The Laws of Motion

- The description of an object in motion included its position, velocity, and acceleration (kinematics). There was no consideration of what might influence the motion.
- The area of such kind of studies is called Dynamics (a branch of mechanics that deals with forces and their relation primarily to the motion but sometimes also to the equilibrium of bodies-Merriam-Webster dictionary)
- It comes to the famous three Newton's laws of motion.

Newton's law	Statement
1st	In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity
2nd	F = ma
3rd	Action & reaction

Bridgeman-Giraudon/Art Resource, NY

Isaac Newton
English physicist and mathematician
(1642–1727)

Newton's First Law

- In the absence of external forces, AND when viewed from an inertial reference frame, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.
  - Newton's First Law describes what happens in the absence of a force (constant velocity => no acceleration).
- The First Law also allows the definition of ***force*** as ***that which causes a change in the motion of an object.***

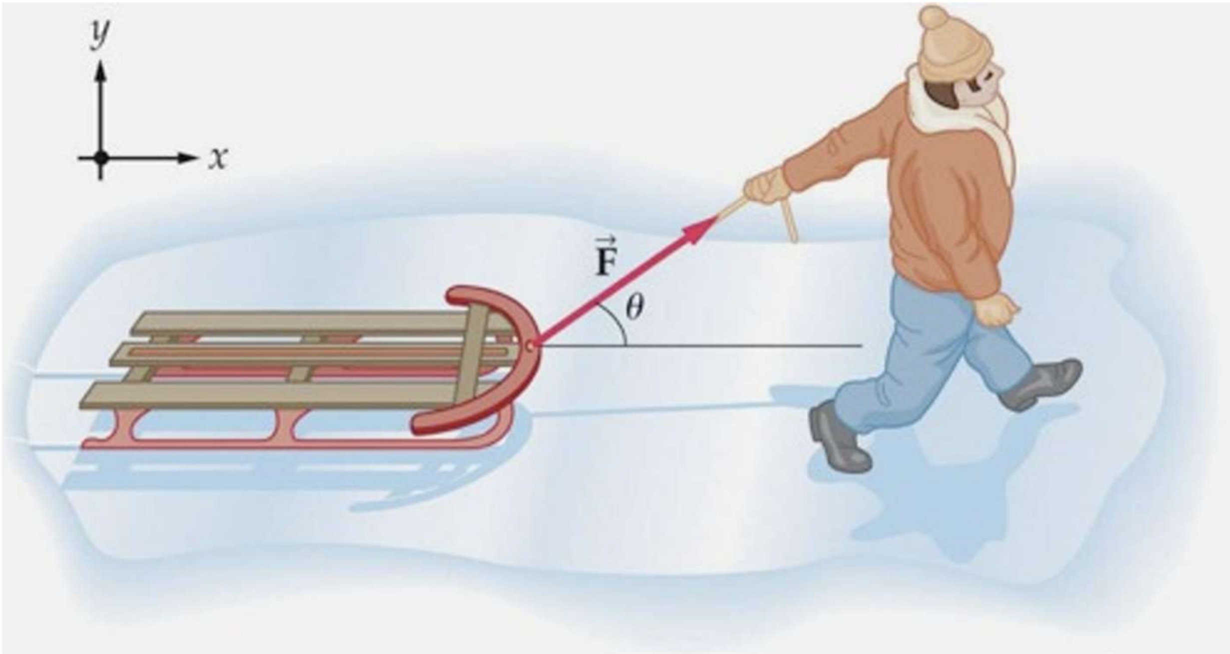
Force

Kinematics vs. dynamics: what <u>causes</u> acceleration?

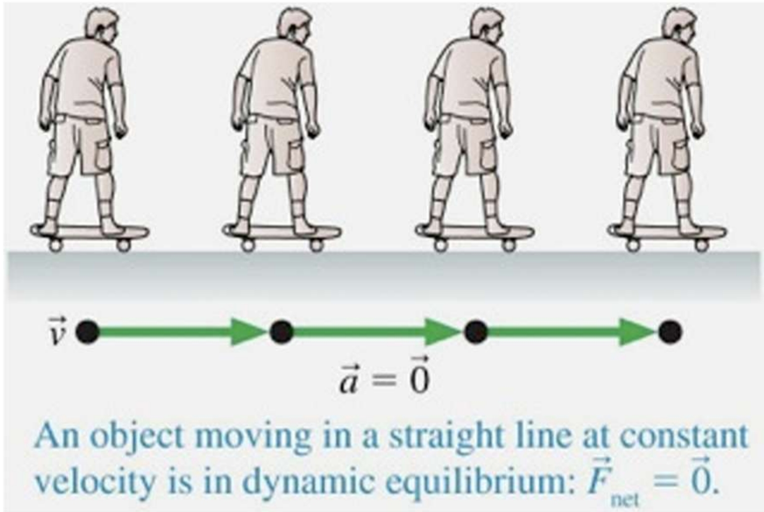
Answer: force.

Force: push or pull

Force is a vector – it has magnitude and direction



Newton's First Law of Motion



Newton's 1st Law:

In the absence of external forces, an object at rest remains at rest; an object in motion remains in motion.

Newton's Second Law of Motion

Newton's 2nd Law:

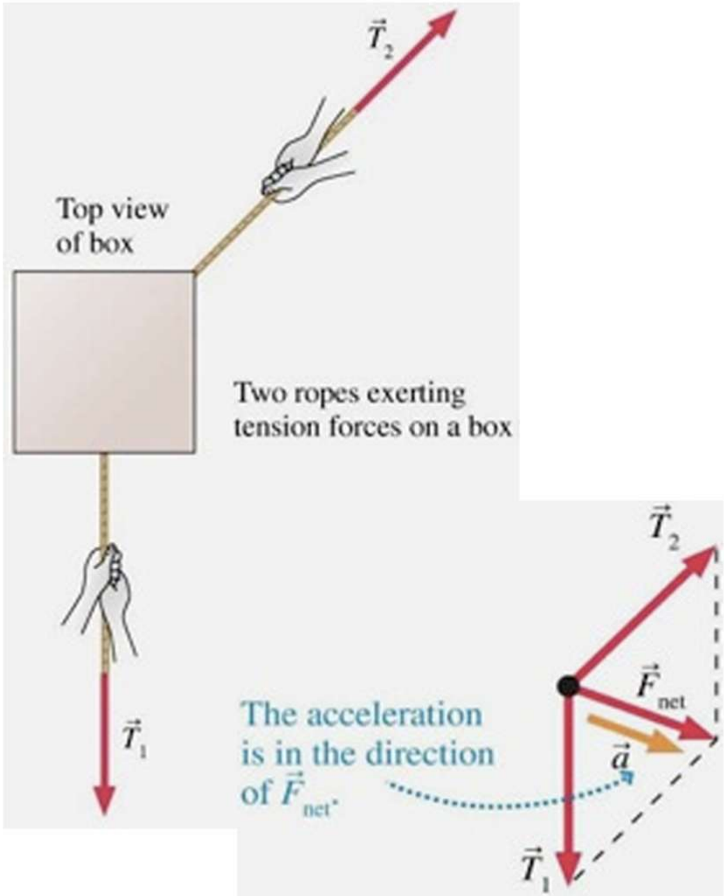
An object of a given mass m subjected to forces $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, … will undergo an acceleration $\vec{a}$ given by:

$$\vec{a} = \vec{F}_{net}/m$$

where

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots$$

The mass m is positive, force and acceleration are in the same direction.



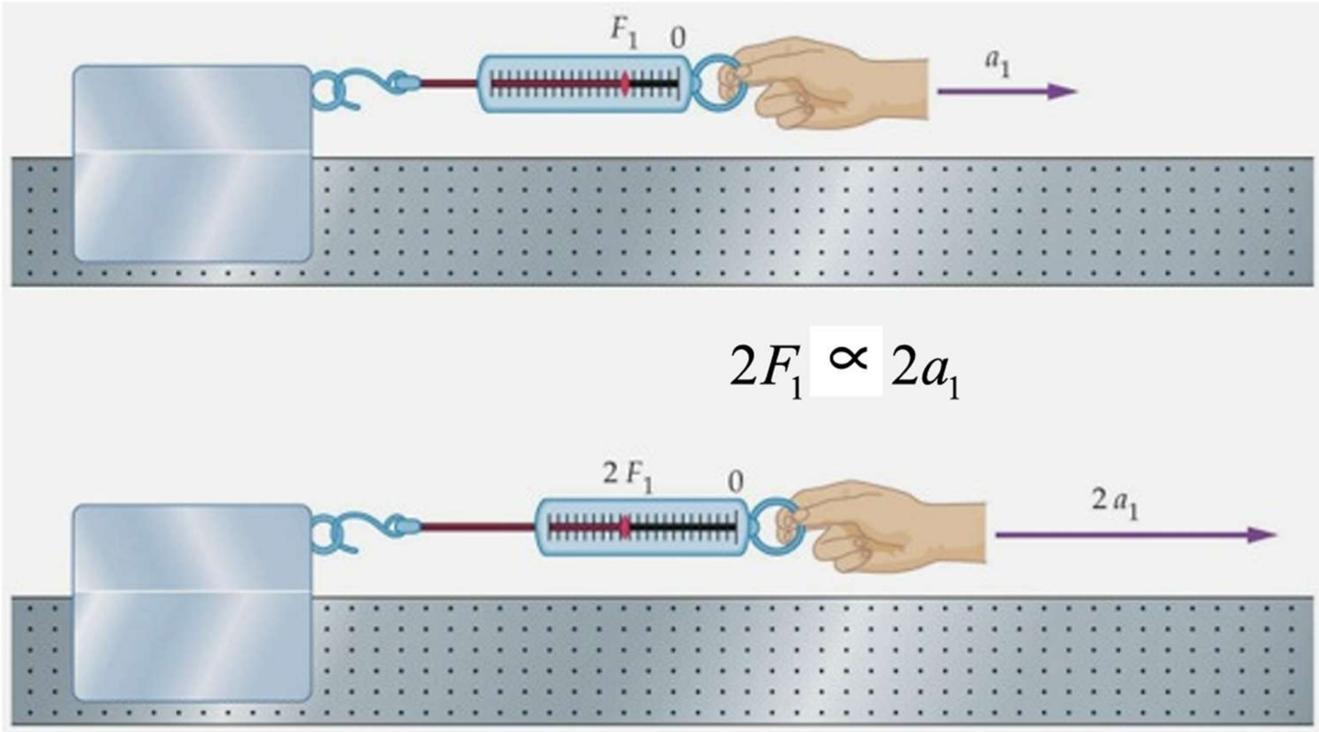
Acceleration vs. Force

Now that we have a calibrated spring, we can do more experiments.

Acceleration is proportional to force:

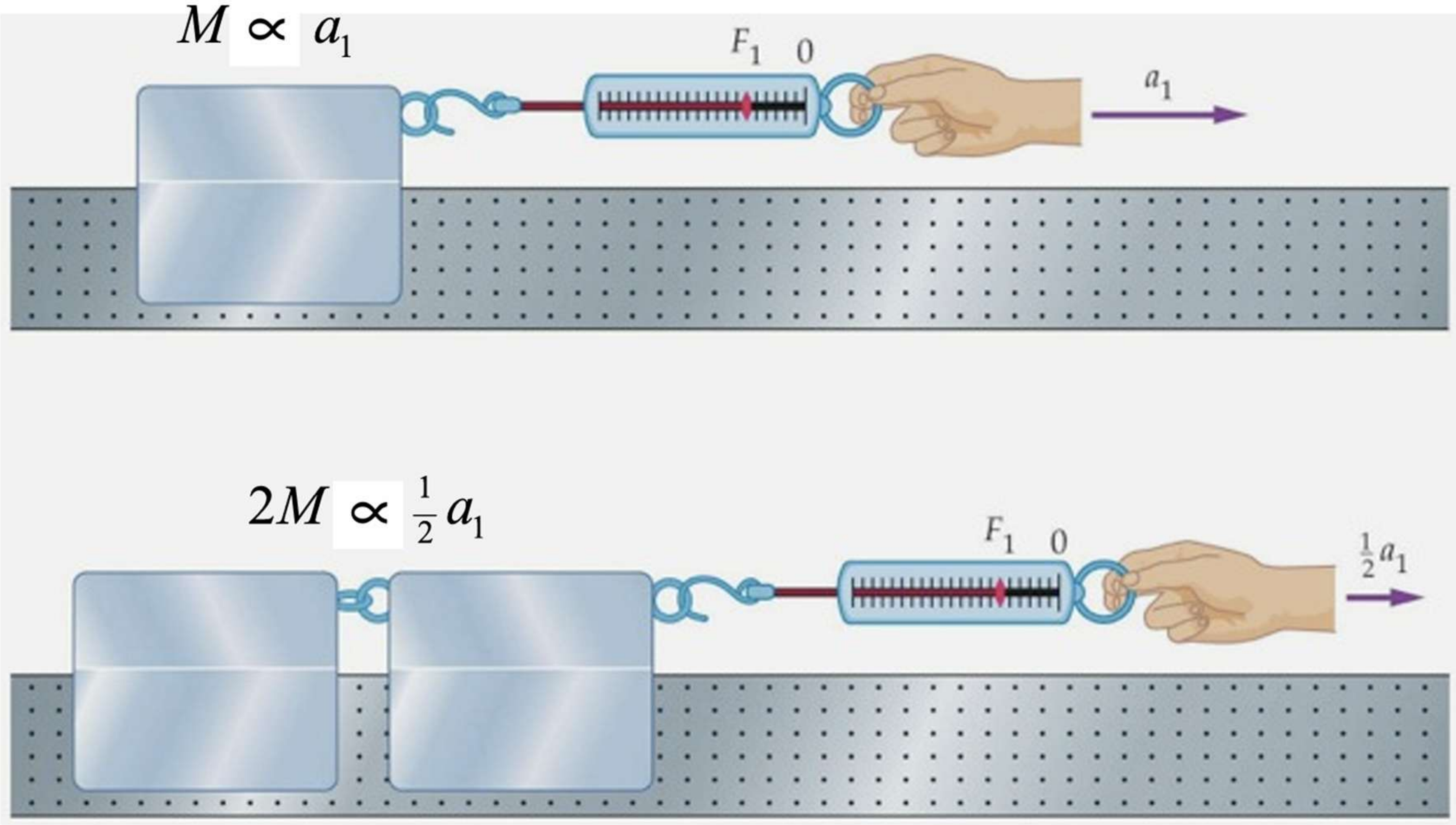
$$F_1 \propto a_1$$

$$2F_1 \propto 2a_1$$



Acceleration vs. Mass

Acceleration is inversely proportional to mass:



Newton's Second Law of Motion

Combining these two observations gives: $\boxed{a = \dfrac{F}{m}}$

Or, more familiarly:

$$\boxed{F = ma \quad \text{or} \quad \vec{F} = m\vec{a}}$$

This is the mathematical expression of ***Newton's 2nd Law of Motion***.

Units: Mass has SI units of kg, and acceleration has SI units of m/s^2.
We define the SI unit of force as:
1 newton = 1 N $\equiv$ 1 kg m/s^2.

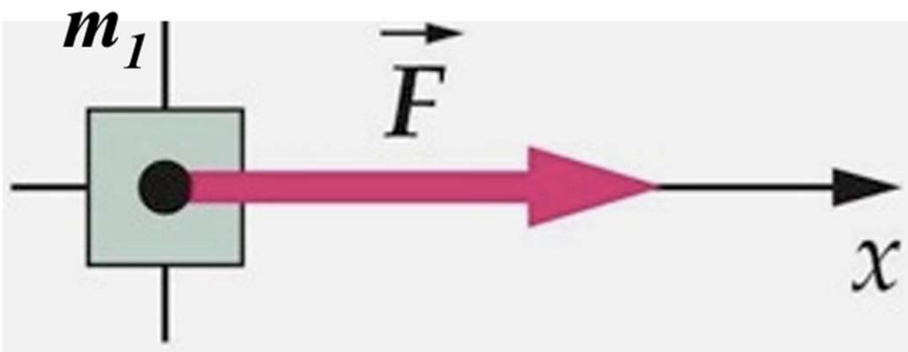
Inertia and Mass

- The tendency of an object to resist any attempt to change its velocity is called ***inertia.***
- ***Mass*** is that property of an object that specifies how much resistance an object exhibits to changes in its velocity.
- Masses can be defined in terms of the accelerations produced by a given force acting on them:

$$m_1/m_2 \equiv a_2/a_1$$

  - The magnitude of the acceleration acting on an object is inversely proportional to its mass.

Example: Accelerated Mass

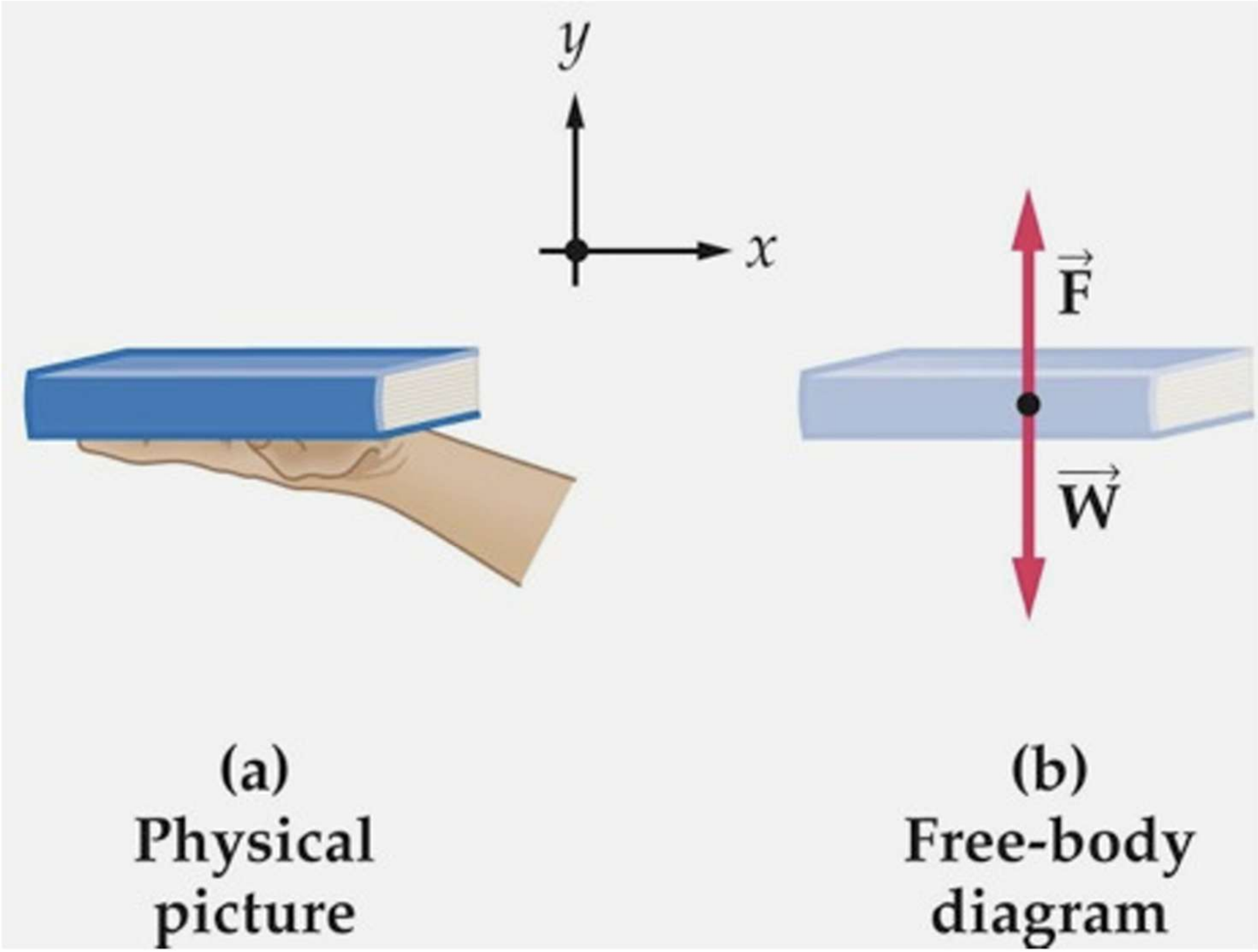


A net force of 3.0 N produces an acceleration of 2.0 m/s^2 on an object of unknown mass.

What is the mass of the object?

$$m_1 = \frac{F}{a_1} = \frac{(3.0\ \mathrm{N})}{(2.0\ \mathrm{m/s^2})} = 1.5\ \mathrm{kg}$$

Newton's Second Law



Newton's Second Law of Motion

Newton's 2nd Law:

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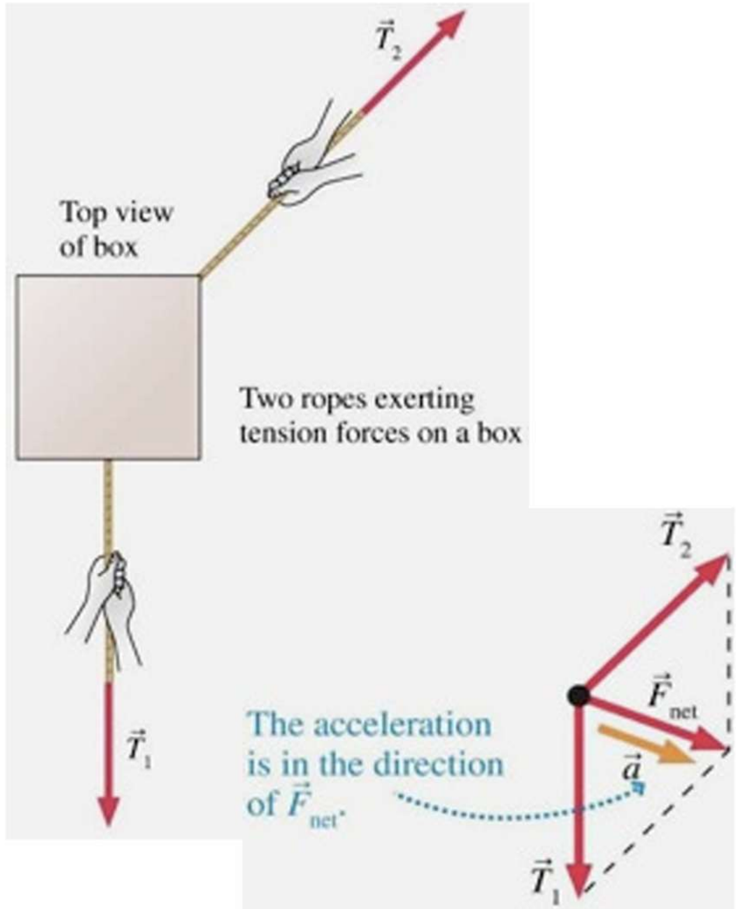
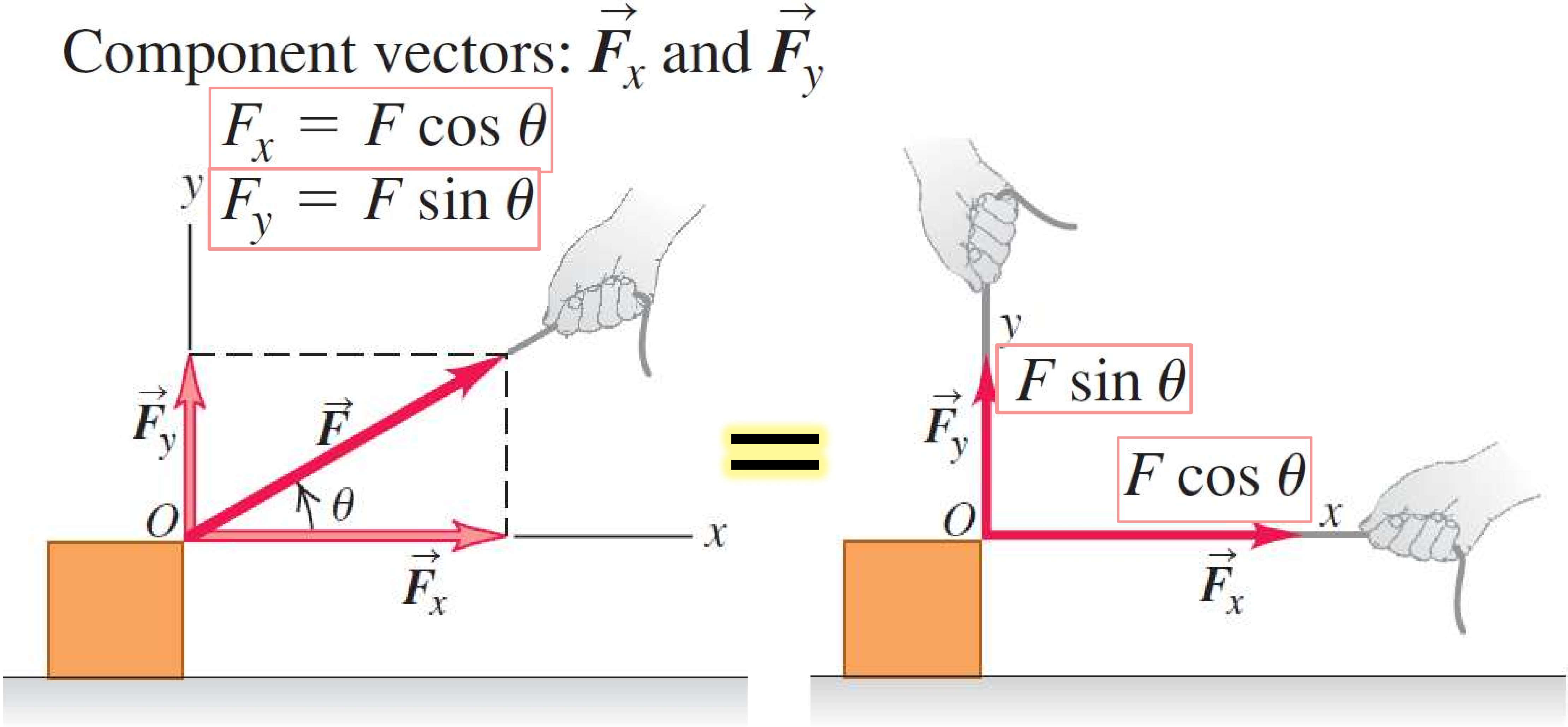
$$\vec{a} = \vec{F}_{net}/m$$

where
$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots$

The mass m is positive, force and acceleration are in the same direction.



Superposition of Forces



Superposition of Forces

Resultant force: sum of multiple forces

Symbol for sum

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

Net Force

Resultant force: also consists of x and y components

$$R_x = \sum F_x$$

$$R_y = \sum F_y$$

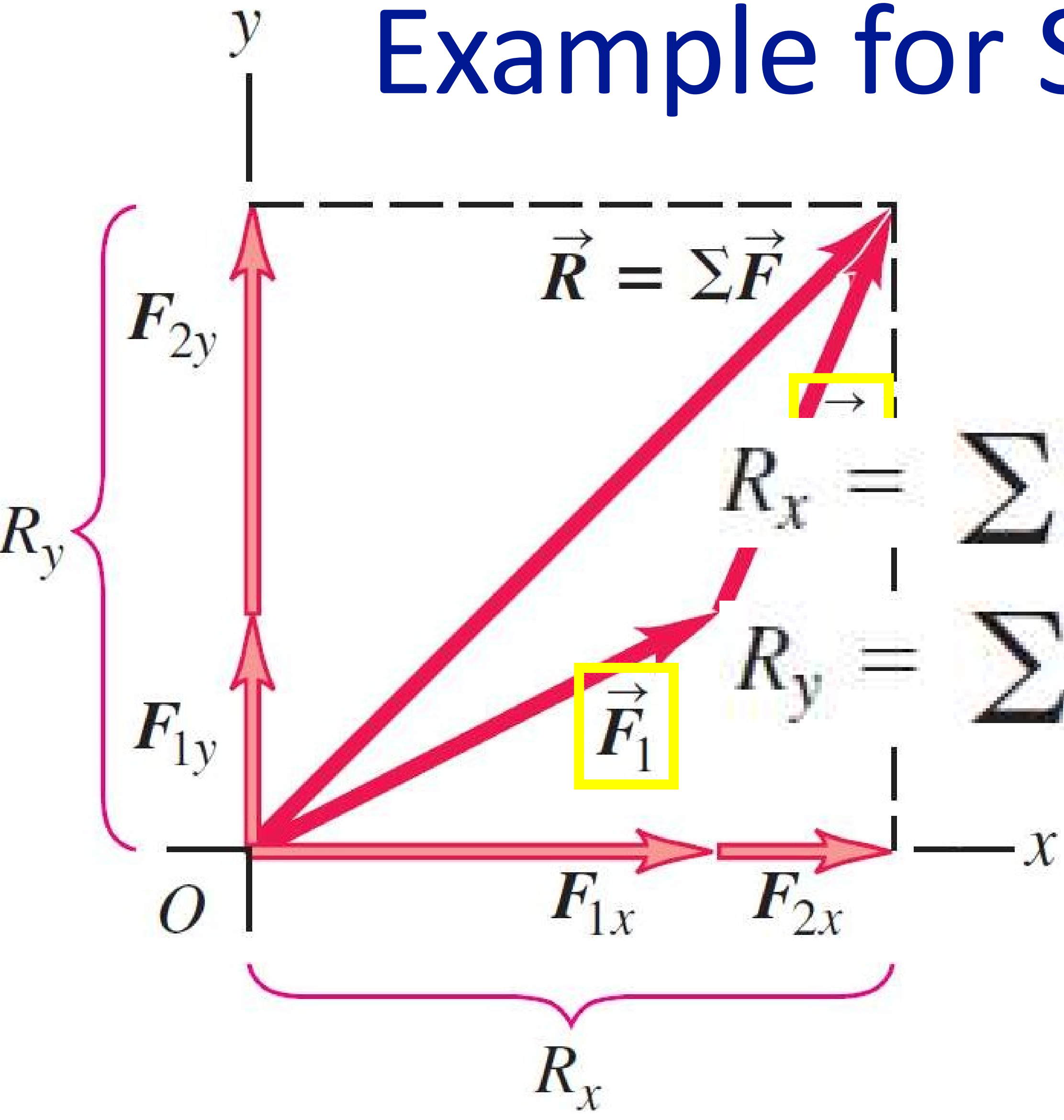
2D space: magnitude calculation

$$R = \sqrt{R_x^2 + R_y^2}$$

3D space: $R_z = \sum F_z$

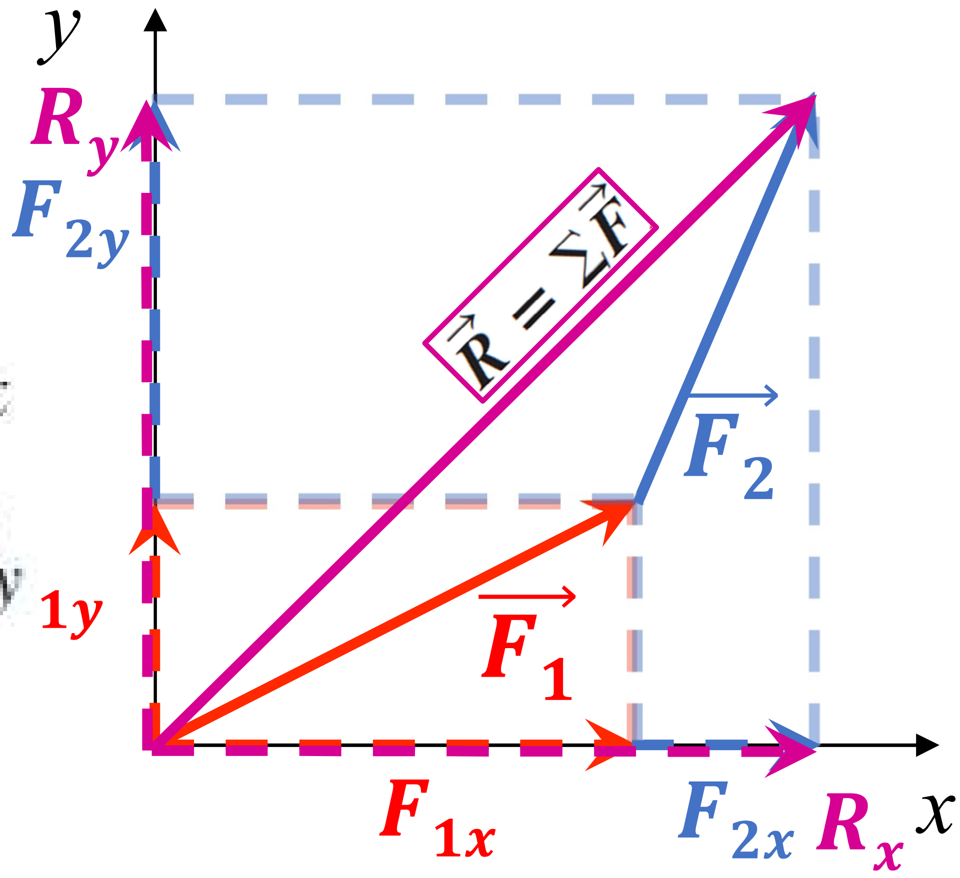
$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

Example for Superposition



$$R_x = \sum F_x$$

$$R_y = \sum F_y$$



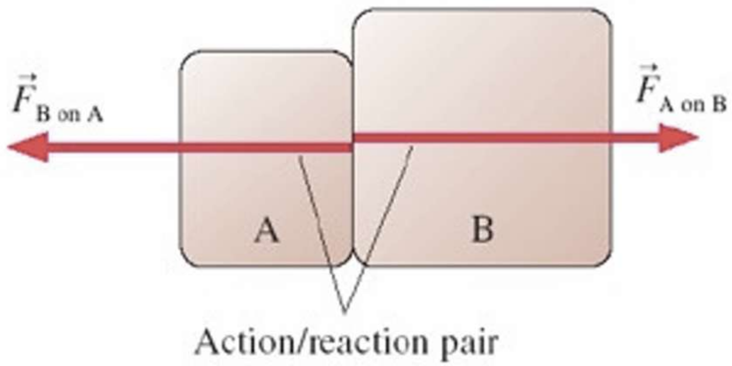
$$\vec{R} = (R_x, R_y)$$

Newton's Third Law of Motion

Forces always come in pairs, acting on different objects:

If Object A exerts a force $\boldsymbol{F}$ on Object B, then Object B exerts a force $-\boldsymbol{F}$ on Object A.

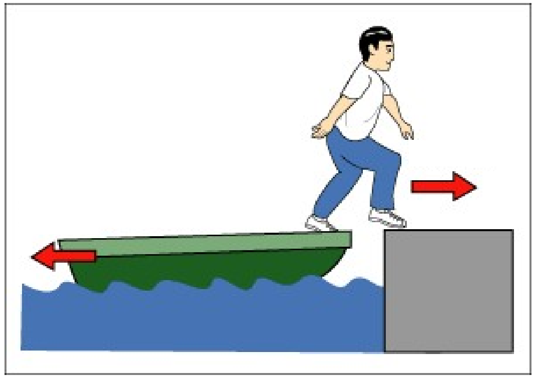
These forces are called action-reaction pairs.

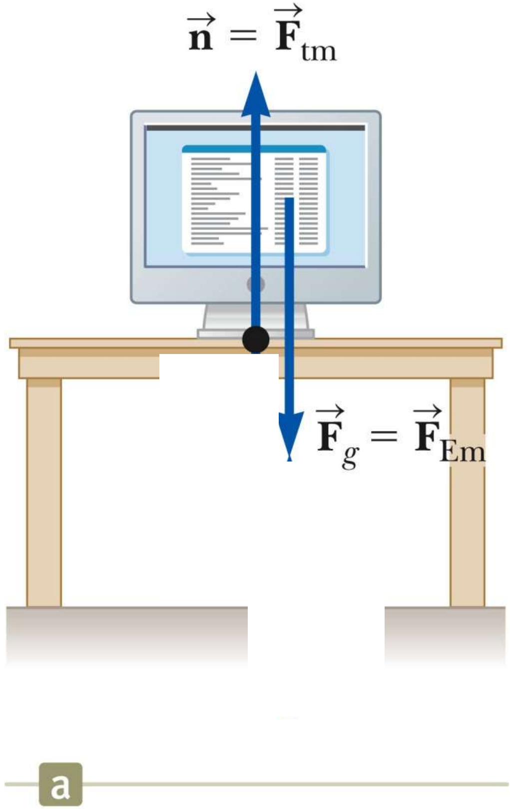


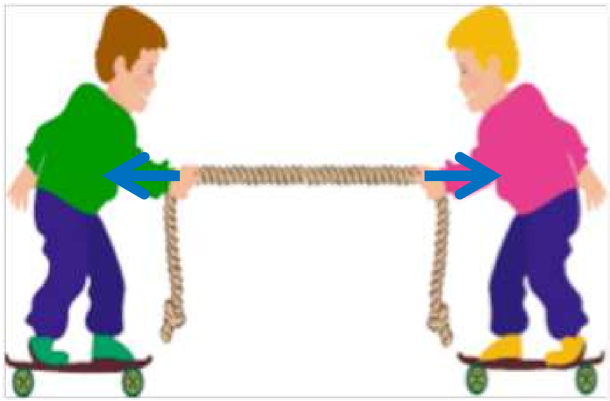
$$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$$

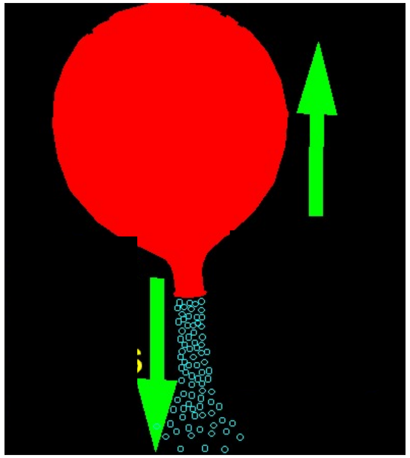
Alternate Wording: "For every action there is an equal and opposite reaction."

Action-Reaction pairs?

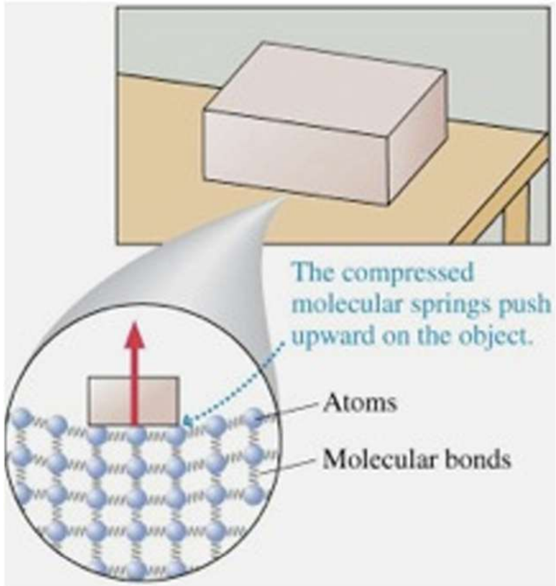


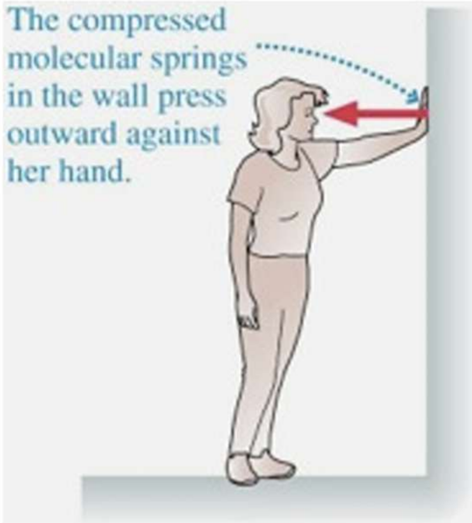


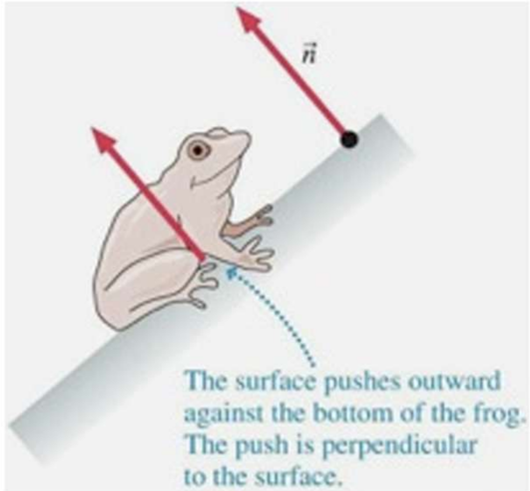




A Catalog of Forces: (1. Normal)







Normal Force

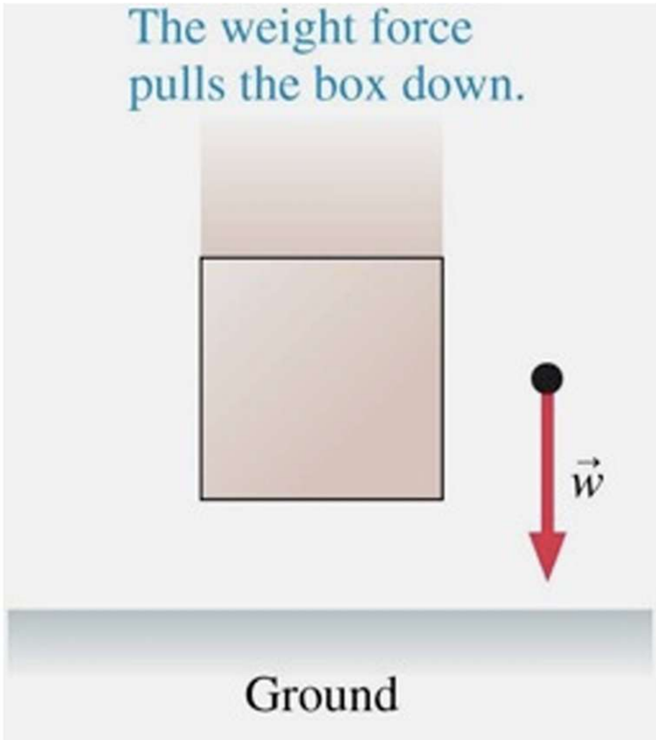
A solid object resists the action of another force which compresses it with what we call the **normal force**. The normal force always acts outward and perpendicular to the surface of the compressed object. The symbol for the normal force is $\vec{\mathbf{n}}$.

A Catalog of Forces: (2. Weight)

Weight

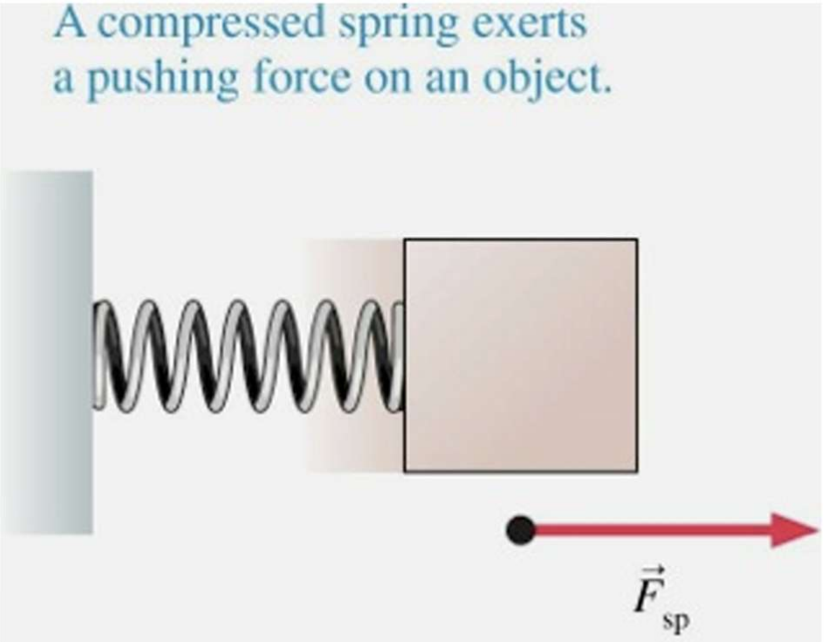
The falling box is pulled toward the Earth by the long-range force of gravity. The gravitational pull on a object on or near the surface of the Earth is called **weight**, for which we use the symbol $\vec{w}$.

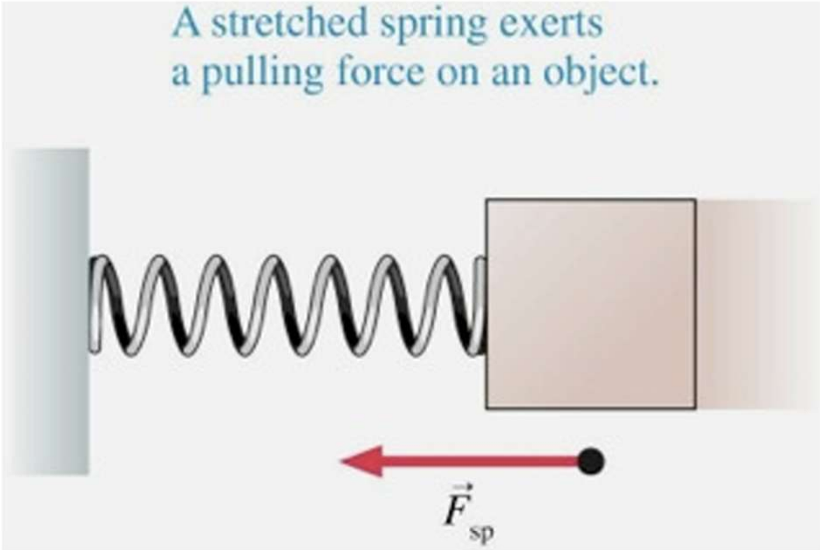
Weight is the only long range force that we will consider in this course. The agent for the weight force is the entire Earth pulling on the object. Weight acts equally on objects at rest or in motion. The weight vector always points vertically downward, and it can be considered to act at the center of mass of the object.



$$\boxed{g = 9.81\ \mathrm{m/s^2}; \quad \vec{w} = m\vec{g}}$$

A Catalog of Forces: (3. Spring)

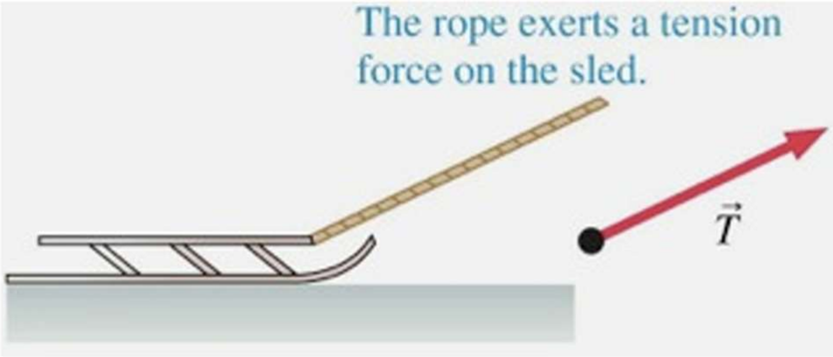




Spring Force

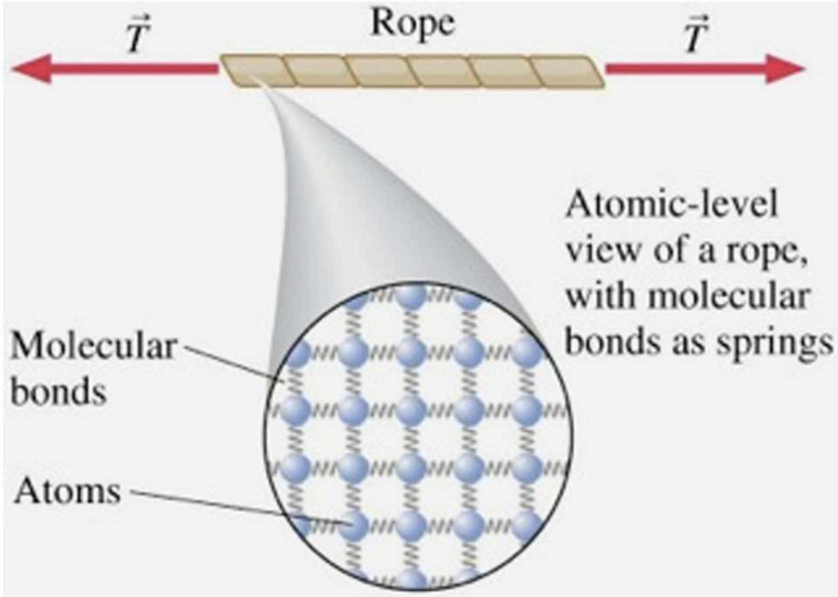
A stretched or compressed spring exerts one of the most common contact forces. A spring can either push (when compressed) or pull (when stretched). In either case, the tail of the vector force is attached to the contact point. There is no special symbol for the spring force, but we can use $\mathbf{F}_{sp}$.

A Catalog of Forces: (4. Tension)



Tension Force

A string or rope exerts a contact force on an object when it pulls on it. We call this a tension force, represented by the symbol $\vec{\mathbf{T}}$. Tension is always directed **along the line of the rope** or string, with no component perpendicular to it.



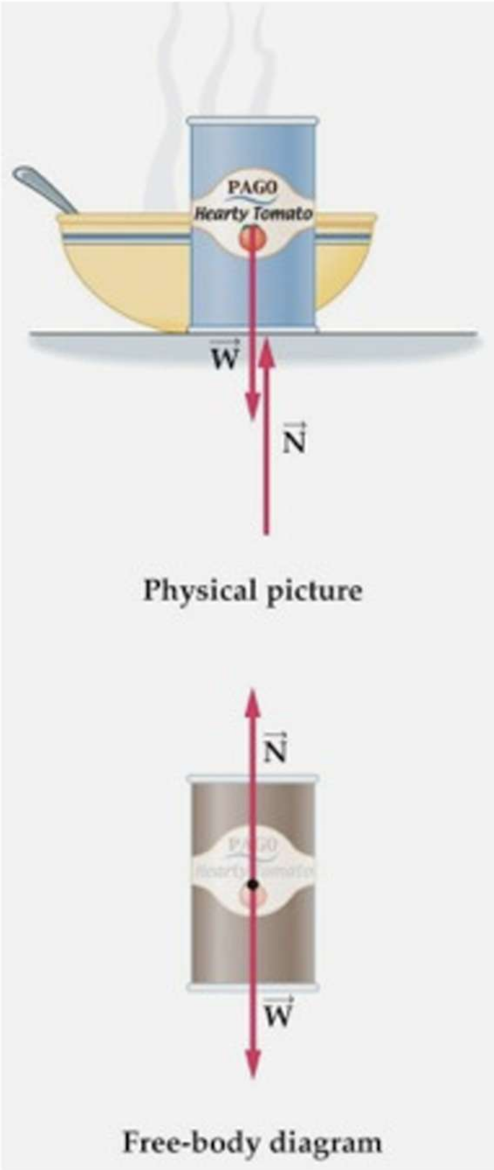
Microscopic View

A powerful microscope would see that the string was made up of atoms connected by molecular bonds, that can be thought of as tiny springs holding the atoms together.

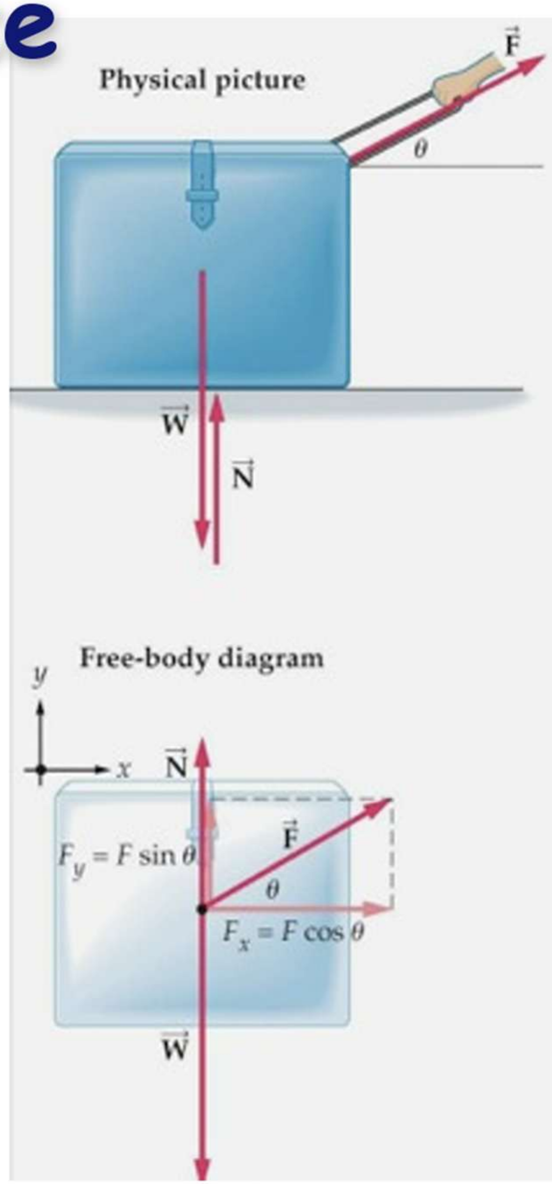
Identifying Forces

- **Identify "the system" and "the environment."** The system is the object whose motion you wish to study; the environment is everything else.
- **Draw a picture of the situation.** Show the object—the system—and everything in the environment that touches the system. Ropes, springs, and surfaces are all parts of the environment.
- **Consider a system at the time.**
- **Locate every point where the environment exerts contact forces on the object.**
- **Name and label each contact force acting on the object.** There is at least one force at each point of contact; there may be more than one. When necessary, use subscripts to distinguish forces of the same type.
- **Name and label each long-range force acting on the object.** For now, the only long-range force is weight.

Normal Force



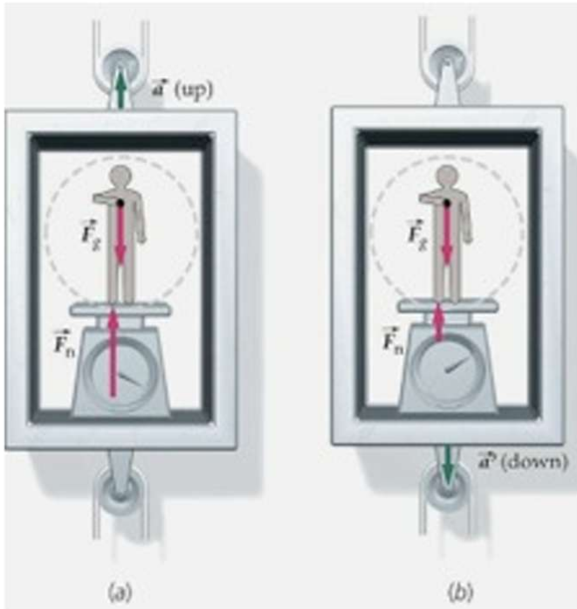
The normal force is the force exerted by a surface on an object.



Normal Force

The normal force is always perpendicular to the surface that produces it.

Example: Weighing Yourself in an Elevator



Suppose that your mass is 80 kg, and you are standing on a scale fastened to the floor of an elevator. The scale measures force and is calibrated in newtons.

(a) What does the scale read when the elevator is rising with acceleration $\vec{a}$?

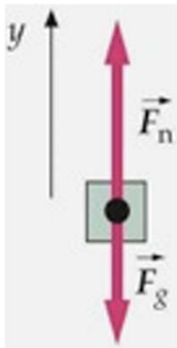
(b) When it is descending with acceleration $\vec{a}'$?

$$\vec{F}_n + \vec{F}_g = m\vec{a}$$

$$F_n - mg = ma$$

$F_{na} = m(g + a)$ the $\vec{a}$ push up so the apparent weight is more

$F_{nb} = m(g - a')$ the $\vec{a}$ push down so the apparent weight is less

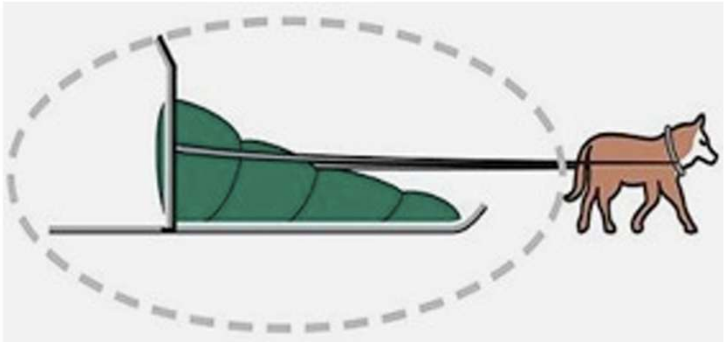


Using Free-Body Diagrams

$$\sum F_x = F_{nx} + F_{gx} + F_x = ma_x$$

$$0 + 0 + F_x = ma_x$$

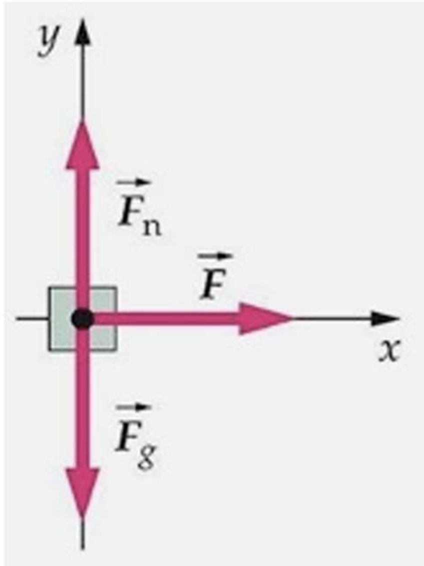
$$\boxed{a_x = \frac{F_x}{m}}$$



$$\sum F_y = F_{ny} + F_{gy} + F_y = ma_y$$

$$F_n - F_g + 0 = ma_y = 0$$

$$\boxed{F_n = F_g}$$

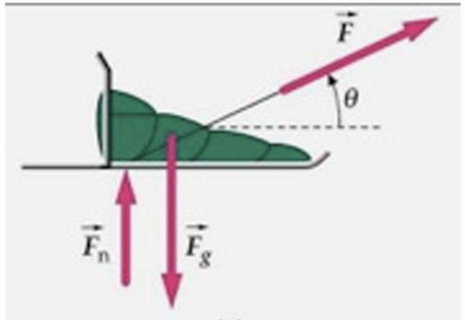


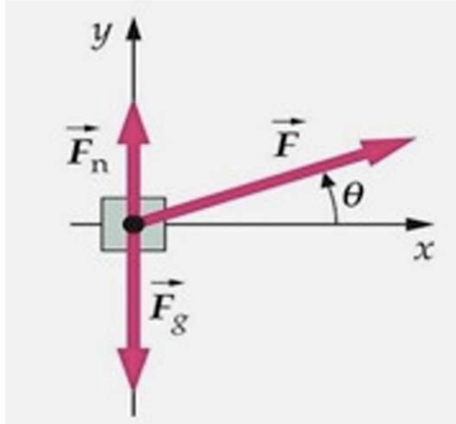
Example: A sled

During your winter break, you pull a rope attached to the sled with a force of 150 N at 25° above the horizontal. The mass of the sled-rope system is 80 kg, and there is negligible friction between the sled runners and the ice.

(a) Find the acceleration of the sled

(b) Find the magnitude of the normal force exerted on the surface by the sled.





$$\vec{F}_n + \vec{F}_g + \vec{F} = m\vec{a}$$

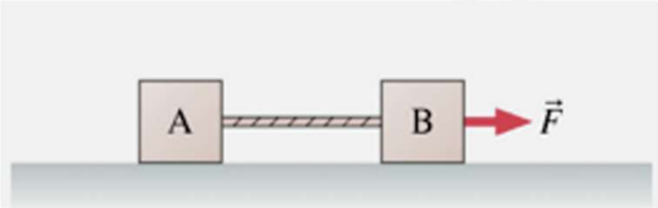
x: $F_x = F\cos\theta = ma_x$

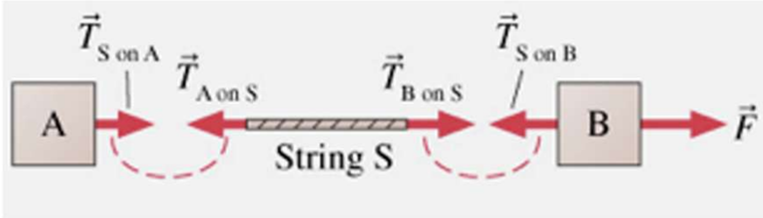
$$a_x = F\cos\theta / m = (150\text{ N})(\cos\ 25°)/(80\text{ kg}) = 1.7\text{ m/s}^2$$

y: $F_y = F_n - mg + F\sin\theta = ma_y = 0$

$$F_n = mg - F\sin\theta = (80\text{ kg})(9.81\text{ m/s}^2) - (150\text{ N})(\sin\ 25°) = 720\text{ N}$$

The Massless String Approximation

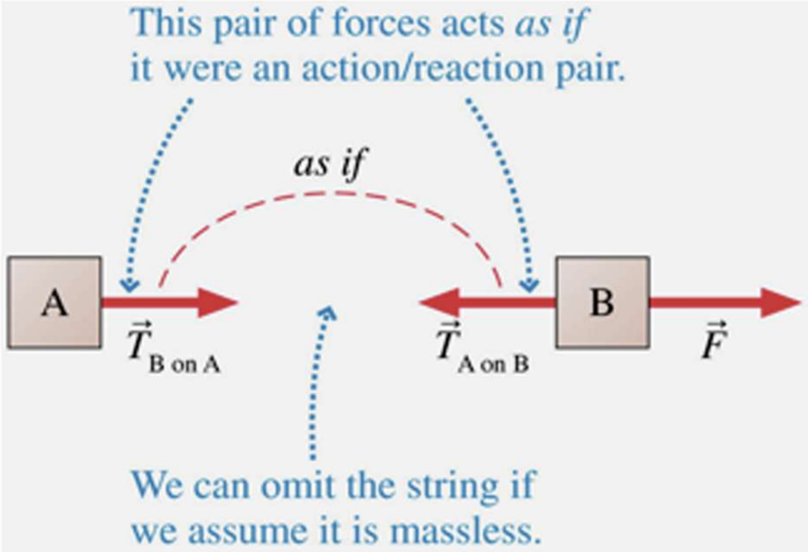




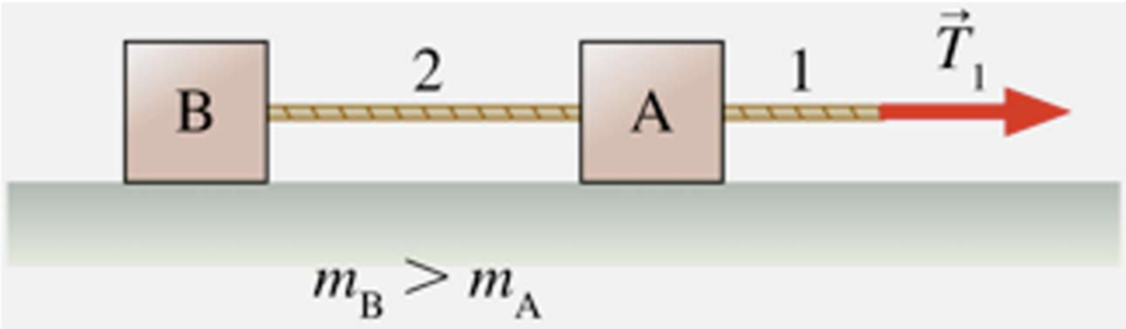
A horizontal force F acts on a block that is connected to another block by a string. Consider the constraints and the forces.

Massless String Approximation:

$$T_{\text{A on S}} = T_{\text{B on S}}$$



Example: Comparing Tensions



Blocks A and B are connected by massless String 2 and pulled across a frictionless surface by massless String 1. The mass of B is larger than the mass of A.

Is the tension in String 2 smaller, equal, or larger than the tension in String 1?

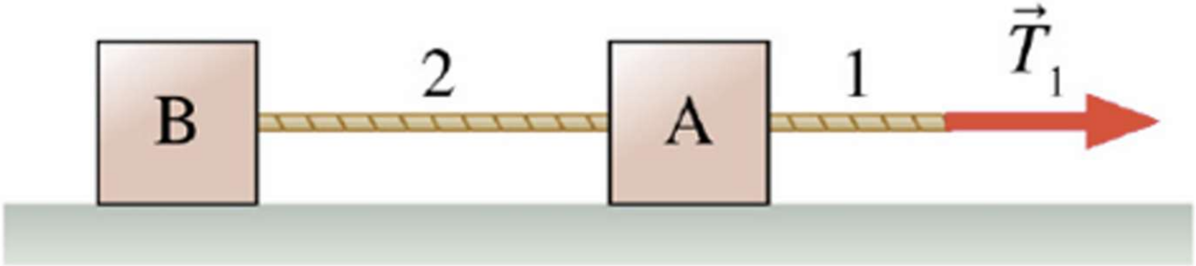
Find tension of rope 2 $T_2 = ?$

$T_1 = 100N$

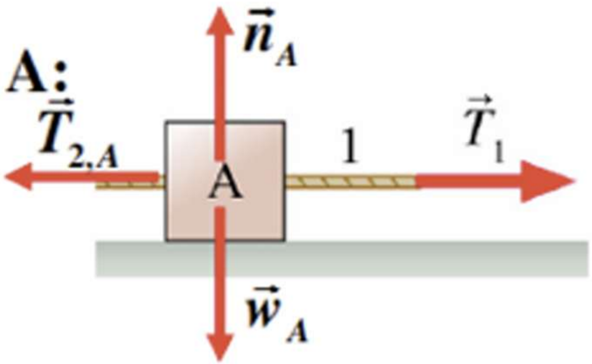
$m_A = 1kg$

$m_B = 4kg$

No friction



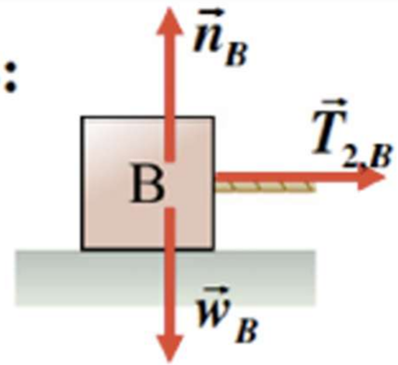
System A:



$$\vec{F}_{A,net} = \vec{T}_{2,A} + \vec{n}_A + \vec{w}_A + \vec{T}_1 = m_A\vec{a}$$

$$T_1 - T_{2,A} = m_A a$$

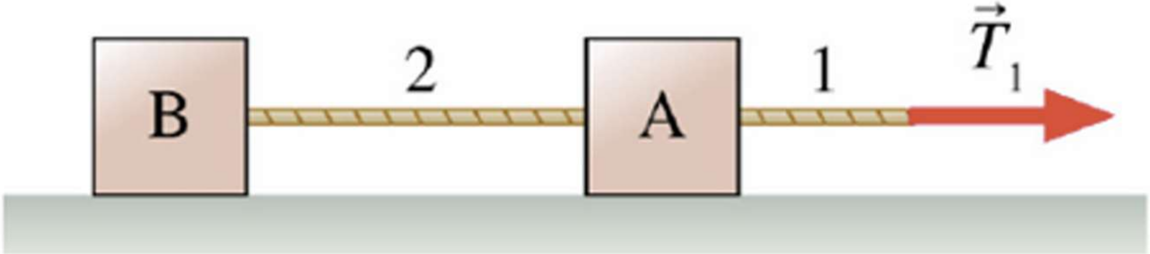
System B:



$$\vec{F}_{B,net} = \vec{T}_{2,B} + \vec{n}_B + \vec{w}_B = m_B\vec{a}$$

$$T_{2,B} = m_B a$$

Find tension of rope 2 $T_2 = ?$



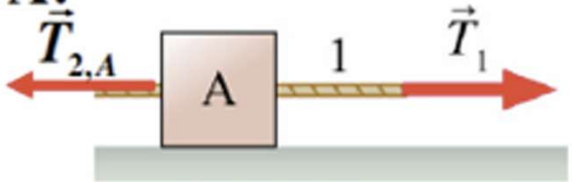
$T_1 = 100N$

$m_A = 1kg$

$m_B = 4kg$

No friction

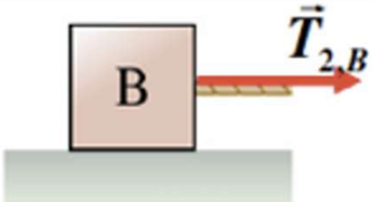
System A:



$\vec{a}$

$$T_1 - T_{2,A} = m_A a$$

System B:



$\vec{a}$

$$T_{2,B} = m_B a$$

Newton's Third Law: $T_{2,B} = T_{2,A} = T_2$

$$T_1 - T_2 = m_A a$$
$$T_2 = m_B a$$

$$a = \frac{T_1}{m_A + m_B}$$

$$T_2 = \frac{m_B T_1}{m_A + m_B} = \frac{4 \cdot 100}{5} = 80N$$

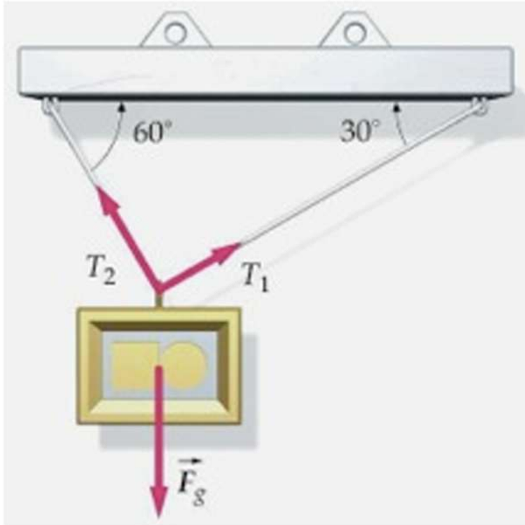
Example: Picture Hanging

A picture weighing 8.0 N is supported by two wires with tensions T_2 and T_2.

Find each tension.

$$\vec{T_1} + \vec{T_2} + \vec{F_g} = \vec{ma} = 0$$

$$\boxed{\begin{aligned} \sin 30° &= \cos 60° = \tfrac{1}{2} \\ \cos 30° &= \sin 60° = \tfrac{\sqrt{3}}{2} \end{aligned}}$$



X: $T_1 \cos 30° - T_2 \cos 60° + 0 = 0$

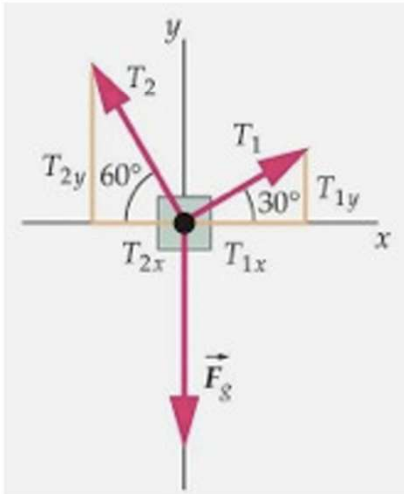
$$T_2 = T_1 \left(\cos 30° / \cos 60°\right) = T_1 \sqrt{3}$$

y: $T_1 \sin 30° + T_2 \sin 60° - F_g = 0$

$$F_g = T_1 \sin 30° + T_2 \sin 60° = T_1 (\tfrac{1}{2} + \sqrt{3}\,\tfrac{\sqrt{3}}{2}) = 2T_1$$

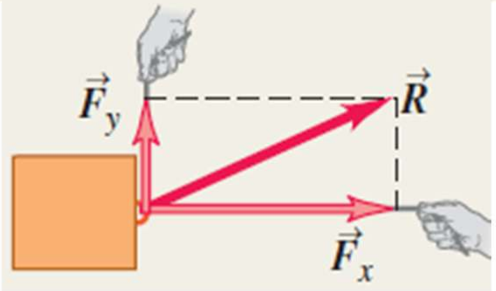
$$\boxed{T_1 = \tfrac{1}{2} F_g = \tfrac{1}{2}(8.0 \text{ N}) = 4.00 \text{ N}}$$

$$\boxed{T_2 = T_1 \sqrt{3} = 6.93 \text{ N}}$$

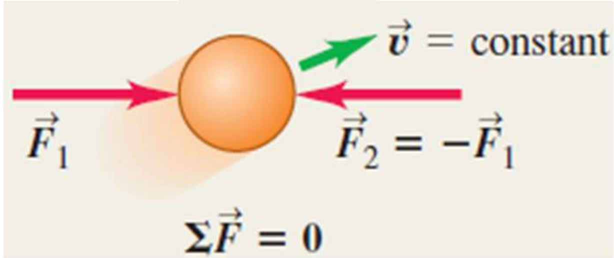


Summary

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = \sum \vec{F} \quad (4.1)$$



$$\sum \vec{F} = 0$$

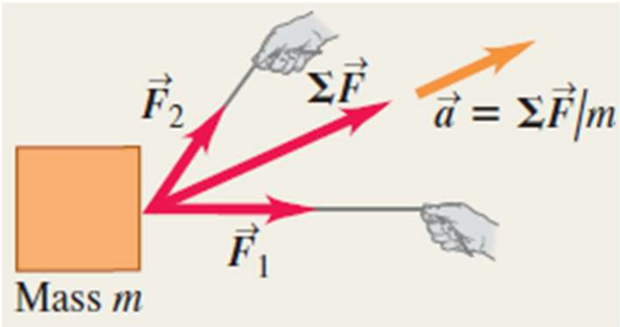


$$\sum \vec{F} = m\vec{a}$$

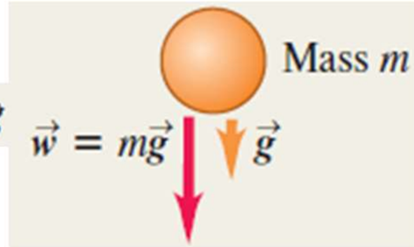
$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

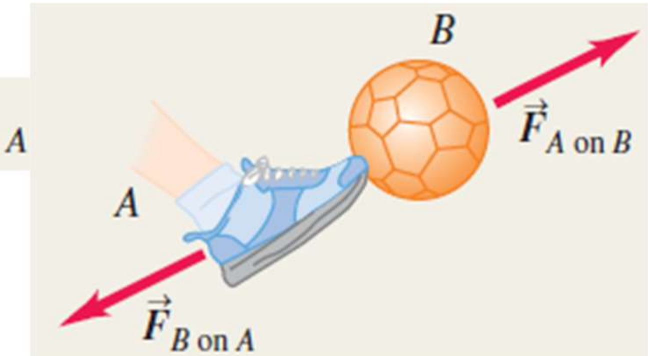
$$\sum F_z = ma_z$$



$$w = mg$$



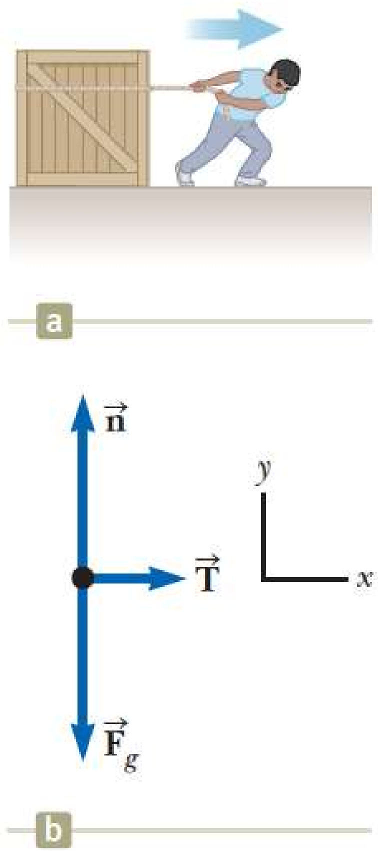
$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$$



Applying Newton's 2nd law:

- A man pulls a cargo box of mass 20 kg with an acceleration of 2 ms^{-2} on a smooth surface. Find the tension of the rope and the normal force acting on the cargo box.

1. Draw the free body diagram.
2. Let rightwards and upwards be positive in x- and y-directions respectively.



For x-direction,

$$F_{net,x} = ma_x$$

$$T = ma_x$$

$$= 20 \times 2$$

$$= 40(N)$$

For y-direction,

$$F_{net,y} = ma_y$$

$$n - W = 0$$

$$n = W = mg = 20 \times 9.8$$

$$n = 196(N)$$

Example 5.6 **The Runaway Car**

Objects under net force

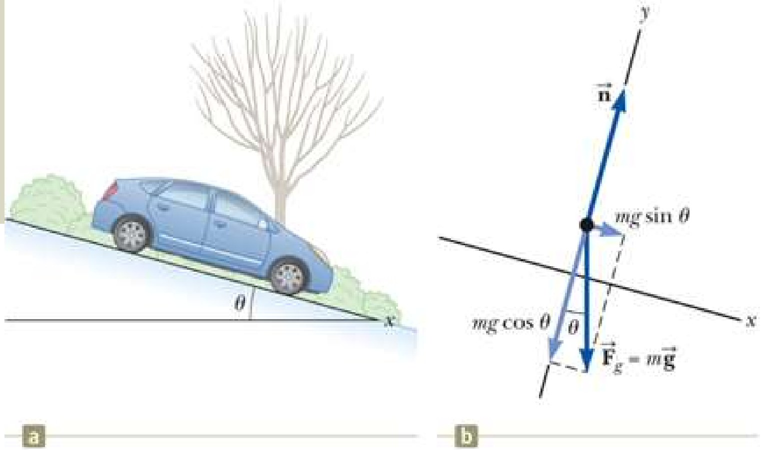
A car of mass m is on an icy driveway inclined at an angle θ as in Figure 5.11a.

(A) Find the acceleration of the car, assuming the driveway is frictionless.

$$\sum F_x = mg\sin\theta = ma_x$$

$$\sum F_y = n - mg\cos\theta = 0$$

$$a_x = \boxed{g\sin\theta}$$



(B) Suppose the car is released from rest at the top of the incline and the distance from the car's front bumper to the bottom of the incline is d. How long does it take the front bumper to reach the bottom of the hill, and what is the car's speed as it arrives there?

$$d = \tfrac{1}{2}a_x t^2$$

$$(4)\quad t = \sqrt{\frac{2d}{a_x}} = \boxed{\sqrt{\frac{2d}{g\sin\theta}}}$$

$$v_{xf}^{\,2} = 2a_x d$$

$$(5)\quad v_{xf} = \sqrt{2a_x d} = \boxed{\sqrt{2gd\sin\theta}}$$

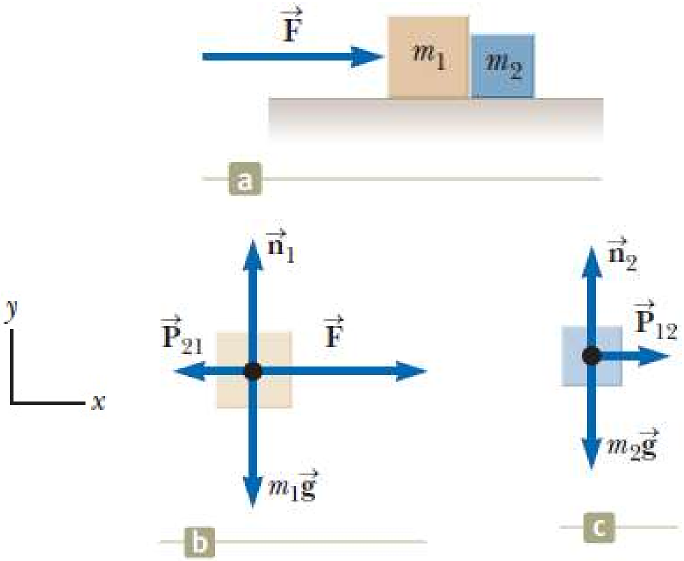
Example 5.7 One Block Pushes Another

Two blocks of masses m_1 and m_2, with $m_1 > m_2$, are placed in contact with each other on a frictionless, horizontal surface as in Active Figure 5.12a. A constant horizontal force $\vec{\mathbf{F}}$ is applied to m_1 as shown.

(A) Find the magnitude of the acceleration of the system.

$$\sum F_x = F = (m_1 + m_2)a_x$$

$$(1)\quad a_x = \frac{F}{m_1 + m_2}$$



(B) Determine the magnitude of the contact force between the two blocks.

$$(2)\quad \sum F_x = P_{12} = m_2 a_x$$

$$(3)\quad P_{12} = m_2 a_x = \left(\frac{m_2}{m_1 + m_2}\right)F$$

$$(4)\quad \sum F_x = F - P_{21} = F - P_{12} = m_1 a_x$$

$$P_{12} = F - m_1 a_x = F - m_1\left(\frac{F}{m_1 + m_2}\right) = \left(\frac{m_2}{m_1 + m_2}\right)F$$

Example 5.8 Weighing a Fish in an Elevator

A person weighs a fish of mass m on a spring scale attached to the ceiling of an elevator as illustrated in Figure 5.13.

(A) Show that if the elevator accelerates either upward or downward, the spring scale gives a reading that is different from the weight of the fish.

$$\sum F_y = T - mg = ma_y$$

$$(1)\ T = ma_y + mg = mg\left(\frac{a_y}{g} + 1\right) = F_g\left(\frac{a_y}{g} + 1\right)$$

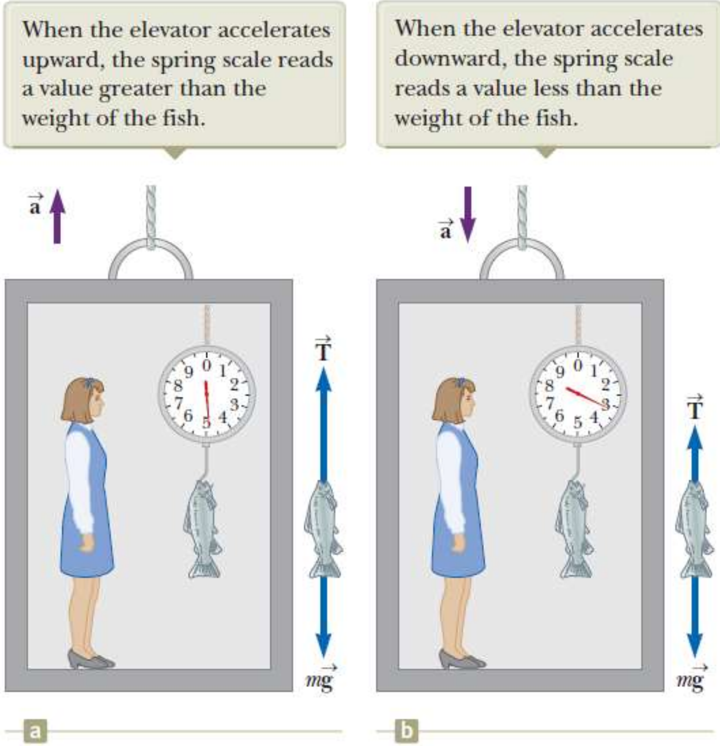


Figure 5.13 (Example 5.8) A fish is weighed on a spring scale in an accelerating elevator car.

Example 5.8 Weighing a Fish in an Elevator

(B) Evaluate the scale readings for a 40.0-N fish if the elevator moves with an acceleration $a_y = \pm 2.00 \text{ m/s}^2$.

$$(1)\ T = ma_y + mg = mg\left(\frac{a_y}{g} + 1\right) = F_g\left(\frac{a_y}{g} + 1\right)$$

$$T = (40.0 \text{ N})\left(\frac{2.00 \text{ m/s}^2}{9.80 \text{ m/s}^2} + 1\right) = \boxed{48.2 \text{ N}}$$

$$T = (40.0 \text{ N})\left(\frac{-2.00 \text{ m/s}^2}{9.80 \text{ m/s}^2} + 1\right) = \boxed{31.8 \text{ N}}$$

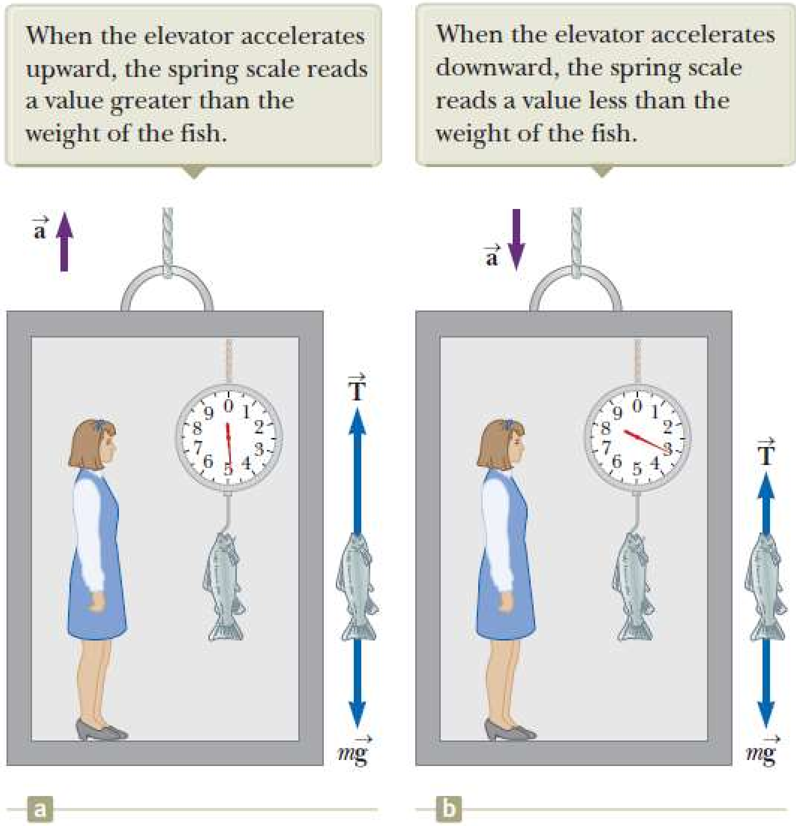


Figure 5.13 (Example 5.8) A fish is weighed on a spring scale in an accelerating elevator car.

Example 5.8 Weighing a Fish in an Elevator

WHAT IF? Suppose the elevator cable breaks and the elevator and its contents are in free fall. What happens to the reading on the scale?

$$(1)\ T = ma_y + mg = mg\left(\frac{a_y}{g} + 1\right) = F_g\left(\frac{a_y}{g} + 1\right)$$

$$a_y = -g$$

$$T = 0$$

Weightless!

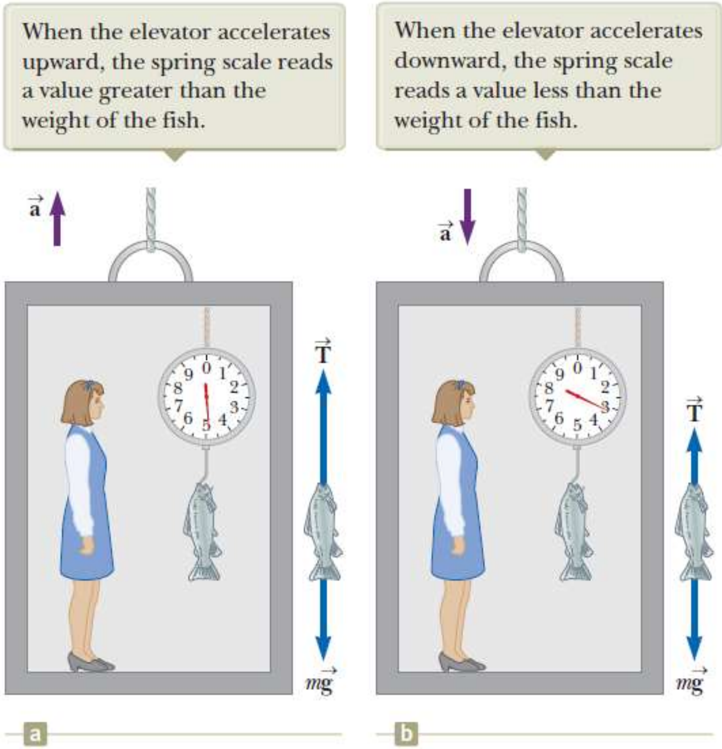


Figure 5.13 (Example 5.8) A fish is weighed on a spring scale in an accelerating elevator car.

Example **5.9** **The Atwood Machine**

Multiple Objects

Determine the magnitude of the acceleration of the two objects and the tension in the lightweight cord.

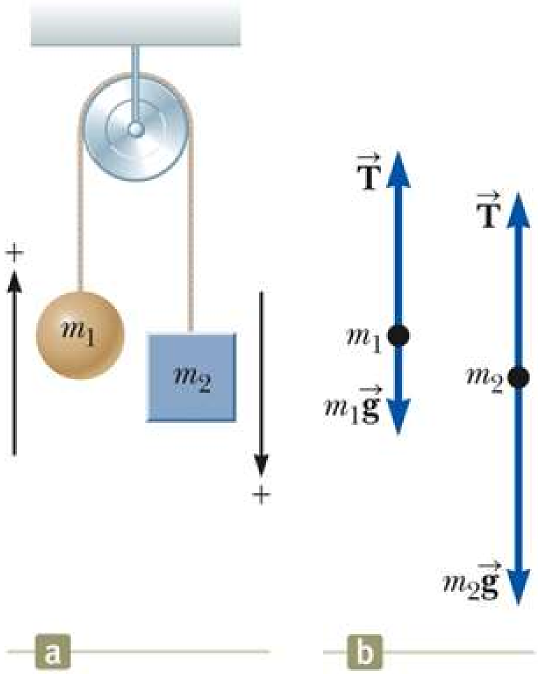
$$(1)\ \sum F_y = T - m_1 g = m_1 a_y$$

$$(2)\ \sum F_y = m_2 g - T = m_2 a_y$$

$$-m_1 g + m_2 g = m_1 a_y + m_2 a_y$$

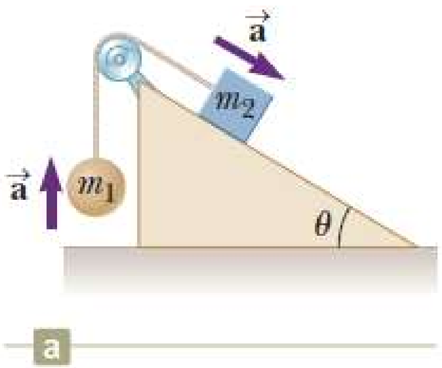
$$(3)\ a_y = \boxed{\left(\frac{m_2 - m_1}{m_1 + m_2}\right) g}$$

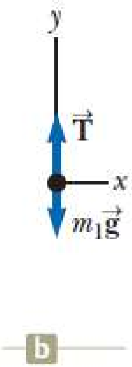
$$(4)\ T = m_1(g + a_y) = \boxed{\left(\frac{2m_1 m_2}{m_1 + m_2}\right) g}$$



Example 5.10 Acceleration of Two Objects Connected by a Cord

A ball of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass as in Figure 5.15a. The block lies on a frictionless incline of angle θ. Find the magnitude of the acceleration of the two objects and the tension in the cord.

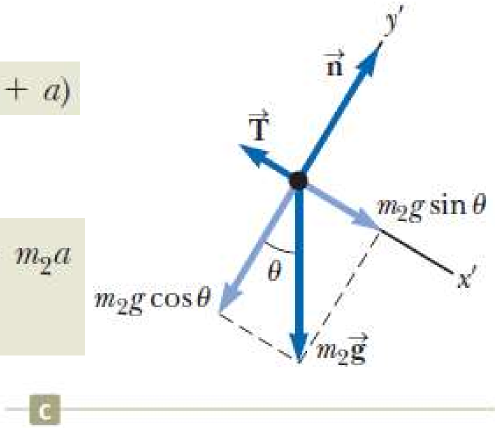




$$(1)\ \sum F_x = 0$$

$$(2)\ \sum F_y = T - m_1 g = m_1 a_y = m_1 a$$

$$(5)\ T = m_1(g + a)$$



$$(3)\ \sum F_{x'} = m_2 g \sin\theta - T = m_2 a_{x'} = m_2 a$$

$$(4)\ \sum F_{y'} = n - m_2 g \cos\theta = 0$$

$$m_2 g \sin\theta - m_1(g + a) = m_2 a$$

$$(6)\ a = \left(\frac{m_2 \sin\theta - m_1}{m_1 + m_2}\right) g$$

$$(7)\ T = \left(\frac{m_1 m_2(\sin\theta + 1)}{m_1 + m_2}\right) g$$

WHAT IF? What happens in this situation if $\theta = 90°$?

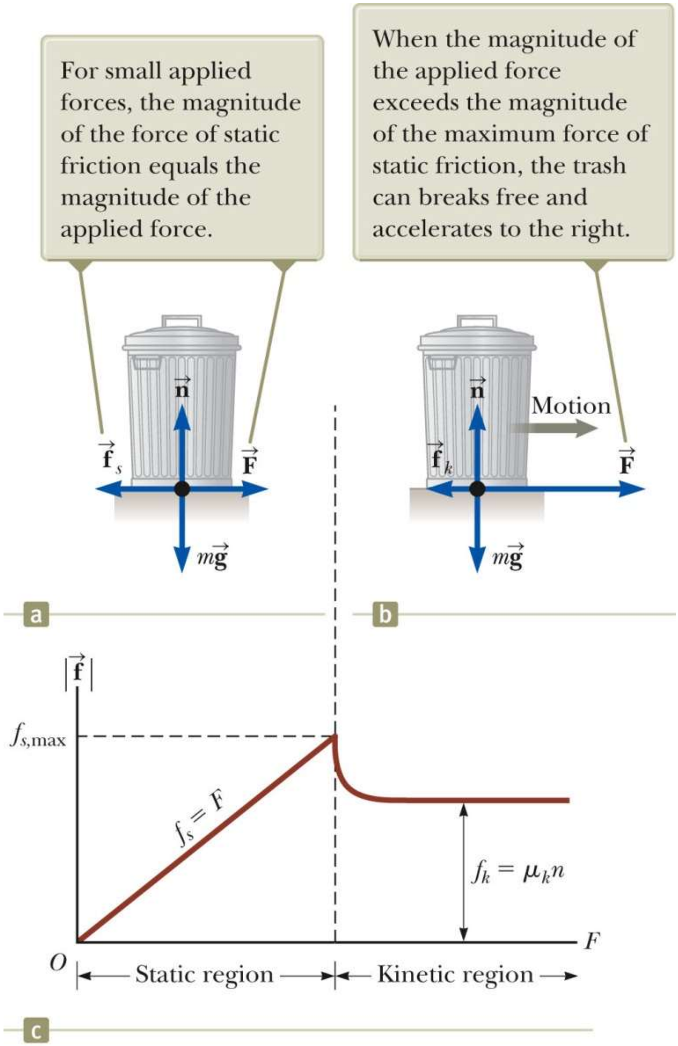
WHAT IF? What if $m_1 = 0$?

Forces of Friction

- When an object is in motion on a surface or through a viscous medium, there will be a resistance to the motion. This resistance is called the *force of friction.*
- Experiments show that friction is proportional to the normal force, is opposite the direction of motion and parallel to the surfaces in contact.
  - $f_s \leq \mu_s n$ and $f_k = \mu_k n$
    - μ is the **coefficient of friction**; n is the normal force
  - These equations relate the **magnitudes** of the forces; they are not vector equations.

Static Friction

- Static friction acts to keep the object from moving.
- As long as the object is not moving, $f_s = F$
- If $\vec{\mathbf{F}}$ increases, so does $\vec{f}_s$
- If $\vec{\mathbf{F}}$ decreases, so does $\vec{f}_s$
- $f_s \le \mu_s n$
  - Remember, the equality holds when the surfaces are on the verge of slipping.

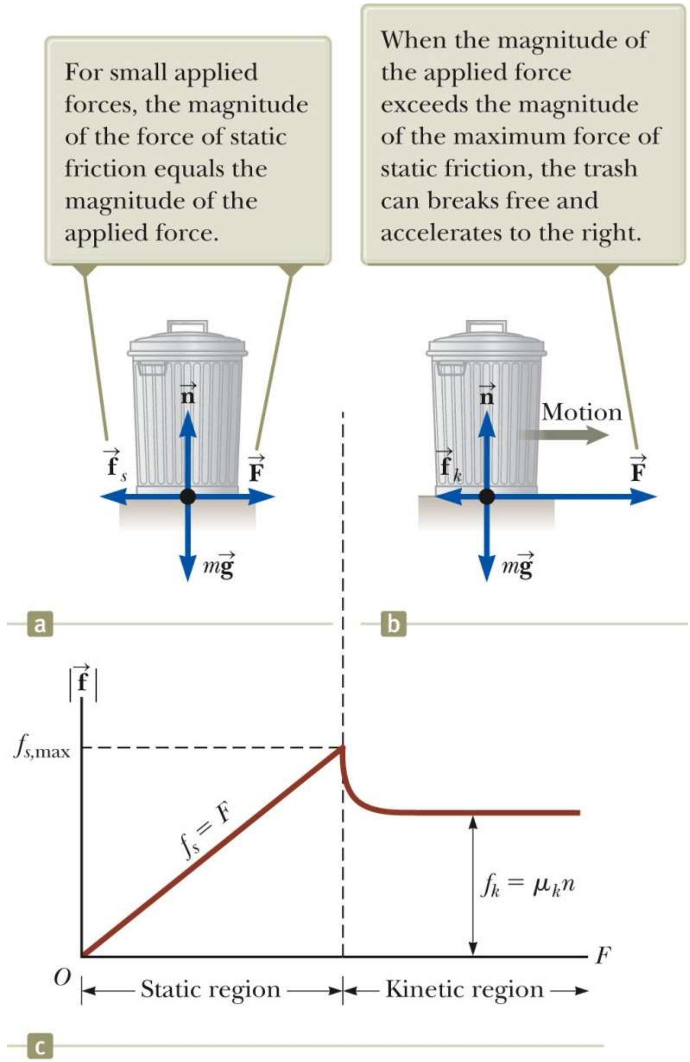


Kinetic Friction

- The force of kinetic friction acts when the object is in motion.
- Although μ_k can vary with speed, we shall neglect any such variations.
- $f_k = \mu_k n$

Pitfall Prevention 5.11

The Direction of the Friction Force
Sometimes, an incorrect statement about the friction force between an object and a surface is made—"the friction force on an object is opposite to its motion or impending motion"—rather than the correct phrasing, "the friction force on an object is opposite to its motion or impending motion *relative to the surface*."



Some Coefficients of Friction

TABLE 5.1 *Coefficients of Friction*

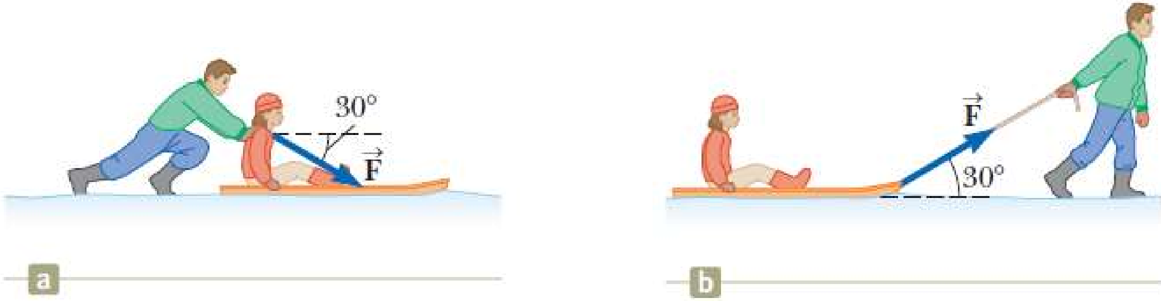
	μ_s	μ_k
Rubber on concrete	1.0	0.8
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Glass on glass	0.94	0.4
Copper on steel	0.53	0.36
Wood on wood	0.25–0.5	0.2
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	—	0.04
Metal on metal (lubricated)	0.15	0.06
Teflon on Teflon	0.04	0.04
Ice on ice	0.1	0.03
Synovial joints in humans	0.01	0.003

Note: All values are approximate. In some cases, the coefficient of friction can exceed 1.0.

Think about it!

***Quick Quiz* 5.6** You press your physics textbook flat against a vertical wall with your hand. What is the direction of the friction force exerted by the wall on the book? **(a)** downward **(b)** upward **(c)** out from the wall **(d)** into the wall

***Quick Quiz* 5.7** You are playing with your daughter in the snow. She sits on a sled and asks you to slide her across a flat, horizontal field. You have a choice of **(a)** pushing her from behind by applying a force downward on her shoulders at 30° below the horizontal (Fig. 5.17a) or **(b)** attaching a rope to the front of the sled and pulling with a force at 30° above the horizontal (Fig. 5.17b). Which would be easier for you and why?



Example 5.12 The Sliding Hockey Puck

A hockey puck on a frozen pond is given an initial speed of 20.0 m/s. If the puck always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the puck and ice.

$$(1) \quad \sum F_x = -f_k = ma_x$$

$$(2) \quad \sum F_y = n - mg = 0$$

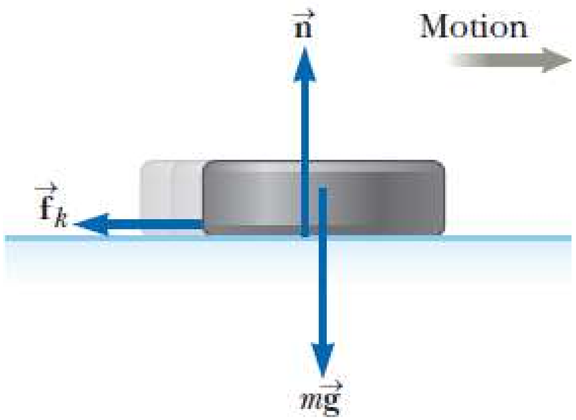
$$-\mu_k n = -\mu_k mg = ma_x$$

$$a_x = -\mu_k g$$

$$0 = v_{xi}^2 + 2a_x x_f = v_{xi}^2 - 2\mu_k g x_f$$

$$\mu_k = \frac{v_{xi}^2}{2gx_f}$$

$$\mu_k = \frac{(20.0\ \mathrm{m/s})^2}{2(9.80\ \mathrm{m/s^2})(115\ \mathrm{m})} = \boxed{0.177}$$



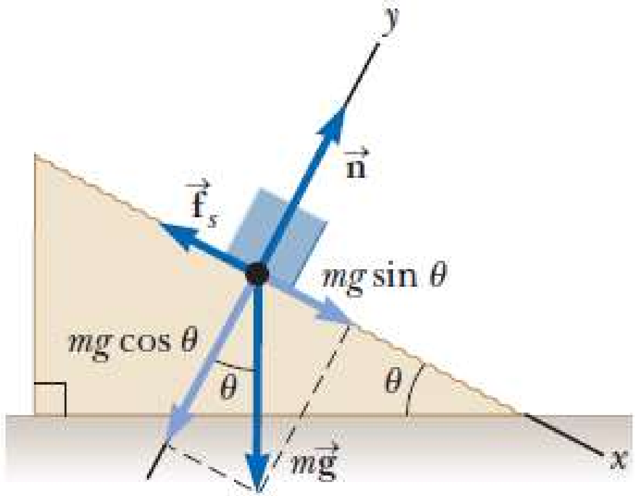
Example 5.11 Experimental Determination of μ_s

The following is a simple method of measuring coefficients of friction. Suppose a block is placed on a rough surface inclined relative to the horizontal as shown in Active Figure 5.18. The incline angle is increased until the block starts to move. Show that you can obtain μ_s by measuring the critical angle θ_c at which this slipping just occurs.

$$(1)\quad \sum F_x = mg\sin\theta - f_s = 0$$

$$(2)\quad \sum F_y = n - mg\cos\theta = 0$$

$$(3)\quad f_s = mg\sin\theta = \left(\frac{n}{\cos\theta}\right)\sin\theta = n\tan\theta$$



$$\mu_s n = n\tan\theta_c$$

$$\mu_s = \tan\theta_c$$

For example, if the block just slips at $\theta_c = 20.0°$, we find that $\mu_s = \tan 20.0° = 0.364$.

Example 5.13 Acceleration of Two Connected Objects When Friction Is Present

A block of mass m_2 on a rough, horizontal surface is connected to a ball of mass m_1 by a lightweight cord over a lightweight, frictionless pulley as shown in Figure 5.20a. A force of magnitude F at an angle θ with the horizontal is applied to the block as shown, and the block slides to the right. The coefficient of kinetic friction between the block and surface is μ_k. Determine the magnitude of the acceleration of the two objects.

(1) $\sum F_x = F\cos\theta - f_k - T = m_2 a_x = m_2 a$

(2) $\sum F_y = n + F\sin\theta - m_2 g = 0$

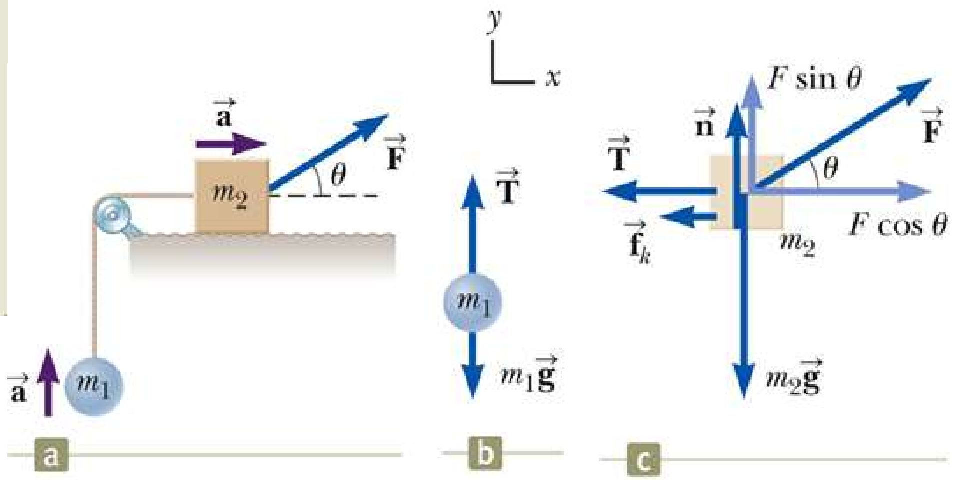
(3) $\sum F_y = T - m_1 g = m_1 a_y = m_1 a$

$n = m_2 g - F\sin\theta$

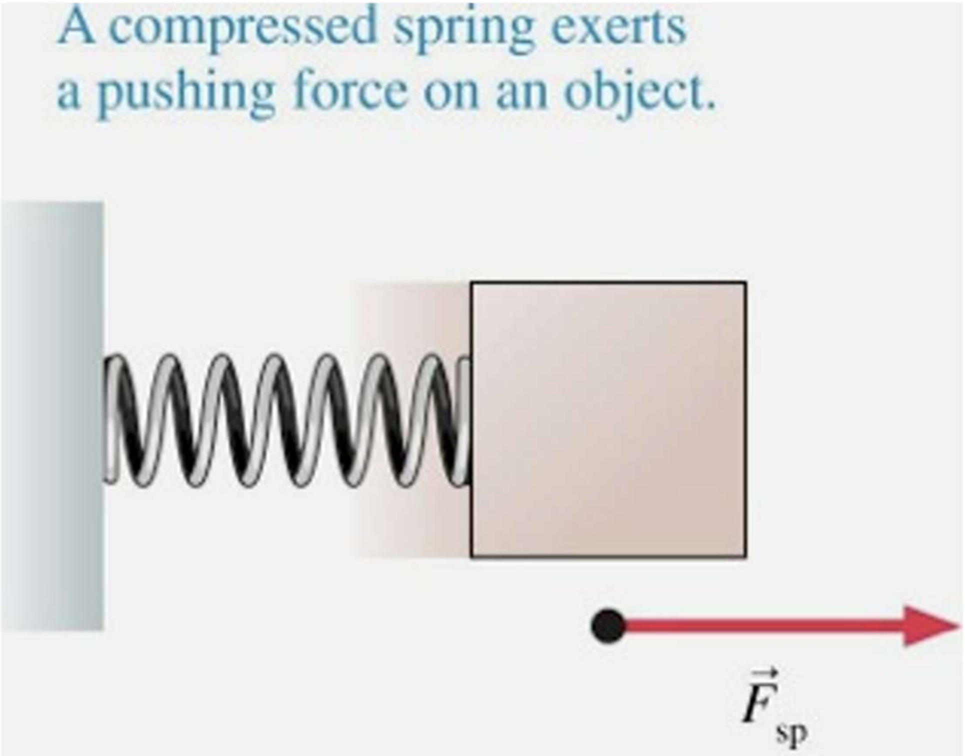
(4) $f_k = \mu_k(m_2 g - F\sin\theta)$

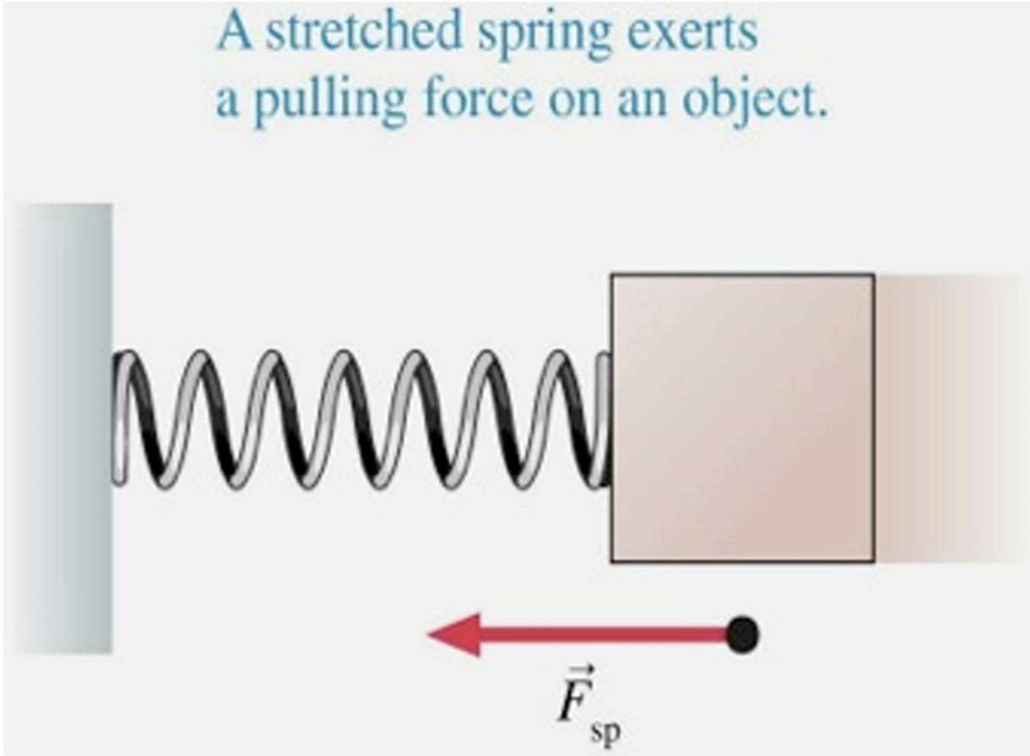
$F\cos\theta - \mu_k(m_2 g - F\sin\theta) - m_1(a + g) = m_2 a$

(5) $a = \dfrac{F(\cos\theta + \mu_k \sin\theta) - (m_1 + \mu_k m_2)g}{m_1 + m_2}$

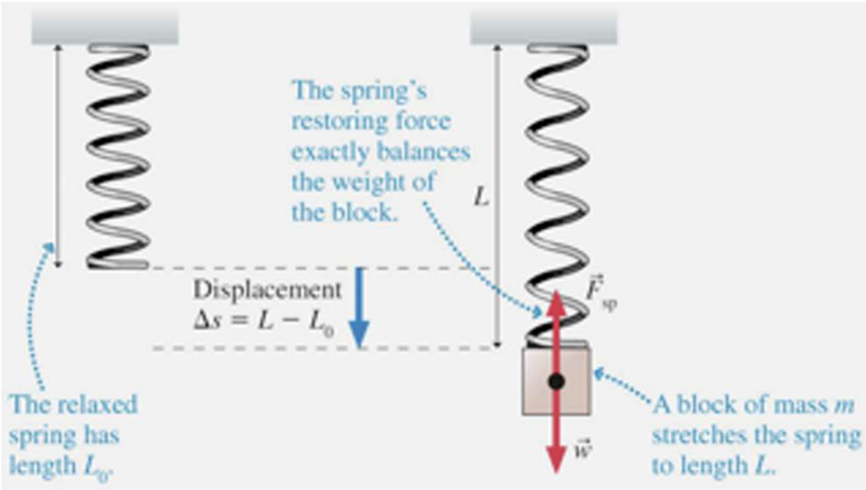


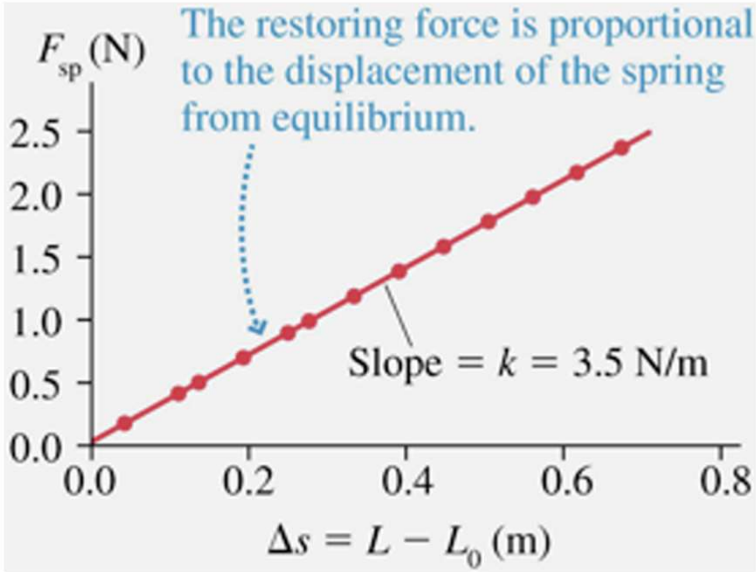
Springs


A compressed spring exerts
a pushing force on an object.
$\vec{F}_{sp}$


A stretched spring exerts
a pulling force on an object.
$\vec{F}_{sp}$

Stretching a Spring



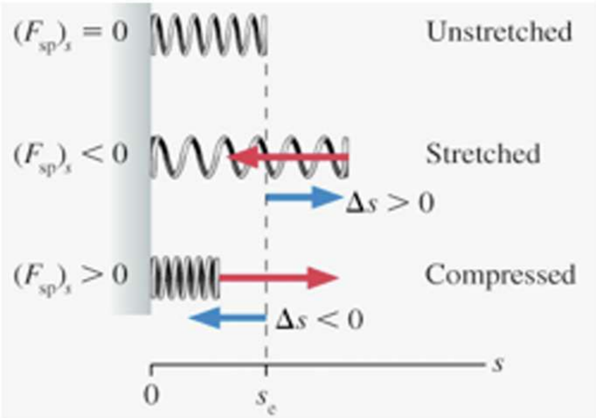


The unloaded spring has a length L_0. Hang a weight of mass m on it and it stretches to a new length L. $\Delta s = L - L_0$ vs. the applied force $F_{sp} = mg$.

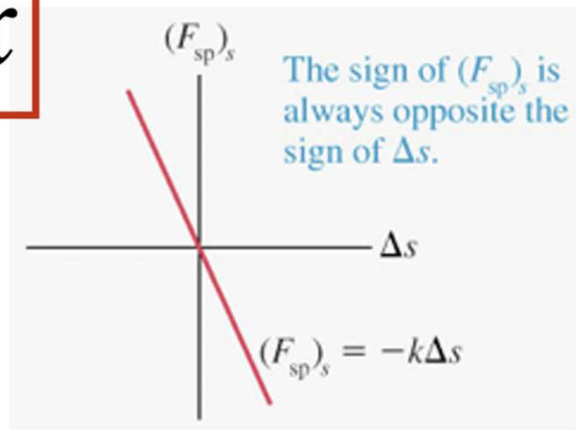
We find that $F_{sp} = k\Delta s$, where k is the "spring constant".

Hooke's Law for Springs

force increases linearly with the amount the spring is stretched or compressed:



$$\boxed{F = -kx}$$



The constant k is called the **spring constant**.

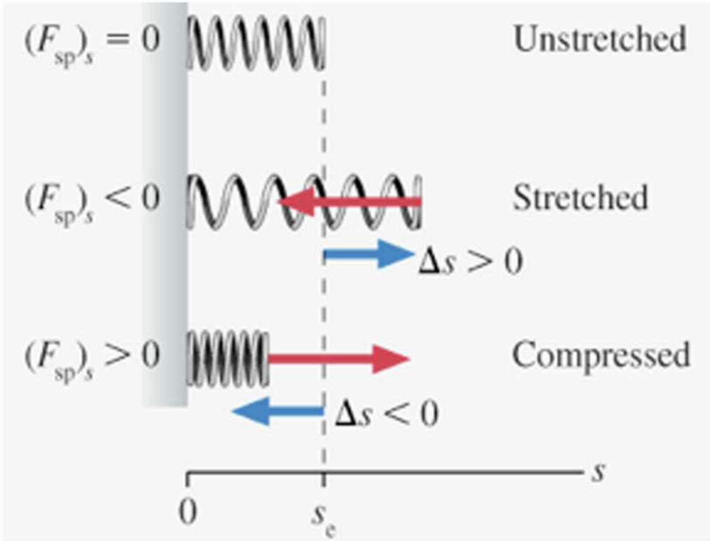
k has units of N/m or kg/s^2.

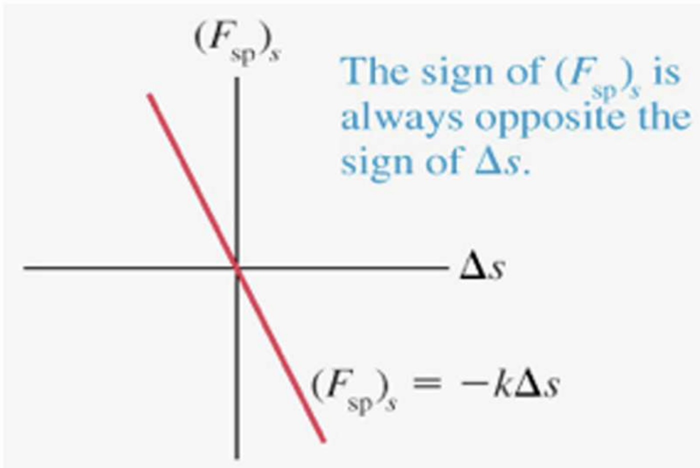
Hooke's Law

The linear proportionality between force and displacement is found to be valid whether the spring is stretched or compressed, and the force and displacement are always in opposite directions.

Therefore, we write the force-displacement relation as:

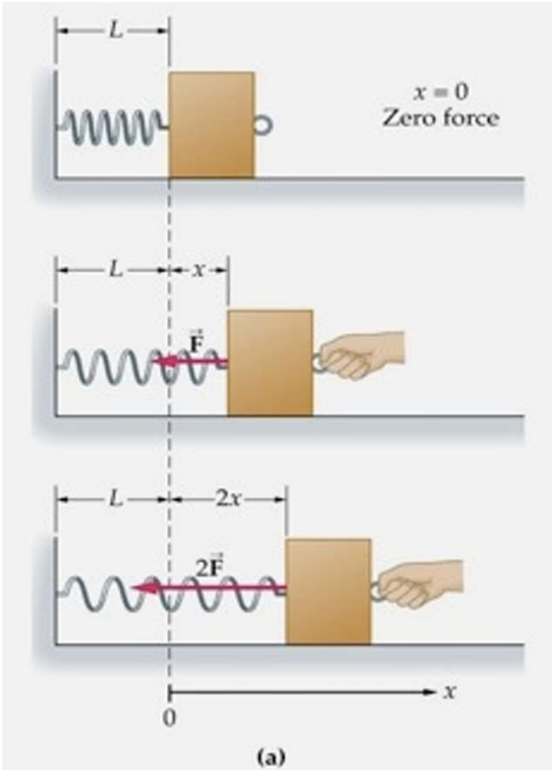
$$\boxed{(F_{sp})_s = -k\Delta s}$$

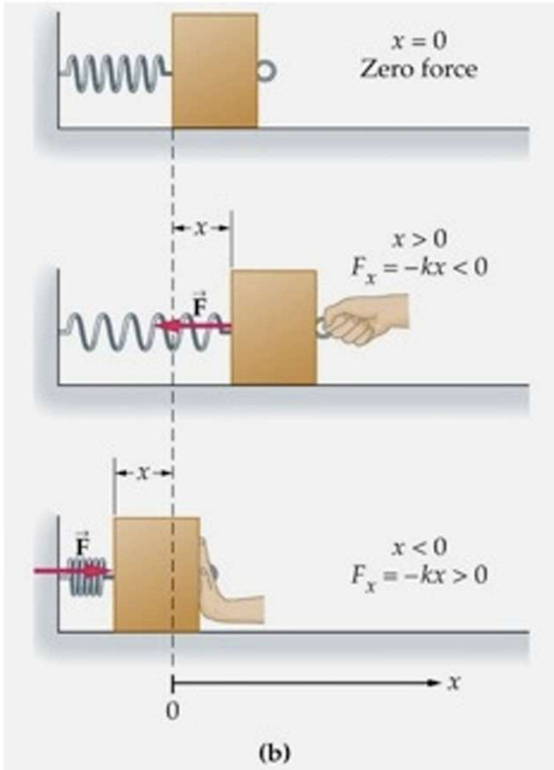




Spring Forces

The force always **opposes** the compression or extension of the spring.





$$\boxed{F = -kx}$$

Centripetal Force

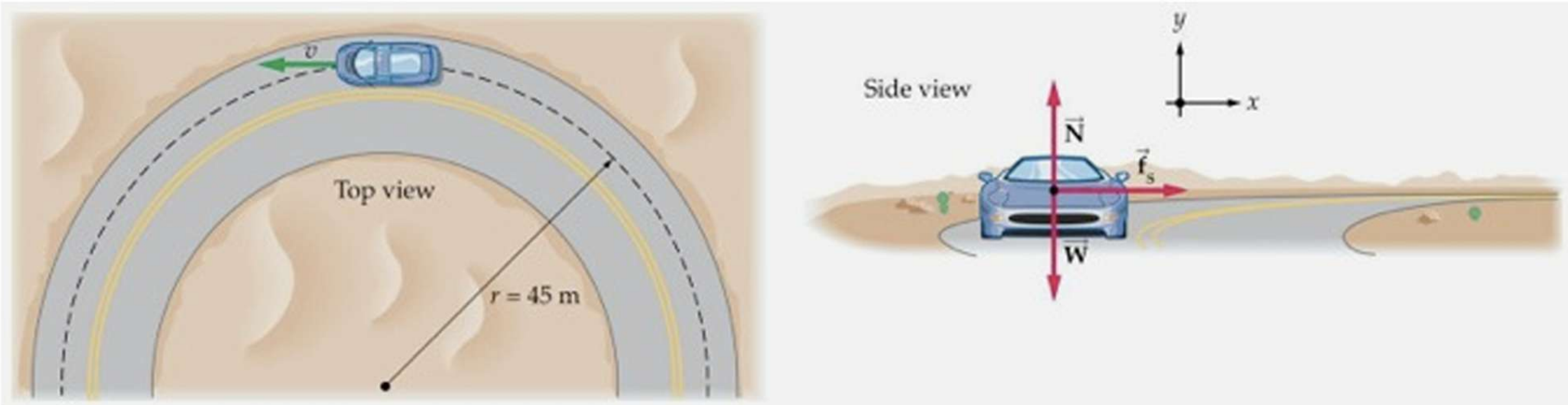
From the centripetal acceleration a_{cp}, we find the centripetal force f_{cp}, required to keep an object of mass m moving in a circle of radius r.

The magnitude of the force f_{cp} , called **centripetal force** because it points toward the center of rotation, is given by:

$$\boxed{f_{cp} = ma_{cp} = m\frac{v^2}{r} = m\omega^2 r}$$

Banked Curves

Example: Rounding a Corner



A 1,200 kg car rounds a corner of radius r = 45.0 m. If the coefficient of friction between the tires and the road is μ_s = 0.82, what is the maximum speed the car can have on the curve without skidding?

$$\sum F_x = f_s = \mu_s N = ma_x = m\frac{v^2}{r} \qquad \sum F_y = 0 = N - W = N - mg \longrightarrow N = mg$$

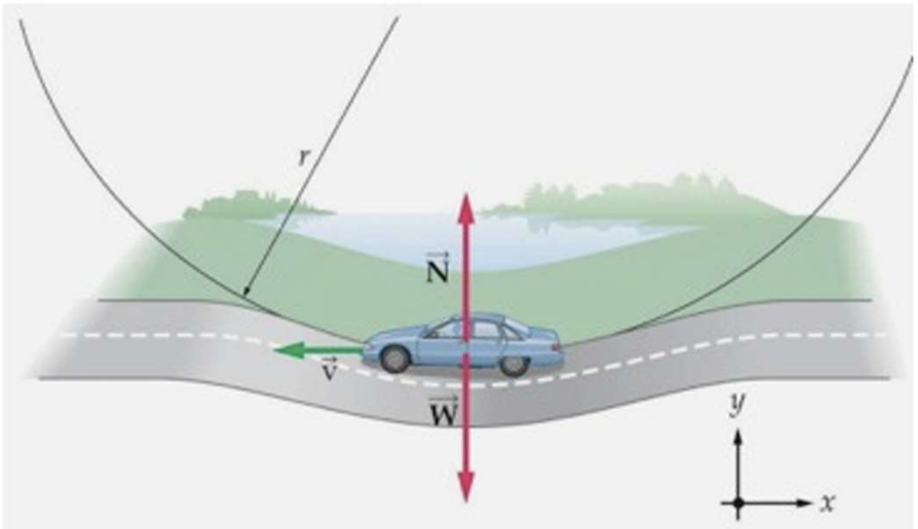
$$\sum F_x = \mu_s \cancel{m} g = \cancel{m}\frac{v^2}{r} \qquad v = \sqrt{\mu_s r g} = \sqrt{(0.82)(45.0\text{ m})(9.81\text{ m/s}^2)} = 19.0\text{ m/s}$$

Question: How does this result depend on the weight of the car?

Example: Normal Force in a Dip

While driving on a country road at a constant speed of 17.0 m/s, you encounter a dip in the road. The dip can be approximated by a circular arc with a radius of 65.0 m.

What is the normal force exerted by the car seat on an 80.0 kg passenger at the bottom of the dip?



$$\sum F_y = N - mg = ma_y = mv^2 / r$$

$$N = mg + mv^2 / r = m(g + v^2 / r)$$

$$N = (80.0\ \text{kg})\left[(9.81\ \text{m/s}^2) + (17.0\ \text{m/s})^2 / (65.0\ \text{m})\right]$$

$$= (80.0\ \text{kg})(14.26\ \text{m/s}^2) = 1140\ \text{N}$$ **(about 45% more)**

Note that **(80 kg)(9.81 m/s²) = 785 N**

Summary

Laws of Motion

Newton's first law states that it is possible to find an inertial frame in which an object that does not interact with other objects experiences zero acceleration, or, equivalently, in the absence of an external force, when viewed from an inertial frame, an object at rest remains at rest and an object in uniform motion in a straight line maintains that motion.

Newton's second law states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

Newton's third law states that if two objects interact, the force exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force exerted by object 2 on object 1.

The maximum **force of static friction** $\vec{\mathbf{f}}_{s,\max}$ between an object and a surface is proportional to the normal force acting on the object. In general, $f_s \le \mu_s n$, where μ_s is the **coefficient of static friction** and n is the magnitude of the normal force.

When an object slides over a surface, the magnitude of the **force of kinetic friction** $\vec{\mathbf{f}}_k$ is given by $f_k = \mu_k n$, where μ_k is the **coefficient of kinetic friction.**