

Problem 1. A point charge of magnitude $+q$ produces an electric field pattern with a certain strength at distance r .

- If another point charge produces an electric field that is twice as strong at the same distance r , what is the magnitude and sign of this second charge? (5 pts)
- If a third point charge produces an electric field that is half as strong and directed inward at the same distance r , what is the magnitude and sign of this charge? (5 pts)

Problem 2.

The electric potential along a line is described by the function:

$$V(x) = 80 - 12x + 3x^2 \quad (\text{with } V \text{ in volts, } x \text{ in meters}).$$

- Write the expression for the electric field $E(x)$ along the x -axis. (3 pts)
- Calculate the electric field at $x = 0.0 \text{ m}, 1.0 \text{ m}, 2.0 \text{ m}, 3.0 \text{ m}$. (7 pts)

Problem 3 (10 pts) — Fill in the blanks

A uniform electric field of 900 N/C points to the right.

- What is the potential difference $V_D - V_C$ if point D is 0.20 m to the right of point C? (3 pts)
- What is the potential difference $V_C - V_A$ if A is 0.35 m to the left of C? (3 pts)
- What is the potential difference $V_D - V_A$? (2 pts)
- Can we determine the absolute value of potential at C? Explain. (2 pts)

Problem 4 (10 pts) — Fill in the blanks

The electric potential along the x -axis is given by

$$V(x) = 100 - 20x + 5x^2 \quad (\text{in volts, with } x \text{ in meters}).$$

Find the electric field at positions $x = 0.5, 1.0, 1.5$, and 2.0 m .

Problem 5 (10 pts) — Fill in the blanks

A steady current I flows through a cylindrical copper wire of radius r .

- a) How much charge passes through the wire in a time t ? (4 pts)
- b) What is the current density J ? (3 pts)
- c) If the resistivity of copper is ρ , what is the electric field inside the wire? (3 pts)

Problem 6 (10 pts) — Fill in the blanks

A parallel-plate capacitor consists of two circular plates of radius R separated by distance d .

- a) Find its capacitance in vacuum. (4 pts)
- b) If the capacitor holds charge Q , what is the potential difference? (3 pts)
- c) If a dielectric with dielectric constant K is inserted fully between the plates, what is the stored energy? (3 pts)

Problem 7 (10 pts) — Long Question

Three resistors of $3\ \Omega$, $6\ \Omega$, and $12\ \Omega$ are connected in parallel across a 9 V battery.

- a) What is the equivalent resistance? (3 pts)
- b) What is the current through each resistor? (3 pts)
- c) What is the power dissipated in each resistor? (2 pts)
- d) What is the total power dissipated, and how does it compare with the power dissipated in the equivalent resistance? (2 pts)

Problem 8 (10pts) Long question

A circuit consists of a 12 V battery connected to two resistors $R_1 = 3\ \Omega$ and $R_2 = 6\ \Omega$ in series. In parallel with R_2 , there is another resistor $R_3 = 4\ \Omega$.

- a) Write down the loop and junction equations needed to analyze the circuit. (4 pts)
- b) Solve for the current through each resistor (I_1, I_2, I_3). (6 pts)

(Hint: Treat the branch containing R_2 and R_3 as a parallel combination.)

Problem 9 (10 pts) — Capacitance with moving dielectric

A parallel-plate capacitor has plates of area A and separation d . A dielectric slab of constant $K = 4$ is inserted halfway between the plates.

- a) Derive the expression for the effective capacitance. (6 pts)
- b) If the dielectric is pulled out with velocity u , find the current in the external circuit. (4 pts)

Problem 10 (10 pts) — Gauss's Law

A solid sphere of radius a carries a uniform charge density ρ . Concentric with it is a conducting spherical shell of inner radius b and outer radius c , which initially has no net charge.

- a) Find the electric field in the regions: $r < a$, $a < r < b$, $b < r < c$, and $r > c$. (6 pts)
- b) Find the induced charges on the inner and outer surfaces of the conducting shell. (4 pts)

Problem 1 — Text-based (Solution)

Given: A reference point charge $+q$ produces field $E = \frac{k|q|}{r^2}$ at distance r .

- a) If another charge gives **twice** the field magnitude at the **same r** , then

$$\frac{k|q'|}{r^2} = 2 \frac{k|q|}{r^2} \Rightarrow |q'| = 2|q|.$$

Direction wasn't stated to change, so keep the **same sign** as the reference.

Answer: $q' = +2q$.

- b) If a third charge gives **half** the magnitude and the field points **inward** (opposite the $+q$ case), then

$$|q''| = \frac{1}{2}|q|, \quad \text{sign negative (to point inward).}$$

Answer: $q'' = -\frac{q}{2}$.

Problem 3 (uniform field → potential differences)

$\vec{E} = 900 \text{ N/C}$ to the right. $V_B - V_A = - \int \vec{E} \cdot d\vec{\ell}$.

- $V_D - V_C = -(900)(0.20) = \boxed{-180 \text{ V}}$
- $V_C - V_A = -(900)(0.35) = \boxed{-315 \text{ V}}$
- $V_D - V_A = -180 - 315 = \boxed{-495 \text{ V}}$
- Absolute V at C ? No — need a reference.

Problem2

Given: $V(x) = 80 - 12x + 3x^2$ (volts), with x in meters.

Electric field along x :

$$E_x(x) = -\frac{dV}{dx} = -(-12 + 6x) = 12 - 6x \quad (\text{V/m}).$$

Now evaluate:

- $x = 0.0: E_x = 12 - 6(0) = \boxed{12 \text{ V/m}} \text{ (along } +\hat{x}\text{)}$
- $x = 1.0: E_x = 12 - 6(1) = \boxed{6 \text{ V/m}} \text{ (along } +\hat{x}\text{)}$
- $x = 2.0: E_x = 12 - 6(2) = \boxed{0 \text{ V/m}}$
- $x = 3.0: E_x = 12 - 6(3) = \boxed{-6 \text{ V/m}} \text{ (i.e., } 6 \text{ V/m along } -\hat{x}\text{)}$

Problem 3 (uniform field → potential differences)

$\vec{E} = 900 \text{ N/C}$ to the right. $V_B - V_A = - \int \vec{E} \cdot d\vec{\ell}$.

- $V_D - V_C = -(900)(0.20) =$ -180 V
- $V_C - V_A = -(900)(0.35) =$ -315 V
- $V_D - V_A = -180 - 315 =$ -495 V
- Absolute V at C ? No — need a reference.

Problem 5 (current, current density, field)

Wire radius r , resistivity ρ .

- $Q = I t$
- $J = \frac{I}{\pi r^2}$
- $E = \rho J = \frac{\rho I}{\pi r^2}$

Problem 6 (circular parallel-plate capacitor)

Area $A = \pi R^2$, separation d .

- $C = \frac{\epsilon_0 \pi R^2}{d}$
- $V = \frac{Q}{C} = \frac{Qd}{\epsilon_0 \pi R^2}$
- With dielectric K : $C_K = K \epsilon_0 \pi R^2 / d$, so
$$U = \frac{Q^2}{2C_K} = \frac{Q^2 d}{2K \epsilon_0 \pi R^2}$$

Problem 7 (three parallel resistors on 9 V)

$R_1 = 3 \Omega$, $R_2 = 6 \Omega$, $R_3 = 12 \Omega$, $V = 9 \text{ V}$.

- $R_{\text{eq}}^{-1} = 1/3 + 1/6 + 1/12 = 7/12 \Rightarrow R_{\text{eq}} = 12/7 \Omega \approx 1.714 \Omega$
- Currents: $I_i = V/R_i$:
$$I_1 = 3.00 \text{ A}, I_2 = 1.50 \text{ A}, I_3 = 0.75 \text{ A}$$
 (total 5.25 A)
- Powers $P_i = VI_i$:
$$P_1 = 27.0 \text{ W}, P_2 = 13.5 \text{ W}, P_3 = 6.75 \text{ W}$$
 (sum 47.25 W = V^2/R_{eq})

Problem 8 — Solution (new text-based circuit)

Circuit description: A 12 V battery → $R_1 = 3 \Omega$ in series
→ node splits into two parallel branches: $R_2 = 6 \Omega$ and
 $R_3 = 4 \Omega$ → rejoin → back to battery.

(a) Required equations

- Parallel equivalent:

$$R_{23} = \left(\frac{1}{R_2} + \frac{1}{R_3} \right)^{-1} = \left(\frac{1}{6} + \frac{1}{4} \right)^{-1} = \left(\frac{5}{12} \right)^{-1} = 2.4 \Omega.$$

- Total resistance:

$$\underline{R_{\text{tot}}} = R_1 + R_{23} = 3 + 2.4 = 5.4 \Omega.$$

- Total current (through R_1) (Ohm's law):

$$I_{\text{tot}} = \frac{V}{R_{\text{tot}}} = \frac{12}{5.4} = \frac{20}{9} \text{ A} \approx 2.222 \text{ A.}$$

- Voltage across the parallel branch:

$$V_{23} = V - I_{\text{tot}}R_1 = 12 - \left(\frac{20}{9} \right)(3) = 12 - \frac{20}{3} = \frac{16}{3} \text{ V} \approx 5.333 \text{ V.}$$

- Branch currents (Ohm's law in each branch):
[3333 A]

$$I_2 = \frac{V_{23}}{R_2} = \frac{16/3}{6} = \frac{8}{9} \text{ A} \approx 0.889 \text{ A,}$$

$$I_3 = \frac{V_{23}}{R_3} = \frac{16/3}{4} = \frac{4}{3} \text{ A} \approx 1.333 \text{ A.}$$

- Junction rule check:
.3333 A.

$$I_{\text{tot}} = I_2 + I_3 = \frac{8}{9} + \frac{4}{3} = \frac{8}{9} + \frac{12}{9} = \frac{20}{9} \text{ A} \quad \checkmark$$

(b) Final answers

- Current through R_1 :

$$I_1 = I_{\text{tot}} = \frac{20}{9} \text{ A} \approx 2.222 \text{ A}$$

- Current through R_2 : $I_2 = \frac{8}{9} \text{ A} \approx 0.889 \text{ A}$
- Current through R_3 : $I_3 = \frac{4}{3} \text{ A} \approx 1.333 \text{ A}$

(Optional power check: $P_{\text{tot}} = VI_{\text{tot}} = 12 \times 20/9 = 80/3 \approx 26.67 \text{ W}$, which equals the sum of branch powers.)

Problem 9 (partially inserted dielectric)

Side-by-side (parallel) regions: vacuum area $A - A_d$, dielectric area A_d , gap d , constant K .

$$(a) C = \frac{\epsilon_0}{d} [(A - A_d) + KA_d] = \frac{\epsilon_0}{d} [A + (K - 1)A_d].$$

If half area filled $A_d = A/2$ and $K = 4$: $C = \frac{5}{2} \frac{\epsilon_0 A}{d}$.

(b) Constant-voltage source V . With $A_d = A_d(t)$:

$$I = \frac{dQ}{dt} = V \frac{dC}{dt} = V \frac{\epsilon_0(K-1)}{d} \frac{dA_d}{dt}.$$

If slab edge moves at speed u across width W : $dA_d/dt = Wu$, so

$$I = V \frac{\epsilon_0(K-1)}{d} Wu.$$

Problem 10 (insulating sphere + conducting shell)

Insulator radius a with ρ ($Q = \frac{4}{3}\pi a^3 \rho$). Concentric conducting shell $b < c$, net shell charge 0.

(a) Field by region

- $r < a$: $Q_{\text{enc}} = \frac{4}{3}\pi r^3 \rho \Rightarrow E = \frac{\rho r}{3\epsilon_0}$ (outward)
- $a < r < b$: $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$
- $b < r < c$: $E = 0$
- $r > c$: $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$

(b) Induced charges

- Inner surface must cancel $+Q$ inside: $q_{\text{inner}} = -Q, \sigma_{\text{inner}} = -\frac{Q}{4\pi b^2}$
- Net shell charge 0 \Rightarrow outer surface: $q_{\text{outer}} = +Q, \sigma_{\text{outer}} = \frac{Q}{4\pi c^2}$