Fundamentals of Electric Circuits

CHAPTER 6 Capacitors and inductors

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CHAPTER 6 Capacitors and inductors

6.2 Capacitors

6.3 Series and parallel capacitors

6.4 Inductors

6.5 Series and parallel inductors

Introduction

- Resistor: a passive element which dissipates energy only
- Two important passive linear circuit elements:

  (1) Capacitor

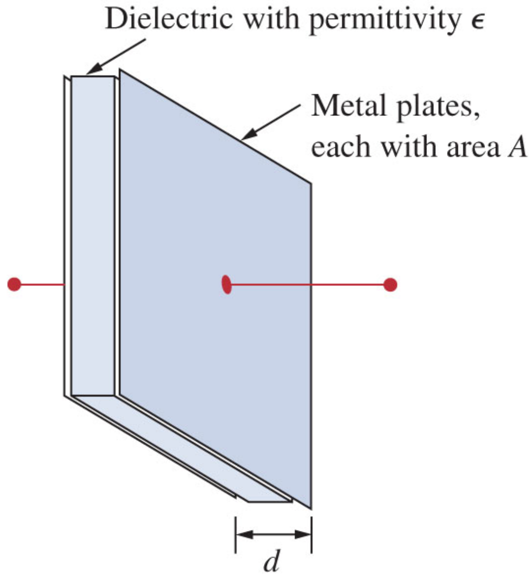
  (2) Inductor

- Capacitors and inductors can neither generate nor dissipate energy but store energy, which can be retrieved at a later time.
- Capacitors and inductors are called storage elements.

6.2 Capacitors

- A capacitor consists of two conducting plates separated by an insulator (or dielectric). The plates are typically aluminum foil, the dielectric is often air, ceramic, paper, plastic, or mica.

- It is a passive element that stores energy in the electric field that exists between the plates of the capacitor.



Capacitors

- When a voltage source v is connected to the capacitor, the source deposits a positive charge q on one plate and a negative charge $-q$ on the other.

- The amount of charge q is proportional to the applied voltage v :

$$q = Cv$$

where C is the capacitance of the capacitor.

- The capacitance is a measure of a capacitor's ability to store charge.

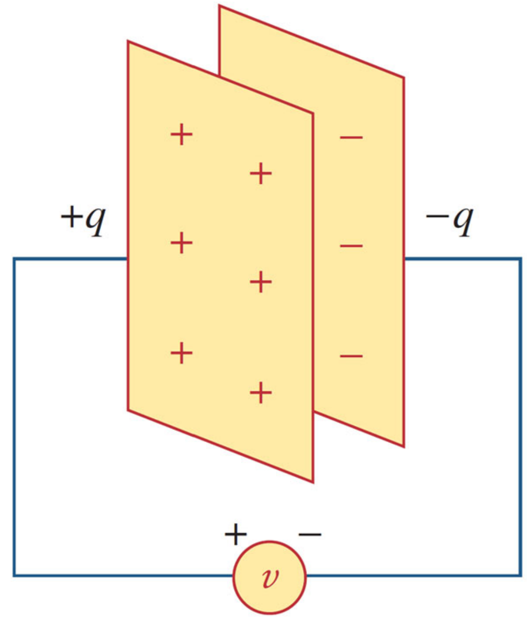
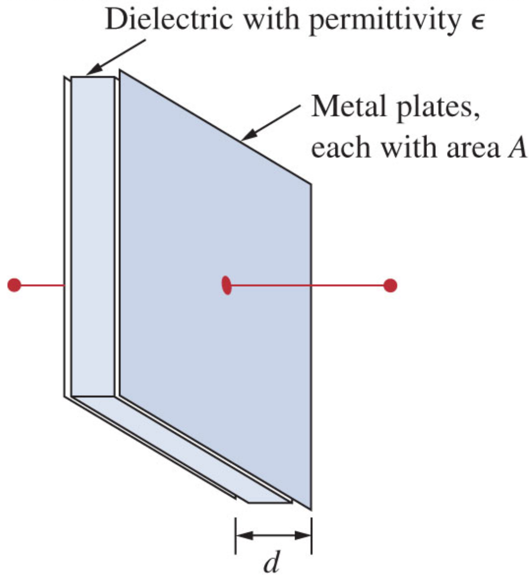


Figure 6.2
A capacitor with applied voltage v.

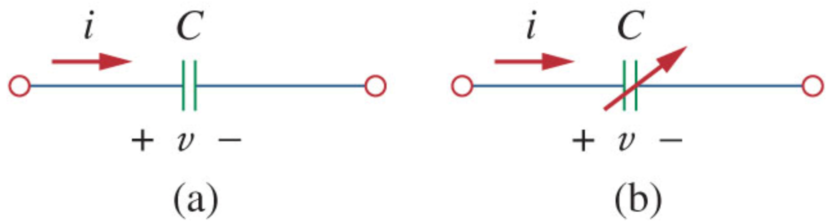
Capacitors

- The unit of capacitance is the Farad (F)
- One Farad is 1 Coulomb/Volt ($q = Cv$)
- Typically, capacitors have values in picofarad (pF) to microfarad (μF) range.
- Capacitance is determined by the physical dimensions of the capacitor:
  - the surface area of each plate (A)
  - the distance between the plates (d)
  - ε is the permittivity of the dielectric material between the plates

$$C = \frac{\varepsilon A}{d}$$



Types of Capacitors



If $vi > 0$, the capacitor is being charged

if $vi < 0$, the capacitor is discharged

Circuit symbols for (a) fixed and (b) variable capacitors

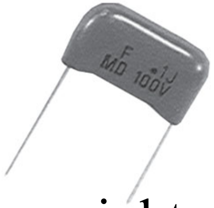
The capacitance can be adjusted by turning the shaft

- The variable capacitor is often placed in parallel with another capacitor so that the equivalent capacitance can be varied slightly.
- Variable capacitors are used in radio receivers allowing one to tune the various stations.

Types of Capacitors

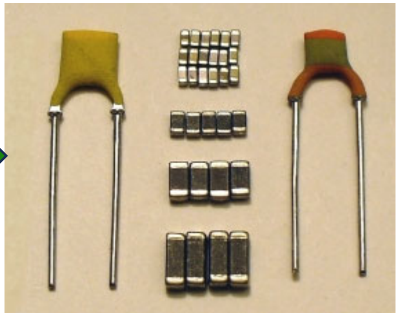
Dielectric material

Polyester capacitor



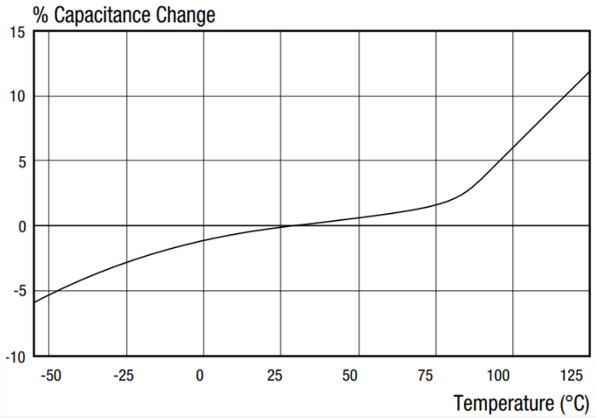
High frequency and small capacitance

Ceramic capacitor



Electrolytic capacitor

- light in weight
- stable
- their change with temperature is predictable

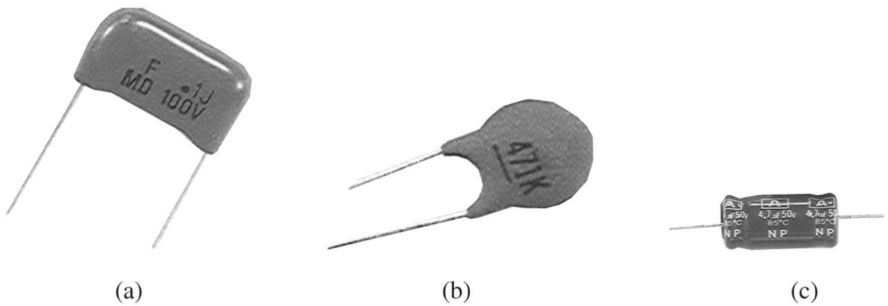


- High stability and low losses for resonant circuit applications
- The most produced and used capacitors in electronic equipment

- Low frequency and high capacitance

Applications of Capacitors

- Capacitors have a wide range of applications, such as:
  - Blocking DC
  - Passing AC
  - Shift phase
  - Store energy
  - Suppress noise
  - Start motors



Courtesy of Tech America

Current-Voltage Relationship

- Using the formula for the charge stored in a capacitor, we can find the current-voltage relationship
- Take the derivative with respect to time gives:

$$q = Cv \qquad i = C\frac{dv}{dt} = \frac{dq}{dt}$$

- Linear capacitors: the capacitance is independent of voltage

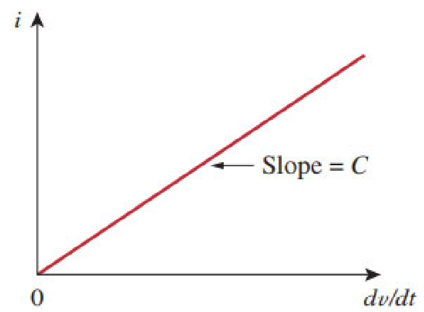


Figure 6.6
Current-voltage relationship of a capacitor.

Stored Charge

- Similarly, the voltage-current relationship can be obtained by integrating both sides:

$$i = C\frac{dv}{dt} \quad \Rightarrow \quad v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

- $v(t_0)$ is the voltage across the capacitor at time t_0. It shows that capacitor voltage depends on the past history of the capacitor current. Hence, the capacitor has memory.

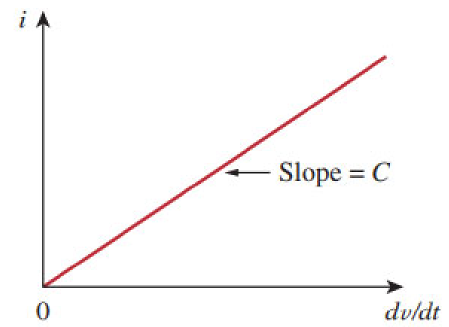


Figure 6.6
Current-voltage relationship of a capacitor.

- The instantaneous power delivered to the capacitor is

$$p = vi = Cv\frac{dv}{dt}$$

- The energy stored in a capacitor is:

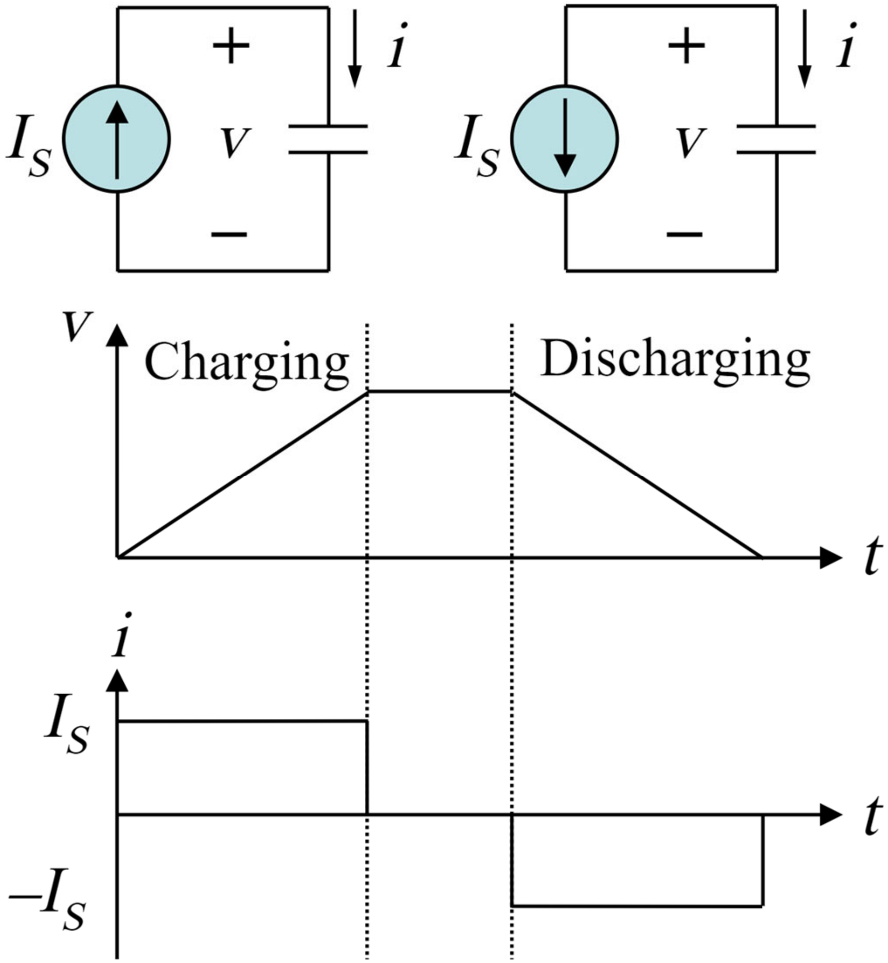
$$W = \int_{-\infty}^{t} p(\tau)d\tau = C\int_{-\infty}^{t} v\frac{dv}{d\tau}d\tau = C\int_{v(-\infty)}^{v(t)} vdv = \frac{1}{2}Cv^2\Big|_{v(-\infty)}^{v(t)} = \frac{1}{2}Cv(t)^2 - \frac{1}{2}Cv(-\infty)^2$$

$$\boldsymbol{W = \frac{1}{2}CV^2 = q^2/2C}$$

$i = C\frac{dv}{dt}$ $\qquad v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$

- The energy stored in a capacitor is:

$W = \frac{1}{2}CV^2 = q^2/2C$



Properties of Capacitors

$$i = C\frac{dv}{dt}$$

1. When the voltage across the capacitor is not changing with time (i.e., dc voltage), the current through the cap is zero.

   If DC voltage is applied, no current will flow.

2. Voltage across a capacitor cannot change instantaneously since injecting or extracting energy can only be done over some time.

3. An abrupt change in voltage would require an infinite current!

4. If the voltage on the cap does not equal the applied voltage, charge will flow and the voltage will finally reach the applied voltage.

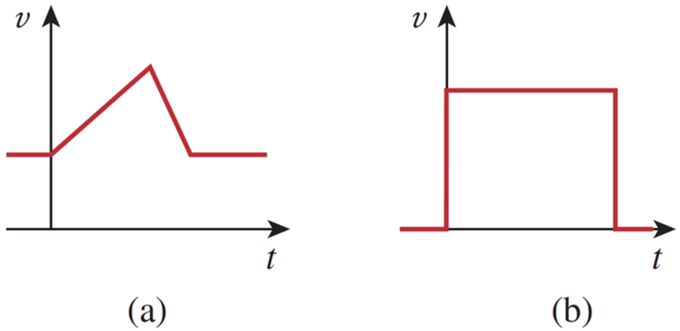


Figure 6.7
Voltage across a capacitor: (a) allowed, (b) not allowable; an abrupt change is not possible.

Properties of Capacitors

$i = C\frac{dv}{dt}$

5. An ideal capacitor does not dissipate energy. It takes power from the circuit when storing energy in the field and returns the stored energy when delivering power to the circuit.

6. A real capacitor has a parallel-model leakage resistance, this resistance is typically very high, on the order of 100 MΩ and thus can be ignored for many circuit applications.

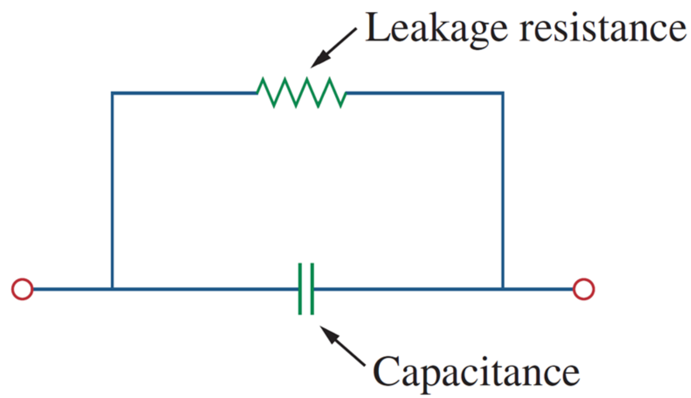


Figure 6.8
Circuit model of a nonideal capacitor.

Example

Example 6.1

(a) Calculate the charge stored on a 3-pF capacitor with 20 V across it.
(b) Find the energy stored in the capacitor.

Solution:

(a) Since $q = Cv$,

$$q = 3 \times 10^{-12} \times 20 = 60 \text{ pC}$$

(b) The energy stored is

$$w = \frac{1}{2}Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = 600 \text{ pJ}$$

Example

Example 6.2

The voltage across a 5-μF capacitor is

$$v(t) = 10 \cos 6000t \text{ V}$$

Calculate the current through it.

Solution:

By definition, the current is

$$\begin{aligned} i(t) &= C\frac{dv}{dt} = 5 \times 10^{-6}\frac{d}{dt}(10 \cos 6000t) \\ &= -5 \times 10^{-6} \times 6000 \times 10 \sin 6000t = -0.3 \sin 6000t \text{ A} \end{aligned}$$

Example

Example 6.3

Determine the voltage across a 2-μF capacitor if the current through it is

$$i(t) = 6e^{-3000t} \text{ mA}$$

Assume that the initial capacitor voltage is zero.

Solution:

Since $v = \dfrac{1}{C}\displaystyle\int_0^t i\,dt + v(0)$ and $v(0) = 0$,

$$v = \frac{1}{2 \times 10^{-6}} \int_0^t 6e^{-3000t}\,dt \cdot 10^{-3}$$

$$= \frac{3 \times 10^3}{-3000} e^{-3000t}\Big|_0^t = (1 - e^{-3000t}) \text{ V}$$

Example

Example 6.4

Determine the current through a 200-μF capacitor whose voltage is shown in Fig. 6.9.

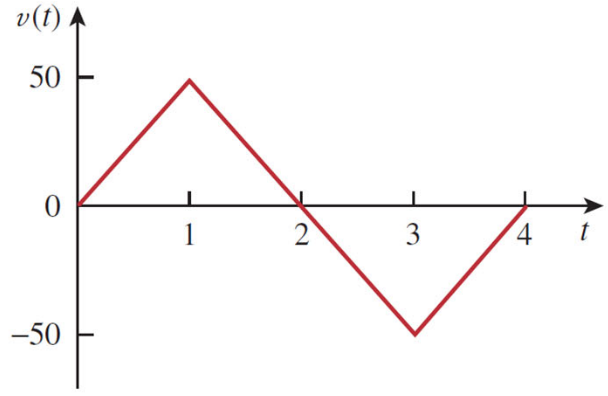


Figure 6.9
For Example 6.4.

Solution:
The voltage waveform can be described mathematically as

$$v(t) = \begin{cases} 50t \text{ V} & 0 < t < 1 \\ 100 - 50t \text{ V} & 1 < t < 3 \\ -200 + 50t \text{ V} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Since $i = C\,dv/dt$ and $C = 200\ \mu$F, we take the derivative of v to obtain

$$i(t) = 200 \times 10^{-6} \times \begin{cases} 50 & 0 < t < 1 \\ -50 & 1 < t < 3 \\ 50 & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 10 \text{ mA} & 0 < t < 1 \\ -10 \text{ mA} & 1 < t < 3 \\ 10 \text{ mA} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Thus the current waveform is as shown in Fig. 6.10.

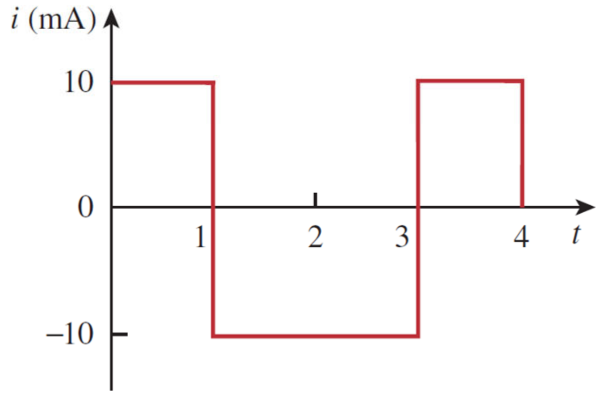


Figure 6.10
For Example 6.4.

Example

Practice Problem 6.4

An initially uncharged 1-mF capacitor has the current shown in Fig. 6.11 across it. Calculate the voltage across it at $t = 2$ ms and $t = 5$ ms.

Answer: 100 mV, 400 mV.

$$i = C\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

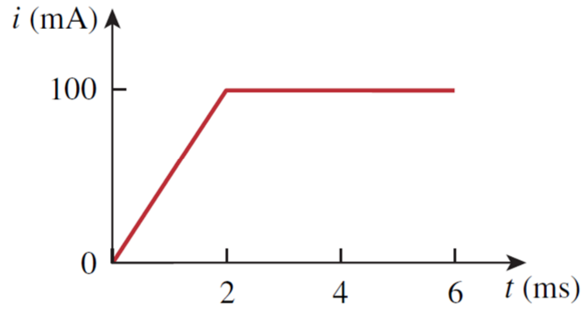


Figure 6.11
For Practice Prob. 6.4.

Example 6.5

Example

Obtain the energy stored in each capacitor in Fig. 6.12(a) under dc conditions.

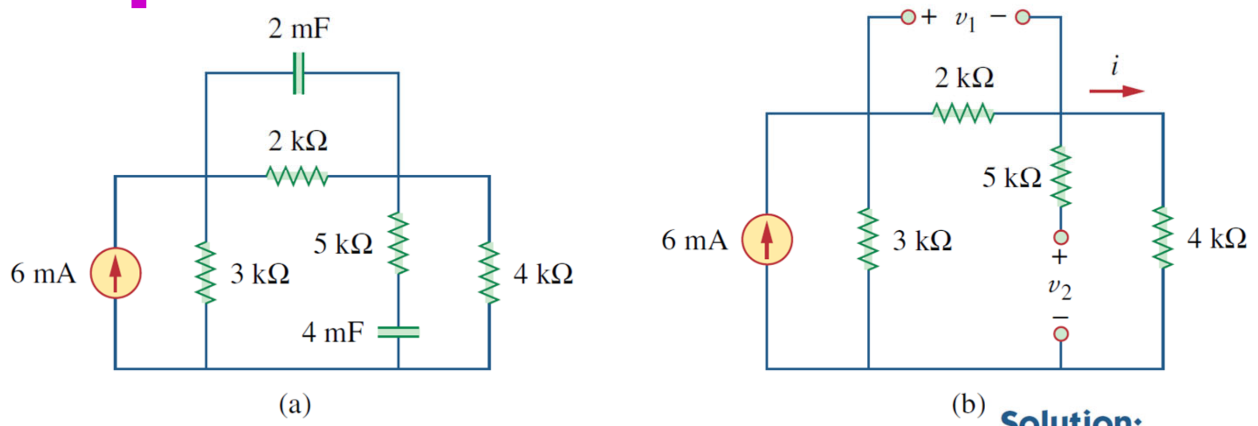


Figure 6.12
For Example 6.5.

Solution:

Under dc conditions, we replace each capacitor with an open circuit, as shown in Fig. 6.12(b). The current through the series combination of the 2-kΩ and 4-kΩ resistors is obtained by current division as

$$i = \frac{3}{3 + 2 + 4}(6\text{ mA}) = 2\text{ mA}$$

Hence, the voltages v_1 and v_2 across the capacitors are

$$v_1 = 2000i = 4\text{ V} \qquad v_2 = 4000i = 8\text{ V}$$

and the energies stored in them are

$$w_1 = \frac{1}{2}C_1 v_1^2 = \frac{1}{2}(2 \times 10^{-3})(4)^2 = 16\text{ mJ}$$

$$w_2 = \frac{1}{2}C_2 v_2^2 = \frac{1}{2}(4 \times 10^{-3})(8)^2 = 128\text{ mJ}$$

6.3 Series and Parallel Capacitors

Parallel Capacitors

- Starting with N parallel capacitors, one can note that the voltages on all the caps are the same.
- Applying KCL:

$$i = i_1 + i_2 + i_3 + \cdots + i_N \qquad i = C\frac{dv}{dt}$$

- Taking into consideration the current voltage relationship of each capacitor:

$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt} + \cdots + C_N\frac{dv}{dt}$$

$$= \left(\sum_{k=1}^{N} C_k\right)\frac{dv}{dt} = C_{eq}\frac{dv}{dt}$$

- Where

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$$

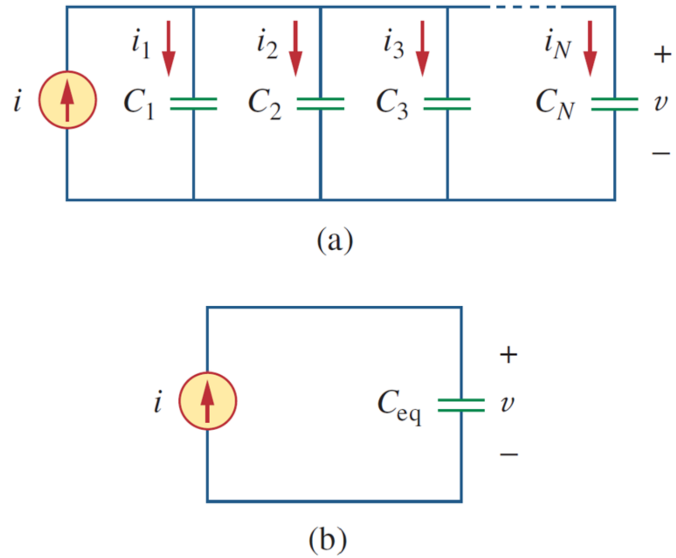


Figure 6.14
(a) Parallel-connected N capacitors, (b) equivalent circuit for the parallel capacitors.

The equivalent capacitance of N parallel-connected capacitors is the sum of the individual capacitances.

Series Capacitors

- Here each capacitor shares the same current.
- Applying KVL to the loop:

$$v = v_1 + v_2 + v_3 + \cdots + v_N \qquad v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

- Now apply the voltage current relationship

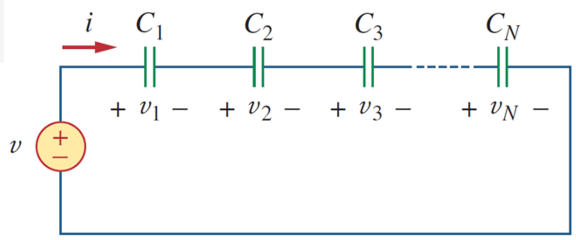
$$v = \frac{1}{C_1}\int_{t_0}^{t} i(\tau)d\tau + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(\tau)d\tau + v_2(t_0) + \frac{1}{C_3}\int_{t_0}^{t} i(\tau)d\tau + v_3(t_0) + \ldots + \frac{1}{C_N}\int_{t_0}^{t} i(\tau)d\tau + v_N(t_0)$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \frac{1}{C_N}\right)\int_{t_0}^{t} i(\tau)d\tau + v_1(t_0) + v_2(t_0) + v_3(t_0) + \cdots + v_N(t_0)$$

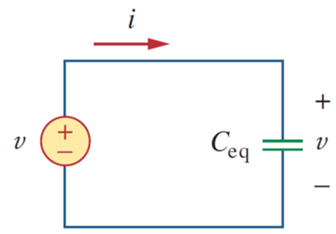
$$= \left(\frac{1}{C_{eq}}\right)\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

- Where

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$



(a)



(b)

Figure 6.15
(a) Series-connected N capacitors, (b) equivalent circuit for the series capacitor.

The equivalent capacitance of series-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.

Example

Example 6.6

Find the equivalent capacitance seen between terminals a and b of the circuit in Fig. 6.16.

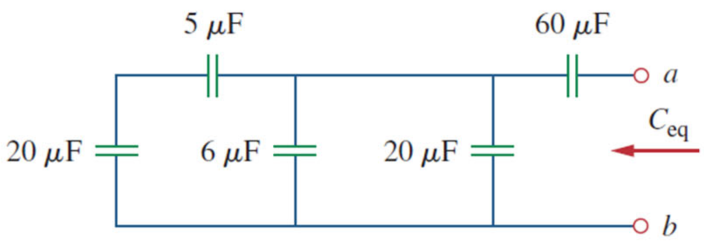


Figure 6.16
For Example 6.6.

Solution:

The 20-μF and 5-μF capacitors are in series; their equivalent capacitance is

$$\frac{20 \times 5}{20 + 5} = 4\ \mu\text{F}$$

This 4-μF capacitor is in parallel with the 6-μF and 20-μF capacitors; their combined capacitance is

$$4 + 6 + 20 = 30\ \mu\text{F}$$

This 30-μF capacitor is in series with the 60-μF capacitor. Hence, the equivalent capacitance for the entire circuit is

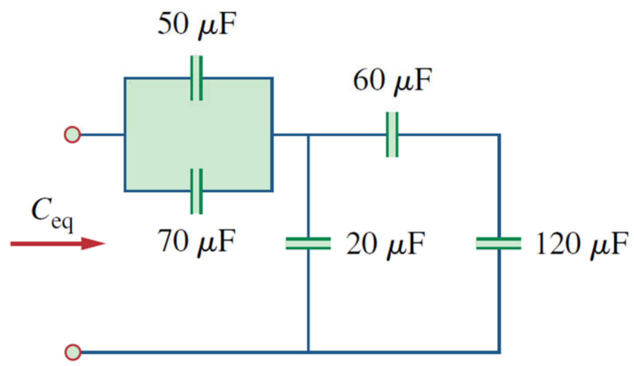
$$C_{eq} = \frac{30 \times 60}{30 + 60} = 20\ \mu\text{F}$$

Example

Find the equivalent capacitance seen at the terminals of the circuit in Fig. 6.17.

Practice Problem 6.6

Answer: 40 μF.



Example

For the circuit in Fig. 6.18, find the voltage across each capacitor.

Example 6.7

Solution:

We first find the equivalent capacitance C_{eq}, shown in Fig. 6.19. The two parallel capacitors in Fig. 6.18 can be combined to get $40 + 20 = 60$ mF. This 60-mF capacitor is in series with the 20-mF and 30-mF capacitors. Thus,

$$C_{eq} = \frac{1}{\frac{1}{60} + \frac{1}{30} + \frac{1}{20}} \text{ mF} = 10 \text{ mF}$$

The total charge is

$$q = C_{eq}v = 10 \times 10^{-3} \times 30 = 0.3 \text{ C}$$

This is the charge on the 20-mF and 30-mF capacitors, because they are in series with the 30-V source. (A crude way to see this is to imagine that charge acts like current, since $i = dq/dt$.) Therefore,

$$v_1 = \frac{q}{C_1} = \frac{0.3}{20 \times 10^{-3}} = 15 \text{ V} \qquad v_2 = \frac{q}{C_2} = \frac{0.3}{30 \times 10^{-3}} = 10 \text{ V}$$

Having determined v_1 and v_2, we now use KVL to determine v_3 by

$$v_3 = 30 - v_1 - v_2 = 5 \text{ V}$$

Alternatively, since the 40-mF and 20-mF capacitors are in parallel, they have the same voltage v_3 and their combined capacitance is $40 + 20 = 60$ mF. This combined capacitance is in series with the 20-mF and 30-mF capacitors and consequently has the same charge on it. Hence,

$$v_3 = \frac{q}{60 \text{ mF}} = \frac{0.3}{60 \times 10^{-3}} = 5 \text{ V}$$

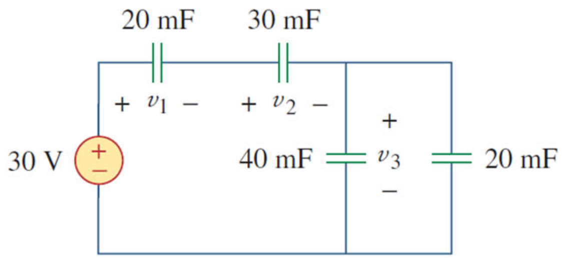


Figure 6.18
For Example 6.7.

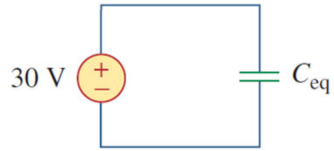


Figure 6.19
Equivalent circuit for Fig. 6.18.

6.4 Inductors

- An inductor is a passive element that stores energy in its magnetic field.
- They have applications in power supplies, transformers, radios, TVs, radars, and electric motors.
- Any conductor of electric current has inductance, but the inductive effect is typically enhanced by wrapping wires around the magnetic core.

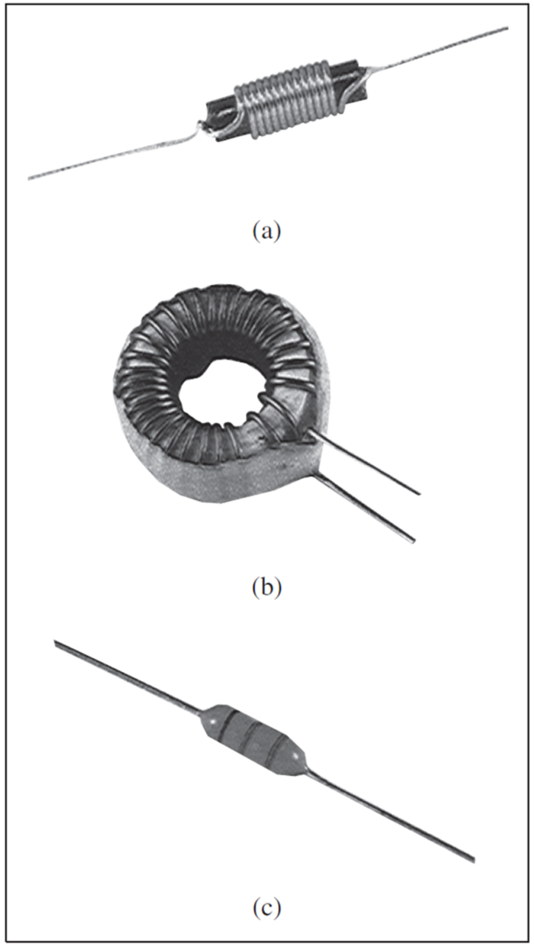


Figure 6.22
Various types of inductors: (a) solenoidal wound inductor, (b) toroidal inductor, (c) chip inductor.

Inductors

- If a current is passed through an inductor, the voltage across it is directly proportional to the time rate of change of the current

$$v = L\frac{di}{dt}$$

- Where L is the inductance, measured in Henry (H).

- The current-voltage relationship for an inductor is:

$$I = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$$

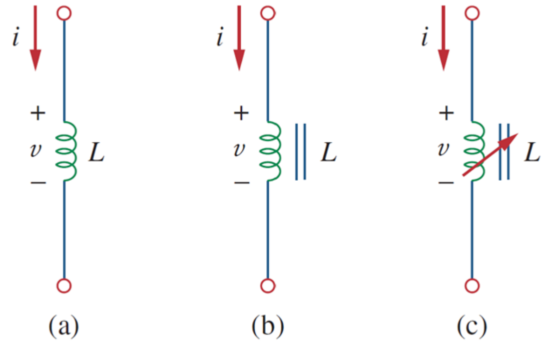


Figure 6.23
Circuit symbols for inductors: (a) air-core, (b) iron-core, (c) variable iron-core.

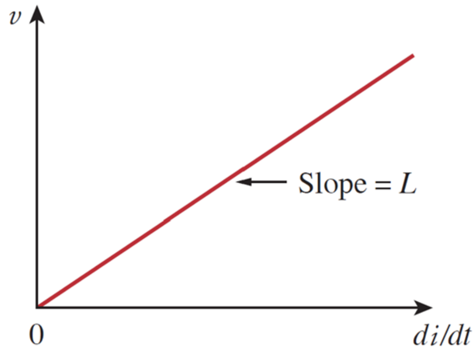


Figure 6.24
Voltage-current relationship of an inductor.

Inductors

- The energy stored is: $v = L\dfrac{di}{dt}$

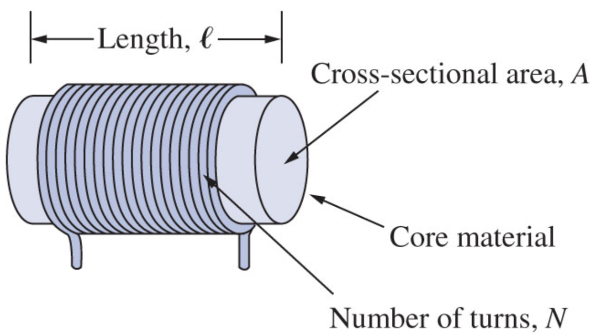
$$W = \int_{-\infty}^{t} p(\tau)d\tau = \int_{-\infty}^{t} vid\tau = L\int_{-\infty}^{t} i\frac{di}{d\tau}d\tau = L\int_{i(-\infty)}^{i(t)} idi = \frac{1}{2}Li^2 \Big|_{i(-\infty)}^{i(t)} = \frac{1}{2}Li(t)^2 - \frac{1}{2}Li(-\infty)^2$$

$$w = \frac{1}{2}Li^2$$

- The inductance depends on its physical dimension and construction.
- The inductance is:

$$L = \frac{N^2\mu A}{l}$$

- Where N is the number of turns, l is the length. And μ is permeability of the core.



Properties of Inductors

- If the current through an inductor is constant, the voltage across it is zero $v = L\frac{di}{dt}$
- Thus an inductor acts like a short circuit to DC
- The current through an inductor cannot change instantaneously. An abrupt current change requires an infinite voltage change, which is not possible.
- This is an important consideration if an inductor is to be turned off abruptly; it will produce a high voltage.

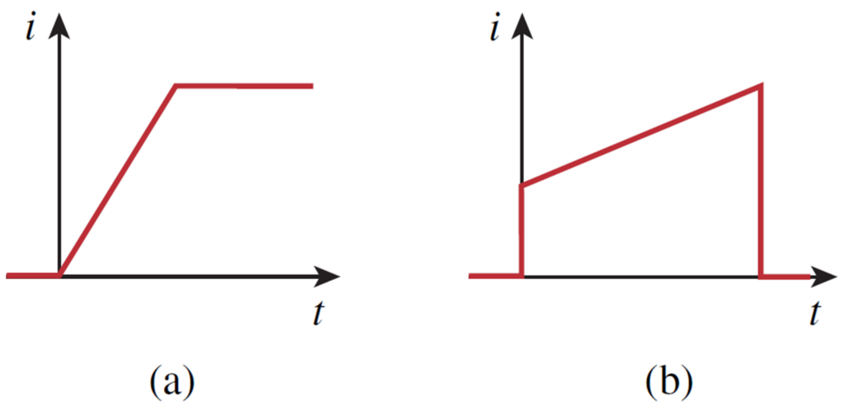


Figure 6.25
Current through an inductor: (a) allowed, (b) not allowable; an abrupt change is not possible.

Properties of Inductors

- Like the ideal capacitor, the ideal inductor does not dissipate energy.
- The energy stored will be returned to the circuit later.
- In reality, an inductor has internal resistance because it is made of conducting material such as copper.
- A real inductor thus has a winding resistance (R_w) in series with it.
- There is also a small winding capacitance (C_w) due to the capacitive coupling between the conducting coils
- R_w and C_w are typically small, which can be ignored in most cases.

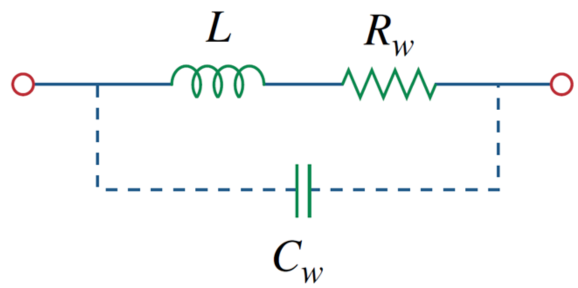


Figure 6.26
Circuit model for a practical inductor.

6.5 Series and Parallel Inductors

Series Inductors

- Consider a series connection of N inductors
- Applying KVL to the loop, and the inductors have the same current through them:

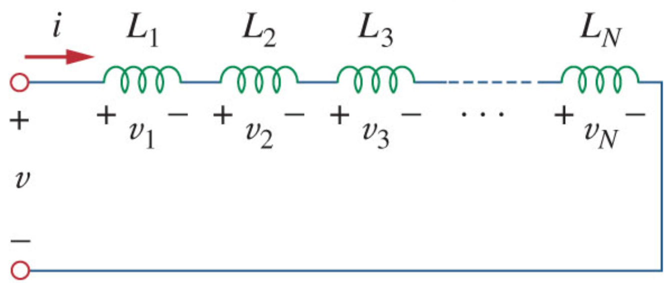
$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

$$v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} + \cdots + L_N \frac{di}{dt}$$

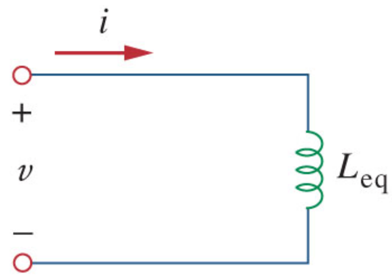
$$= \left(\sum_{k=1}^{N} L_k \right) \frac{di}{dt} = L_{eq} \frac{di}{dt}$$

- Where

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$$



(a)



(b)

The equivalent inductances of series-connected inductors is the sum of the individual inductances.

Parallel Inductors

- Consider a parallel connection of N inductors
- Applying KCL to the circuit, and the inductors have the same voltage across them :

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

- When the current voltage relationship is considered, we have:

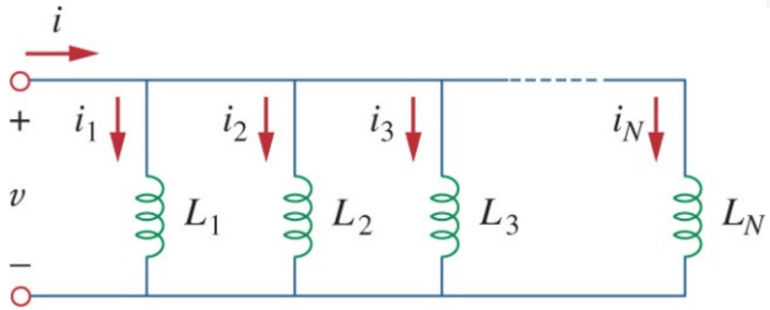
$$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$$

$$i = \frac{1}{L_1}\int_{t_0}^{t} v(\tau)d\tau + i_1(t_0) + \frac{1}{L_2}\int_{t_0}^{t} v(\tau)d\tau + i_{2(t_0)} + \cdots + \frac{1}{LN}\int_{t_0}^{t} v(\tau)d\tau + iN(t_0)$$

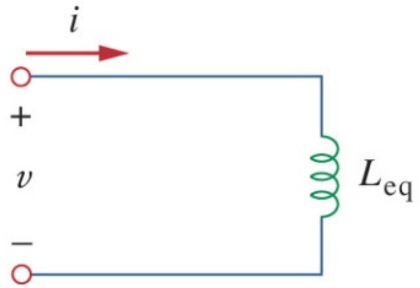
$$= \left(\sum_{k=1}^{N}\frac{1}{L_k}\right)\int_{t_0}^{t} vdt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}}\int_{t_0}^{t} vdt + i(t_0)$$

- The equivalent inductance is thus:

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$



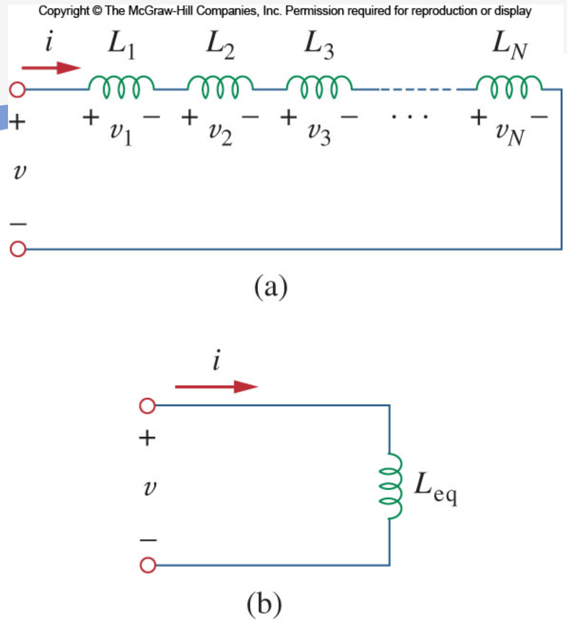
(a)



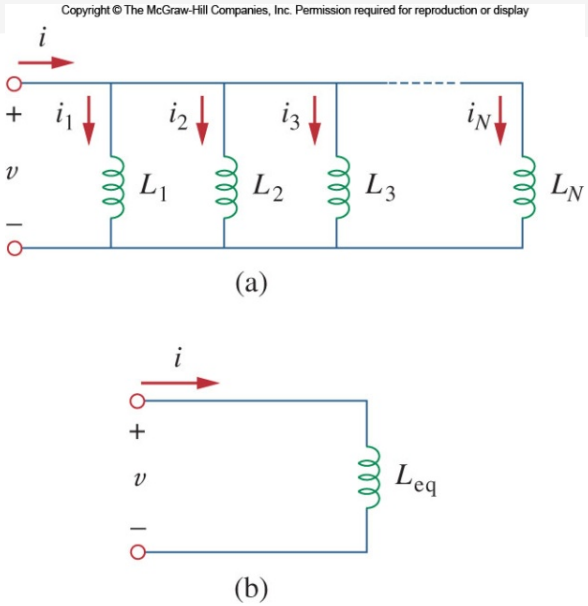
(b)

The equivalent inductances of parallel-connected inductors is the reciprocal of the sum of the reciprocals of the individual inductances.

Series Inductors



Parallel Inductors



$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

- The inductors in series/parallel are combined in the same way as resistors in series/parallel.
- If all the elements are of the same type, the Delta-Wye transformation for resistors can be applied to inductors and capacitors.

Example

Example 6.11

Find the equivalent inductance of the circuit shown in Fig. 6.31.

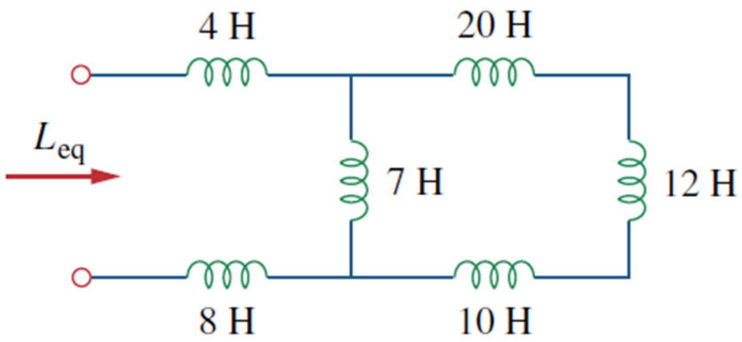


Figure 6.31
For Example 6.11.

Solution:

The 10-H, 12-H, and 20-H inductors are in series; thus, combining them gives a 42-H inductance. This 42-H inductor is in parallel with the 7-H inductor so that they are combined, to give

$$\frac{7 \times 42}{7 + 42} = 6 \text{ H}$$

This 6-H inductor is in series with the 4-H and 8-H inductors. Hence,

$$L_{eq} = 4 + 6 + 8 = 18 \text{ H}$$

Summary of Capacitors and Inductors

Important characteristics of the basic elements.

Relation	**Resistor (R)**	**Capacitor *(C)***	**Inductor *(L)***
$v-i$:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$v = L\frac{di}{dt}$
$i-v$:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$
P or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	*v*	*i*

Special properties of capacitors and inductors

- The capacity to store energy makes them useful as temporary voltage or current source.
- Capacitors and inductors are frequency sensitive. This property makes them useful for frequency discrimination.
- Due to the bulky size, inductors are less frequently used as compared to capacitors.

Applications: Differentiator

- Capacitors, in combination with op-amps can be made to perform advanced mathematical functions
- One such function is the integrator or differentiator.
- By replacing the input resistor with a capacitor, the resulting circuit is a differentiator

$$i_R = -\frac{u_o}{R}$$

$$i_C = C\frac{du_C}{dt} = C\frac{du_i}{dt}$$

$$i_C = i_R \quad \Rightarrow \quad \boxed{u_o = -RC\frac{du_i}{dt}}$$

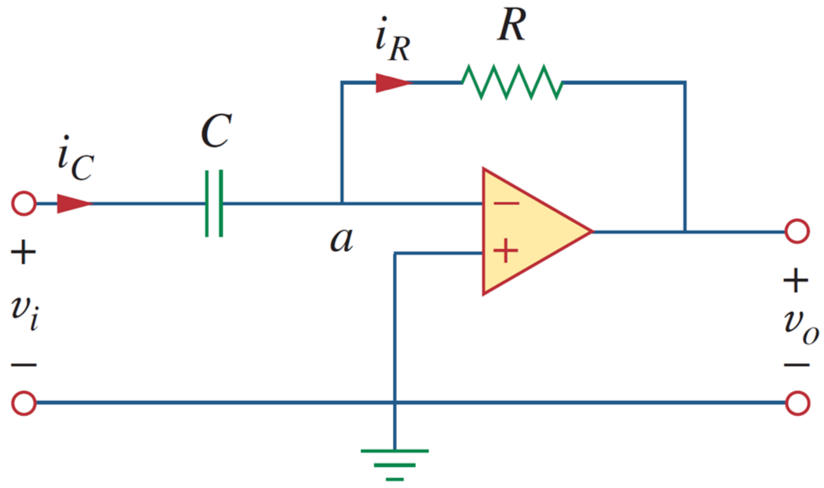


Figure 6.37
An op amp differentiator.

The output voltage is proportional to the rate of change of the input signal.

$$\boxed{u_{\mathrm{o}} = -RC\frac{\mathrm{d}u_{i}}{\mathrm{d}t}}$$

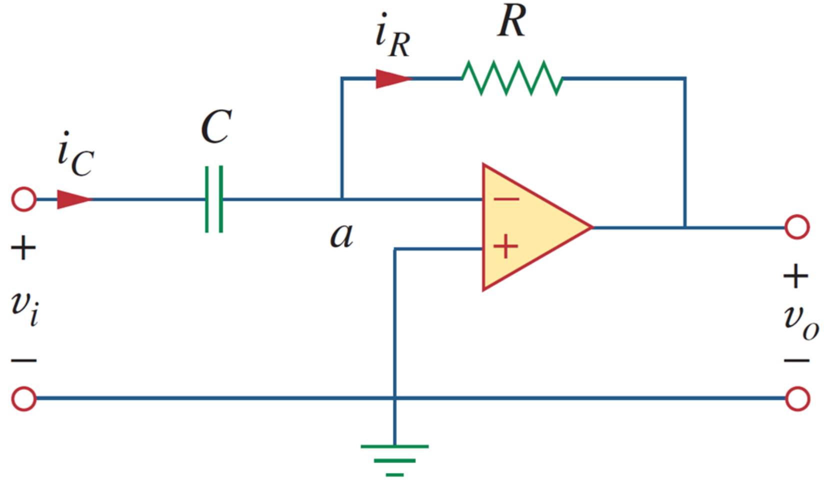
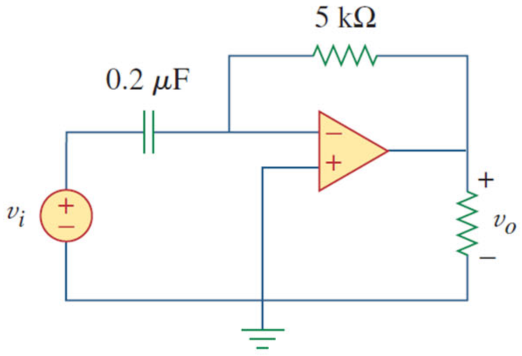


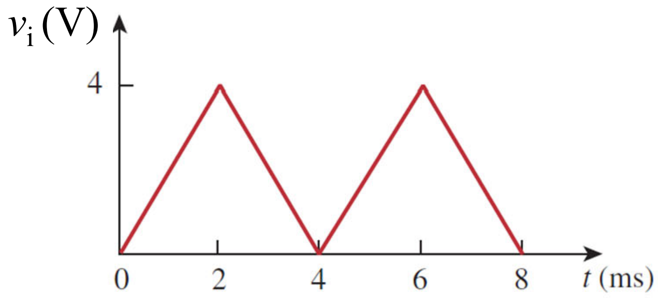
Figure 6.37
An op amp differentiator.

Example 6.14

Sketch the output voltage for the circuit in Fig. 6.38(a), given the input voltage in Fig. 6.38(b). Take $v_o = 0$ at $t = 0$.



(a)



(b)

Figure 6.38
For Example 6.14.

$$\boxed{u_o = -RC\frac{du_i}{dt}}$$

Solution:

This is a differentiator with

$$RC = 5 \times 10^3 \times 0.2 \times 10^{-6} = 10^{-3}\ \text{s}$$

For $0 < t < 4$ ms, we can express the input voltage in Fig. 6.38(b) as

$$v_i = \begin{cases} 2000t & 0 < t < 2\ \text{ms} \\ 8 - 2000t & 2 < t < 4\ \text{ms} \end{cases}$$

This is repeated for $4 < t < 8$ ms. Using Eq. (6.37), the output is obtained as

$$v_o = -RC\frac{dv_i}{dt} = \begin{cases} -2\ \text{V} & 0 < t < 2\ \text{ms} \\ 2\ \text{V} & 2 < t < 4\ \text{ms} \end{cases}$$

Thus, the output is as sketched in Fig. 6.39.