Direct-Current Circuits

- Electromotive force

- Resistors in series and parallel

- Kirchhoff's rules

- *RC* circuits

- Household wiring and electrical safety (optional)

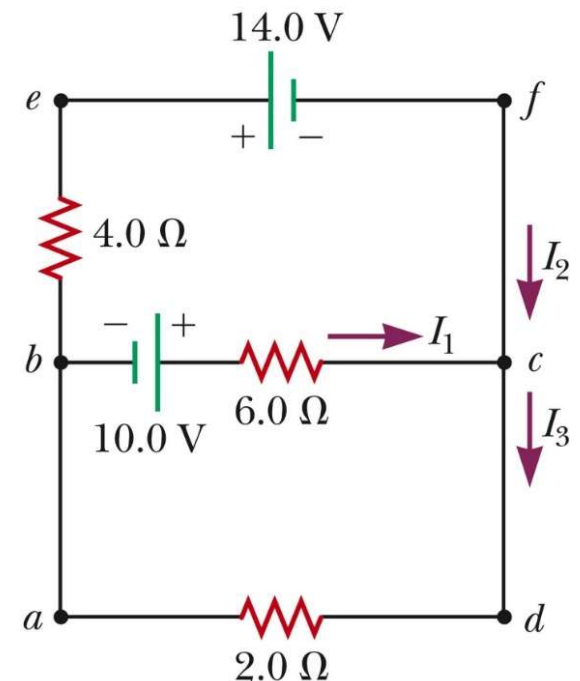Introduction

Circuit Analysis

Simple electric circuits may contain batteries, resistors, and capacitors in various combinations.

For some circuits, analysis may consist of combining resistors.

In more complex circuits, Kirchhoff's Rules may be used for analysis.

- These Rules are based on conservation of energy and conservation of electric charge for isolated systems.

Circuits may involve direct current or alternating current.

Introduction

Direct Current & Electromotive Force

When the current in a circuit has a constant direction, the current is called **direct current.**

- Most of the circuits analyzed will be assumed to be in *steady state*, with constant magnitude and direction.

Because the potential difference between the terminals of a battery is constant, the battery produces direct current.

The battery is known as a source of electromotive force (emf).

The **electromotive force** ε, of a battery is **the maximum possible _voltage_** that the battery can provide between its terminals.

- The emf supplies energy, and it does not apply a force.

The positive terminal of the battery is at a higher potential than the negative terminal. In the circuit analysis we consider the wires to have no resistance.

Internal Battery Resistance

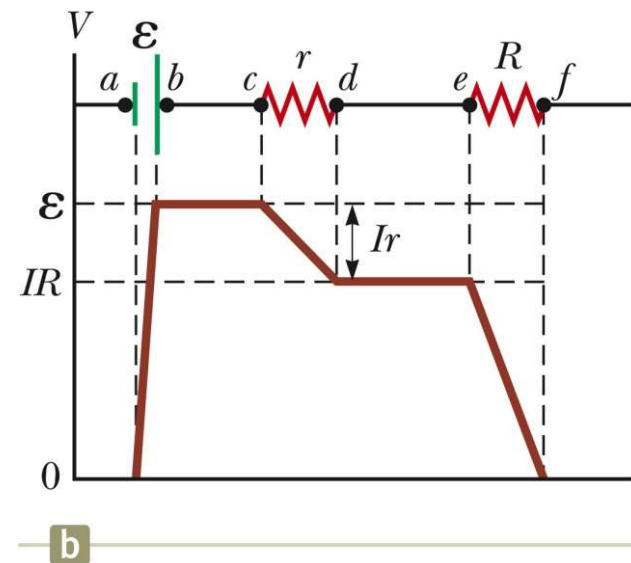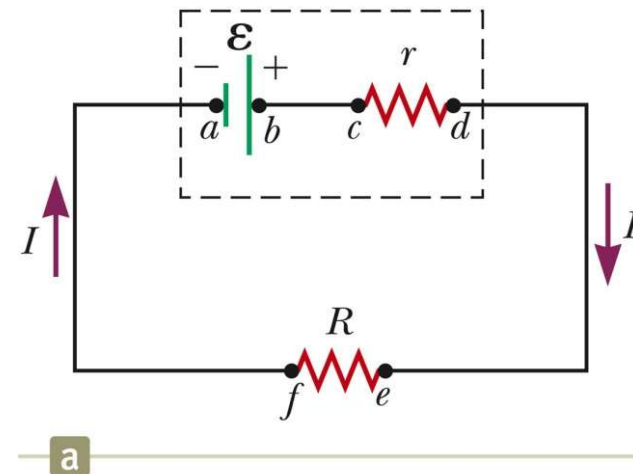If the internal resistance is zero, the terminal voltage equals the emf.
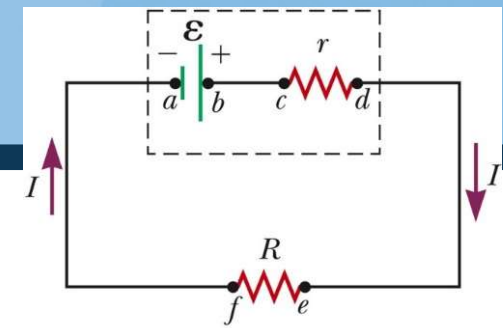
In a real battery, there is internal resistance, r.

The terminal voltage, $\Delta V = \varepsilon - Ir$

The emf is equivalent to the *open-circuit* voltage.

- This is the terminal voltage when no current is in the circuit.

- This is the voltage labeled on the battery.

The actual potential difference between the terminals of the battery depends on the current in the circuit.

Load Resistance

The terminal voltage also equals the voltage across the external resistance.

- This external resistor is called the *load resistance.*

- In the previous circuit, the load resistance is just the external resistor.

- In general, the load resistance could be any electrical device.
  - These resistances represent *loads* on the battery since it supplies the energy to operate the device containing the resistance.

Power

The total power output of the battery is $P = I\Delta V_{bat} = I\mathcal{E}$

This power is delivered to the external resistor ($I^2 R$) and to the internal resistor($I^2 r$), so $P = I^2 R + I^2 r$

The battery is a supply of constant emf (voltage supply), but doesn't supply a constant current (since the current in the circuit depends on the resistance connected to the battery) or a constant terminal voltage.

5

Resistors in Series

Currents through the resistors are the same

- $I = I_1 = I_2$

Total potential drop from a to b is

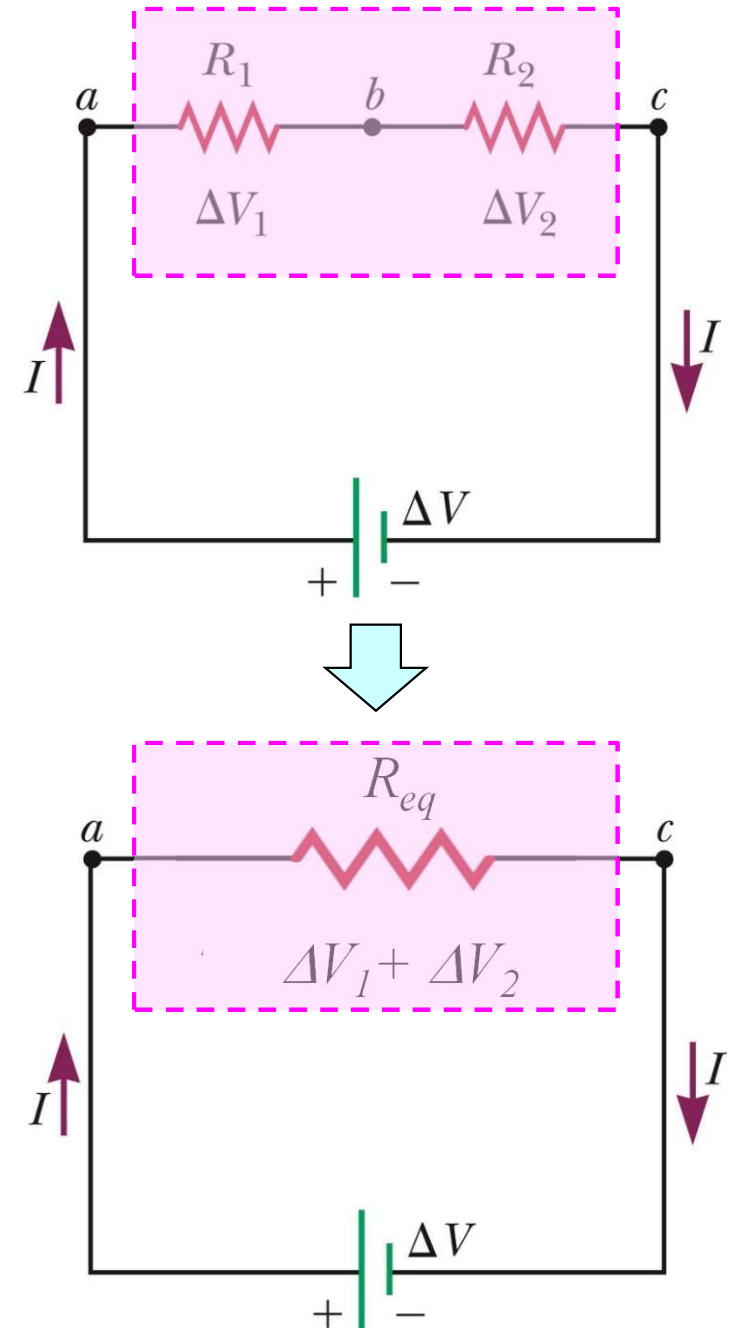- $\Delta V = \Delta V_1 + \Delta V_2 = IR_1 + IR_2 = I(R_1 + R_2)$

Equivalent resistance:

$$R_{eq} = R_1 + R_2 + R_3 + ...$$

The equivalent resistance has the same effect on the circuit as the original combination of resistors.

An algebraic sum of the individual resistances and is always greater than any individual resistance.

If one device in the series circuit creates an open circuit, all devices are inoperative.

6

Resistors in Parallel

The potential difference across each resistor is the same because each is connected directly across the battery terminals.

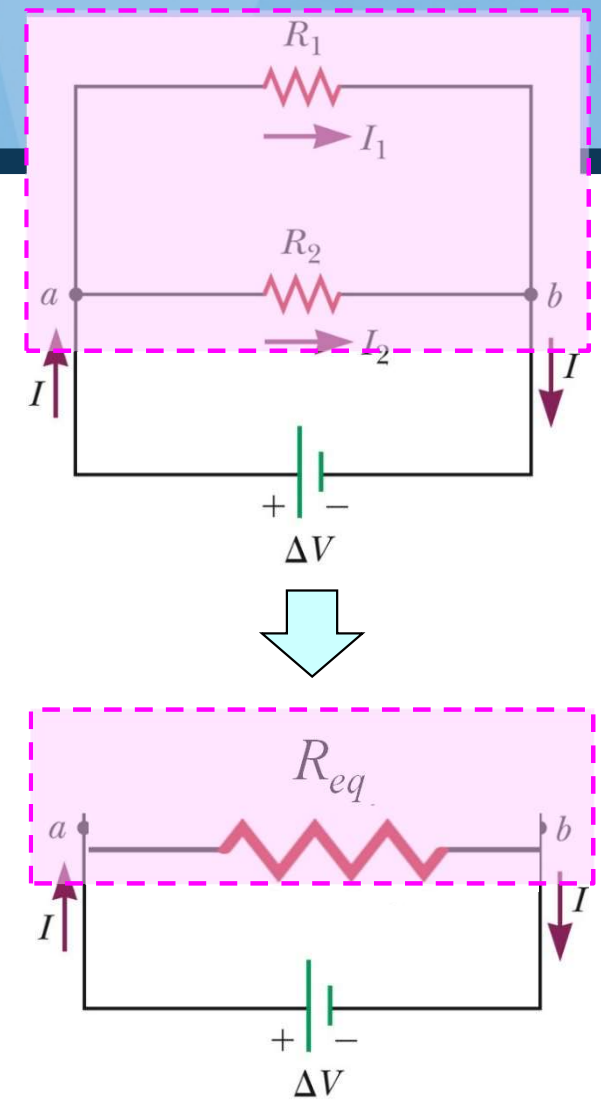$$\Delta V = \Delta V_1 = \Delta V_2$$

A **junction** is a point where the current can split.

The current, I, that enters the junction must be equal to the total current leaving that junction.

- $I = I_1 + I_2 = (\Delta V_1/R_1) + (\Delta V_2/R_2)$

- The currents are generally not the same.

- It's the consequence of conservation of electric charge.

Equivalent resistance:

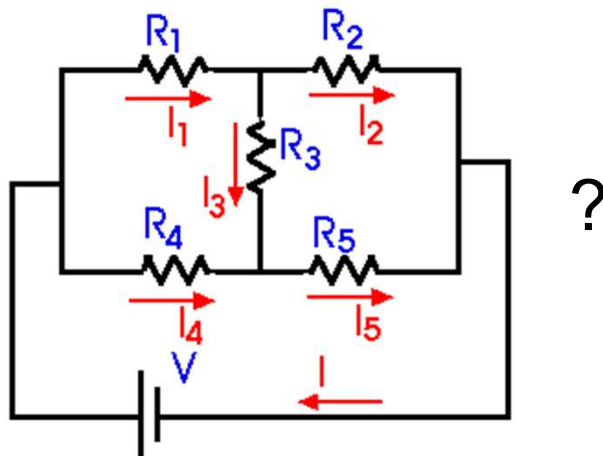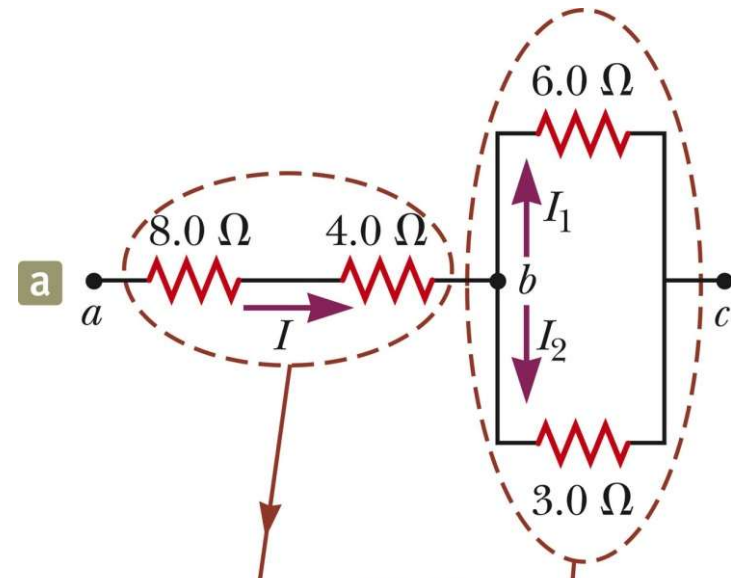$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

In parallel, each device operates independently of the others so that if one is switched off, the others remain on. *Household circuits* are wired so that electrical devices are connected in parallel.
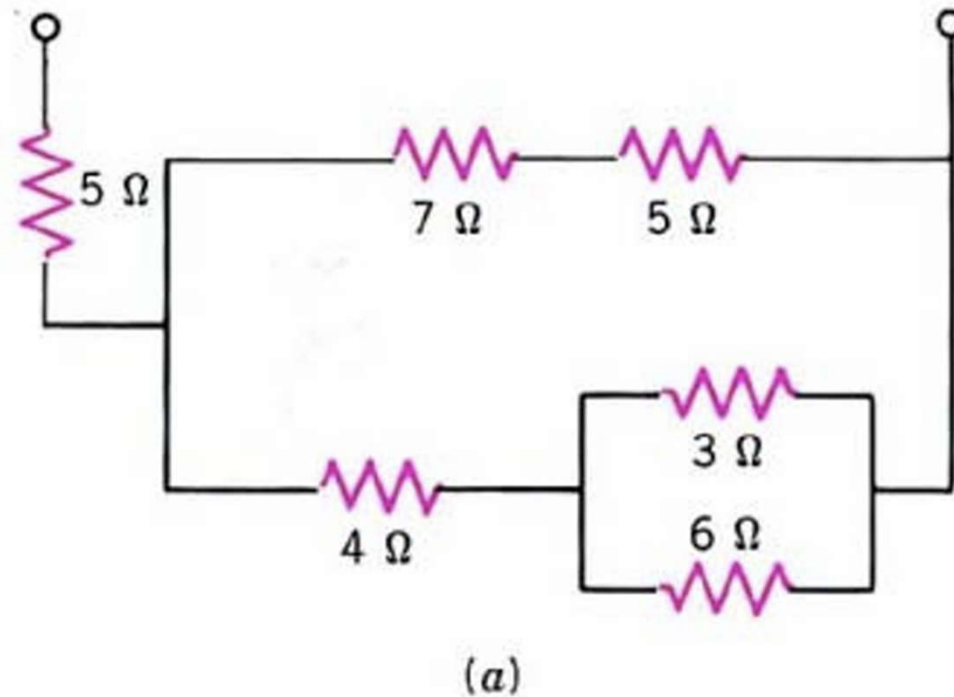
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Combinations of Resistors

The 8.0-Ω and 4.0-Ω resistors are in series and can be replaced with their equivalent, 12.0 Ω.

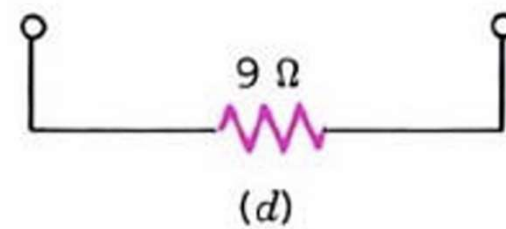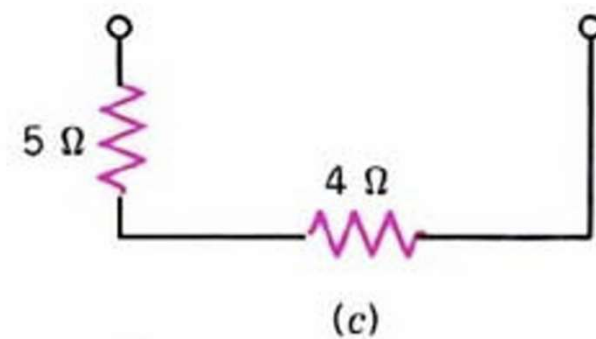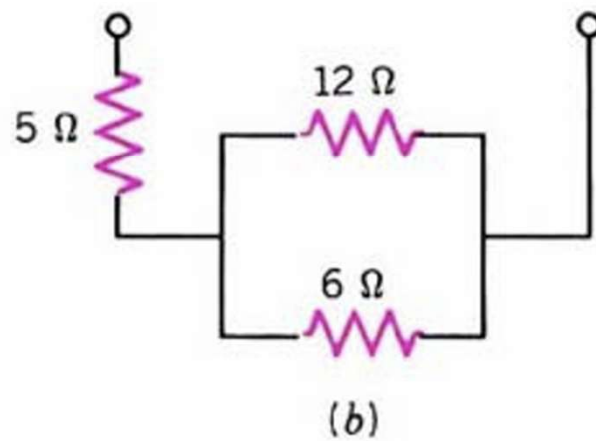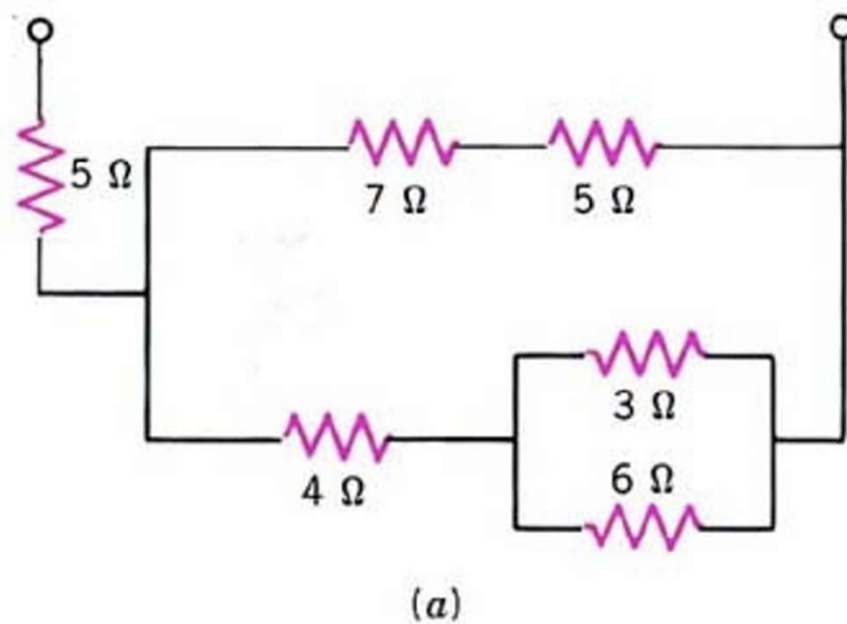The 6.0-Ω and 3.0-Ω resistors are in parallel and can be replaced with their equivalent, 2.0 Ω.

These equivalent resistances are in series and can be replaced with their equivalent resistance, 14.0 Ω.

Find the equivalent resistance of the combination of resistors shown in Fig. 28.10a.

(a)

Solution:

(a)

(b)

(c)

(d)

Kirchhoff's Rules

There are ways in which resistors can be connected so that the circuits formed cannot be reduced to a single equivalent resistor.

Two rules, called **Kirchhoff's rules**, can be used instead.

(1) **Junction Rule:** $$\sum_{junction} I = 0$$

- The sum of the currents entering any junction must equal zero.

(2) **Loop Rule:** $$\sum_{closed\ loop} \Delta V = 0$$

The sum of the potential differences across all elements around any closed circuit loop must be zero.

Junction Rule

- The sum of the currents flowing into any junction must equal zero:
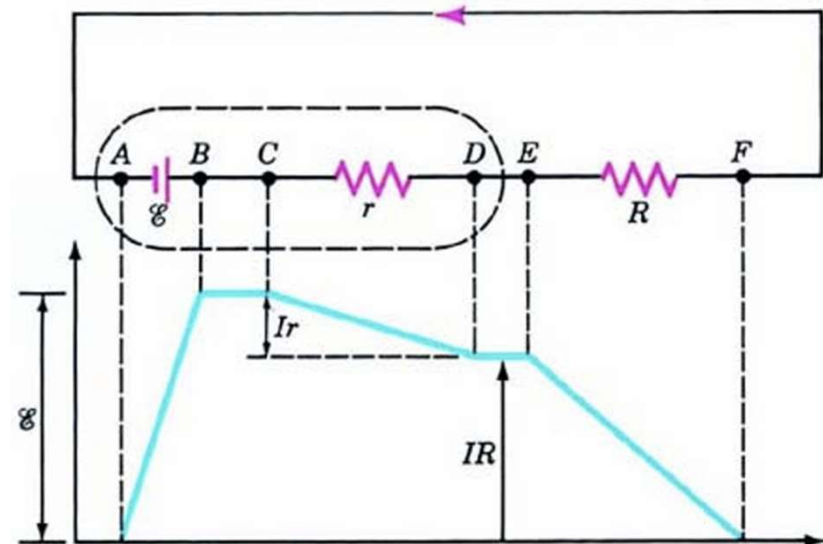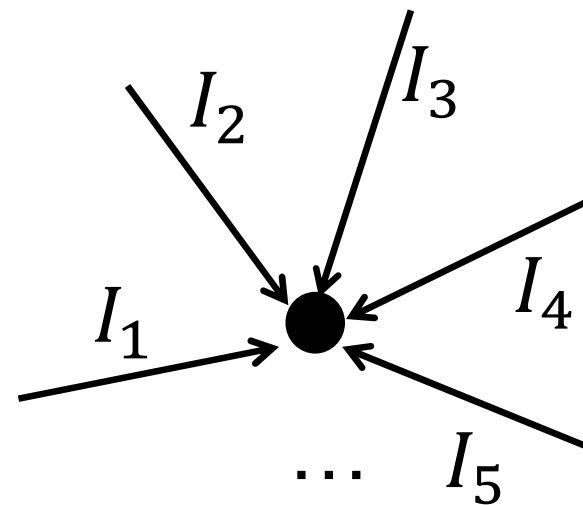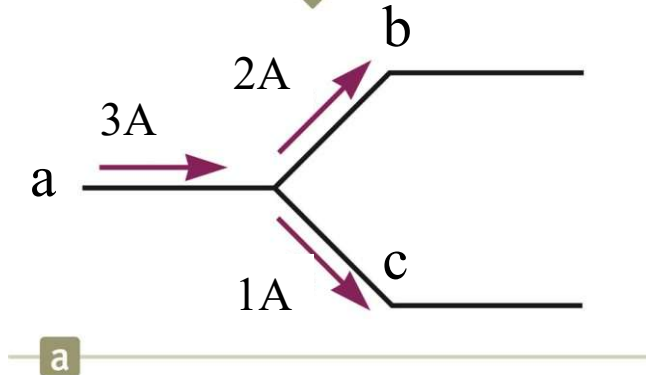
$$\sum_n I_n = 0$$

- It is a statement of Conservation of Charge

- The current I_n has a positive value if the direction is towards the junction.

- The current I_n has a negative value if the direction is away from the junction.

I_2 I_3

I_1 I_4

$\cdots$ I_5

Junction Rule - Example

$$I_a + I_b + I_c = 0$$

- Current into junction from (a) is $I_a = 3$ A
- Current into junction from (b) is $I_b = -2$ A
- Current into junction from (c) is $I_c = -1$ A

Required by Conservation of Charge

Bottom diagram shows a mechanical analog

The amount of charge flowing out of the branches on the right must equal the amount flowing into the single branch on the left.

b

2A

3A

a

c
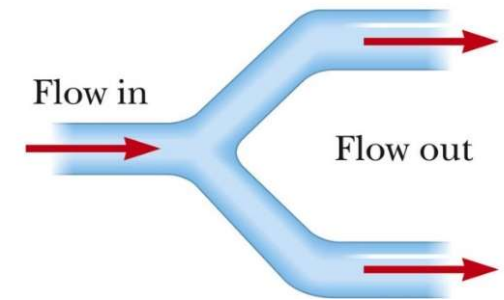
1A

a

The amount of water flowing out of the branches on the right must equal the amount flowing into the single branch on the left.

Flow in

Flow out

Loop Rule

- The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_n \Delta V_n = 0$$

- It is a statement of Conservation of Energy

- To choose a sequence of points in a circuit forming a closed loop

- ΔV_n is the potential difference between two adjacent points in the sequence.

- The sign of ΔV_n depends on the chosen direction of loop

  - The direction may not follow the direction of currents.

More about the Loop Rule (sign of ΔV) [*Important*]

In each diagram, $\Delta V = V_b - V_a$ and the circuit element is traversed from a to b, left to right.

Chosen direction: left to right

a

I

$\Delta V = -IR$

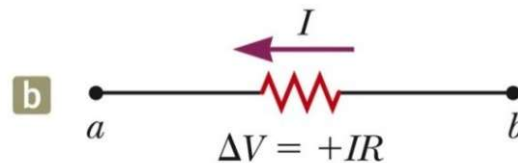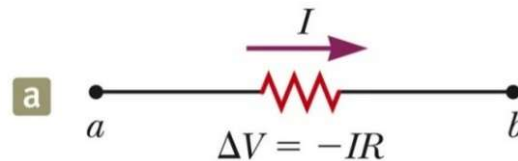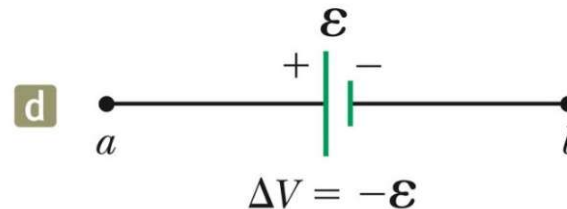**b**

I

$\Delta V = +IR$

c

$\mathcal{E}$

$-$ $+$

a b

$\Delta V = +\mathcal{E}$

d

$\mathcal{E}$

$+$ $-$

a b

$\Delta V = -\mathcal{E}$

In (a), the resistor is traversed in the direction of the current, the potential across the resistor is $-IR$; in (b), the resistor traversed in the opposite direction of the current, thus the potential across the resistor is $+IR$.

In (c), the source of emf is traversed in the direction of the emf (from $-$ to $+$), and the change in the potential difference is $+\mathcal{E}$; in (d), the source of emf traversed in the opposite direction of the emf (from $+$ to $-$), thus the potential difference is $-\mathcal{E}$.

15

Equations from Kirchhoff's Rules

Use the junction rule as often as needed, so long as each time you write an equation, you include in it a current that has not been used in a previous junction rule equation.

- In general, the number of times the junction rule can be used is one fewer than the number of junction points in the circuit.

The loop rule can be used as often as needed so long as a new circuit element (resistor or battery) or a new current appears in each new equation.

In order to solve a particular circuit problem, the number of independent equations you need to obtain from the two rules equals the number of unknown currents.

Any capacitor acts as an open branch in a circuit.

- The current in the branch containing the capacitor is zero under steady-state conditions.

Problem-Solving Strategy – Kirchhoff's Rules

Conceptualize

- Study the circuit diagram and identify all the elements.

- Identify the polarity of each battery.

- Imagine the directions of the currents in each battery.

Categorize

- Determine if the circuit can be reduced by combining series and parallel resistors.

    - If so, proceed with those techniques

    - If not, apply Kirchhoff's Rules

Problem-Solving Strategy, cont.

Analyze

- Assign labels and symbols to all known and unknown quantities.

- Assign directions to the currents.
    - The direction is arbitrary, but you must adhere to the assigned directions when applying Kirchhoff's rules.

- Apply the junction rule to any junction in the circuit that provides new relationships among the various currents.

- Apply the loop rule to as many loops as are needed to solve for the unknowns.
    - To apply the loop rule, you must choose a direction in which to travel around the loop.
    - You must also correctly identify the potential difference as you cross various elements.

- Solve the equations simultaneously for the unknown quantities.

Example: a multiloop circuit

Find the currents I_1, I_2 and I_3 in the circuit.

Conceptualize and categorize: we cannot simplify the circuit by rules associated with combining resistances in series and in parallel.

Apply Kirchhoff's junction rule to junction c:

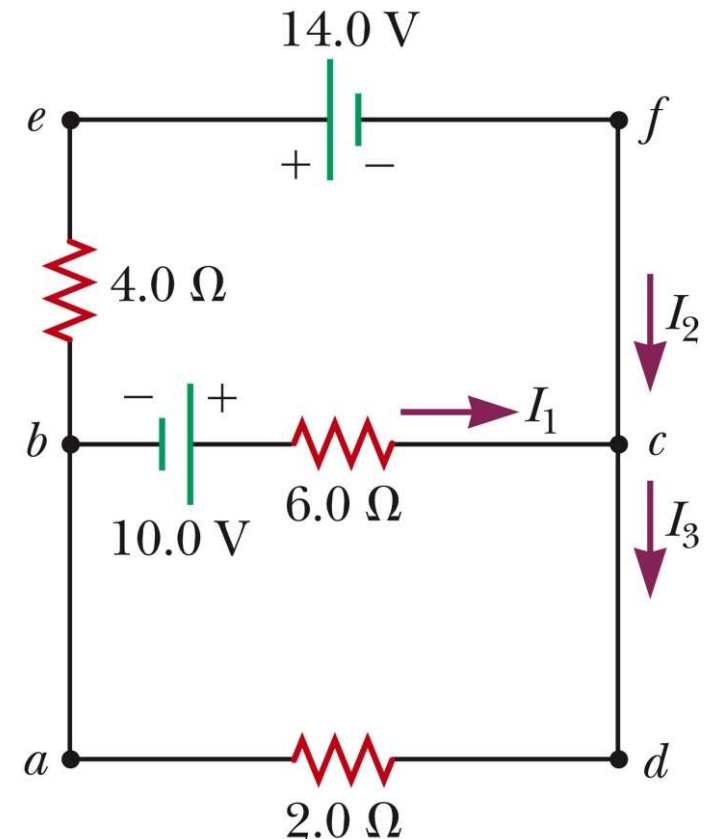(1) $I_1 + I_2 + (-I_3) = 0$

Apply Kirchhoff's loop rule for *abcda*:

(2) $10.0V - (6.0\Omega)I_1 - (2.0\Omega)I_3 = 0$

Apply Kirchhoff's loop for *befcb*:

(3) $-(4.0\Omega)I_2 - 14.0V + (6.0\Omega)I_1 - 10.0V = 0$

Solve Eq. (1) for I_3, use the solution to replace I_3 in Eq. (2), and combine the new equation with (3).

$I_1 = 2.0A$, $I_2 = -3.0A$, $I_3 = -1.0A$.

The circuit in Fig. 28.14 has two loops and three sources of emf. (a) determine the currents given that $r_1=r_2=2$ ohm, $r_3=1$ ohm, $R_1=4$ ohm, $R_2=4$ ohm, $E_1=15V$, $E_2=6V$, and $E_3=4V$. (b) What is the change in potential Va-Vb?
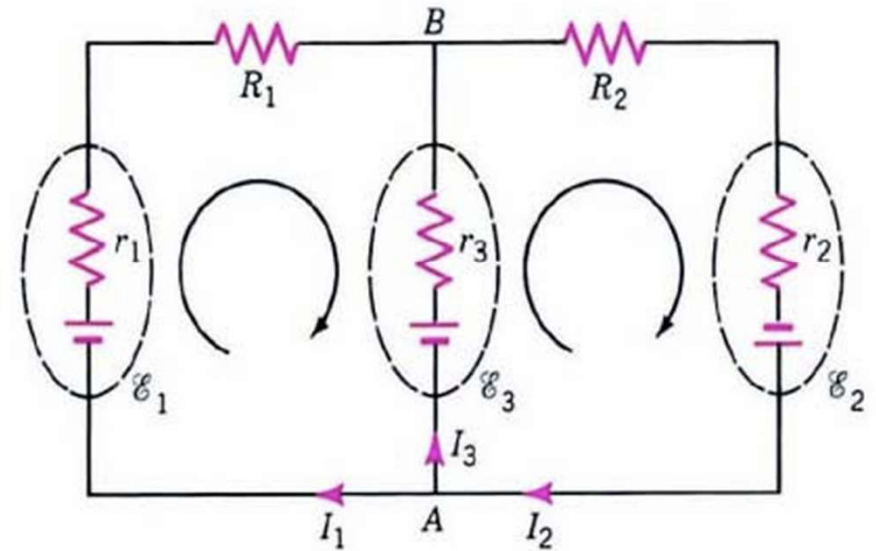
Solution:

Left loop $\qquad 15 - 2I_1 - 4I_1 + I_3 - 4 = 0$

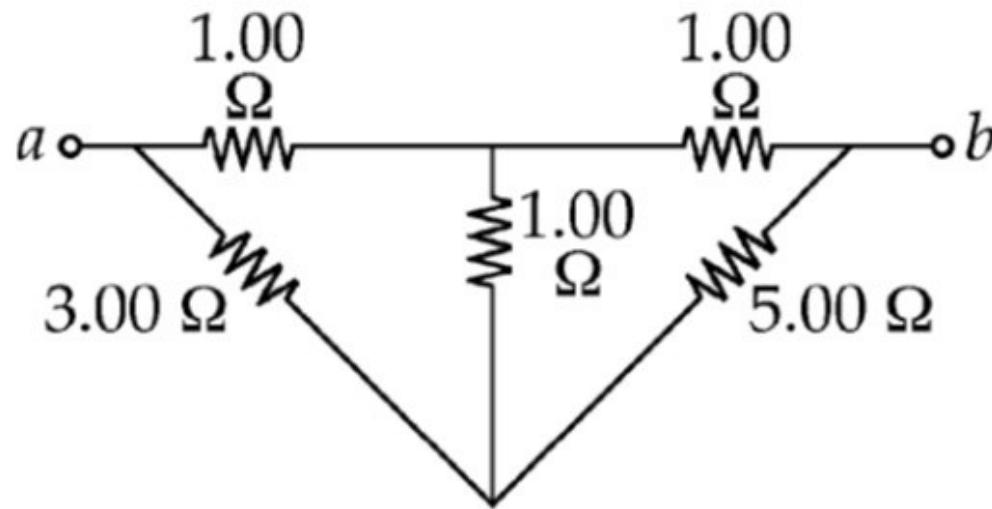right loop $\qquad 4 - I_3 - 3I_2 + 6 - 2I_2 = 0$

junction rule $\qquad I_1 - I_2 + I_3 = 0$
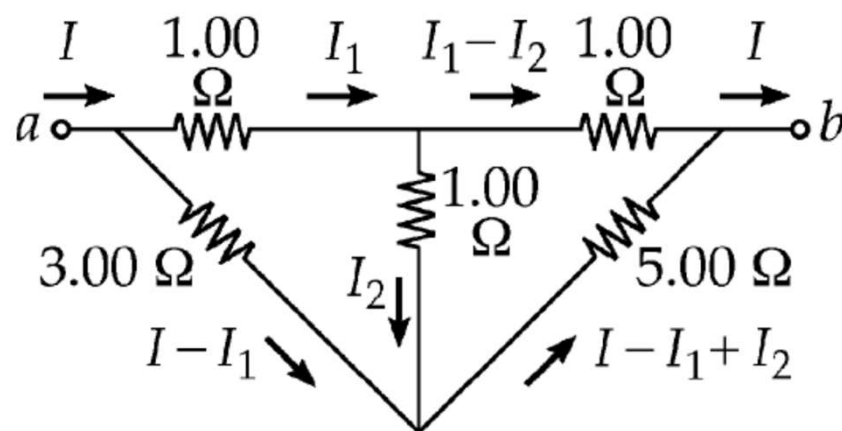

When analyzing a circuit, the currents may be drawn in arbitrary directions.

For the network shown in Figure 1, show that the resistance is R_{ab} = 27/17 Ω.

$$V_{ab} = (1.0)I_1 + (1.0)(I_1 - I_2) = 2I_1 - I_2 \tag{1}$$

$$V_{ab} = (1.0)I_1 + (1.0)I_2 + (5.0)(I - I_1 + I_2) = 5I - 4I_1 + 6I_2 \tag{2}$$

$$V_{ab} = (3.0)(I - I_1) + (5.0)(I - I_1 + I_2) = 8I - 8I_1 + 5I_2 \tag{3}$$

Eliminate I_2 using (1) and (2), and (1) and (3)

$$V_{ab} = 5I - 4I_1 + 6(2I_1 - V_{ab}) \Rightarrow \quad 7V_{ab} = 5I + 8I_1 \tag{4}$$

$$V_{ab} = 8I - 8I_1 + 5(2I_1 - V_{ab}) \Rightarrow \quad 6V_{ab} = 8I + 2I_1 \tag{5}$$

Eliminate I_1 using (4) and (5)

$$7V_{ab} = 5I + 8\left(\frac{6V_{ab} - 8I}{2}\right) \Rightarrow \quad 17V_{ab} = 27I \quad \therefore R = \frac{V_{ab}}{I} = \frac{27}{17}\Omega$$

RC Circuits and Charging a Capacitor

In direct current circuits containing capacitors, the current may vary with time.

- The current is still in the same direction.

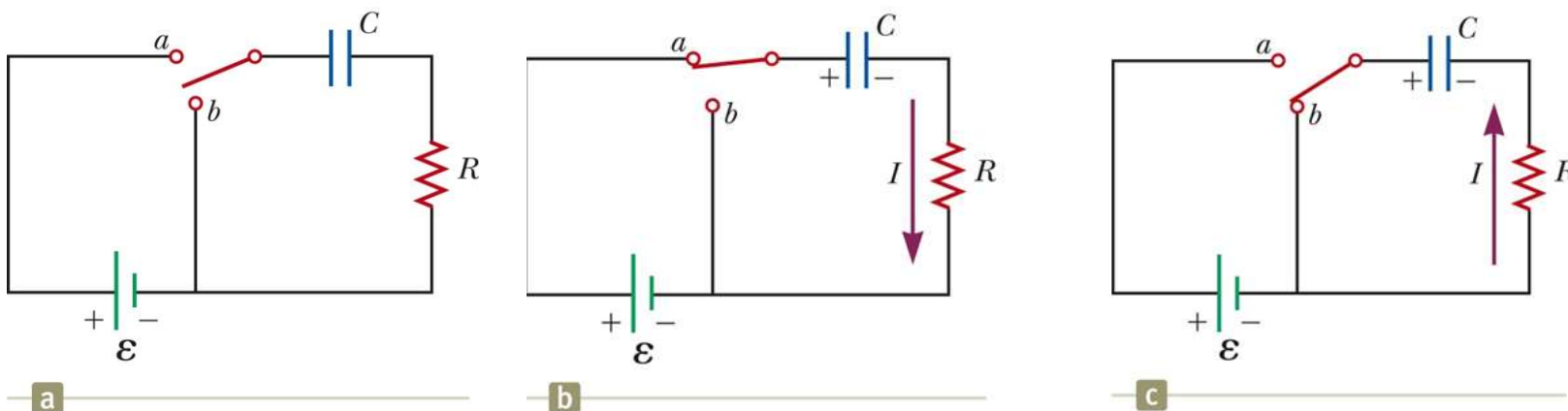An RC circuit will contain a series combination of a resistor and a capacitor.

When the circuit is completed, the capacitor starts to charge.

The capacitor continues to charge until it reaches its maximum charge ($Q = C\varepsilon$).

Once the capacitor is fully charged, the current in the circuit is zero.

As the plates are being charged, the potential difference across the capacitor increases.

Charging a Capacitor in an RC Circuit (Optional)

Apply Kirchhoff loop rule after

the switch is thrown to position a :

$\varepsilon - q/C - IR = 0$

Because $I = dq/dt$, so :

$$\frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC} = -\frac{q - C\varepsilon}{RC}$$

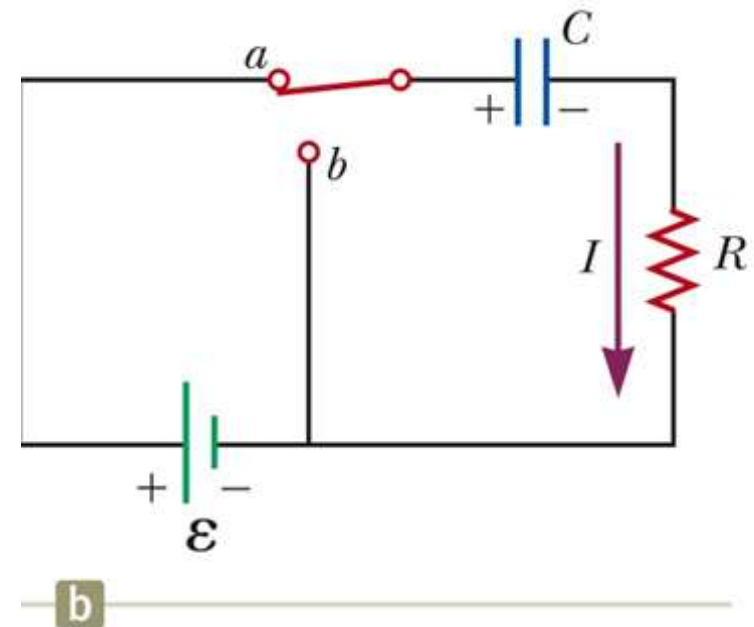Multiply this equation by dt and divide by q-$C\varepsilon$,

$$\frac{dq}{q - C\varepsilon} = -\frac{1}{RC}dt$$

Integrate this expression, using $q = 0$ at $t = 0$,

$$\int_0^q \frac{dq}{q - C\varepsilon} = -\frac{1}{RC}\int_0^t dt \rightarrow \ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = -\frac{t}{RC}$$
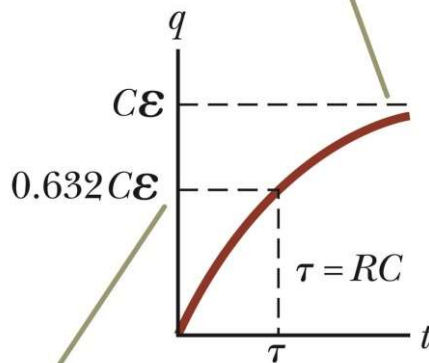
We can write this experission as

$$q(t) = C\varepsilon(1 - e^{-t/RC}) = Q(1 - e^{-t/RC})$$

Charging a Capacitor in an RC Circuit, cont. (Optional)

The charge approaches its maximum value $C\mathcal{E}$ as t approaches infinity.

After a time interval equal to one time constant τ has passed, the charge is 63.2% of the maximum value $C\mathcal{E}$.

a

The charge on the capacitor varies with time.

$$q(t) = C\mathcal{E}(1 - e^{-t/RC}) = Q(1 - e^{-t/RC})$$

The current can be found

$$I(t) = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC} = \frac{\mathcal{E}}{R}e^{-t/\tau}$$

- τ is the *time constant* and $\tau = RC$

The time constant represents the time required for the charge to increase from zero to 63.2% of its maximum.

Discharging a Capacitor in an RC Circuit (Optional)

Apply Kirchhoff loop rule after
the switch is thrown to position b :
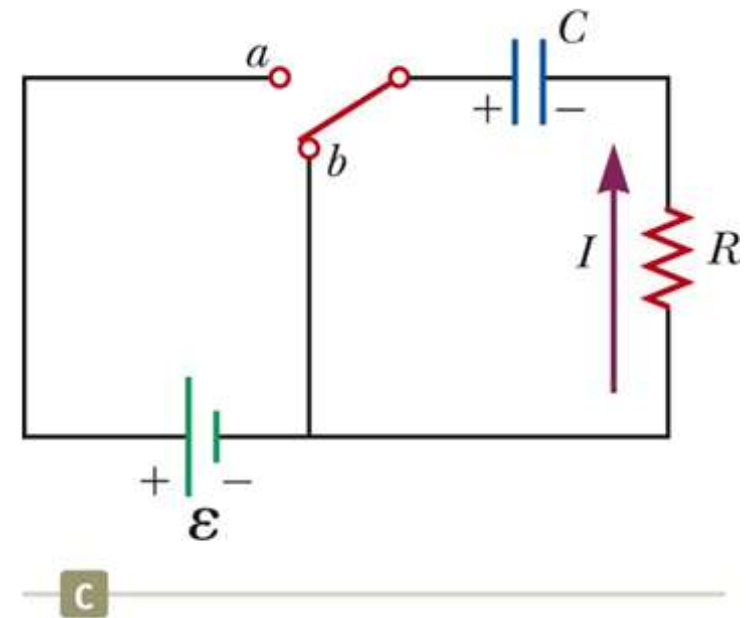
$$-q/C - IR = 0$$

Because $I = dq/dt$, so :

$$-R\frac{dq}{dt} = \frac{q}{C} \rightarrow \frac{dq}{q} = -\frac{1}{RC}dt$$

Integrate this expression, using $q = Q$ at $t = 0$,

$$\int_0^q \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt \rightarrow \ln\left(\frac{q}{Q}\right) = -\frac{t}{RC}$$

We can write this experission as

$$q(t) = Qe^{-t/RC}$$

Discharging a Capacitor in an RC Circuit, cont. (Optional)

When a charged capacitor is placed in the circuit, it can be discharged.

$$q(t) = Qe^{-t/RC} = Qe^{-t/\tau}$$

The charge decreases exponentially.

At $t = \tau = RC$, the charge decreases to $0.368Q_{max}$.

- In other words, in one time constant, the capacitor loses 63.2% of its initial charge.

The current can be found

$$I(t) = \frac{dq}{dt} = -\frac{Q}{RC}e^{-t/RC}$$

Both charge and current decay exponentially at a rate characterized by $\tau = RC$.

The current has its maximum value $I_i = \mathcal{E}/R$ at $t = 0$ and decays to zero exponentially as t approaches infinity.

$$I_i = \frac{\mathcal{E}}{R}$$

After a time interval equal to one time constant τ has passed, the current is 36.8% of its initial value.

b

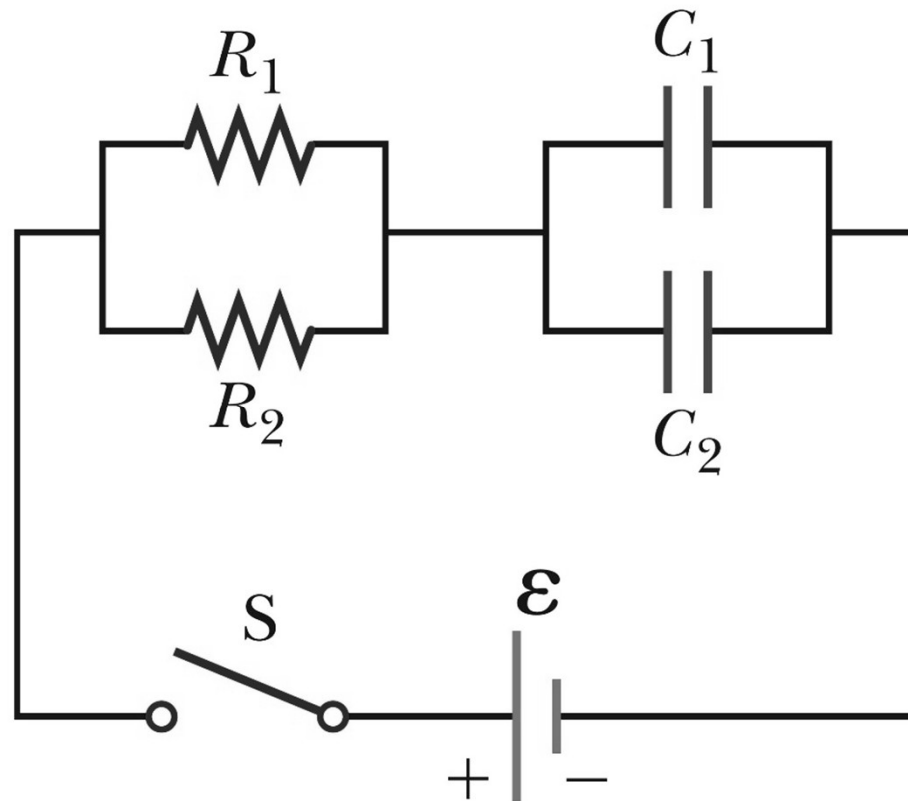The circuit in Figure 2 contains two resistors, $R_1 = 2.00$ kV and $R_2 = 3.00$ kV, and two capacitors, $C_1 = 2.00$ μF and $C_2 = 3.00$ μF, connected to a battery with e.m.f. = 120 V. If there are no charges on the capacitors before switch S is closed, determine the charges on capacitors (a) C_1 and (b) C_2 as functions of time, after the switch is closed.

The total resistance in the circuit is

$$R = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \left(\frac{1}{2.00 \text{ k}\Omega} + \frac{1}{3.00 \text{ k}\Omega}\right)^{-1} = 1.20 \text{ k}\Omega$$

and the total capacitance is $C = C_1 + C_2 = 2.00 \ \mu F + 3.00 \ \mu F = 5.00 \ \mu F$

Thus, $Q_{max} = C\mathcal{E} = (5.0 \ \mu F)(120 \text{ V}) = 600 \ \mu C$

and $\quad \tau = RC = \left(1.2 \times 10^3 \ \Omega\right)\left(5.0 \times 10^{-6} \text{ F}\right) = 6.0 \times 10^{-3} \text{ s} = \dfrac{6.0 \text{ s}}{1\ 000}$

The total stored charge at any time t is then

$$q = q_1 + q_2 = Q_{max}\left(1 - e^{-t/\tau}\right) \quad \text{or} \quad q_1 + q_2 = (600 \ \mu C)\left(1 - e^{-1\ 000\, t/6.0 \text{ s}}\right) \quad [1]$$

Since the capacitors are in parallel with each other, the same potential difference exists across both at any time.

Therefore, $$(\Delta V)_C = \frac{q_1}{C_1} = \frac{q_2}{C_2} \qquad \rightarrow \qquad q_2 = \left(\frac{C_2}{C_1}\right) q_1 = 1.5\, q_1 \qquad [2]$$

(a) Substituting Equation [2] into [1] gives

$$2.5 q_1 = (600\ \mu C)\left(1 - e^{-1\,000\, t / 6.0\ s}\right)$$

$$q_1 = \left(\frac{600\ \mu C}{2.5}\right)\left(1 - e^{-t/(6.0\ s/1\,000)}\right) \qquad \rightarrow \qquad q_1 = 240\ \mu C\left(1 - e^{-t/6\ ms}\right)$$

or $\boxed{q = 240\left(1 - e^{-t/6}\right)}$, $\boxed{\text{where } q \text{ is in microcoulombs and } t \text{ is in milliseconds.}}$

(b) and From Equation [2],

$$q_2 = 1.5\, q_1 = 1.5\left[240\ \mu C\left(1 - e^{-t/6\ ms}\right)\right] = 360\ \mu C\left(1 - e^{-t/6\ ms}\right)$$

or, $q = \boxed{360\left(1 - e^{-t/6}\right)}$, $\boxed{\text{where } q \text{ is in microcoulombs and } t \text{ is in milliseconds.}}$

Short Circuit & Electrical Safety (Optional)

A *short circuit* occurs when almost zero resistance exists between two points at different potentials, which results in a very large current.

In a household circuit, a circuit breaker will open the circuit in the case of an accidental short circuit, which prevents any damage.

A person in contact with ground can be electrocuted by touching the live wire.

Electric shock can result in fatal burns and can cause the muscles of vital organs (such as the heart) to malfunction.

The degree of damage depends on:

- The magnitude of the current

- The length of time it acts

- The part of the body touched by the live wire

- The part of the body in which the current exists

Effects of Various Currents (Optional)

5 mA or less

- Can cause a sensation of shock
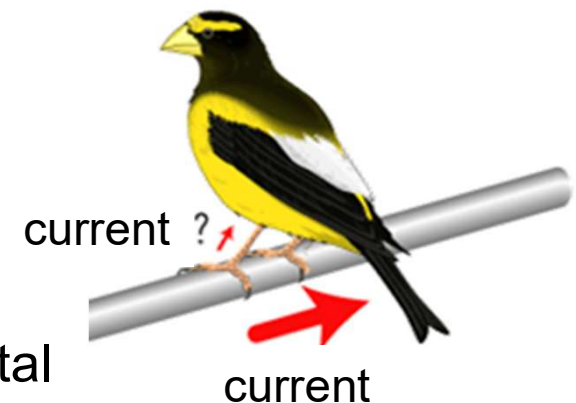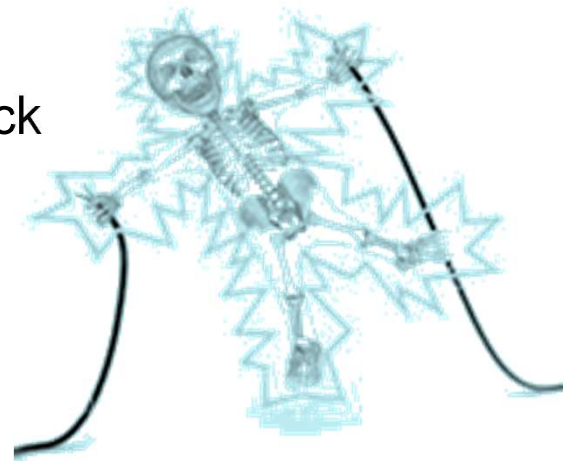
- Generally little or no damage

10 mA

- Muscles contract

- May be unable to let go of a live wire

100 mA

- If passing through the body for a few seconds can be fatal

- Paralyzes the respiratory muscles and prevents breathing

In some cases, currents of 1 A can produce serious burns.

No contact with live wires is considered safe whenever the voltage is greater than 24 V.

current ?

current
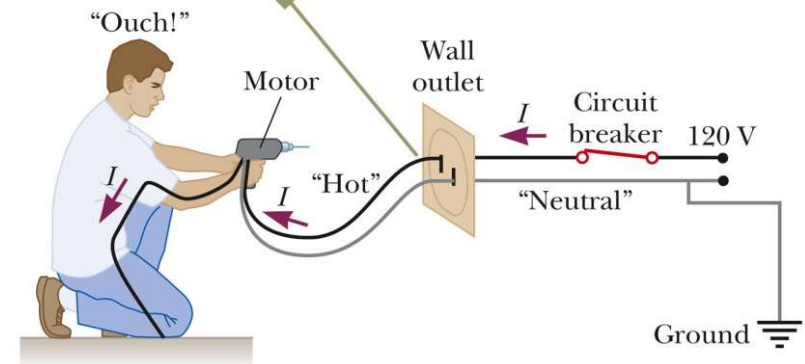
33

Ground Wire (Optional)

Electrical equipment manufacturers use electrical cords that have a third wire, called a ground.

This safety ground normally carries no current and is both grounded and connected to the appliance.

If the live wire is accidentally shorted to the casing, most of the current takes the low-resistance path through the appliance to the ground.
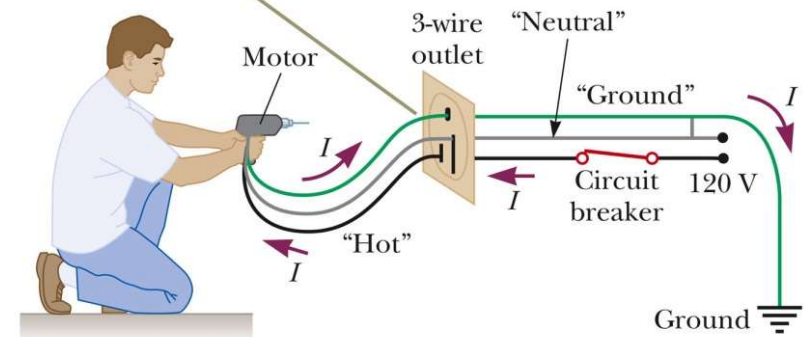
If it was not properly grounded, anyone in contact with the appliance could be shocked because the body produces a low-resistance path to ground.

In the situation shown, the live wire has come into contact with the drill case. As a result, the person holding the drill acts as a current path to ground and receives an electric shock.

"Ouch!"

Motor

Wall outlet

Circuit breaker   120 V

I

"Hot"

"Neutral"

I

I

Ground

a

In this situation, the drill case remains at ground potential and no current exists in the person.

Motor

3-wire outlet   "Neutral"

"Ground"

I

I

Circuit breaker   120 V

I

"Hot"

I

Ground

b