the Electron, Proton, and Neutron*

Particle	Charge (C)	Mass (kg)
Electron (e)	$-1.602\,176\,5 \times 10^{-19}$ $(-e)$	$9.109\,4 \times 10^{-31}$
Proton (p)	$+1.602\,176\,5 \times 10^{-19}$ $(+e)$	$1.672\,62 \times 10^{-27}$
Neutron (n)	0	$1.674\,93 \times 10^{-27}$

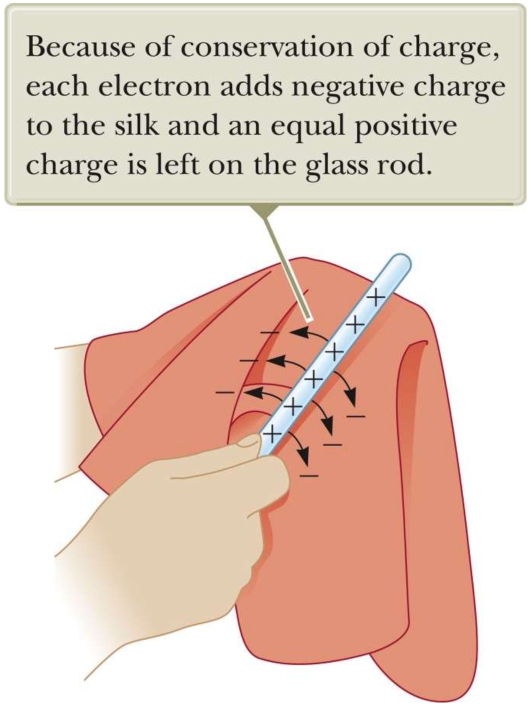
Electric charge exists as discrete packets in most situations. (Charges are quantized.)

It was discovered that the charge of an object must be $q = \pm Ne$, where

- N is an integer
- $e = 1.6 \text{ x } 10^{-19}$ C is the fundamental unit of charge

Charging an Object

- Electric charge is always conserved in an isolated system.
  - We cannot create charges but transfer charges from one object to another.



Example: A glass rod is rubbed with silk

- Electrons (i.e., negative charges) are transferred from the glass to the silk.
- An equal positive charge is left on the rod.

Conductors

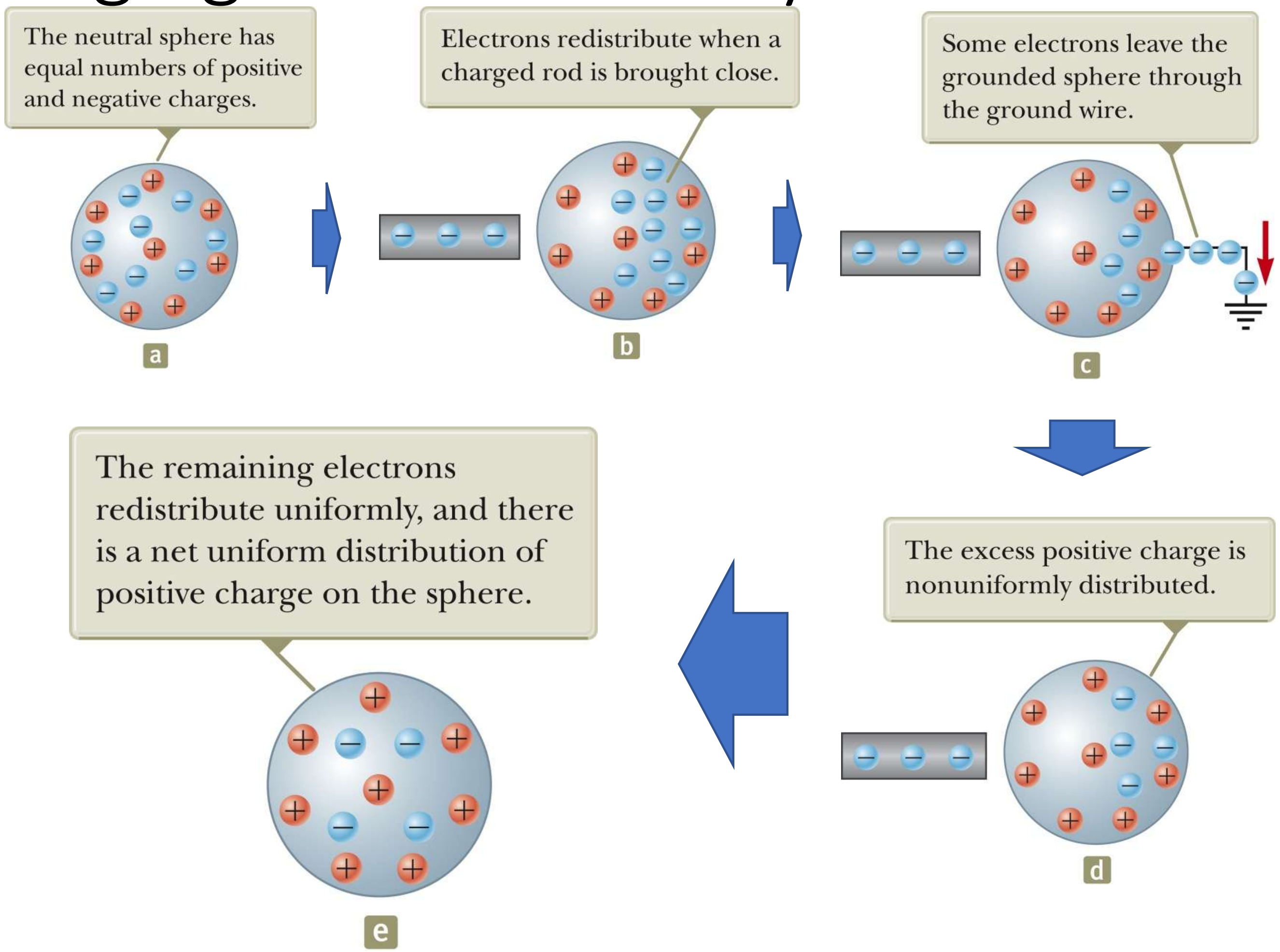
- Electrical conductors are materials in which some of the electrons are free electrons.
  - Free electrons are not bound to the atoms and can move relatively freely through the material. Examples of good conductors include copper, aluminum and silver.
  - When a good conductor is charged in a small region, the charge readily distributes itself over the entire surface of the material.

Insulators

Electrical insulators are materials in which all of the electrons are bound to atoms.

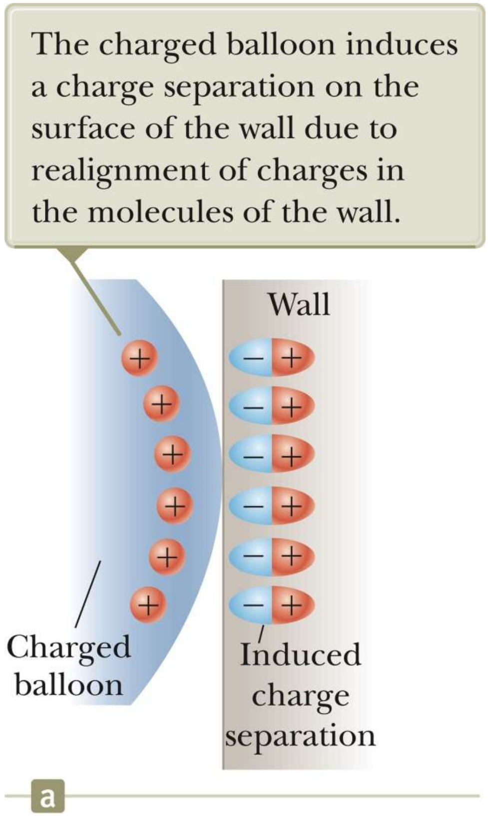
- These electrons can not move relatively freely through the material.
- Examples of good insulators include glass, rubber and wood.
- When a good insulator is charged in a small region, the charge is unable to move to other regions of the material.

Charging a Conductor by Induction



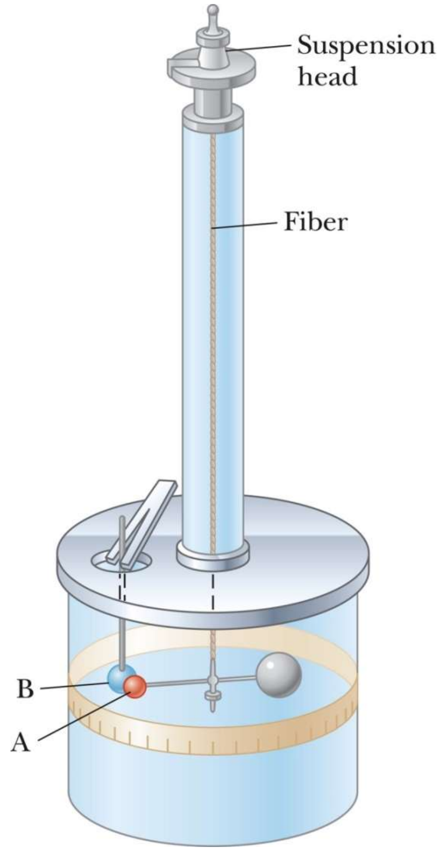
Charge Rearrangement in Insulators

- A process similar to induction can take place in insulators.
- The charges within the molecules of the material are rearranged.
- The proximity of the positive charges on the surface of the object and the negative charges on the surface of the insulator results in an attractive force between the object and the insulator.



Coulomb’s Law

- Charles Coulomb measured the magnitudes of electric forces between two small charged spheres.
- The force is inversely proportional to the square of the separation *r* between the charges and directed along the line joining them.
- The force is proportional to the product of the charges, q_1 and q_2, on the two particles.
- The electrical force between two stationary point charges is given by Coulomb’s Law.



Coulomb’s torsion balance, used to establish the inverse square law.

Coulomb's Law, cont.

Mathematically, the magnitude of the force between two stationary charges is

$$F_e = k_e \frac{|q_1|\,|q_2|}{r^2}$$

The SI unit of charge is the **coulomb** C.

- $k_e = 8.9876 \times 10^9\ \mathrm{N{\cdot}m^2/C^2} = 1/(4\pi\varepsilon_o)$ is called the **Coulomb constant.**
- $\varepsilon_o = 8.8542 \times 10^{-12}\ \mathrm{C^2 / N{\cdot}m^2}$ is the **permittivity of free space.**

Remember the charges need to be in coulombs. e is the smallest unit of charge except quarks. Since $e = 1.6 \times 10^{-19}$ C, so 1 C needs 6.24×10^{18} electrons or protons.

Typical charges can be in the μC range.

Remember that force is a *vector* quantity.

Vector Nature of Electric Forces

- In vector form, the force between charges is

$$\vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}_{12}$$

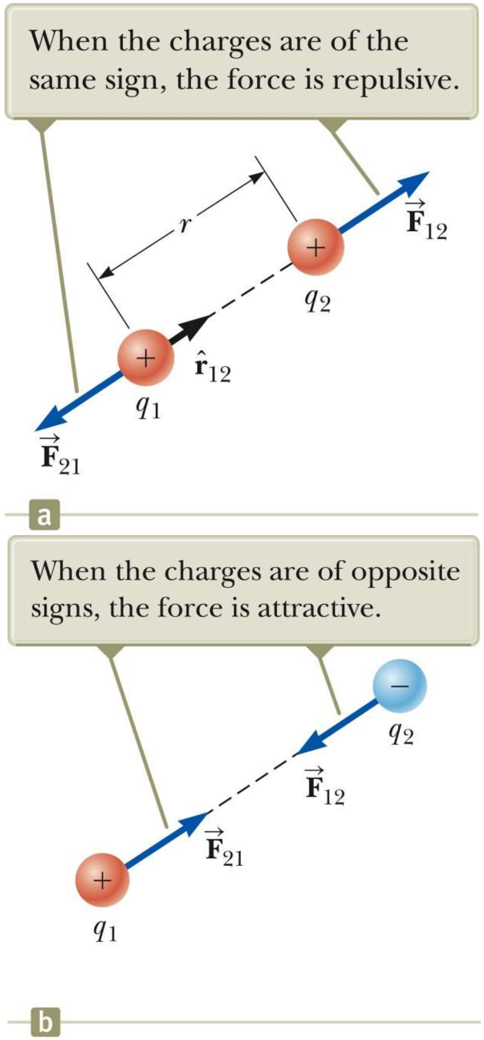
$\hat{\mathbf{r}}_{12}$ is a unit vector directed from q_1 to q_2.

- (a) The like charges produce a repulsive force between them.
- (b) The unlike charges produce an attractive force between them.

Electrical forces obey Newton's Third Law.
The force on q_1 is equal in magnitude and opposite in direction to the force on q_2

$$\vec{\mathbf{F}}_{21} = -\vec{\mathbf{F}}_{12}$$

With like signs for the charges, the product $q_1 q_2$ is positive and the force is repulsive.



Multiple Charges

•The resultant force on any one charge equals the vector sum of the forces exerted by the other individual charges that are present.

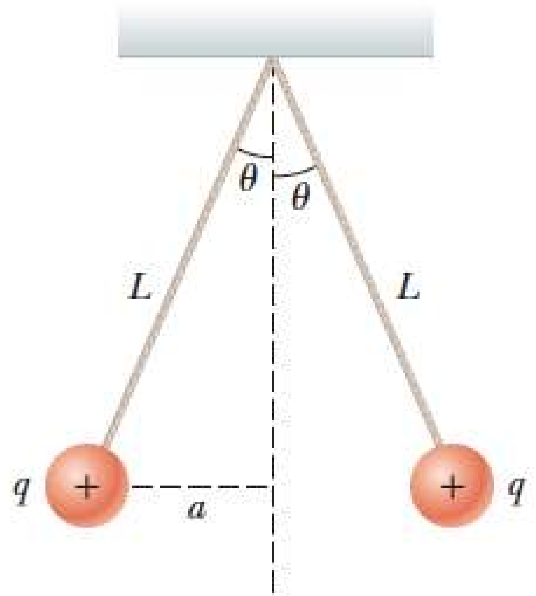
- Remember to add the forces *as vectors.*

•For example, if four charges are present, the resultant force on one of these equals the vector sum of the forces exerted on it by each of the other charges.

$$\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_{21} + \vec{\mathbf{F}}_{31} + \vec{\mathbf{F}}_{41}$$

Example Finding the Charge on the Spheres

Two identical small charged spheres, each having a mass of 3.00×10^{-2} kg, hang in equilibrium as shown in Figure 23.9a. The length L of each string is 0.150 m, and the angle θ is $5.00°$. Find the magnitude of the charge on each sphere.



The spheres are in equilibrium.

Since they are separated, they exert a repulsive force on each other.

- Charges are like charges

Model each sphere as a particle in equilibrium.

Proceed as usual with equilibrium problems, noting one force is an electrical force.

Electrical Force with Other Forces Example

- The force diagram includes the components of the tension, the electric force, and the gravity force.
- Solve for $|q|$
- Note that you cannot determine the sign of *q*, only that they both have the same sign.

$$(1)\quad \sum F_x = T\sin\theta - F_e = 0 \;\rightarrow\; T\sin\theta = F_e$$

$$(2)\quad \sum F_y = T\cos\theta - mg = 0 \;\rightarrow\; T\cos\theta = mg$$

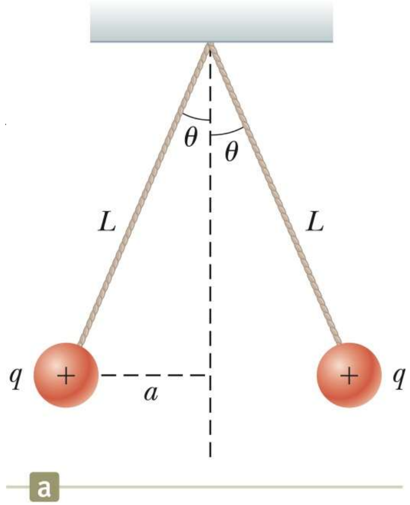
$$\tan\theta = \frac{F_e}{mg} \;\rightarrow\; F_e = mg\tan\theta$$

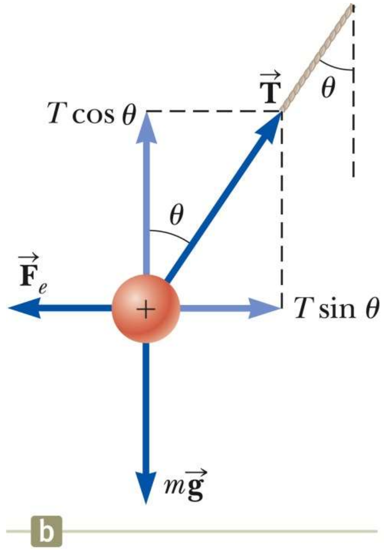
$$\sin\theta = \frac{a}{L} \;\rightarrow\; a = L\sin\theta$$

$$|q| = \sqrt{\frac{F_e r^2}{k_e}} = \sqrt{\frac{F_e(2a)^2}{k_e}} = \sqrt{\frac{mg\tan\theta(2L\sin\theta)^2}{k_e}}$$

$$|q| = \sqrt{\frac{(3.00\times10^{-2}\text{ kg})(9.80\text{ m/s}^2)\tan(5.00°)[2(0.150\text{ m})\sin(5.00°)]^2}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2}}$$

$$= \boxed{4.42\times10^{-8}\text{ C}}$$





Electric Field – Definition

An **electric field** is said to exist in the region of space around a charged object.

- This charged object is the **source charge.**

When another charged object, the **test charge**, enters this electric field, an electric force acts on it.

The electric field is defined as the electric force on the test charge per unit charge.

The electric field vector, $\vec{E}$, at a point in space is defined as the electric force acting on a positive test charge, q_o, placed at that point divided by the test charge:

$$\vec{\mathbf{E}} \equiv \frac{\vec{\mathbf{F}}}{q_o}$$

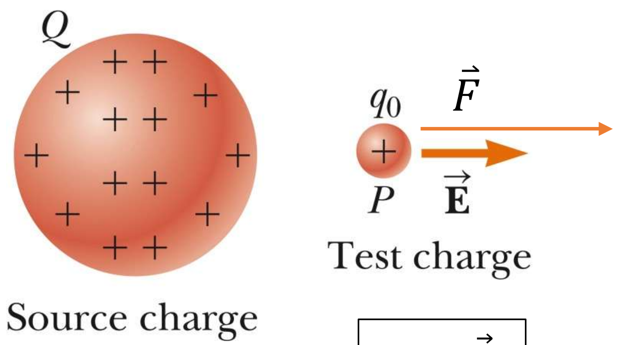
The SI unit of $\vec{\mathbf{E}}$ is N/C (or more often V/m)

Electric Field, Notes

- $\vec{E}$ is the field produced by some charge or charge distribution, separate from the test charge.
- The existence of an electric field is a property of the source charge.
  - The presence of the test charge is not necessary for the field to exist.
- The test charge serves as a detector of the field.

The direction of $\vec{\mathbf{E}}$ is that of the force on a positive test charge.

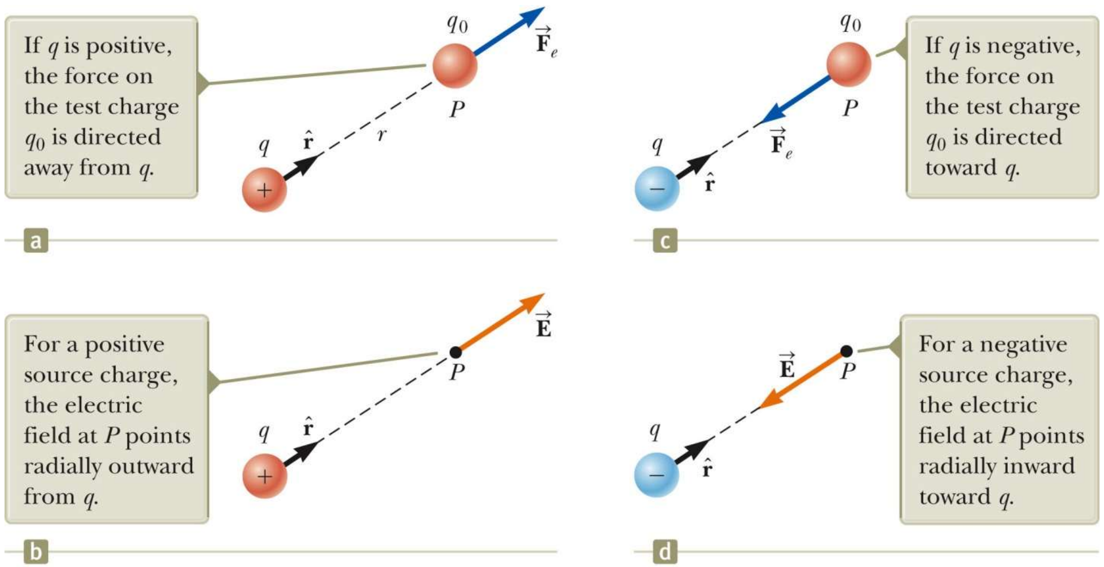
We can also say that an electric field exists at a point if a test charge at that point experiences an electric force.



$$\vec{\mathbf{E}} \equiv \frac{\vec{\mathbf{F}}}{q_o}$$

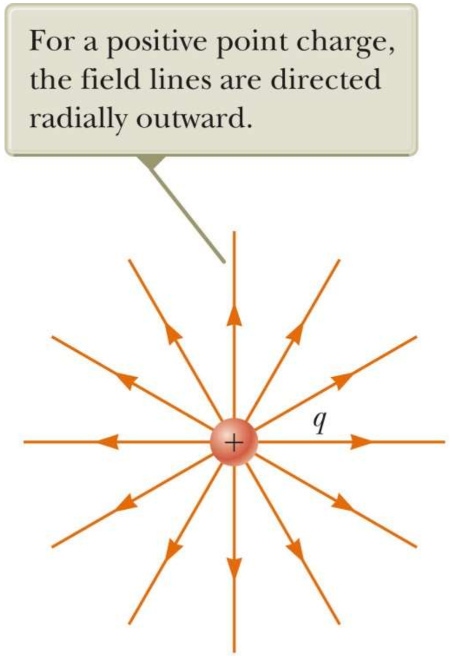
More About Electric Field Direction

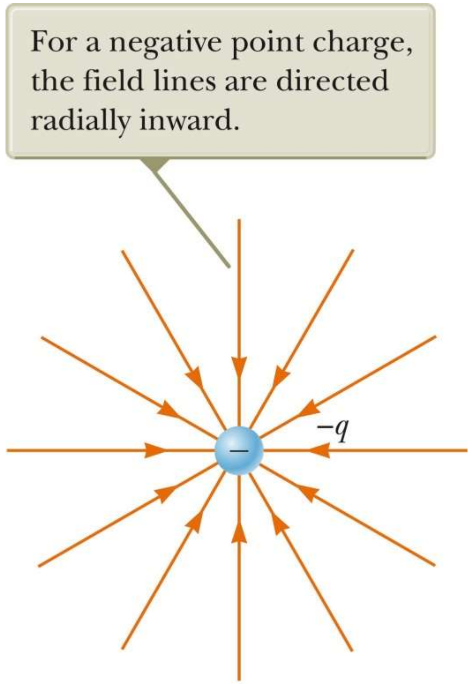
- a) *q* is positive, the force is directed away from *q.*
- b) The direction of the field is also away from the positive source charge.
- c) *q* is negative, the force is directed toward *q.*
- d) The field is also toward the negative source charge.



Electric Field Lines, Point Charge

- The field lines radiate outward in all directions.
  - In three dimensions, the distribution is spherical.
- The lines are directed away from the source charge.
  - A positive test charge would be repelled away from the positive **source** charge.





The field lines radiate inward in all directions.

The lines are directed toward the source charge.

- A positive test charge would be attracted toward the negative source charge.

Relationship Between F and E

- $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$
- If q is positive, the force and the field are in the same direction.
- If q is negative, the force and the field are in opposite directions.

Electric Field, Vector Form

Remember Coulomb's law, between the source and test charges, can be expressed as

$$\vec{\mathbf{F}}_e = k_e \frac{q q_o}{r^2} \hat{\mathbf{r}}$$

Then, the electric field of a point charge will be

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}_e}{q_o} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$

Electric Fields from Multiple Charges

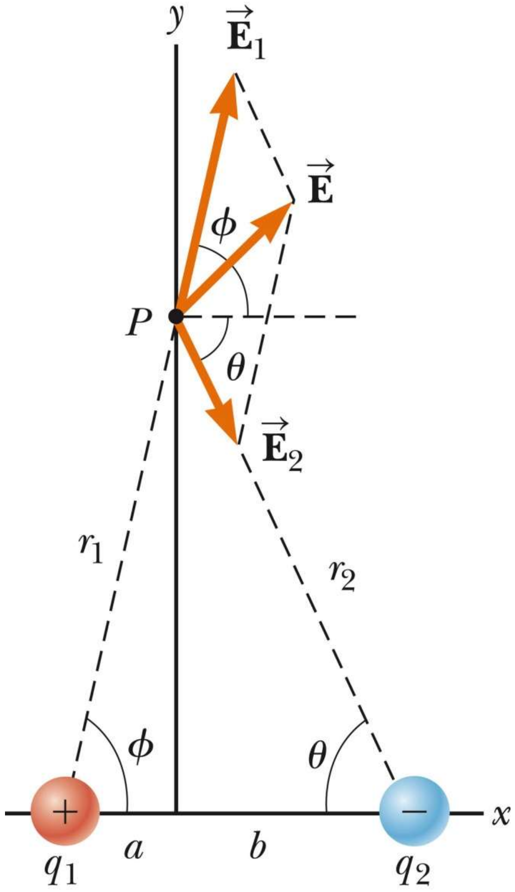
- At any point P, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges.

$$\vec{\mathbf{E}} = \sum_i \vec{\mathbf{E}}_i$$

$$= k_e \sum_i \frac{q_i}{{r_i}^2} \hat{\mathbf{r}}_i$$

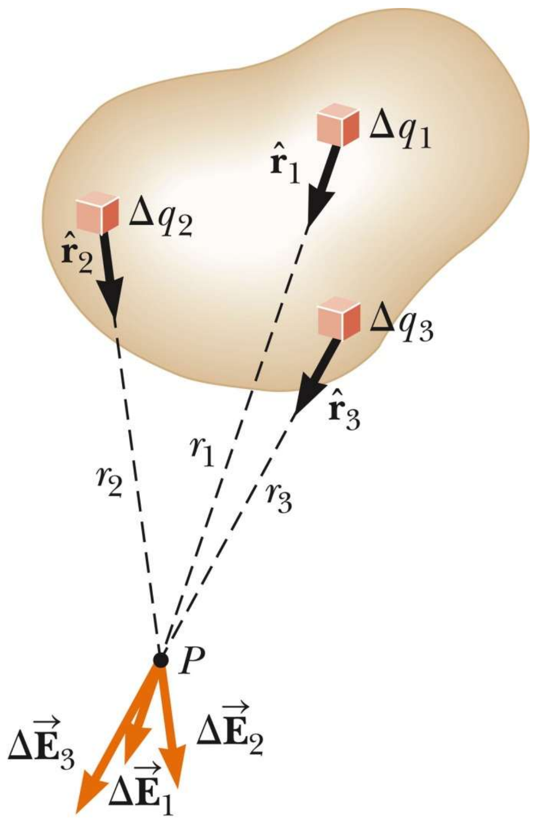
The figure represent the vector sum of the electric field produced by two points charges.

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = \frac{k_e q_1}{{r_1}^2} \hat{\mathbf{r}}_1 + \frac{k_e q_2}{{r_2}^2} \hat{\mathbf{r}}_2$$



Electric Field – Continuous Charge Distribution, cont

A continuous object can have distributed charges with varying charge densities at different locations.



- Procedures in finding the total electric field at point P:
  - Divide the charge distribution into small elements, each of which contains Δq.
  - Calculate the electric field due to one of these elements at point P.
  - Evaluate the total field by summing the contributions of all the charge elements.

- Equations:
- For the individual charge elements

$$\Delta\vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2}\hat{\mathbf{r}}$$

- Because the charge distribution is continuous

$$\vec{\mathbf{E}} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta \mathrm{q_i}}{r_i^2}\hat{\mathbf{r}}_\mathrm{i} = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}$$

This becomes an integration.
More details are provided at the end of this chapter.

Motion of Charged Particles

- When a charged particle is placed in an electric field, it experiences an electrical force.
- If this is the only force on the particle, it must be the net force.
- The net force will cause the particle to accelerate according to Newton's second law.

If the field is uniform, then the acceleration is constant.

The particle under the constant acceleration model can be applied to the motion of the particle.

- The electric force causes a particle to move according to the models of forces and motion.

If the particle has a positive charge, its acceleration is in the direction of the field.

If the particle has a negative charge, its acceleration is in the direction opposite the electric field.

Electron in a Uniform Field

An electron enters the region of a uniform electric field as shown in Active Figure 23.24, with $v_i = 3.00 \times 10^6$ m/s and $E = 200$ N/C. The horizontal length of the plates is $\ell = 0.100$ m.

(A) Find the acceleration of the electron while it is in the electric field.

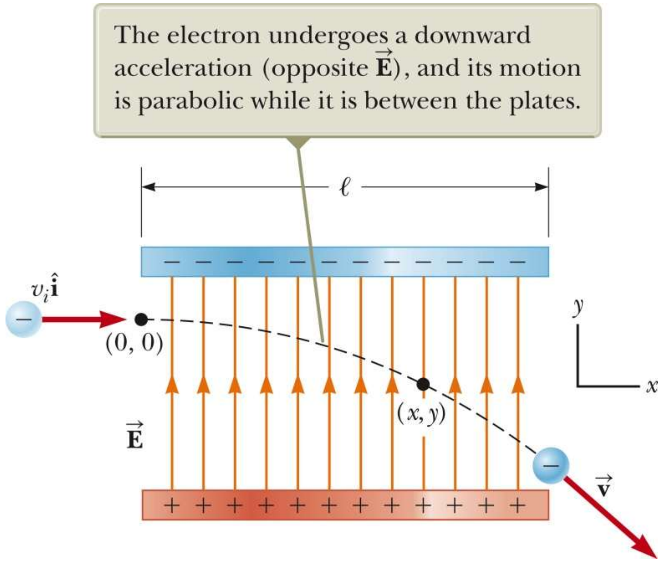
$$\sum F_y = ma_y \rightarrow a_y = \frac{\sum F_y}{m} = -\frac{eE}{m_e}$$

$$a_y = -\frac{(1.60 \times 10^{-19}\text{ C})(200\text{ N/C})}{9.11 \times 10^{-31}\text{ kg}} = \boxed{-3.51 \times 10^{13}\text{ m/s}^2}$$

(B) Assuming the electron enters the field at time $t = 0$, find the time at which it leaves the field.

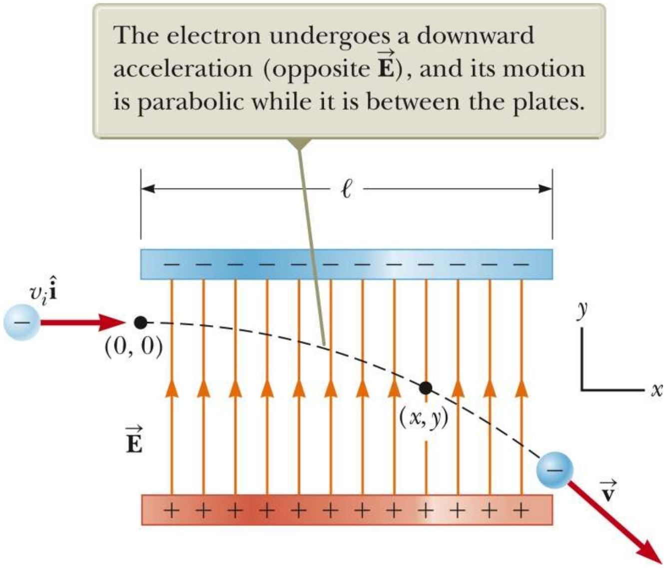
$$x_f = x_i + v_x t \rightarrow t = \frac{x_f - x_i}{v_x}$$

$$t = \frac{\ell - 0}{v_x} = \frac{0.100\text{ m}}{3.00 \times 10^6\text{ m/s}} = \boxed{3.33 \times 10^{-8}\text{ s}}$$



Cont. Electron in a Uniform Field

(C) Assuming the vertical position of the electron as it enters the field is $y_i = 0$, what is its vertical position when it leaves the field?



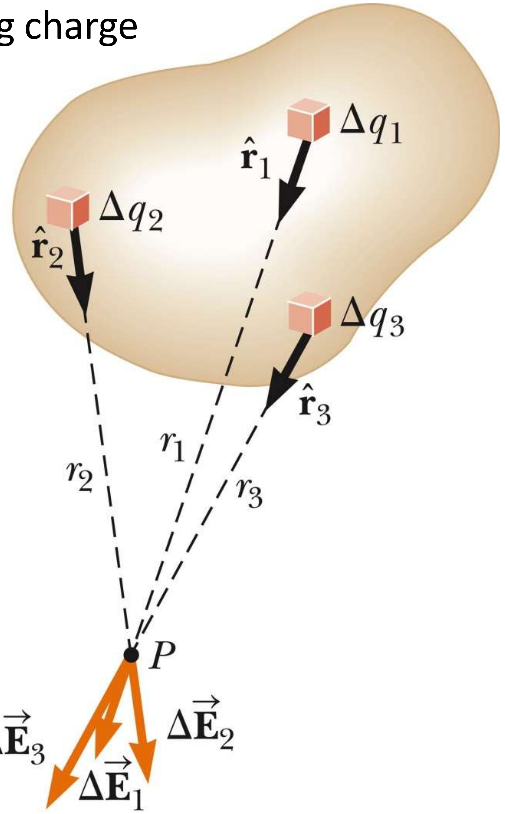
- The electron is projected horizontally into a uniform electric field.
- The electron undergoes a downward acceleration.
  - It is negative, so the acceleration is opposite the direction of the field.
- Its motion is parabolic while between the plates.

$$y_f = y_i + v_{yi}t + \tfrac{1}{2}a_y t^2$$

$$y_f = 0 + 0 + \tfrac{1}{2}(-3.51 \times 10^{13}\ \text{m/s}^2)(3.33 \times 10^{-8}\ \text{s})^2$$
$$= -0.019\,5\ \text{m} = \boxed{-1.95\ \text{cm}}$$

Electric Field – Continuous Charge Distribution, cont

A continuous object can have distributed charges with varying charge densities at different locations.



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- Because the charge distribution is continuous

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This becomes an integration.
More details are provided at the end of this chapter.

Charge Densities

•ρ is the **Volume charge density**.
When a charge Q is distributed evenly throughout a volume V

- $\rho \equiv Q / V$ (in units of $\mathrm{C/m^3}$)

•σ is the **Surface charge density**.
When a charge Q is distributed evenly over a surface area A

- $\sigma \equiv Q / A$ (in units of $\mathrm{C/m^2}$)

•λ is the **Linear charge density**.
When a charge Q is distributed evenly along a line of length ℓ

- $\lambda \equiv Q / \ell$ (in units of $\mathrm{C/m}$)

Amount of Charge in an Infinitesimally Small Volume/Area/Length

When charges are NOT uniformly distributed over a volume, surface, or line, the amount of charge, dq, in a small element is given by

- $dq = \rho dV$ for a small volume of dV ($dq = \rho\, dx\, dy\, dz$ in Cartesian coordinates)
- $dq = \sigma dA$ for a small area of dA ($dq = \sigma\, dx\, dy$ for surface in xy-plane)
- $dq = \lambda d\ell$ for a small area of $d\ell$ ($dq = \lambda\, dx$ for line along x-direction)

Problem-Solving Strategy

•*Conceptualize*

- Establish a mental representation of the problem.
- Imagine the electric field produced by the charges or charge distribution.

•*Categorize*

- Individual charge? Group of individual charges? Continuous distribution of charges?

Analyze

- **Analyzing a group of individual charges**:
  - Use the superposition principle, find the fields due to the individual charges at the point of interest and then add them as vectors to find the resultant field.
- **Analyzing a continuous charge distribution:**
  - The vector sums for evaluating the total electric field at some point must be replaced with vector integrals.
  - Divide the charge distribution into infinitesimal pieces, calculate the vector sum by integrating over the entire charge distribution.
- **Symmetry:** Taking advantage of any symmetry to simplify calculations.

A few terminologies in Definite Integral and a few examples of indefinite integral

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

$f(x)$ is called integrand;

$F(x)$ is called integral function;

$f(x)$ is the derivative of $F(x)$;

$[a,\ b]$ is the integral range.

$$\int x^n dx = \frac{1}{n+1} x^{n+1}$$

$$\int \frac{1}{x} dx = \ln(x)$$

$$\int \sin x dx = -\cos x$$

$$\int \cos x dx = \sin x$$

Example – Charged Rod

[Video: Derivation](#)

•The rod of length l has a uniform charge density λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of rod and a distance a from one end.

•Conceptualize: The field $d\vec{\mathbf{E}}$ at point P due to each segment of charge on the rod is in the negative *x* direction because every segment carries a positive charge.

•Categorize: The continuous rod results in the field produced by a continuous charge distribution; each segment produces a field in the same direction, -*x*.

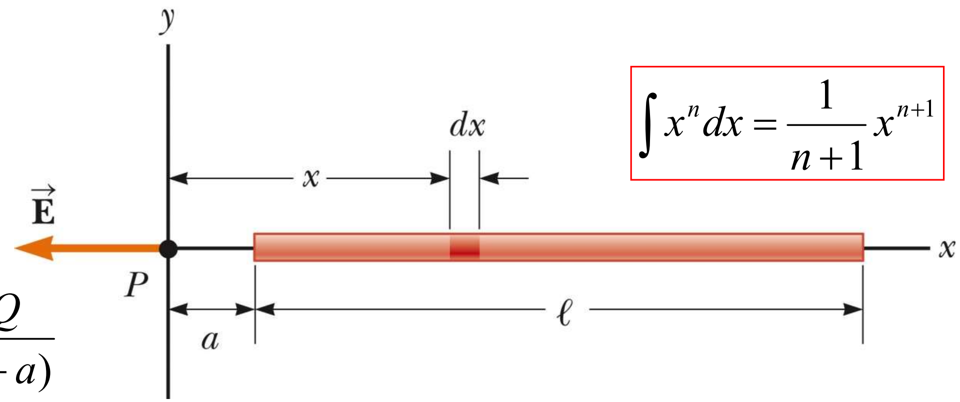
•Analyze: Choose *dq* as a segment of the rod. The segment has a length of *dx.* By symmetry, the field at an axial point must be along the central axis.

$$dq = \lambda dx$$

$$dE = k_e \frac{dq}{x^2} = k_e \frac{\lambda dx}{x^2}$$

$$E = \int_a^{l+a} k_e \lambda \frac{dx}{x^2} = k_e \lambda \int_a^{l+a} \frac{dx}{x^2}$$

$$= k_e \lambda \left[-\frac{1}{x} \right]\Big|_a^{l+a} = k_e \frac{Q}{l} \left[\frac{1}{a} - \frac{1}{l+a} \right] = \frac{k_e Q}{a(l+a)}$$

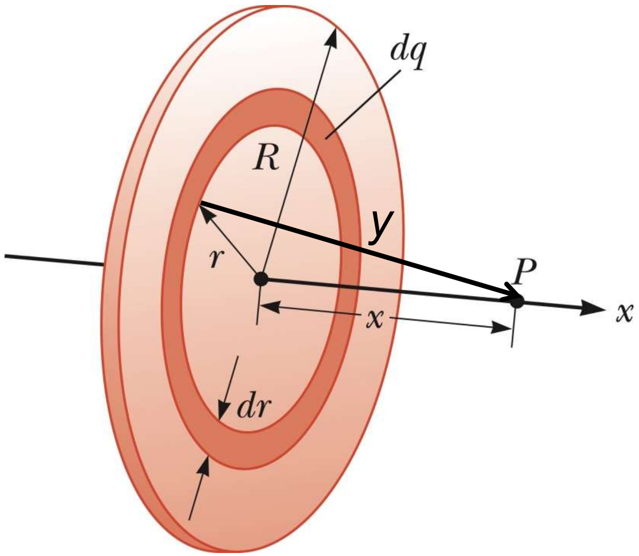


$$\boxed{\int x^n dx = \frac{1}{n+1} x^{n+1}}$$

Charged Disk

[Video: Derivation]

- The disk has a radius *R* and a uniform charge density *σ*. Calculate the electric field at position P on the central axis.
- Choose *dq* as a ring of radius *r.* The ring has a surface area 2*πr dr.*
- By symmetry, the field at an axial point must be along the central axis.



$$dq = \sigma dA = \sigma(2\pi r dr) = 2\pi\sigma r dr$$

$$dE_x = k_e \frac{dq}{y^2}\cos\theta$$

$$\cos\theta = x/y = x/(x^2+r^2)^{1/2}$$

$$\boxed{\int x^n dx = \frac{1}{n+1}x^{n+1}}$$

$$dE_x = k_e \frac{dq}{y^2}\cos\theta = k_e x \frac{2\pi\sigma r dr}{(x^2+r^2)^{3/2}}$$

$$E_x = k_e x\pi\sigma \int_0^R \frac{2r dr}{(x^2+r^2)^{3/2}} = k_e x\pi\sigma \int_0^R (x^2+r^2)^{-3/2} d(r^2)$$

$$= k_e x\pi\sigma \frac{(x^2+r^2)^{-1/2}}{-1/2}\Bigg|_0^R = 2\pi k_e\sigma\left[1-\frac{x}{(x^2+R^2)^{1/2}}\right]$$