

Problem 1. A point charge of magnitude  $+q$  produces an electric field pattern with a certain strength at distance  $r$ .

- a) If another point charge produces an electric field that is twice as strong at the same distance  $r$ , what is the magnitude and sign of this second charge? (5 pts)
- b) If a third point charge produces an electric field that is half as strong and directed inward at the same distance  $r$ , what is the magnitude and sign of this charge? (5 pts)

Problem 2.

The electric potential along a line is described by the function:

$$V(x) = 80 - 12x + 3x^2 \quad (\text{with } V \text{ in volts, } x \text{ in meters}).$$

- a) Write the expression for the electric field  $E(x)$  along the  $x$ -axis. (3 pts)
- b) Calculate the electric field at  $x = 0.0 \text{ m}$ ,  $1.0 \text{ m}$ ,  $2.0 \text{ m}$ ,  $3.0 \text{ m}$ . (7 pts)

### Problem 3 (10 pts) — Fill in the blanks

A uniform electric field of  $900 \text{ N/C}$  points to the right.

- What is the potential difference  $V_D - V_C$  if point D is  $0.20 \text{ m}$  to the right of point C? (3 pts)
- What is the potential difference  $V_C - V_A$  if A is  $0.35 \text{ m}$  to the left of C? (3 pts)
- What is the potential difference  $V_D - V_A$ ? (2 pts)
- Can we determine the absolute value of potential at C? Explain. (2 pts)

### Problem 4 (10 pts) — Fill in the blanks

The electric potential along the  $x$ -axis is given by

$$V(x) = 100 - 20x + 5x^2 \quad (\text{in volts, with } x \text{ in meters}).$$

Find the electric field at positions  $x = 0.5$ ,  $1.0$ ,  $1.5$ , and  $2.0 \text{ m}$ .

### Problem 5 (10 pts) — Fill in the blanks

A steady current  $I$  flows through a cylindrical copper wire of radius  $r$ .

- a) How much charge passes through the wire in a time  $t$ ? (4 pts)
- b) What is the current density  $J$ ? (3 pts)
- c) If the resistivity of copper is  $\rho$ , what is the electric field inside the wire? (3 pts)

### Problem 6 (10 pts) — Fill in the blanks

A parallel-plate capacitor consists of two circular plates of radius  $R$  separated by distance  $d$ .

- a) Find its capacitance in vacuum. (4 pts)
- b) If the capacitor holds charge  $Q$ , what is the potential difference? (3 pts)
- c) If a dielectric with dielectric constant  $K$  is inserted fully between the plates, what is the stored energy? (3 pts)

### Problem 7 (10 pts) — Long Question

Three resistors of  $3\ \Omega$ ,  $6\ \Omega$ , and  $12\ \Omega$  are connected in parallel across a 9 V battery.

- a) What is the equivalent resistance? (3 pts)
- b) What is the current through each resistor? (3 pts)
- c) What is the power dissipated in each resistor? (2 pts)
- d) What is the total power dissipated, and how does it compare with the power dissipated in the equivalent resistance? (2 pts)

### Problem 8 (10pts) Long question

A circuit consists of a 12 V battery connected to two resistors  $R_1 = 3\ \Omega$  and  $R_2 = 6\ \Omega$  in series. In parallel with  $R_2$ , there is another resistor  $R_3 = 4\ \Omega$ .

- a) Write down the loop and junction equations needed to analyze the circuit. (4 pts)
- b) Solve for the current through each resistor ( $I_1$ ,  $I_2$ ,  $I_3$ ). (6 pts)

*(Hint: Treat the branch containing  $R_2$  and  $R_3$  as a parallel combination.)*

**Problem 9 (10 pts) — Capacitance with moving dielectric**

A parallel-plate capacitor has plates of area  $A$  and separation  $d$ . A dielectric slab of constant  $K = 4$  is inserted halfway between the plates.

- a) Derive the expression for the effective capacitance. (6 pts)
- b) If the dielectric is pulled out with velocity  $u$ , find the current in the external circuit. (4 pts)

**Problem 10 (10 pts) — Gauss's Law**

A solid sphere of radius  $a$  carries a uniform charge density  $\rho$ . Concentric with it is a conducting spherical shell of inner radius  $b$  and outer radius  $c$ , which initially has no net charge.

- a) Find the electric field in the regions:  $r < a$ ,  $a < r < b$ ,  $b < r < c$ , and  $r > c$ . (6 pts)
- b) Find the induced charges on the inner and outer surfaces of the conducting shell. (4 pts)

## Problem 1 — Text-based (Solution)

**Given:** A reference point charge  $+q$  produces field  $E = \frac{k|q|}{r^2}$  at distance  $r$ .

a) If another charge gives **twice** the field magnitude at the **same**  $r$ , then

$$\frac{k|q'|}{r^2} = 2 \frac{k|q|}{r^2} \Rightarrow |q'| = 2|q|.$$

Direction wasn't stated to change, so keep the **same sign** as the reference.

**Answer:**  $q' = +2q$ .

b) If a third charge gives **half** the magnitude **and** the field points **inward** (opposite the  $+q$  case), then

$$|q''| = \frac{1}{2}|q|, \quad \text{sign negative (to point inward).}$$

**Answer:**  $q'' = -\frac{q}{2}$ .

### Problem 3 (uniform field → potential differences)

$$\vec{E} = 900 \text{ N/C to the right. } V_B - V_A = - \int \vec{E} \cdot d\vec{\ell}.$$

- $V_D - V_C = -(900)(0.20) = \boxed{-180 \text{ V}}$
- $V_C - V_A = -(900)(0.35) = \boxed{-315 \text{ V}}$
- $V_D - V_A = -180 - 315 = \boxed{-495 \text{ V}}$
- Absolute  $V$  at  $C$ ? No — need a reference.

### Problem2

**Given:**  $V(x) = 80 - 12x + 3x^2$  (volts), with  $x$  in meters.

Electric field along  $x$ :

$$E_x(x) = -\frac{dV}{dx} = -(-12 + 6x) = 12 - 6x \quad (\text{V/m}).$$

Now evaluate:

- $x = 0.0$ :  $E_x = 12 - 6(0) = \boxed{12 \text{ V/m}}$  (along  $+\hat{x}$ )
- $x = 1.0$ :  $E_x = 12 - 6(1) = \boxed{6 \text{ V/m}}$  (along  $+\hat{x}$ )
- $x = 2.0$ :  $E_x = 12 - 6(2) = \boxed{0 \text{ V/m}}$
- $x = 3.0$ :  $E_x = 12 - 6(3) = \boxed{-6 \text{ V/m}}$  (i.e.,  
6 V/m along  $-\hat{x}$ )

### Problem 3 (uniform field $\rightarrow$ potential differences)

$\vec{E} = 900 \text{ N/C}$  to the right.  $V_B - V_A = - \int \vec{E} \cdot d\vec{\ell}$ .

- $V_D - V_C = -(900)(0.20) = \boxed{-180 \text{ V}}$
- $V_C - V_A = -(900)(0.35) = \boxed{-315 \text{ V}}$
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- Absolute  $V$  at  $C$ ? No — need a reference.

### Problem 5 (current, current density, field)

Wire radius  $r$ , resistivity  $\rho$ .

- $Q = I t$
- $J = \frac{I}{\pi r^2}$
- $E = \rho J = \frac{\rho I}{\pi r^2}$

## Problem 6 (circular parallel-plate capacitor)

Area  $A = \pi R^2$ , separation  $d$ .

- $C = \frac{\varepsilon_0 \pi R^2}{d}$
- $V = \frac{Q}{C} = \frac{Qd}{\varepsilon_0 \pi R^2}$
- With dielectric  $K$ :  $C_K = K \varepsilon_0 \pi R^2 / d$ , so
$$U = \frac{Q^2}{2C_K} = \frac{Q^2 d}{2K \varepsilon_0 \pi R^2}$$

## Problem 7 (three parallel resistors on 9 V)

$R_1 = 3 \Omega$ ,  $R_2 = 6 \Omega$ ,  $R_3 = 12 \Omega$ ,  $V = 9 \text{ V}$ .

- $R_{\text{eq}}^{-1} = 1/3 + 1/6 + 1/12 = 7/12 \Rightarrow R_{\text{eq}} = 12/7 \Omega \approx 1.714 \Omega$
- Currents:  $I_i = V/R_i$ :
$$I_1 = 3.00 \text{ A}, I_2 = 1.50 \text{ A}, I_3 = 0.75 \text{ A} \text{ (total } 5.25 \text{ A)}$$
- Powers  $P_i = VI_i$ :
$$P_1 = 27.0 \text{ W}, P_2 = 13.5 \text{ W}, P_3 = 6.75 \text{ W} \text{ (sum } 47.25 \text{ W} = V^2/R_{\text{eq}})$$

## Problem 8 — Solution (new text-based circuit)

Circuit description: A 12 V battery  $\rightarrow R_1 = 3\ \Omega$  in series  $\rightarrow$  node splits into **two parallel branches**:  $R_2 = 6\ \Omega$  and  $R_3 = 4\ \Omega \rightarrow$  rejoin  $\rightarrow$  back to battery.

### (a) Required equations

- Parallel equivalent:

$$R_{23} = \left( \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1} = \left( \frac{1}{6} + \frac{1}{4} \right)^{-1} = \left( \frac{5}{12} \right)^{-1} = 2.4\ \Omega.$$

- Total resistance:

$$R_{\text{tot}} = R_1 + R_{23} = 3 + 2.4 = 5.4\ \Omega.$$

- Total current (through  $R_1$ ) (Ohm's law):

$$I_{\text{tot}} = \frac{V}{R_{\text{tot}}} = \frac{12}{5.4} = \frac{20}{9}\ \text{A} \approx 2.222\ \text{A}.$$

- Voltage across the parallel branch:

$$V_{23} = V - I_{\text{tot}}R_1 = 12 - \left( \frac{20}{9} \right)(3) = 12 - \frac{20}{3} = \frac{16}{3}\ \text{V} \approx 5.333\ \text{V}.$$

- **Branch currents** (Ohm's law in each branch):

$$I_2 = \frac{V_{23}}{R_2} = \frac{16/3}{6} = \frac{8}{9}\ \text{A} \approx 0.889\ \text{A},$$

$$I_3 = \frac{V_{23}}{R_3} = \frac{16/3}{4} = \frac{4}{3}\ \text{A} \approx 1.333\ \text{A}.$$

• Junction rule check:  
 $2.3333 \text{ A}$ .

$$I_{\text{tot}} = I_2 + I_3 = \frac{8}{9} + \frac{4}{3} = \frac{8}{9} + \frac{12}{9} = \frac{20}{9} \text{ A} \quad \checkmark$$

## (b) Final answers

- Current through  $R_1$ :

$$I_1 = I_{\text{tot}} = \frac{20}{9} \text{ A} \approx 2.222 \text{ A}$$

- Current through  $R_2$ :  $I_2 = \frac{8}{9} \text{ A} \approx 0.889 \text{ A}$

- Current through  $R_3$ :  $I_3 = \frac{4}{3} \text{ A} \approx 1.333 \text{ A}$

(Optional power check:  $P_{\text{tot}} = VI_{\text{tot}} = 12 \times 20/9 = 80/3 \approx 26.67 \text{ W}$ , which equals the sum of branch powers.)

## Problem 9 (partially inserted dielectric)

Side-by-side (parallel) regions: vacuum area  $A - A_d$ , dielectric area  $A_d$ , gap  $d$ , constant  $K$ .

$$(a) C = \frac{\epsilon_0}{d} [(A - A_d) + KA_d] = \frac{\epsilon_0}{d} [A + (K - 1)A_d].$$

If half area filled  $A_d = A/2$  and  $K = 4$ :  $C = \frac{5}{2} \frac{\epsilon_0 A}{d}.$

(b) Constant-voltage source  $V$ . With  $A_d = A_d(t)$ :

$$I = \frac{dQ}{dt} = V \frac{dC}{dt} = V \frac{\epsilon_0(K - 1)}{d} \frac{dA_d}{dt}.$$

If slab edge moves at speed  $u$  across width  $W$ :  $dA_d/dt = Wu$ , so

$$I = V \frac{\epsilon_0(K - 1)}{d} Wu.$$

### Problem 10 (insulating sphere + conducting shell)

Insulator radius  $a$  with  $\rho$  ( $Q = \frac{4}{3}\pi a^3 \rho$ ). Concentric conducting shell  $b < c$ , net shell charge 0.

#### (a) Field by region

- $r < a$ :  $Q_{\text{enc}} = \frac{4}{3}\pi r^3 \rho \Rightarrow E = \frac{\rho r}{3\epsilon_0}$  (outward)
- $a < r < b$ :  $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$
- $b < r < c$ :  $E = 0$
- $r > c$ :  $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$

#### (b) Induced charges

- Inner surface must cancel  $+Q$  inside:  $q_{\text{inner}} = -Q, \sigma_{\text{inner}} = -\frac{Q}{4\pi b^2}$
- Net shell charge 0  $\Rightarrow$  outer surface:  $q_{\text{outer}} = +Q, \sigma_{\text{outer}} = \frac{Q}{4\pi c^2}$