Energy of a System & Conservation of Energy

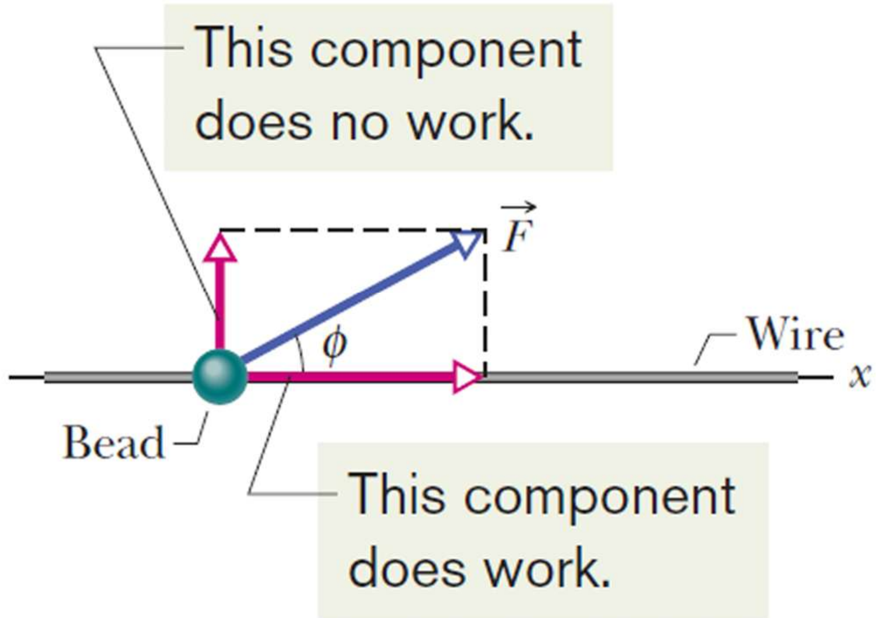
Work and Kinetic Energy

- To calculate the work a force does on an object as the object moves through some displacement, we use only the force component along the object's displacement
  - The force component perpendicular to the displacement does zero work

- For a constant force,

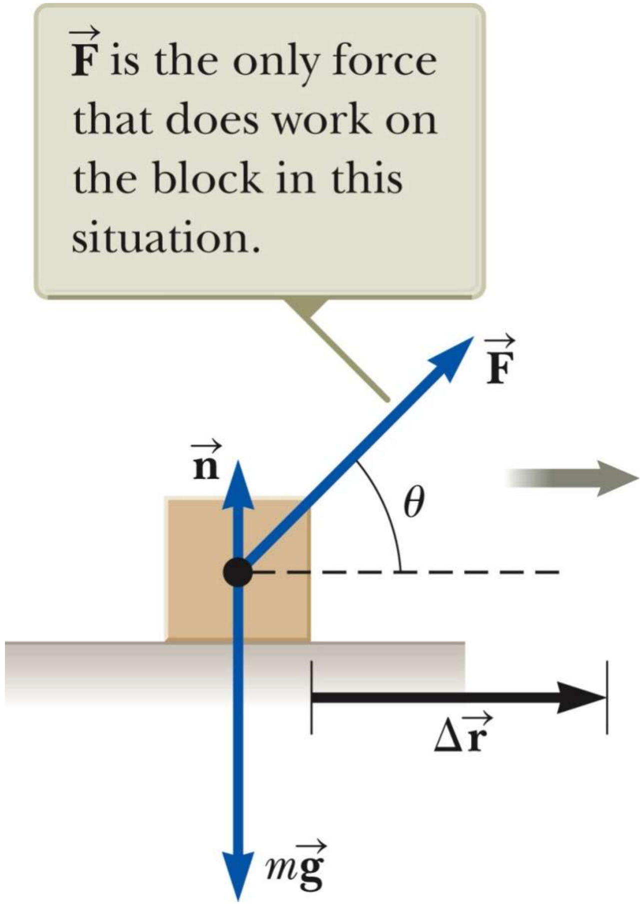
$$W = Fd\cos\phi$$

$$W = \vec{F} \cdot \vec{d}$$



Work Example

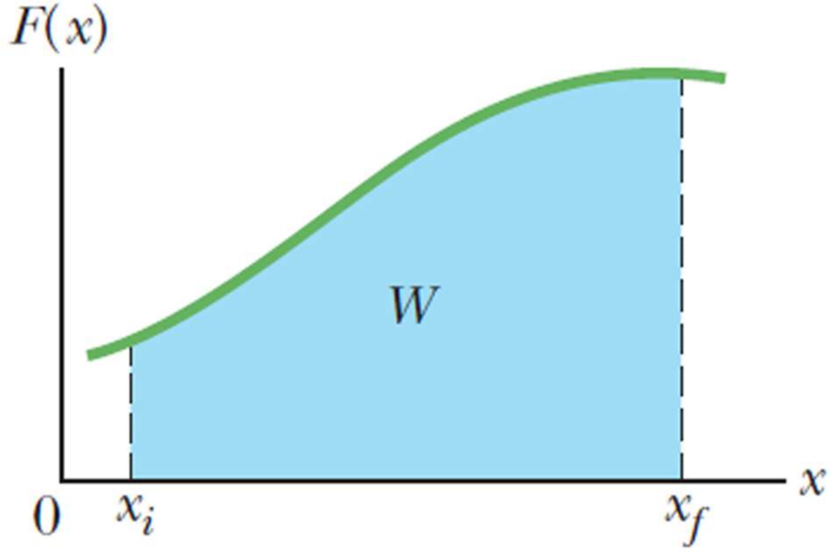
- The normal force and the gravitational force do no work on the object.
  - $\cos\theta = \cos 90° = 0$
- The force $\vec{\mathbf{F}}$ is the only force that does work on the object.



- One-Dimensional Analysis

$$W = \int_{x_i}^{x_f} F(x)\, dx$$

  - x_i: initial position
  - x_f: final position



- Three-Dimensional Analysis

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x\, dx + \int_{y_i}^{y_f} F_y\, dy + \int_{z_i}^{z_f} F_z\, dz$$

  - r_i: initial position
  - r_f: final position

Work Done by a Varying Force

•To calculate the work done by a varying force, assume that during a very small displacement, Δx, F is constant.

•For that displacement, $W \sim F\,\Delta x$

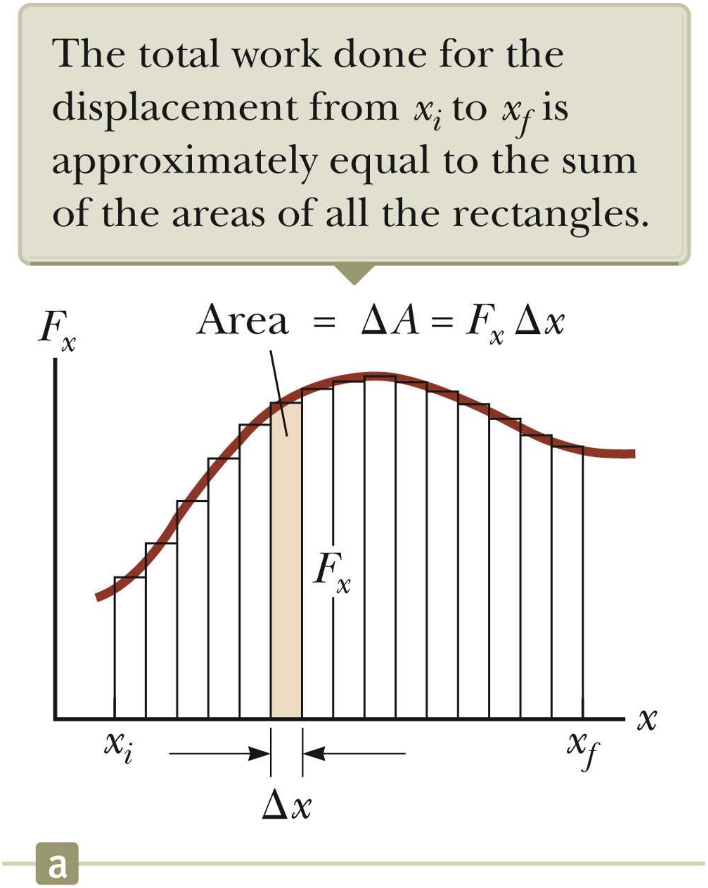
•For all of the intervals,

$$W \approx \sum_{x_i}^{x_f} F_x \Delta x$$

•If $\Delta x \rightarrow 0$

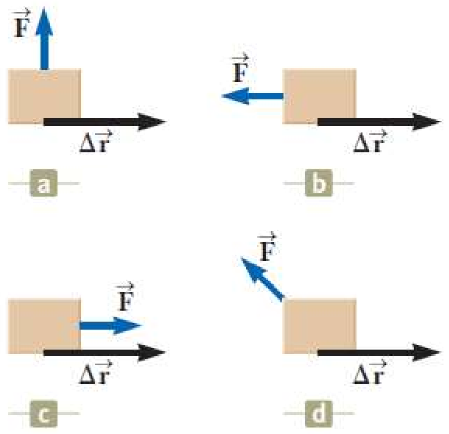
$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x\, dx$$

•The work done is equal to the area under the curve between x_i and x_f.



***Quick Quiz* 7.1** The gravitational force exerted by the Sun on the Earth holds the Earth in an orbit around the Sun. Let us assume that the orbit is perfectly circular. The work done by this gravitational force during a short time interval in which the Earth moves through a displacement in its orbital path is **(a)** zero **(b)** positive **(c)** negative **(d)** impossible to determine

***Quick Quiz* 7.2** Figure 7.4 shows four situations in which a force is applied to an object. In all four cases, the force has the same magnitude, and the displacement of the object is to the right and of the same magnitude. Rank the situations in order of the work done by the force on the object, from most positive to most negative.



Kinetic Energy, cont

- Calculating the work:

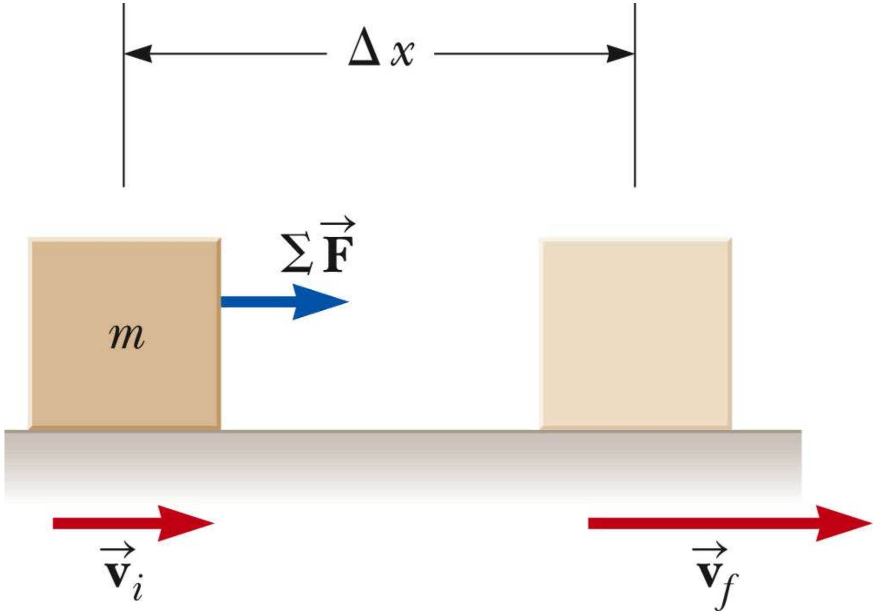
$$W_{ext} = \int_{x_i}^{x_f} \sum F dx = \int_{x_i}^{x_f} ma\, dx$$

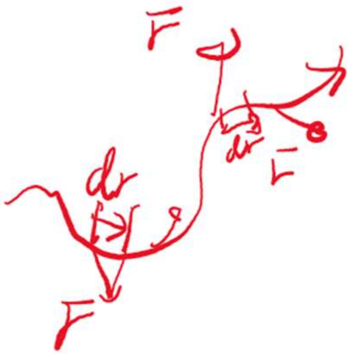
$$= \int_{x_i}^{x_f} m \frac{dv}{dt} dx = \int_{v_i}^{v_f} mv dv$$

$$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$= K_f - K_i = \Delta K$$

where $K = \frac{1}{2} m v^2$





$$W \equiv \int_i^f \vec{F} \cdot d\vec{r}$$

$$= \int F_x dx + \int F_y dy + \int F_z dz$$

$$= \int m a_x dx + \int m a_y dy + \int m a_z dz$$

$$= m\left[\int \frac{dV_x}{dt} dx + \int \frac{dV_y}{dt} dy + \int \frac{dV_z}{dt} dz\right]$$

$$= m\left[\int_i^f dV_x V_x + \int dV_y V_y + \int dV_z V_z\right]$$

$$= \frac{m}{2}\left[V_f^2 - V_i^2\right]$$

$$= KE_f - KE_i \quad (KE = \text{kinetic energy})$$

Example 7.6 A Block Pulled on a Frictionless Surface

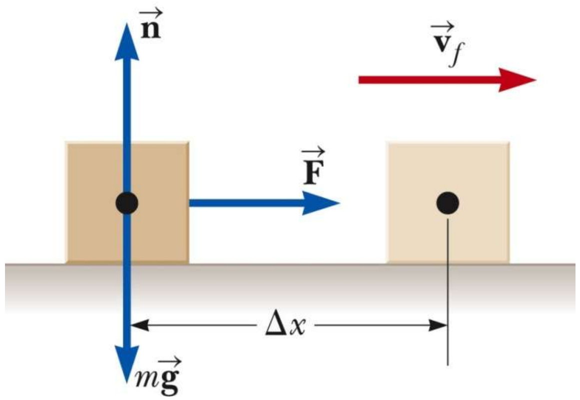
A 6.0-kg block initially at rest is pulled to the right along a frictionless, horizontal surface by a constant horizontal force of 12 N. Find the block's speed after it has moved 3.0 m.

The net external force acting on the block is the horizontal 12-N force.

$$W_{\text{ext}} = K_f - K_i = \tfrac{1}{2}mv_f^2 - 0 = \tfrac{1}{2}mv_f^2$$

$$v_f = \sqrt{\frac{2W_{\text{ext}}}{m}} = \sqrt{\frac{2F\Delta x}{m}}$$

$$v_f = \sqrt{\frac{2(12\text{ N})(3.0\text{ m})}{6.0\text{ kg}}} = \boxed{3.5\text{ m/s}}$$



WHAT IF? Suppose the magnitude of the force in this example is doubled to $F' = 2F$. The 6.0-kg block accelerates to 3.5 m/s due to this applied force while moving through a displacement $\Delta x'$. How does the displacement $\Delta x'$ compare with the original displacement Δx?

$$v_f^2 - v_i^2 = 2as$$

$$v_f^2 - v_i^2 = 2(2a)\left(\frac{s}{2}\right)$$

$$W_{\text{ext}} = F'\Delta x' = \Delta K = F\Delta x$$

$$\Delta x' = \frac{F}{F'}\Delta x = \frac{F}{2F}\Delta x = \tfrac{1}{2}\Delta x$$

Sample Problem

- Force $\vec{F} = (3x^2\ \mathrm{N})\hat{\imath} + (4\ \mathrm{N})\hat{\jmath}$, with x in metres, acts on a particle, changing only the kinetic energy of the particle. How much work is done on the particle as it moves from coordinates (2 m, 3 m) to (3 m, 0 m)? Does the speed of the particle increase, decrease, or remain the same?

Calculation: We set up two integrals, one along each axis:

$$W = \int_2^3 3x^2\,dx + \int_3^0 4\,dy = 3\int_2^3 x^2\,dx + 4\int_3^0 dy$$
$$= 3[\tfrac{1}{3}x^3]_2^3 + 4[y]_3^0 = [3^3 - 2^3] + 4[0 - 3]$$
$$= 7.0\ \mathrm{J}. \qquad \text{(Answer)}$$

The positive result means that energy is transferred to the particle by force $\vec{F}$. Thus, the kinetic energy of the particle increases and, because $K = \frac{1}{2}mv^2$, its speed must also increase. If the work had come out negative, the kinetic energy and speed would have decreased.

Power

- The time rate at which work is done by a force is said to be the power due to the force
  - Average power

$$P_{\text{avg}} = \frac{W}{\Delta t}$$

  - Instantaneous power

$$P = \frac{dW}{dt}$$

$$= \vec{F} \cdot \vec{v}$$

(for a constant force)

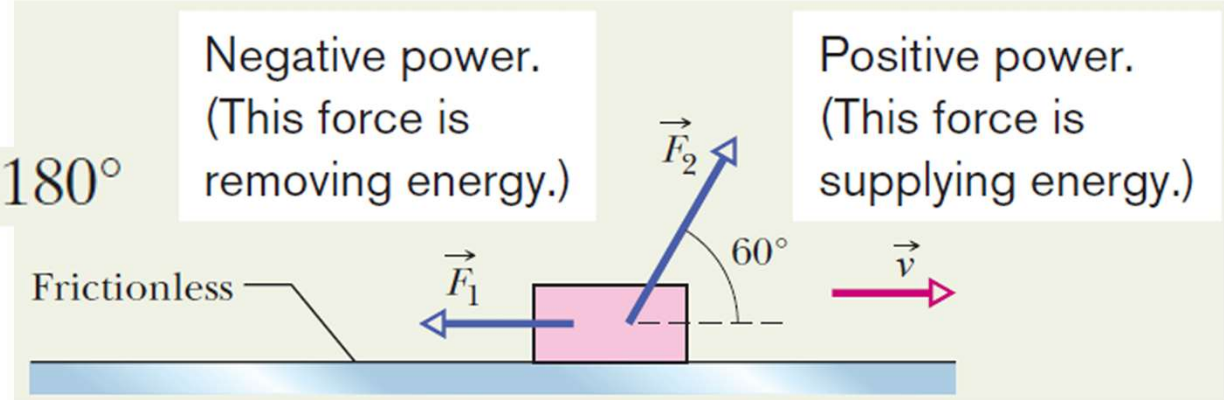
- Unit
  - The SI unit of power is the joule per second (J/s) or watt (W)

$$1 \text{ watt} = 1 \text{ W} = 1 \text{ J/s} = 0.738 \text{ ft} \cdot \text{lb/s}$$

$$1 \text{ horsepower} = 1 \text{ hp} = 550 \text{ ft} \cdot \text{lb/s} = 746 \text{ W}$$

Sample Problem

- Constant forces $\vec{F}_1$ and $\vec{F}_2$ are acting on a box as the box slides rightward across a frictionless floor. Force $\vec{F}_1$ is horizontal, with magnitude 2.0 N; force $\vec{F}_2$ is angled upward by 60° to the floor and has magnitude 4.0 N. The speed *v* of the box at a certain instant is 3.0 m/s. What is the power due to each force acting on the box at that instant, and what is the net power?

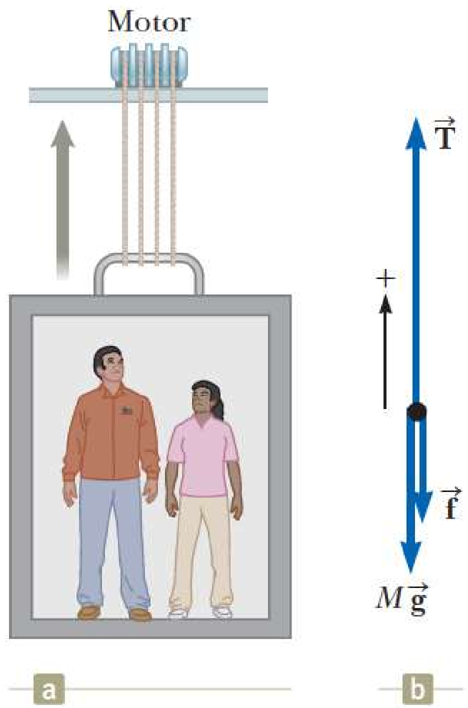


$$P_1 = F_1 v \cos \phi_1 = (2.0\ \text{N})(3.0\ \text{m/s}) \cos 180° = -6.0\ \text{W}.$$

$$P_2 = F_2 v \cos \phi_2 = (4.0\ \text{N})(3.0\ \text{m/s}) \cos 60° = 6.0\ \text{W}.$$

$$P_{\text{net}} = P_1 + P_2 = -6.0\ \text{W} + 6.0\ \text{W} = 0.$$

Example 8.11 Power Delivered by an Elevator Motor



An elevator car (Fig. 8.14a) has a mass of 1 600 kg and is carrying passengers having a combined mass of 200 kg. A constant friction force of 4 000 N retards its motion.

(A) How much power must a motor deliver to lift the elevator car and its passengers at a constant speed of 3.00 m/s?

$$\sum F_y = T - f - Mg = 0$$

$$T = f + Mg$$

$$P = \vec{\mathbf{T}} \cdot \vec{\mathbf{v}} = Tv = (f + Mg)v$$

$$P = [(4\,000\text{ N}) + (1\,800\text{ kg})(9.80\text{ m/s}^2)](3.00\text{ m/s}) = \boxed{6.49 \times 10^4\text{ W}}$$

(B) What power must the motor deliver at the instant the speed of the elevator is v if the motor is designed to provide the elevator car with an upward acceleration of 1.00 m/s²?

$$\sum F_y = T - f - Mg = Ma$$

$$T = M(a + g) + f$$

$$P = Tv = [M(a + g) + f]v$$

$$P = [(1\,800\text{ kg})(1.00\text{ m/s}^2 + 9.80\text{ m/s}^2) + 4\,000\text{ N}]v$$

$$= \boxed{(2.34 \times 10^4)v}$$

Conservative force and Potential energy

A **conservative force** is a force for which the work done on an object is **independent of the path taken** and depends only on the initial and final positions. This means that when an object moves in a closed loop under a conservative force, the **net work done is zero**.

•**Path Independence:**

•The work done by a conservative force only depends on the starting and ending positions, not on the path taken between them.

•**Work Done in a Closed Path is Zero:** If a particle moves in a closed loop, returning to its starting point, the total work done by a conservative force is zero:

$$\oint \mathbf{F} \cdot d\mathbf{r} = 0$$

Potential Energy and Conservative Forces

Potential energy defined in terms of work done by the associated conservative force.

$$U_B - U_A = -\int_A^B \mathbf{F}_c \cdot d\mathbf{s}$$

*Conservative forces tend to ***minimize*** the potential energy within any system: If allowed to, an apple falls to the ground and a spring returns to its natural length.

Non-conservative force does not imply it is dissipative, for example, magnetic force, and also does not mean it will decrease the potential energy, such as hand force.

Distinction Between Conservative and Non-conservative Forces

The distinction between conservative and non-conservative forces is best stated as follows:

A conservative force may be associated with a scalar potential energy function, whereas a non-conservative force cannot.

$$U_B - U_A = -\int_A^B \mathbf{F}_c \cdot d\mathbf{s}$$

$$\mathbf{F}_c = -\nabla U$$

Force and Potential Energy in 3D

$$F_x = -\frac{\Delta U}{\Delta x} \qquad F_y = -\frac{\Delta U}{\Delta y} \qquad F_z = -\frac{\Delta U}{\Delta z}$$

When we take the limits $\Delta x \to 0,\ \Delta y \to 0\ \ \Delta z \to 0$

$$F_x = -\frac{\partial U}{\partial x} \qquad F_y = -\frac{\partial U}{\partial y} \qquad F_z = -\frac{\partial U}{\partial z}$$

This operation is called the **gradient** of U and is often abbreviated as $\vec{\nabla} U$.

$$\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{\imath} + \frac{\partial U}{\partial y}\hat{\jmath} + \frac{\partial U}{\partial z}\hat{k}\right) = -\vec{\nabla} U$$

Potential Energy Functions

Potential energy is clearly a function of position. We now find this function *for the gravitational force near the surface of the earth* and *for the force exerted by an ideal spring*.

Gravitational Potential Energy (near the earth's surface)

$$U_g(y) = mgy$$

Spring Potential Energy

$$U_{sp}(x) = \frac{1}{2}kx^2$$

How can we find a conservative force if the associated potential energy function is given?

A conservative force can be derived from a scalar potential energy function.

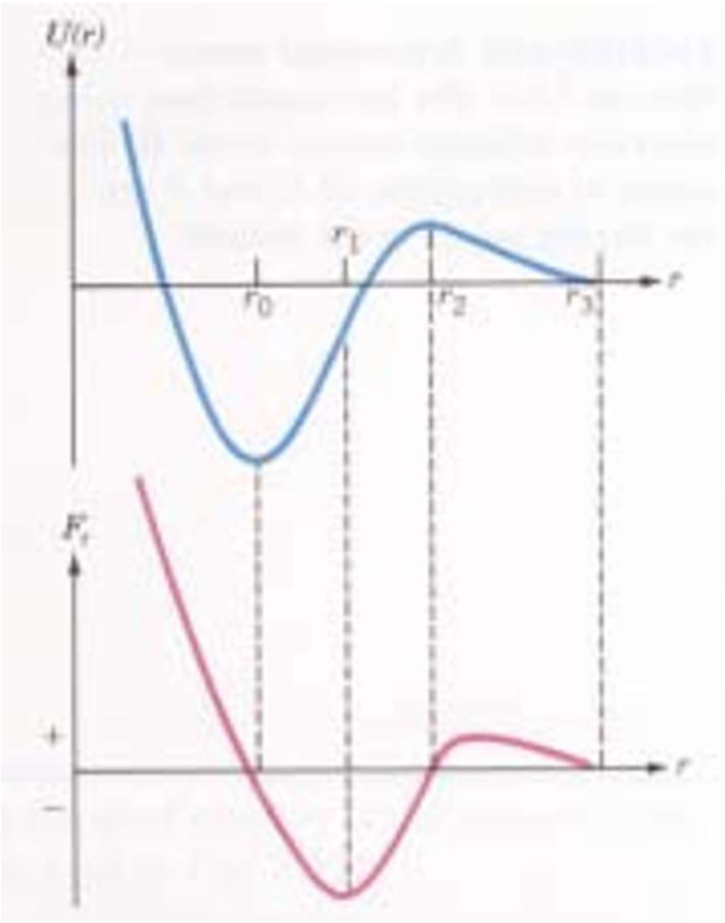
$$\mathbf{F}_c = -\nabla U$$

The negative sign indicates that the force points in the direction of ***decreasing*** potential energy.

$$\textbf{Gravity} \quad U_g = mgy; \qquad F_y = -\frac{dU_g}{dy} = -mg$$

$$\textbf{Spring} \quad U_{sp} = \frac{1}{2}kx^2; \qquad F_x = -\frac{dU_{sp}}{dx} = -kx$$

Conservative Force for a Hypothetical Potential Energy Function



(r > r2): Fr > 0, repulsive force.

(r = r2): Fr = 0, unstable equilibrium.

(r2> r > r0): Fr < 0, attractive force.

(r = r0): Fr = 0, stable equilibrium.

(r0> r): Fr > 0, repulsive force.

Potential Energy

- Potential energy U is energy that can be associated with the configuration (arrangement) of a system of objects that exert forces on one another

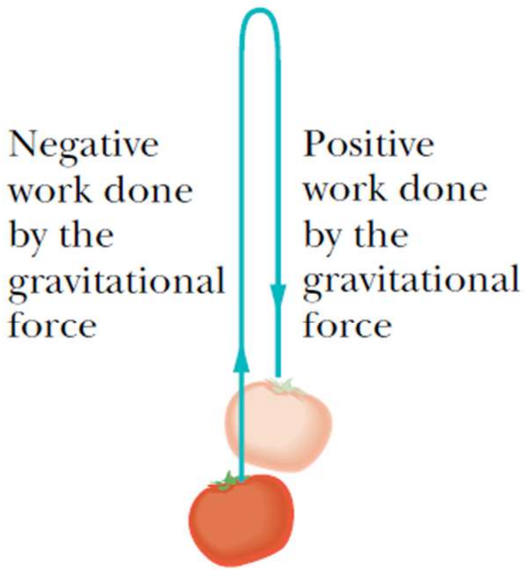
- Gravitational potential energy (重力勢能), U_g
  - associated with the state of separation between two objects that attract each other by the gravitational force

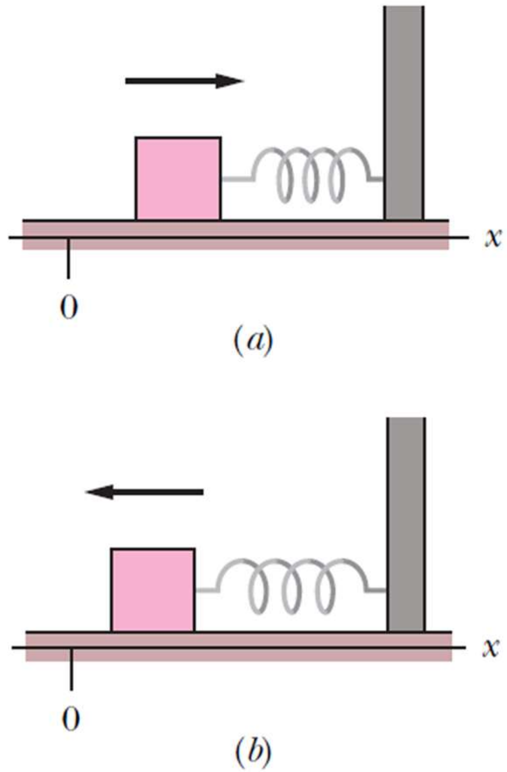
- Elastic potential energy (彈性勢能), U_e
  - associated with the state of compression or extension of an elastic object

Potential Energy

- The change in potential energy ΔU (gravitational or elastic) is defined as the *negative* work done on the object

$$\Delta U = -W.$$





- Gravitational potential energy

$\uparrow y$

  - Consider a particle with mass *m* moving vertically along *y* axis

$$\Delta U = -W$$

$$= -\int_{y_i}^{y_f} (-mg)\, dy = mg \int_{y_i}^{y_f} dy = mg\Big[\, y \,\Big]_{y_i}^{y_f}$$

$$= mg(y_f - y_i) = mg\,\Delta y$$

$$U - U_i = mg(y - y_i)$$

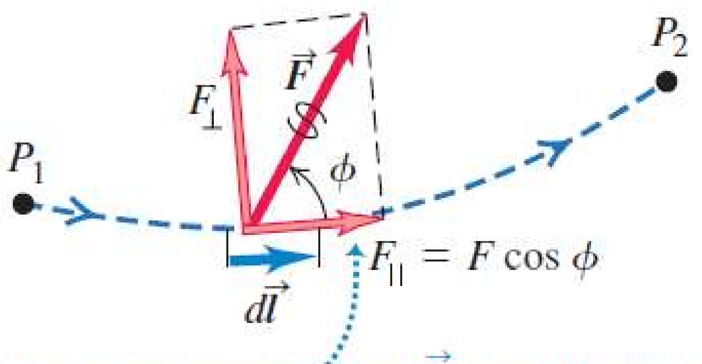
  - Usually we take $U_i = 0$ and $y_i = 0$

$$\boxed{U(y) = mgy}$$

Motion Along a Curve

$$dW = F\cos\phi\, dl = F_{\parallel}\, dl = \vec{F}\cdot d\vec{l}$$

where $F_{\parallel} = F\cos\phi$ is the component of $\vec{F}$ in the direction parallel to $d\vec{l}$ (Fig. 6.23b). The total work done by $\vec{F}$ on the particle as it moves from P_1 to P_2 is



Only the component of $\vec{F}$ parallel to the displacement, $F_{\parallel} = F\cos\phi$, contributes to the work done by $\vec{F}$.

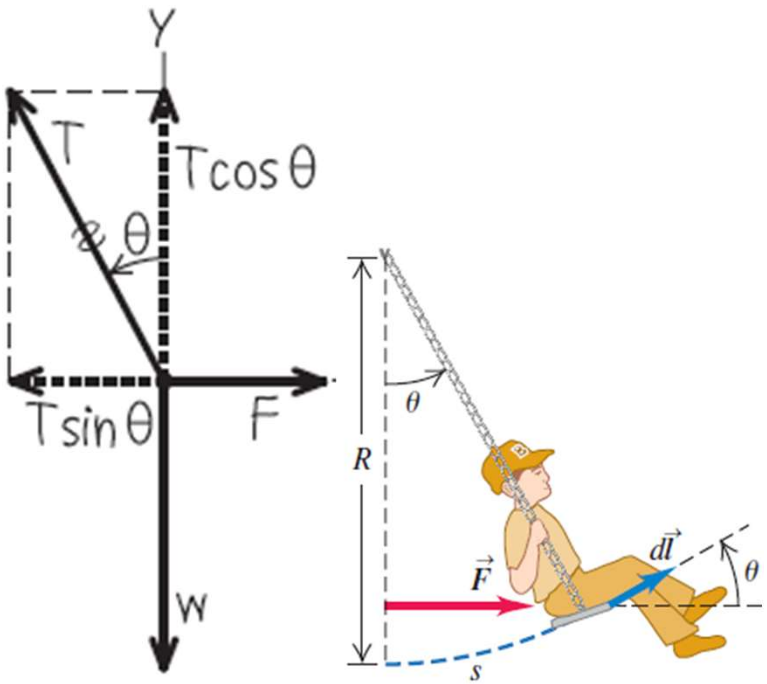
$$W = \int_{P_1}^{P_2} F\cos\phi\, dl = \int_{P_1}^{P_2} F_{\parallel}\, dl = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{l}$$

Example 6.8: Motion on a curved path

At a family picnic you are appointed to push your obnoxious cousin Throckmorton in a swing (Fig. 6.24a). His weight is w, the length of the chains is R, and you push Throcky until the chains make an angle θ_0 with the vertical. To do this, you exert a varying horizontal force $\vec{F}$ that starts at zero and gradually increases just enough that Throcky and the swing move very slowly and remain very nearly in equilibrium throughout the process. What is the total work done on Throcky by all forces? What is the work done by the tension T in the chains? What is the work you do by exerting the force $\vec{F}$? (Neglect the weight of the chains and seat.)

Example 6.8: Motion on a curved path

To compute the work done by $\vec{F}$, we need to know how this force varies with the angle θ. The net force on Throcky is zero, so $\sum F_x = 0$ and $\sum F_y = 0$. From Fig. 6.24b,



$$\sum F_x = F + (-T\sin\theta) = 0$$
$$\sum F_y = T\cos\theta + (-w) = 0$$

Example 6.8: Motion on a curved path

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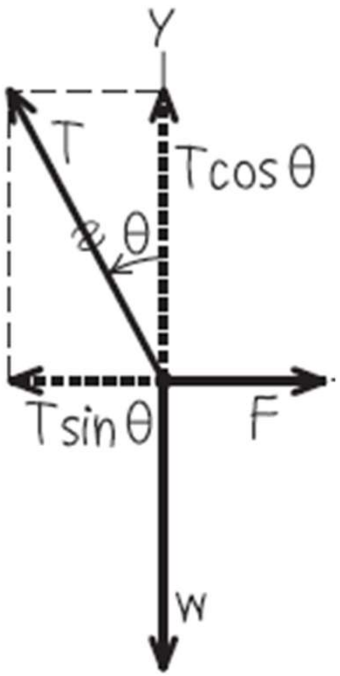
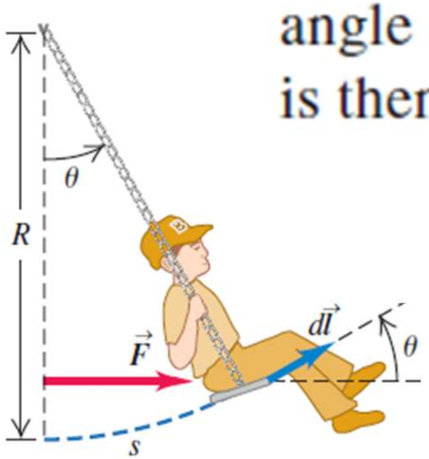


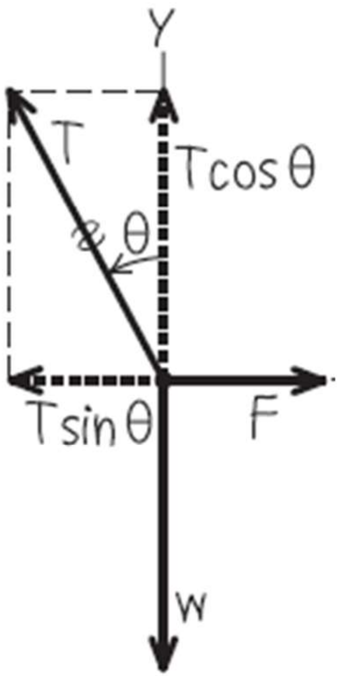


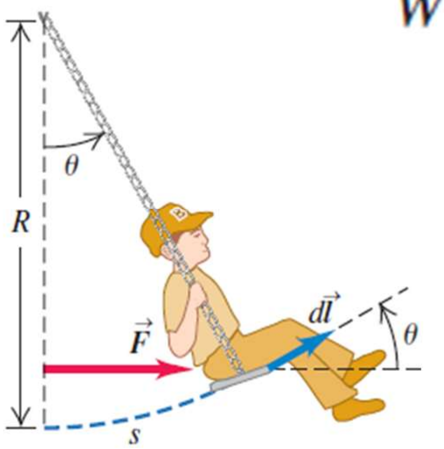
The point where $\vec{F}$ is applied moves through the arc s (Fig. 6.24a). The arc length s equals the radius R of the circular path multiplied by the length θ (in radians), so $s = R\theta$. Therefore the displacement $d\vec{l}$ corresponding to a small change of angle $d\theta$ has a magnitude $dl = ds = R\,d\theta$. The work done by $\vec{F}$ is then

$$W = \int \vec{F} \cdot d\vec{l} = \int F \cos\theta \, ds$$

Example 6.8: Motion on a curved path

To compute the work done by $\vec{F}$, we need to know how this force varies with the angle θ. The net force on Throcky is zero, so $\sum F_x = 0$ and $\sum F_y = 0$. From Fig. 6.24b,



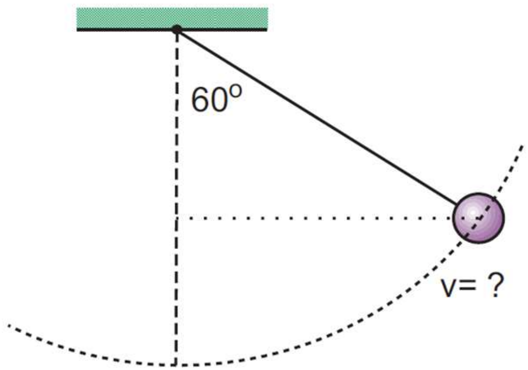


Now we express F and ds in terms of the angle θ, whose value increases from 0 to θ_0:

$$W = \int_0^{\theta_0} (w \tan\theta)\cos\theta\ (R\ d\theta) = wR\int_0^{\theta_0} \sin\theta\, d\theta$$

$$= wR(1 - \cos\theta_0)$$

Now in this system there are only a *conservative force* acting on the particle of interest, i.e. the stone. (We should note that the string tension also acts on the stone, but since it always pulls perpendicularly to the motion of the stone, it does no work.) So the total mechanical energy of the stone is conserved:



$$K_i + U_i = K_f + U_f$$

We can substitute the values found above to get:

$$64.0\,\mathrm{J} + 0 = \tfrac{1}{2}(2.0\,\mathrm{kg})v_f^2 + 39.2\,\mathrm{J}$$

which we can solve for v_f:

$$(1.0\,\mathrm{kg})v_f^2 = 64.0\,\mathrm{J} - 39.2\,\mathrm{J} = 24.8\,\mathrm{J} \qquad \Longrightarrow \qquad v_f^2 = 24.8\,\tfrac{\mathrm{m}^2}{\mathrm{s}^2}$$

and then:

$$v_f = 5.0\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

The speed of the stone at the 60° position will be $5.0\,\frac{\mathrm{m}}{\mathrm{s}}$.

Example 8.8 Block–Spring Collision

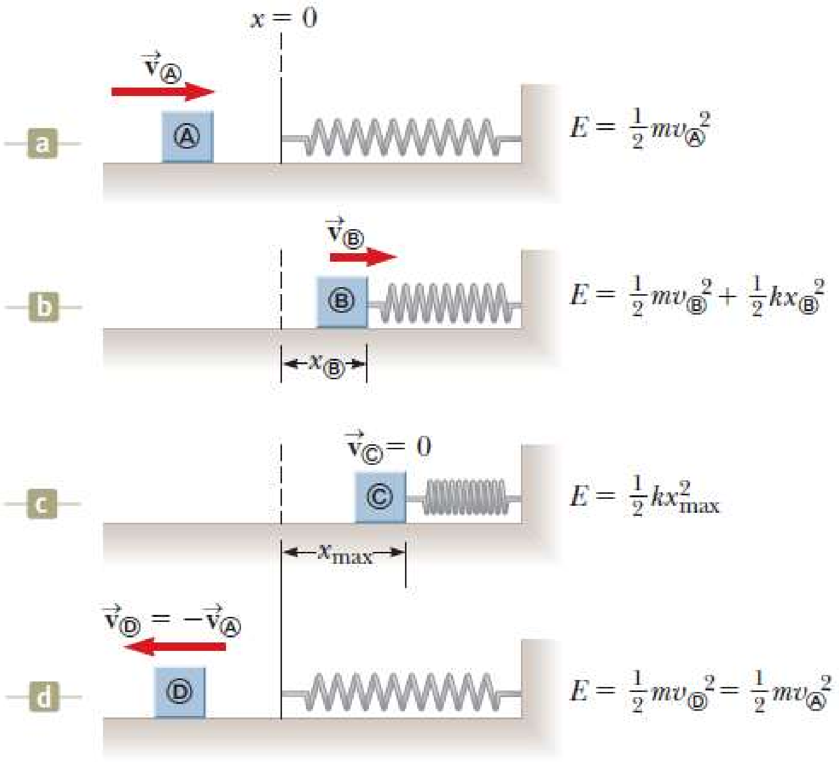
A block having a mass of 0.80 kg is given an initial velocity $v_Ⓐ = 1.2$ m/s to the right and collides with a spring whose mass is negligible and whose force constant is $k = 50$ N/m as shown in Figure 8.11.

(A) Assuming the surface to be frictionless, calculate the maximum compression of the spring after the collision.

$$K_Ⓒ + U_{sⒸ} = K_Ⓐ + U_{sⒶ}$$

$$0 + \frac{1}{2}kx_{max}^2 = \frac{1}{2}mv_Ⓐ{}^2 + 0$$

$$x_{max} = \sqrt{\frac{m}{k}}\,v_Ⓐ = \sqrt{\frac{0.80\text{ kg}}{50\text{ N/m}}}(1.2\text{ m/s}) = \boxed{0.15\text{ m}}$$



(B) Suppose a constant force of kinetic friction acts between the block and the surface, with $\mu_k = 0.50$. If the speed of the block at the moment it collides with the spring is $v_{Ⓐ} = 1.2$ m/s, what is the maximum compression $x_{Ⓒ}$ in the spring?

$$K_f + U_f = K_i + U_i - W_{friction}$$

$$0 + \frac{1}{2}kx_{\max}^2 = \frac{1}{2}mv_i^2 + 0 - \mu mgx_{\max}$$

$$\frac{1}{2}kx_{\max}^2 + \mu mgx_{\max} - \frac{1}{2}mv_i^2 = 0$$

$$x_{\max} = \frac{-\mu mg \pm \sqrt{(\mu mg)^2 - 4\left(\tfrac{1}{2}k\right)\left(-\tfrac{1}{2}mv_i^2\right)}}{2\left(\tfrac{1}{2}k\right)} = 0.0924(m)$$

(C) What is the speed of the mass when it just leave the spring, if friction is present?

$$K_f + U_f = K_i + U_i - W_{friction}$$

$$\frac{1}{2}mv_f^2 + 0 = \frac{1}{2}mv_i^2 + 0 - \mu mg(2x_{\max})$$

$$v_f = \sqrt{v_i^2 - 4\mu gx_{\max}} = \sqrt{-0.37}\ ?$$

Potential Energy

- Consider the block–spring system with the block moving on the end of a spring of spring constant *k*

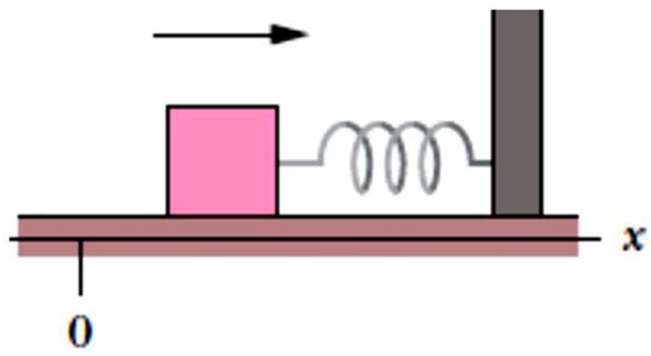
$$\Delta U = -\int_{x_i}^{x_f} (-kx)\,dx = k\int_{x_i}^{x_f} x\,dx = \tfrac{1}{2}k\Big[x^2\Big]_{x_i}^{x_f},$$

$$\Delta U = \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2.$$

- When the spring is at its relaxed length and the block is at x_i = 0,

$$U - 0 = \tfrac{1}{2}kx^2 - 0$$

$$U(x) = \tfrac{1}{2}kx^2$$



Conservation of Mechanical Energy

- Mechanical energy, E_{mec} $$E_{mec} = K + U$$
  - Sum of potential and the kinetic energies of the object

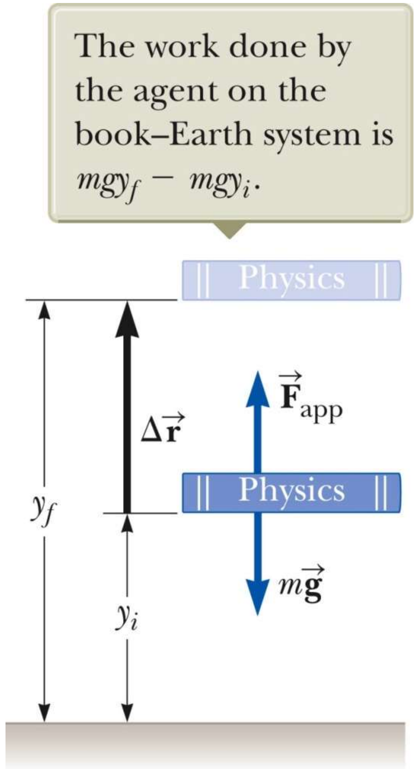
- In an isolated system where only *conservative forces* (friction is non-conservative!) cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change
  - $\Delta K = W$
  - $\Delta U = -W$
  - $\Delta E_{mec} = \Delta K + \Delta U = 0$

Potential Energy

- Potential energy is energy determined by the configuration of a system in which the components of the system interact by forces.
  - The forces are internal to the system.
  - Can be associated with only specific types of (conservative) forces acting between members of a system.

  Examples of conservative forces:
    - Gravity
    - Spring force

Gravitational Potential Energy



•The system is the Earth and the book.

•Do work on the book by lifting it *slowly* through a vertical displacement.

•The work done on the system must appear as an increase in the energy of the system. The energy storage through this mechanism is called *gravitational potential energy.*

$$W_{ext} = \int_{y_i}^{y_f} mg\,dy = mg\left(y_f - y_i\right)$$

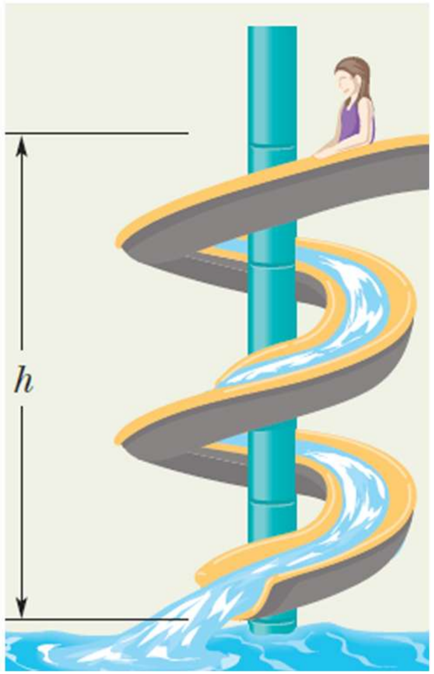
$$= U_{g,f} - U_{g.i} = \Delta U_g$$

where $U_g = mgh$

In order to express *U* explicitly, the reference point (*h* = 0) has to be defined first.

***Quick Quiz* 7.6** Choose the correct answer. The gravitational potential energy of a system **(a)** is always positive **(b)** is always negative **(c)** can be negative or positive

A child of mass m is released from rest at the top of a water slide, at height h = 8.5 m above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.



Calculations: Let the mechanical energy be $E_{\text{mec},t}$ when the child is at the top of the slide and $E_{\text{mec},b}$ when she is at the bottom. Then the conservation principle tells us

$$E_{\text{mec},b} = E_{\text{mec},t}. \qquad (8\text{-}19)$$

To show both kinds of mechanical energy, we have

$$K_b + U_b = K_t + U_t, \qquad (8\text{-}20)$$

or

$$\tfrac{1}{2}mv_b^2 + mgy_b = \tfrac{1}{2}mv_t^2 + mgy_t.$$

Dividing by m and rearranging yield

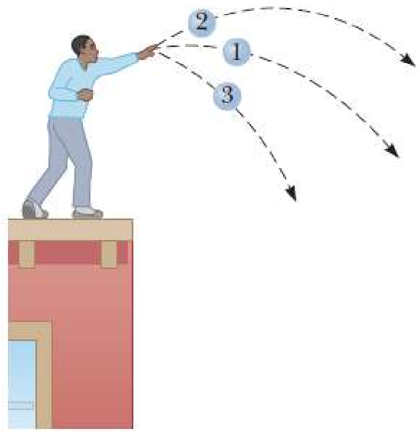
$$v_b^2 = v_t^2 + 2g(y_t - y_b).$$

Putting $v_t = 0$ and $y_t - y_b = h$ leads to

$$v_b = \sqrt{2gh} = \sqrt{(2)(9.8\text{ m/s}^2)(8.5\text{ m})}$$
$$= 13\text{ m/s}. \qquad \text{(Answer)}$$

***Quick Quiz* 8.3** A rock of mass m is dropped to the ground from a height h. A second rock, with mass $2m$, is dropped from the same height. When the second rock strikes the ground, what is its kinetic energy? **(a)** twice that of the first rock **(b)** four times that of the first rock **(c)** the same as that of the first rock **(d)** half as much as that of the first rock **(e)** impossible to determine

***Quick Quiz* 8.4** Three identical balls are thrown from the top of a building, all with the same initial speed. As shown in Active Figure 8.3, the first is thrown horizontally, the second at some angle above the horizontal, and the third at some angle below the horizontal. Neglecting air resistance, rank the speeds of the balls at the instant each hits the ground.



Example 8.1 Ball in Free Fall

A ball of mass m is dropped from a height h above the ground as shown in Active Figure 8.4.

(A) Neglecting air resistance, determine the speed of the ball when it is at a height y above the ground.

$$K_f + U_{gf} = K_i + U_{gi}$$

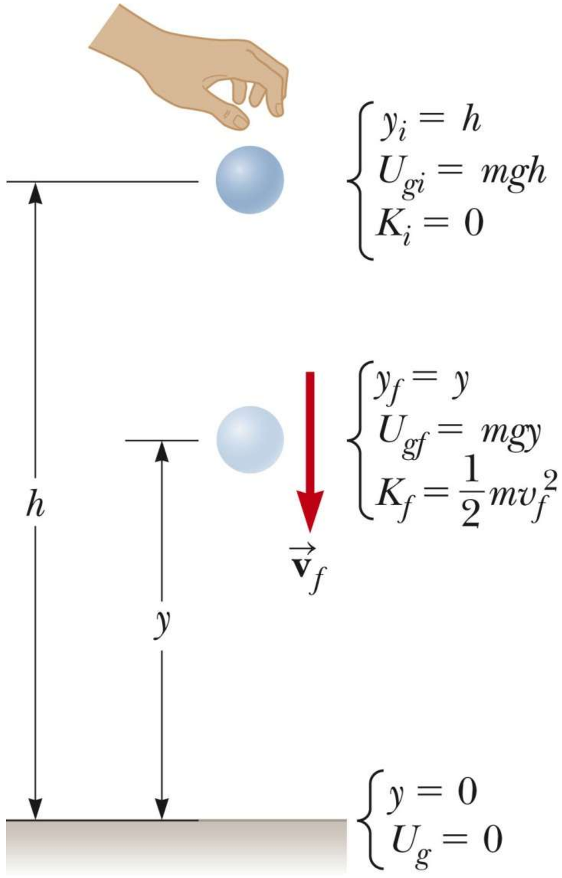
$$\tfrac{1}{2}mv_f^2 + mgy = 0 + mgh$$

$$v_f^2 = 2g(h-y) \quad \rightarrow \quad v_f = \boxed{\sqrt{2g(h-y)}}$$

(B) Determine the speed of the ball at y if at the instant of release it already has an initial upward speed v_i at the initial altitude h.

$$\tfrac{1}{2}mv_f^2 + mgy = \tfrac{1}{2}mv_i^2 + mgh$$

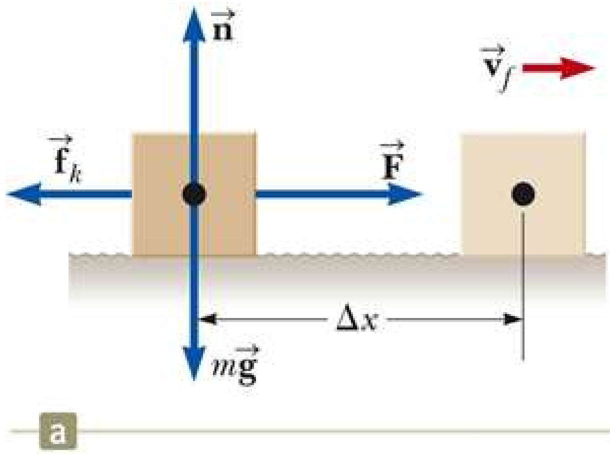
$$v_f^2 = v_i^2 + 2g(h-y) \quad \rightarrow \quad v_f = \boxed{\sqrt{v_i^2 + 2g(h-y)}}$$



Example 8.4 A Block Pulled on a Rough Surface

A 6.0-kg block initially at rest is pulled to the right along a horizontal surface by a constant horizontal force of 12 N.

(A) Find the speed of the block after it has moved 3.0 m if the surfaces in contact have a coefficient of kinetic friction of 0.15.



$$\sum W_{\text{other forces}} = W_F = F\Delta x$$

$$\sum F_y = 0 \;\rightarrow\; n - mg = 0 \;\rightarrow\; n = mg$$

$$f_k = \mu_k n = \mu_k mg = (0.15)(6.0\text{ kg})(9.80\text{ m/s}^2) = 8.82\text{ N}$$

$$\tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_i^2 - f_k d + W_F$$

$$v_f = \sqrt{v_i^2 + \frac{2}{m}(-f_k d + F\Delta x)}$$

$$v_f = \sqrt{0 + \frac{2}{6.0\text{ kg}}[-(8.82\text{ N})(3.0\text{ m}) + (12\text{ N})(3.0\text{ m})]} = \boxed{1.8\text{ m/s}}$$

(B) Suppose the force $\vec{\mathbf{F}}$ is applied at an angle θ as shown in Active Figure 8.8b. At what angle should the force be applied to achieve the largest possible speed after the block has moved 3.0 m to the right?

$$\sum W_{\text{other forces}} = W_F = F\,\Delta x \cos\theta = Fd\cos\theta$$

$$\sum F_y = n + F\sin\theta - mg = 0$$

$$n = mg - F\sin\theta$$

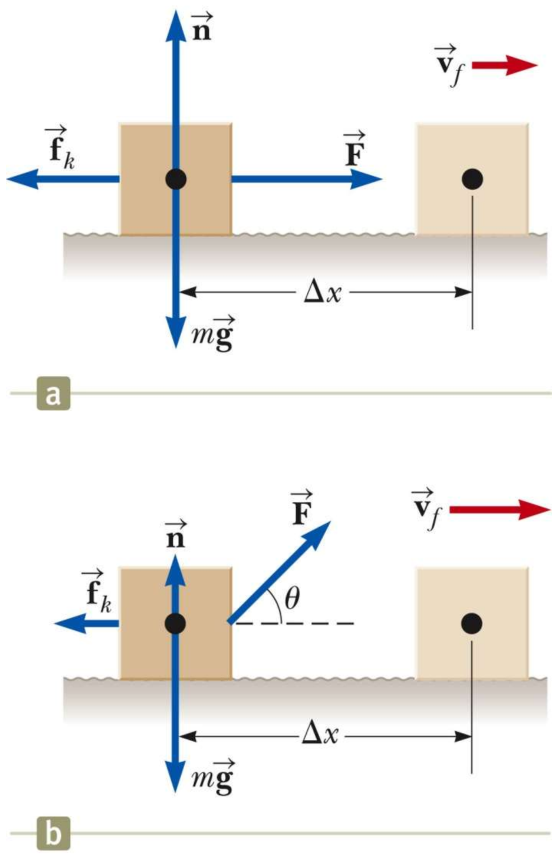
$$K_f = K_i - f_k d + W_F$$
$$= 0 - \mu_k n d + Fd\cos\theta = -\mu_k(mg - F\sin\theta)d + Fd\cos\theta$$

$$\frac{dK_f}{d\theta} = -\mu_k(0 - F\cos\theta)d - Fd\sin\theta = 0$$

$$\mu_k\cos\theta - \sin\theta = 0$$

$$\tan\theta = \mu_k$$

$$\theta = \tan^{-1}(\mu_k) = \tan^{-1}(0.15) = \boxed{8.5^\circ}$$



Conceptual Example 8.5 Useful Physics for Safer Driving

A car traveling at an initial speed v slides a distance d to a halt after its brakes lock. If the car's initial speed is instead $2v$ at the moment the brakes lock, estimate the distance it slides.

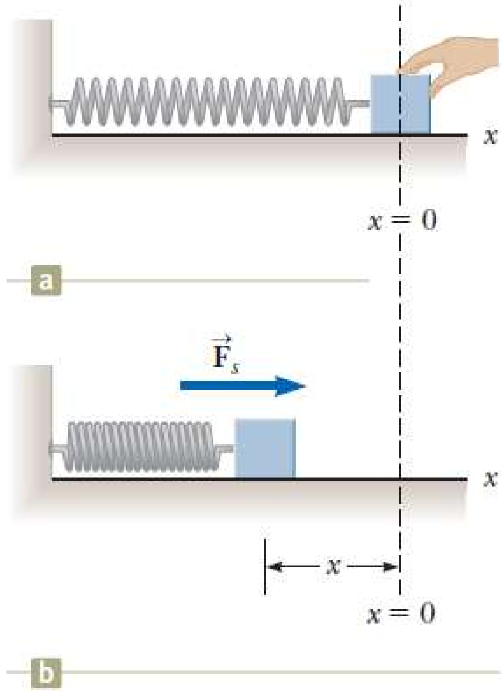
$$K_2 = \frac{1}{2}m(2v)^2 = 4\left(\frac{1}{2}mv^2\right) = 4K_1$$

$$fd_2 = W_{friction,2} = K_2 = 4K_1 = 4W_{friction,1} = 4fd_1 \Rightarrow d_2 = 4d_1$$

Example 8.6 A Block–Spring System

A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 1 000 N/m as shown in Figure 8.9. The spring is compressed 2.0 cm and is then released from rest.

(A) Calculate the speed of the block as it passes through the equilibrium position $x = 0$ if the surface is frictionless.



$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2}mv_f^2 = \frac{1}{2}kx_{\max}^2$$

$$v_f = \sqrt{\frac{k}{m}}x_{\max} = \sqrt{\frac{1000}{1.6}}\left(2\times10^{-2}\right) = 0.5\left(ms^{-1}\right)$$

(B) Calculate the speed of the block as it passes through the equilibrium position if a constant friction force of 4.0 N retards its motion from the moment it is released.

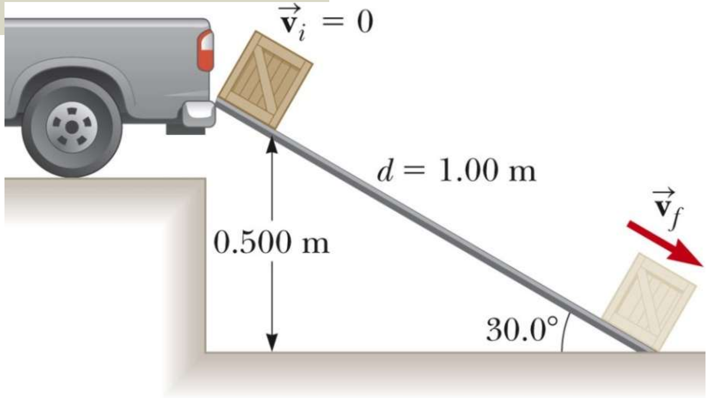
$$K_f + U_f = K_i + U_i - W_{friction}$$

$$\frac{1}{2}mv_f^2 = \frac{1}{2}kx_{\max}^2 - fx_{\max} \qquad v_f = \sqrt{\frac{k}{m}x_{\max}^2 - \frac{2f}{m}x_{\max}} = 0.39\left(ms^{-1}\right)$$

Example 8.7 Crate Sliding Down a Ramp

A 3.00-kg crate slides down a ramp. The ramp is 1.00 m in length and inclined at an angle of 30.0° as shown in Figure 8.10. The crate starts from rest at the top, experiences a constant friction force of magnitude 5.00 N, and continues to move a short distance on the horizontal floor after it leaves the ramp.

(A) Use energy methods to determine the speed of the crate at the bottom of the ramp.



$$E_i = K_i + U_i = 0 + U_i = mgy_i$$

$$E_f = K_f + U_f = \tfrac{1}{2}mv_f^2 + 0$$

$$\Delta E_{mech} = E_f - E_i = \tfrac{1}{2}mv_f^2 - mgy_i = -f_k d$$

$$(1)\ v_f = \sqrt{\frac{2}{m}(mgy_i - f_k d)}$$

$$v_f = \sqrt{\frac{2}{3.00\ \text{kg}}[(3.00\ \text{kg})(9.80\ \text{m/s}^2)(0.500\ \text{m}) - (5.00\ \text{N})(1.00\ \text{m})]} = \boxed{2.54\ \text{m/s}}$$

(B) How far does the crate slide on the horizontal floor if it continues to experience a friction force of magnitude 5.00 N?

$$E_i = K_i = \tfrac{1}{2}mv_i^{\,2}$$

$$E_f - E_i = 0 - \tfrac{1}{2}mv^2 = -f_k d \;\rightarrow\; \tfrac{1}{2}mv^2 = f_k d$$

$$d = \frac{mv^2}{2f_k} = \frac{(3.00\text{ kg})(2.54\text{ m/s})^2}{2(5.00\text{ N})} = \boxed{1.94\text{ m}}$$

WHAT IF? A cautious worker decides that the speed of the crate when it arrives at the bottom of the ramp may be so large that its contents may be damaged. Therefore, he replaces the ramp with a longer one such that the new ramp makes an angle of 25.0° with the ground. Does this new ramp reduce the speed of the crate as it reaches the ground?

Answer Because the ramp is longer, the friction force acts over a longer distance and transforms more of the mechanical energy into internal energy. The result is a reduction in the kinetic energy of the crate, and we expect a lower speed as it reaches the ground.

$$\sin 25.0° = \frac{0.500\text{ m}}{d} \;\rightarrow\; d = \frac{0.500\text{ m}}{\sin 25.0°} = 1.18\text{ m}$$

$$v_f = \sqrt{\frac{2}{3.00\text{ kg}}\left[(3.00\text{ kg})(9.80\text{ m/s}^2)(0.500\text{ m}) - (5.00\text{ N})(1.18\text{ m})\right]} = 2.42\text{ m/s}$$

Example 8.8 Block–Spring Collision

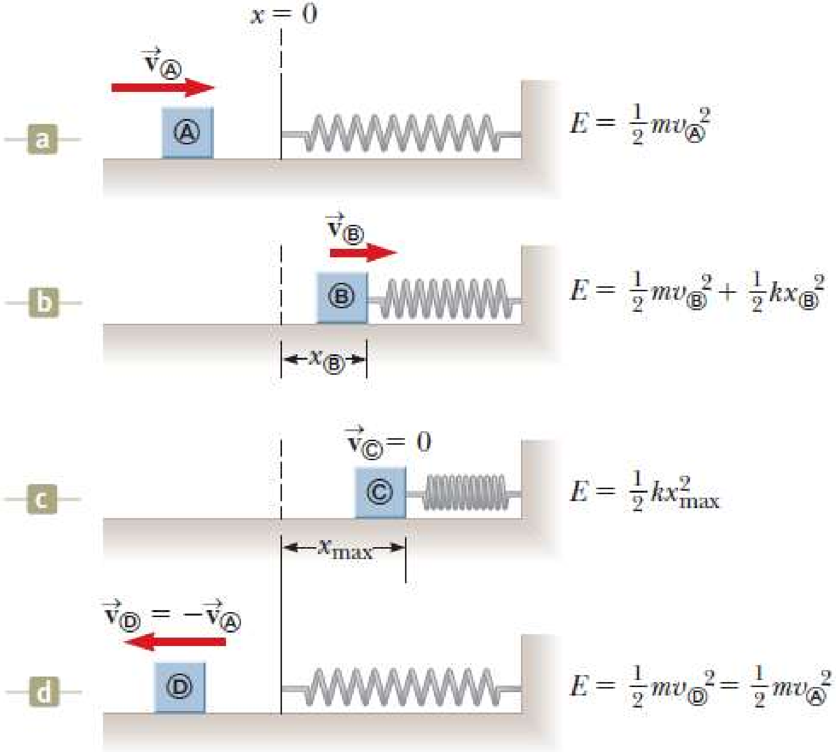
A block having a mass of 0.80 kg is given an initial velocity $v_Ⓐ = 1.2$ m/s to the right and collides with a spring whose mass is negligible and whose force constant is $k = 50$ N/m as shown in Figure 8.11.

(A) Assuming the surface to be frictionless, calculate the maximum compression of the spring after the collision.

$$K_Ⓒ + U_{sⒸ} = K_Ⓐ + U_{sⒶ}$$

$$0 + \tfrac{1}{2}kx_{\max}^2 = \tfrac{1}{2}mv_Ⓐ^2 + 0$$

$$x_{\max} = \sqrt{\frac{m}{k}}\,v_Ⓐ = \sqrt{\frac{0.80\text{ kg}}{50\text{ N/m}}}(1.2\text{ m/s}) = \boxed{0.15\text{ m}}$$



(B) Suppose a constant force of kinetic friction acts between the block and the surface, with $\mu_k = 0.50$. If the speed of the block at the moment it collides with the spring is $v_{Ⓐ} = 1.2$ m/s, what is the maximum compression $x_{©}$ in the spring?

$$K_f + U_f = K_i + U_i - W_{friction}$$

$$0 + \frac{1}{2}kx_{\max}^2 = \frac{1}{2}mv_i^2 + 0 - \mu mgx_{\max}$$

$$\frac{1}{2}kx_{\max}^2 + \mu mgx_{\max} - \frac{1}{2}mv_i^2 = 0$$

$$x_{\max} = \frac{-\mu mg \pm \sqrt{(\mu mg)^2 - 4\left(\frac{1}{2}k\right)\left(-\frac{1}{2}mv_i^2\right)}}{2\left(\frac{1}{2}k\right)} = 0.0924(m)$$

(C) What is the speed of the mass when it just leave the spring, if friction is present?

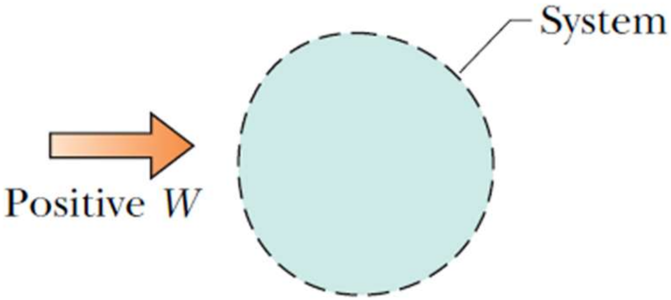
$$K_f + U_f = K_i + U_i - W_{friction}$$

$$\frac{1}{2}mv_f^2 + 0 = \frac{1}{2}mv_i^2 + 0 - \mu mg(2x_{\max})$$

$$v_f = \sqrt{v_i^2 - 4\mu gx_{\max}} = \sqrt{-0.37}\,?$$

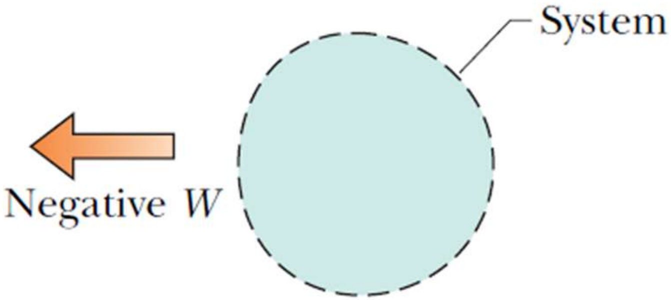
Work Done on a System by an External Force

- Work is energy transferred to or from a system by means of an external force acting on that system



(a)

(a) Positive work W done on an arbitrary system means a transfer of energy <u>*to*</u> the system

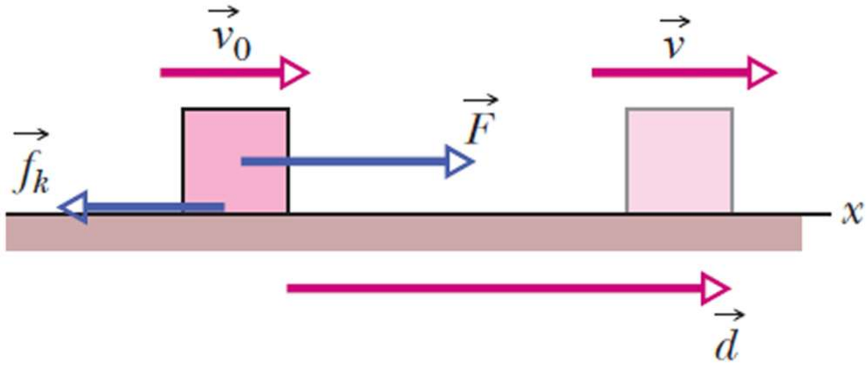


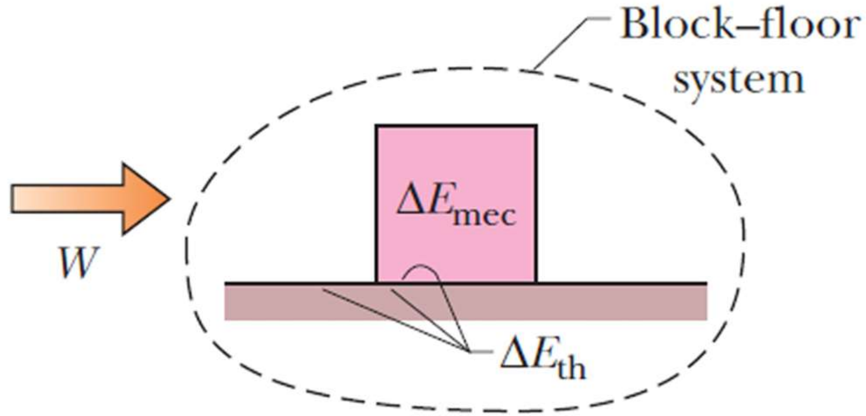
(b)

(b) Negative work W means a transfer of energy <u>*from*</u> the system

The applied force supplies energy. The frictional force transfers some of it to thermal energy.

So, the work done by the applied force goes into kinetic energy and also thermal energy.





Kinetics: $F - f_k = ma$

Kinematics: $v^2 = v_0^2 + 2ad$

$$Fd = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 + f_k d$$

$$Fd = \Delta K + f_k d$$

thermal energy

$$\boxed{W = \Delta E_{mec} + \Delta E_{th}}$$

Conservation of Energy

- The total energy *E* of a system can change only by amounts of energy that are transferred to or from the system

$$W = \Delta E = \Delta E_{\mathrm{mec}} + \Delta E_{\mathrm{th}} + \Delta E_{\mathrm{int}}$$

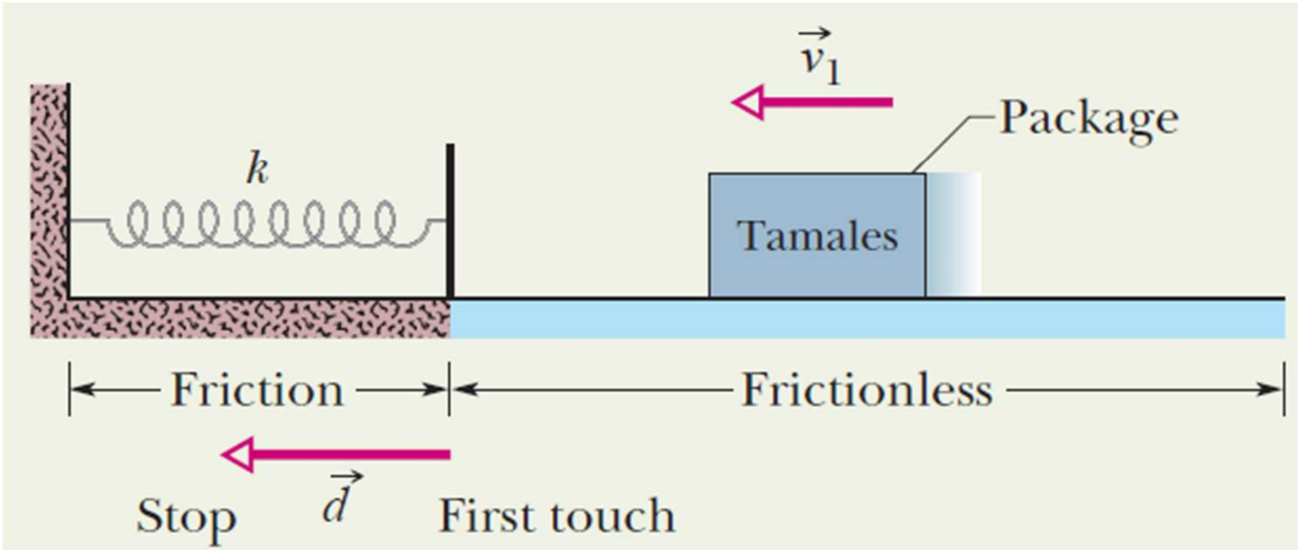
  - ΔE_{mec} is any change in the mechanical energy (機械能)
  - ΔE_{th} is any change in the thermal energy (熱能)
  - ΔE_{int} is any change in any *other type* of internal energy (內能)

- If a system is isolated from its environment, there can be no energy transfers to or from it
  - The total energy E of an *isolated system* cannot change

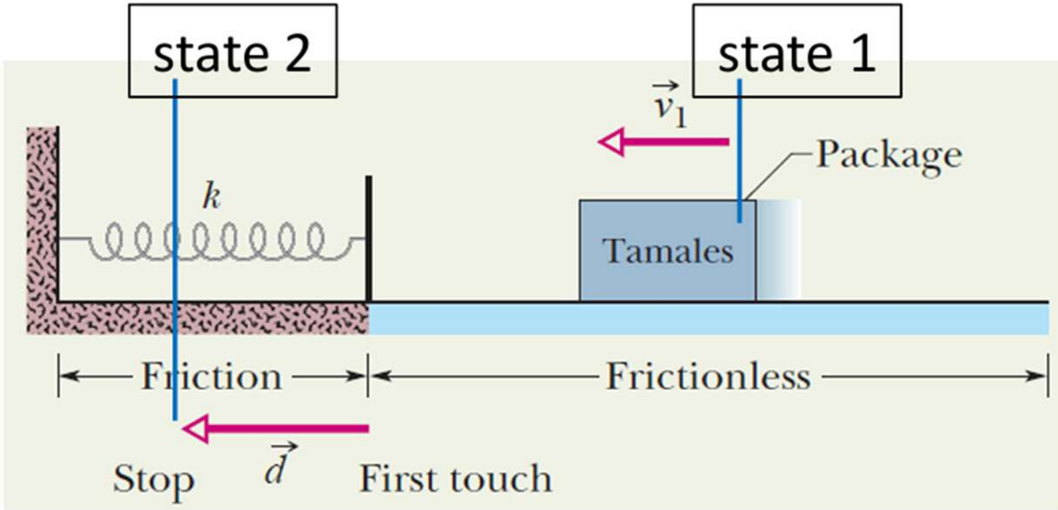
$$\Delta E_{\mathrm{mec}} + \Delta E_{\mathrm{th}} + \Delta E_{\mathrm{int}} = 0 \qquad \text{(isolated system)}$$

Sample Problem

- A 2.0 kg package of tamales slides along a floor with speed v_1 = 4.0 m/s. It then runs into and compresses a spring, until the package momentarily stops. Its path to the initially relaxed spring is frictionless, but as it compresses the spring, a kinetic frictional force from the floor, of magnitude 15 N, acts on the package. If k = 10 000 N/m, by what distance d is the spring compressed when the package stops?

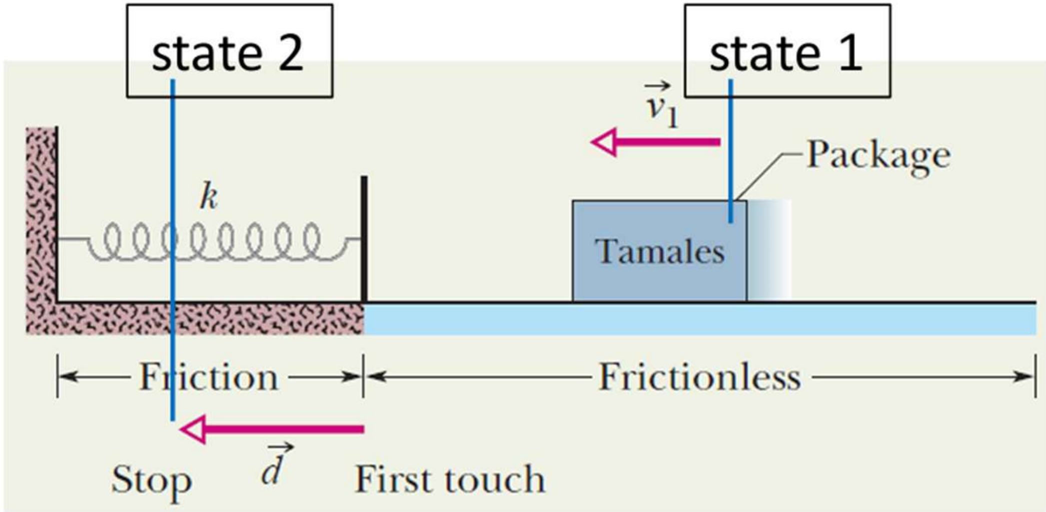


- Approach 1 (package-spring-wall as a system)
  - Friction is an *external* force, considered as a negative *work*



  - By work-energy theorem,
    - $W_{f_k} = \Delta E$
    - $W_{f_k} = \Delta K + \Delta U_g + \Delta U_e$
    - $-f_k d = \left(0 - \frac{1}{2}m{v_1}^2\right) + 0 + \left(\frac{1}{2}kd^2 - 0\right)$
    - $\frac{1}{2}kd^2 + f_k d - \frac{1}{2}m{v_1}^2 = 0$
    - $5000d^2 + 15d - 16 = 0$
    - $d = 0.055\ \mathrm{m}$

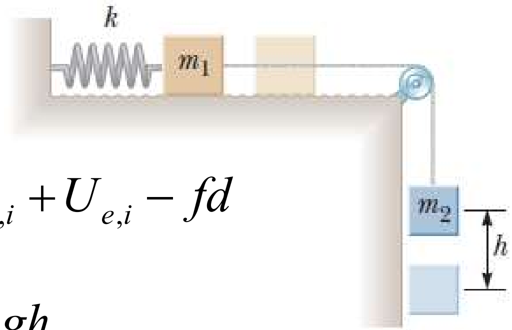
- Approach 2 (package-spring-**floor**-wall as a system)
  - Friction is an *internal* force, considered as a thermal *energy*



  - By conservation of energy,
    - $\Delta E = 0$
    - $\Delta K + \Delta U_g + \Delta U_e + \Delta E_{th} = 0$
    - $\left(0 - \frac{1}{2} m {v_1}^2\right) + 0 + \left(\frac{1}{2} k d^2 - 0\right) + f_k d = 0$
    - $\frac{1}{2} k d^2 + f_k d - \frac{1}{2} m {v_1}^2 = 0$
    - $5000 d^2 + 15 d - 16 = 0$
    - $d = 0.055 \text{ m}$

Example 8.9 Connected Blocks in Motion

Two blocks are connected by a light string that passes over a frictionless pulley as shown in Figure 8.12. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k. The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.



$$K_f + U_f = K_i + U_i - W_{friction}$$

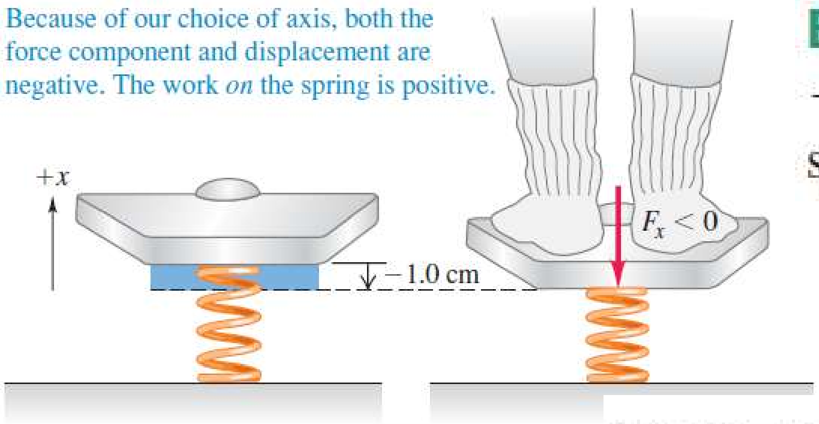
$$K_{1,f} + U_{1,f} + K_{2,f} + U_{2,f} + U_{e,f} = K_{1,i} + U_{1,i} + K_{2,i} + U_{2,i} + U_{e,i} - fd$$

$$0 + 0 + 0 + (-m_2 gh) + \frac{1}{2}kh^2 = 0 + 0 + 0 + 0 + 0 - \mu m_1 gh$$

$$\mu = \frac{m_2 g - \frac{1}{2}kh}{m_1 g}$$

Example 6.6: Work done on a spring scale

A woman weighing 600 N steps on a bathroom scale that contains a stiff spring (Fig. 6.21). In equilibrium, the spring is compressed 1.0 cm under her weight. Find the force constant of the spring and the total work done on it during the compression.



EXECUTE: The top of the spring is displaced by $x = -1.0 \text{ cm} = -0.010 \text{ m}$, and the woman exerts a force $F_x = -600 \text{ N}$ on the spring. From Eq. (6.8) the force constant is then

$$k = \frac{F_x}{x} = \frac{-600 \text{ N}}{-0.010 \text{ m}} = 6.0 \times 10^4 \text{ N/m}$$

Then, using $x_1 = 0$ and $x_2 = -0.010$ m in Eq. (6.10), we have

$$W = \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}(6.0 \times 10^4 \text{ N/m})(-0.010 \text{ m})^2 - 0 = 3.0 \text{ J}$$

Summary

The **work** W done on a system by an agent exerting a constant force $\vec{\mathbf{F}}$ on the system is the product of the magnitude Δr of the displacement of the point of application of the force and the component $F\cos\theta$ of the force along the direction of the displacement $\Delta\vec{\mathbf{r}}$:

$$W \equiv F\Delta r\cos\theta \qquad \textbf{(7.1)}$$

The **kinetic energy** of a particle of mass m moving with a speed v is

$$K \equiv \tfrac{1}{2}mv^2 \qquad \textbf{(7.16)}$$

If a particle of mass m is at a distance y above the Earth's surface, the **gravitational potential energy** of the particle–Earth system is

$$U_g \equiv mgy \qquad \textbf{(7.19)}$$

The **elastic potential energy** stored in a spring of force constant k is

$$U_s \equiv \tfrac{1}{2}kx^2 \qquad \textbf{(7.22)}$$

The **total mechanical energy of a system** is defined as the sum of the kinetic energy and the potential energy:

$$E_{\text{mech}} \equiv K + U \qquad \textbf{(7.24)}$$

$$K_f + U_f = K_i + U_i + W_{done} - W_{friction}$$