University Physics IA – Model Exam

Instructions: Answer all problems. Show your work clearly. Express results in terms of given symbols whenever possible.

Problem 1: Coulomb's Law (10 pts)

Two point charges $q_1=+q$ and $q_2=+2q_0$ are separated by distance a along the x-axis.

1. Find the force exerted on q_1 by q_2. Express in vector form. (5 pts)
2. A third charge $q_3=-q_0$ is placed midway between them. Find the net force on q_3. (5 pts)

Problem 2: Gauss's Law (10 pts)

A spherical shell of inner radius a and outer radius b is made of conducting material and carries net charge Q. A point charge $+q_0$ is placed at the center.

1. What is the electric flux through a Gaussian surface of radius $r<a$? (3 pts)
2. What is the electric flux through a Gaussian surface at radius $a<r<b$? (4 pts)
3. Determine the charge induced on the inner and outer surfaces of the shell. (3 pts)

Problem 3: Electric Potential Energy (10 pts)

Two charges $q_1 = +q_0$ and $q_2 = -2q_0$ are separated by distance d.

1. Write the expression for their potential energy. (4 pts)
2. If a third charge $q_3=+q_0$ is moved from infinity to the midpoint, what is the change in system potential energy? (6 pts)

Problem 4: Capacitance (10 pts)

Two capacitors $C_1=4\ \mu F$ and $C_2=6\mu F$ are connected in parallel across a 12 V battery.

1. Find the equivalent capacitance. (3 pts)
2. Find the total stored charge. (3 pts)
3. Find the charge on each capacitor. (4 pts)

Problem 5: Current Density (10 pts)

A copper wire of cross-sectional area 2 mm^2, carries a current of 10 A. The electron number density is 8.5×10^{28} m^{-3}.

1. Calculate the drift velocity of electrons. (5 pts)
2. The wire is 2 m long. Given the resistivity of copper is 1.7×10^{-8} Ω·m, calculate the resistance of the wire. (5 pts)

Problem 6: RC Circuit (10 pts)

A battery of emf V_0 is connected in series with a resistor R and a capacitor C. At t=0, the capacitor is uncharged.

1. Write the time constant of the circuit. (3 pts)
2. Find the charge on the capacitor at time t. (4 pts)
3. What is the maximum current in the circuit just after closing the switch? (3 pts)

Problem 7: Coulomb Forces in Configuration (10 pts)

Three identical charges $+q_0$are placed at the vertices of an equilateral triangle of side length a.

1. Find the net force on one charge due to the other two. (6 pts)
2. Find the potential energy of the system. (4 pts)

Problem 8: Gauss's Law (10 pts)

A long insulating cylinder of radius R carries uniform charge density ρ.

1. Find the electric field at a distance $r<R$ inside the cylinder. (5 pts)
2. Find the electric field at a distance $r>R$ outside the cylinder. (5 pts)

Problem 9: Capacitor Network (10 pts)

A network consists of three capacitors $C_1 = C_0, C_2 = 2C_0, C_3 = 3C_0.$. Capacitors C_1 and C_2 are in parallel, and the pair is in series with C_3. The system is connected to a voltage V_0.

1. Find the equivalent capacitance. (4 pts)

2. Find the charge on each capacitor. (6 pts)

Problem 10: Electric Potential (10 pts)

A point charge $+q_0$ is placed at the origin.

1. Find the potential at a point P located at distance d. (3 pts)
2. Two identical charges $+q_0$ are placed at (0,0) and (2d,0). Find the potential at the midpoint. (4 pts)
3. A third charge $-q_0$is placed at (d,0). Find the total potential energy of the system. (3 pts)

Ans:

Problem 1: Coulomb's Law

Two charges $q_1 = +q_0$, $q_2 = +2q_0$, separated by a on x-axis.

(1) Force on q_1:

$$F_{12} = k\frac{q_1 q_2}{a^2}\,\hat{\imath} = k\frac{(q_0)(2q_0)}{a^2}\,\hat{\imath} = \frac{2kq_0^2}{a^2}\,\hat{\imath}.$$

(2) Net force on $q_3 = -q_0$ at midpoint:
Distance from each = $a/2$.
Magnitude of each force:

$$F = k\frac{|q_3 q|}{(a/2)^2} = \frac{4kq_0^2}{a^2}.$$

Force directions: both to left (toward + charges). Net:

$$F_3 = \left(\frac{4kq_0^2}{a^2} + \frac{8kq_0^2}{a^2}\right)\hat{\imath} = \frac{12kq_0^2}{a^2}\,\hat{\imath}.$$

Problem 2: Gauss's Law

Spherical shell, radii a, b. Point charge $+q_0$ at center. Net charge Q.

(1) Flux for $r < a$:

Only enclosed charge = $+q_0$.

$$\Phi = \frac{q_0}{\varepsilon_0}.$$

(2) Flux for $a < r < b$:

Still only $+q_0$ enclosed.

$$\Phi = \frac{q_0}{\varepsilon_0}.$$

(3) Induced charges:

Inner surface must carry $-q_0$.

Outer surface must carry $Q + q_0$.

Problem 3: Electric Potential Energy

Two charges: $q_1 = +q_0, q_2 = -2q_0$, distance d.

(1) Potential energy of pair:

$$U_{12} = k\frac{q_1 q_2}{d} = k\frac{(+q_0)(-2q_0)}{d} = -\frac{2kq_0^2}{d}.$$

(2) Add $q_3 = +q_0$ at midpoint:

Distance from q_1 and q_2 = $d/2$.

$$U_{13} = k\frac{q_1 q_3}{d/2} = \frac{2kq_0^2}{d}, \quad U_{23} = k\frac{q_2 q_3}{d/2} = -\frac{4kq_0^2}{d}.$$

So ΔU = $U_{13} + U_{23} = -\frac{2kq_0^2}{d}$.

Problem 4: Capacitance (parallel)

$C_1 = 4\mu F, C_2 = 6\mu F, V = 12V.$

(1) $C_{eq} = C_1 + C_2 = 10\mu F.$
(2) $Q_{tot} = C_{eq}V = 120\,\mu C.$
(3) Charges:
$Q_1 = C_1V = 48\,\mu C, \;\; Q_2 = C_2V = 72\,\mu C.$

Problem 5: Current Density

Area = $2\ \mathrm{mm}^2 = 2\times10^{-6}m^2$. Current $I = 10A$.

(1) Drift velocity:

$$J = \frac{I}{A} = 5\times10^6\,A/m^2,$$

$$v_d = \frac{J}{ne} = \frac{5\times10^6}{(8.5\times10^{28})(1.6\times10^{-19})} \approx 3.7\times10^{-4}\,m/s.$$

(2) Resistance:

$$R = \rho\frac{L}{A} = \frac{1.7\times10^{-8}(2)}{2\times10^{-6}} = 1.7\times10^{-2}\,\Omega.$$

Problem 6: RC Circuit

Series: R, C, V_0.

(1) $\tau = RC.$
(2) Charge at time t:

$$q(t) = CV_0(1 - e^{-t/RC}).$$

(3) Max current (at $t = 0$):

$$I_{max} = \frac{V_0}{R}.$$

Problem 7: Equilateral Triangle Forces

Three charges $+q_0$, side a.

(1) Force on one charge: each other charge exerts $F = kq_0^2/a^2$. Angle = 60°. Net magnitude:

$$F = \sqrt{F^2 + F^2 + 2F^2 \cos 60^\circ} = \sqrt{3}F.$$

$$F = \frac{\sqrt{3}kq_0^2}{a^2}.$$

(2) Potential energy: sum over 3 pairs.

$$U = 3 \cdot \frac{kq_0^2}{a} = \frac{3kq_0^2}{a}.$$

Problem 8: Charged Cylinder

Uniform charge density ρ.

(1) Inside ($r < R$):
Enclosed charge = $\rho\pi r^2 L$.

$$E = \frac{\rho r}{2\varepsilon_0}.$$

(2) Outside ($r > R$):
Acts like line with charge density $\rho\pi R^2$.

$$E = \frac{\rho R^2}{2\varepsilon_0 r}.$$

Problem 9: Capacitor Network

$C_1 = C_0, C_2 = 2C_0$ in parallel → $3C_0$. In series with $C_3 = 3C_0$.

(1) Equivalent capacitance:

$$C_{eq} = \frac{3C_0 \cdot 3C_0}{3C_0 + 3C_0} = \frac{9C_0^2}{6C_0} = \tfrac{3}{2}C_0.$$

(2) Total charge: $Q = C_{eq}V_0 = \tfrac{3}{2}C_0V_0$.

Voltage across parallel branch:
$V_{12} = Q/(3C_0) = V_0/2.$
So charges:
$Q_1 = C_0(V_0/2) = \tfrac{1}{2}C_0V_0,$
$Q_2 = 2C_0(V_0/2) = C_0V_0,$
$Q_3 = \tfrac{3}{2}C_0V_0.$

Problem 10: Electric Potential

Point charge $+q_0$ at origin.

(1) Potential at P (distance d):

$$V = \frac{kq_0}{d}.$$

(2) Two charges at (0,0), (2d,0). Potential at midpoint:

$$V = \frac{kq_0}{d} + \frac{kq_0}{d} = \frac{2kq_0}{d}.$$

(3) Add $-q_0$ at (d,0). Potential energy:

Pairs:
$U_{12} = \frac{kq_0^2}{2d},$
$U_{13} = -\frac{kq_0^2}{d},$
$U_{23} = -\frac{kq_0^2}{d}.$
Total: $-\frac{3}{2}\frac{kq_0^2}{d}.$