

Capacitance & Dielectrics

- Definition of capacitance
- Calculating capacitance of capacitors with simple geometries
- Combination of capacitors
- Circuit symbols to represent capacitors connected to batteries
- Energy stored in a charged capacitor
- Capacitors with dielectrics

Circuits and Circuit Elements

Electric circuits are the basis for the vast majority of the devices used in society.

Circuit elements can be connected with wires to form electric circuits.

Capacitors are one circuit element.

- Others will be introduced in other chapters

Capacitors

Capacitors are devices that store electric charges.

Examples of where capacitors are used include:

- radio receivers
- filters in power supplies
- to eliminate sparking in automobile ignition systems
- energy-storing devices in electronic flashes

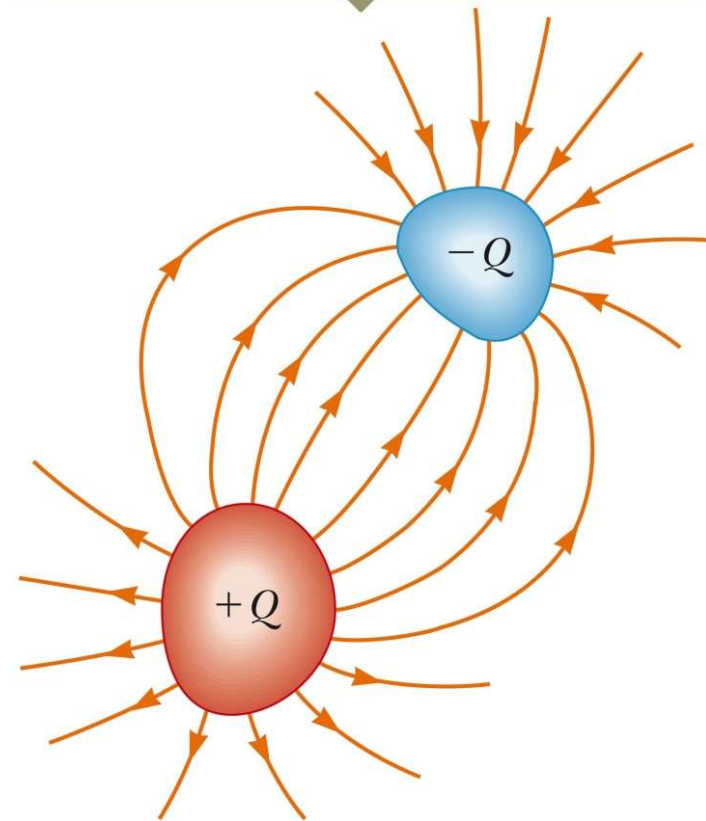
Makeup of a Capacitor

A capacitor consists of **two conductors**.

- These conductors are called plates.
- When the conductor is charged, the plates carry charges of equal magnitude and opposite signs.

A potential difference exists between the plates due to the charges.

When the capacitor is charged, the conductors carry charges of equal magnitude and opposite sign.



Definition of Capacitance

The **capacitance**, C , of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the potential difference between the conductors.

$$C \equiv \frac{Q}{\Delta V}$$

The SI unit of capacitance is the **farad** (F).

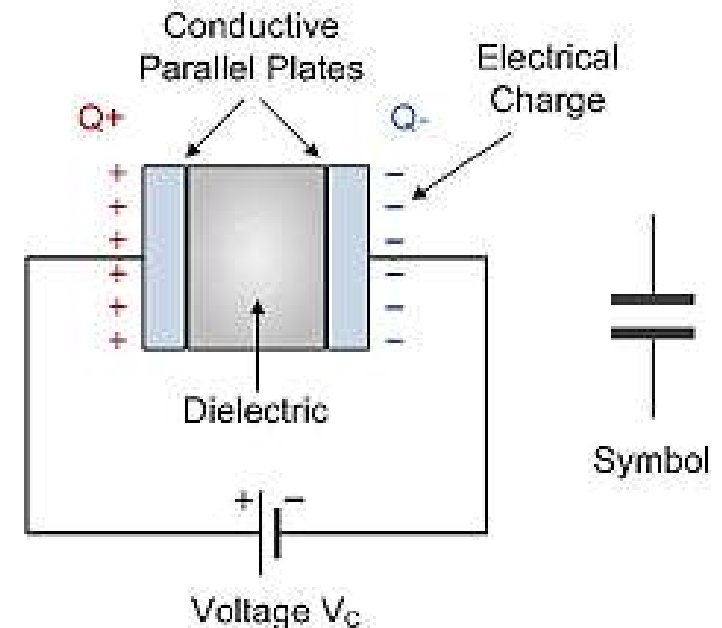
The farad is a large unit, typically you will see microfarads (μF) and picofarads (pF).

Capacitance will always be a **positive** quantity.

The capacitance of a given capacitor is constant.

The capacitance is a measure of the capacitor's ability to store charge.

- The capacitance of a capacitor is **the amount of charge the capacitor can store per unit of potential difference**.



Parallel Plate Capacitor

Each plate is connected to a terminal of the battery.

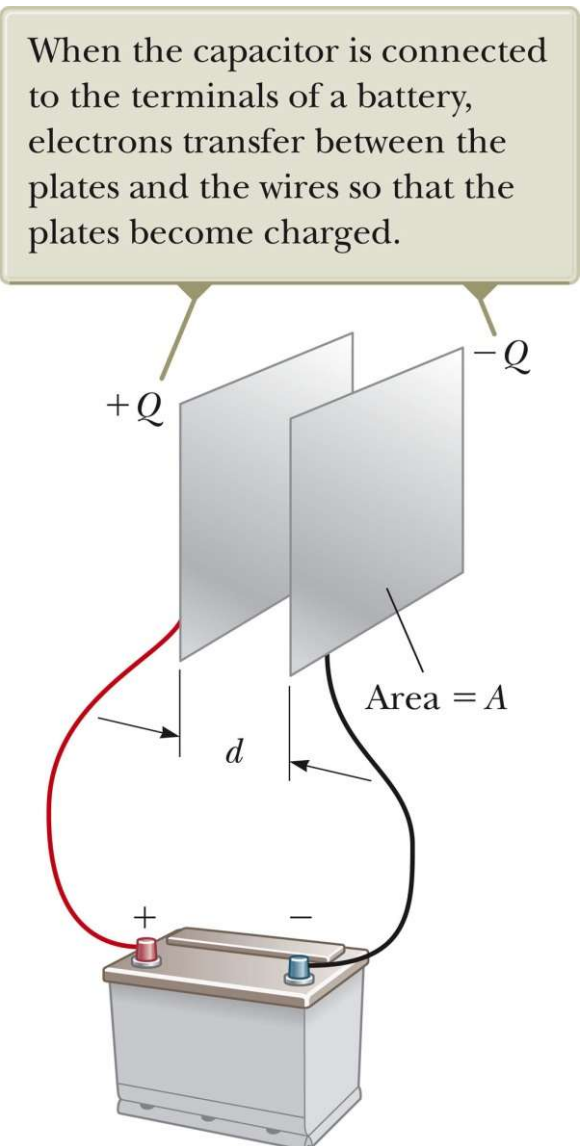
- The battery is a source of potential difference.

If the capacitor is initially uncharged, the battery establishes an electric field in the connecting wires.

This field applies a force on electrons in the wire just outside of the plates -> electrons moving onto the negative plate -> continues until equilibrium is achieved: plate, wire & terminal at the same potential.

At this point, there is no field present in the wire and the movement of the electrons ceases -> plate negatively charged; A similar process occurs at the other plate.

In its final configuration, the **potential difference across the capacitor plates is the same as that between the terminals of the battery.**



Capacitance – Parallel Plates

The charge density on the plates is $\sigma = Q / A$.

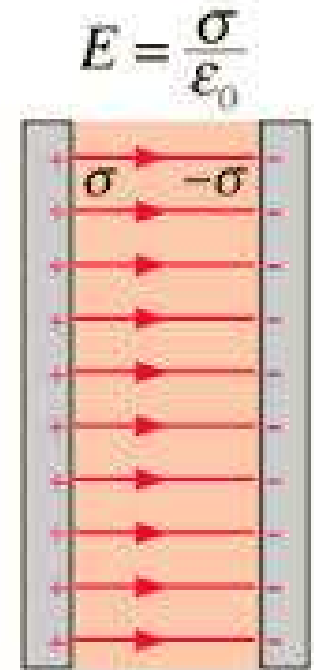
- A is the area of each plate, the area of each plate is equal
- Q is the charge on each plate, equal with opposite signs

The electric field is uniform between the plates and zero elsewhere.

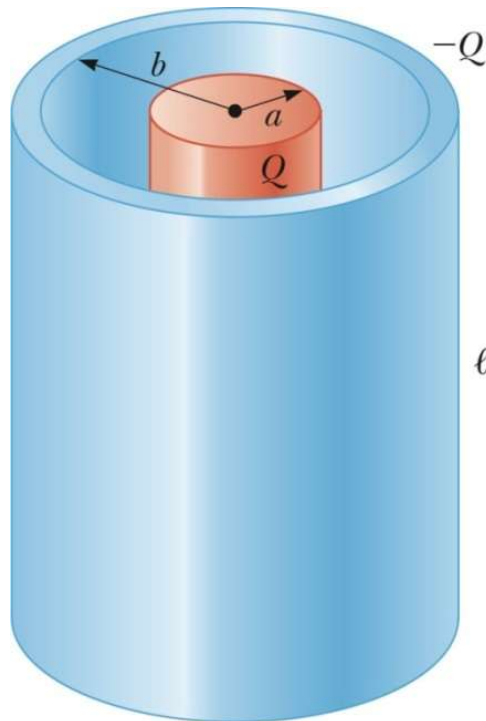
The capacitance is proportional to the area of its plates and inversely proportional to the distance between the plates.

$$C = \frac{Q}{\Delta V} = \frac{Q}{Ed} = \frac{\epsilon_0 Q}{\sigma d} = \frac{\epsilon_0 Q}{(Q/A)d} = \frac{\epsilon_0 A}{d}$$

→ $C = \frac{\epsilon_0 A}{d}$



Capacitance - Cylindrical Capacitors (Optional)



$$V_b - V_a = - \int_a^b \vec{E} \cdot d\vec{s}$$

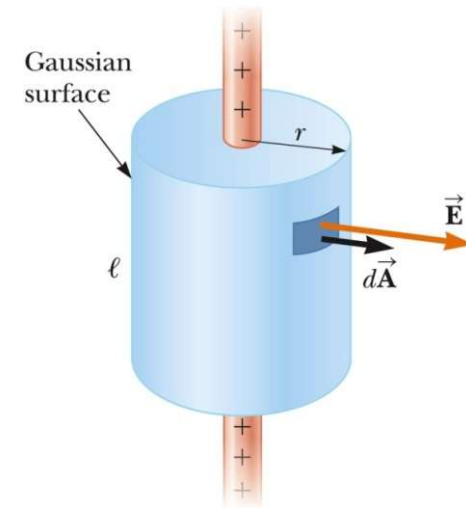
$$= - \int_a^b \underline{E_r} dr$$

$$= -2k_e \lambda \int_a^b \frac{dr}{r}$$

$$= -2k_e \lambda \ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{|V_b - V_a|} = \frac{l\lambda}{2k_e \lambda \ln\left(\frac{b}{a}\right)} = \frac{l}{2k_e \ln\left(\frac{b}{a}\right)}$$

From Chapter 24

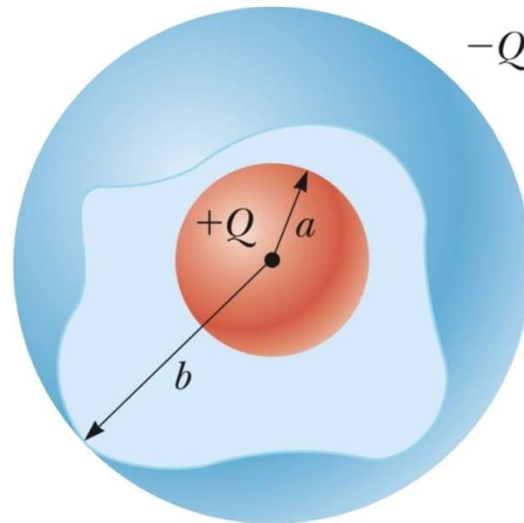


$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = E \oint dA = \frac{q_{in}}{\epsilon_0}$$

$$E(2\pi r l) = \frac{\lambda l}{\epsilon_0}$$

$$E = \frac{\lambda l}{2\pi r l \epsilon_0} = 2k_e \frac{\lambda}{r}$$

Capacitance - Spherical Capacitor (Optional)



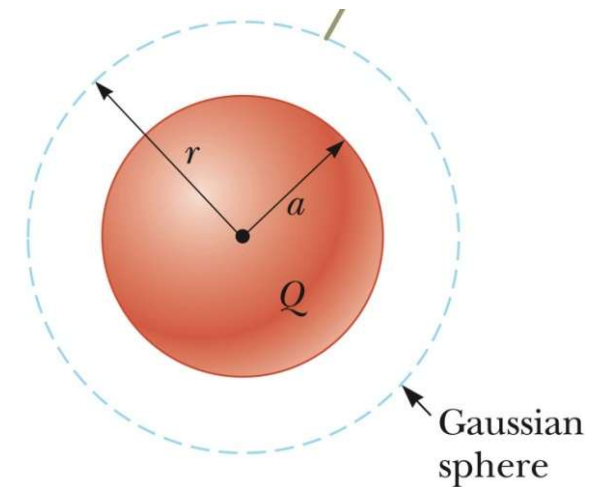
$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s}$$

$$= -\int_a^b \underline{E_r} dr = -k_e Q \int_a^b \frac{dr}{r^2} = k_e Q \left[\frac{1}{r} \right]_a^b$$

$$\Delta V = k_e Q \left(\frac{1}{b} - \frac{1}{a} \right)$$

$$C = \left| \frac{Q}{\Delta V} \right| = \frac{Q}{k_e Q \left(\frac{1}{a} - \frac{1}{b} \right)} = \frac{ab}{k_e (b - a)}$$

From Chapter 24



$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = E \oint dA = \frac{q_{in}}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} = k_e \frac{Q}{r^2}$$

Capacitance – Isolated Sphere (Optional)

Assume a spherical charged conductor with radius a .

The sphere will have the same capacitance as it would if there were a conducting sphere of infinite radius ($b \rightarrow \infty$), concentric with the original sphere.

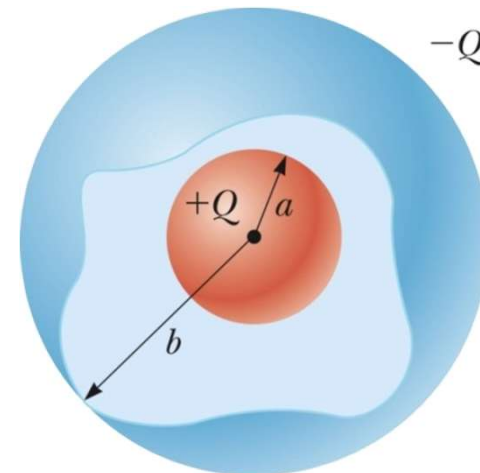
Assume $V = 0$ for the infinitely large shell

$$C = \left. \frac{ab}{k_e(b-a)} \right|_{b \rightarrow +\infty} = \frac{a}{k_e} = 4\pi\epsilon_0 a$$

Note, this is independent of the charge on the sphere and its potential.

From previous page

$$C = \frac{Q}{\Delta V} = \frac{ab}{k_e(b-a)}$$



Circuit Symbols

A circuit diagram is a simplified representation of an actual circuit.

Circuit symbols are used to represent the various elements.

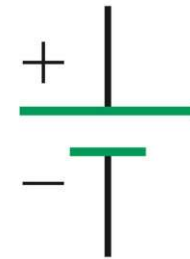
Lines are used to represent wires.

The battery's positive terminal is indicated by the longer line.

Capacitor
symbol



Battery
symbol



Switch
symbol



Capacitors in Parallel

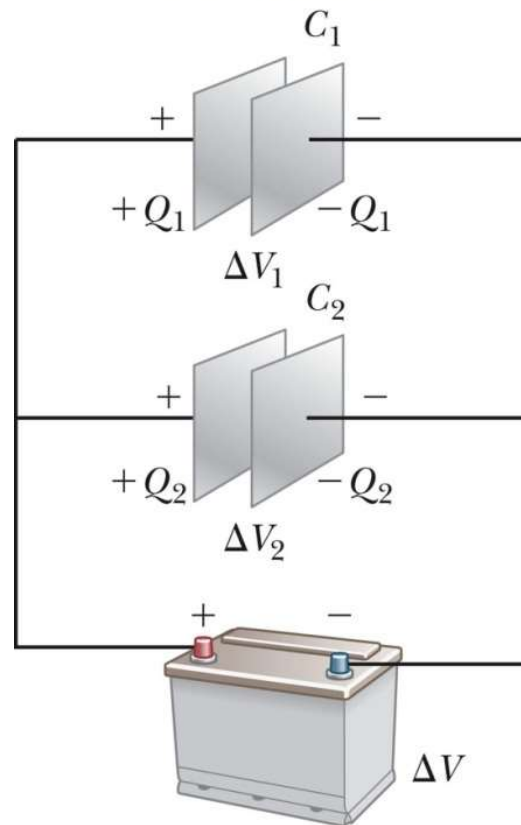
The potential difference across the capacitors is the same. And each is equal to the voltage of the battery

- $\Delta V_1 = \Delta V_2 = \Delta V$
- ΔV is the battery terminal voltage

The total charge is equal to the sum of the charges on the capacitors.

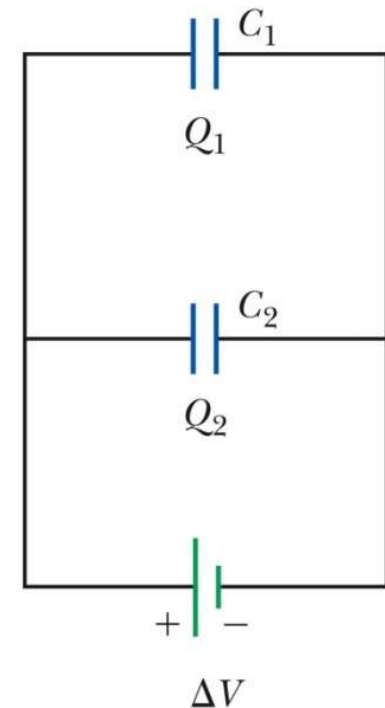
- $Q_{\text{tot}} = Q_1 + Q_2$

A pictorial representation of two capacitors connected in parallel to a battery



a

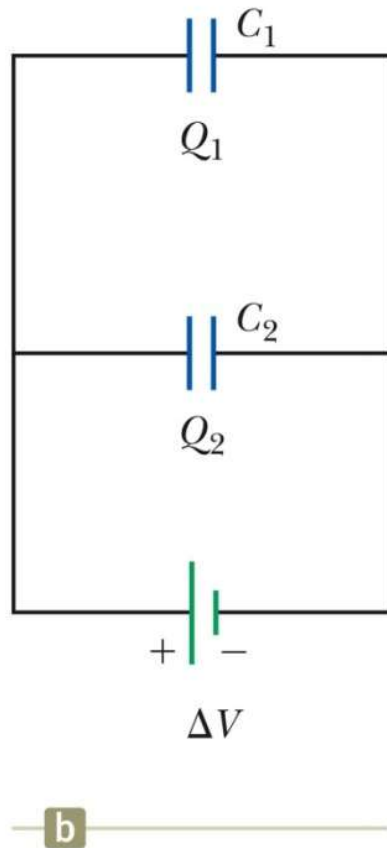
A circuit diagram showing the two capacitors connected in parallel to a battery



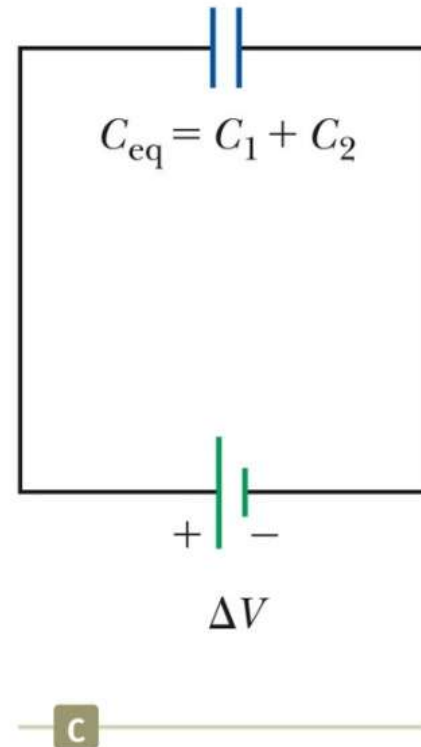
b

Capacitors in Parallel (Equivalent capacitance)

The capacitors can be represented by one capacitor with a capacitance of C_{eq} .



Equivalent to



$$(1) \Delta V_1 = \Delta V_2 = \Delta V$$

$$(2) Q_{tot} = Q_1 + Q_2$$

$$Q_{tot} = Q_1 + Q_2$$

$$= C_1 \Delta V_1 + C_2 \Delta V_2$$

$$= C_1 \Delta V + C_2 \Delta V$$

Therefore,

$$C_{eq} = Q_{tot} / \Delta V = C_1 + C_2$$

$$C_{eq} = C_1 + C_2 + C_3 + \dots \text{ (parallel combination)}$$

Capacitors in Series

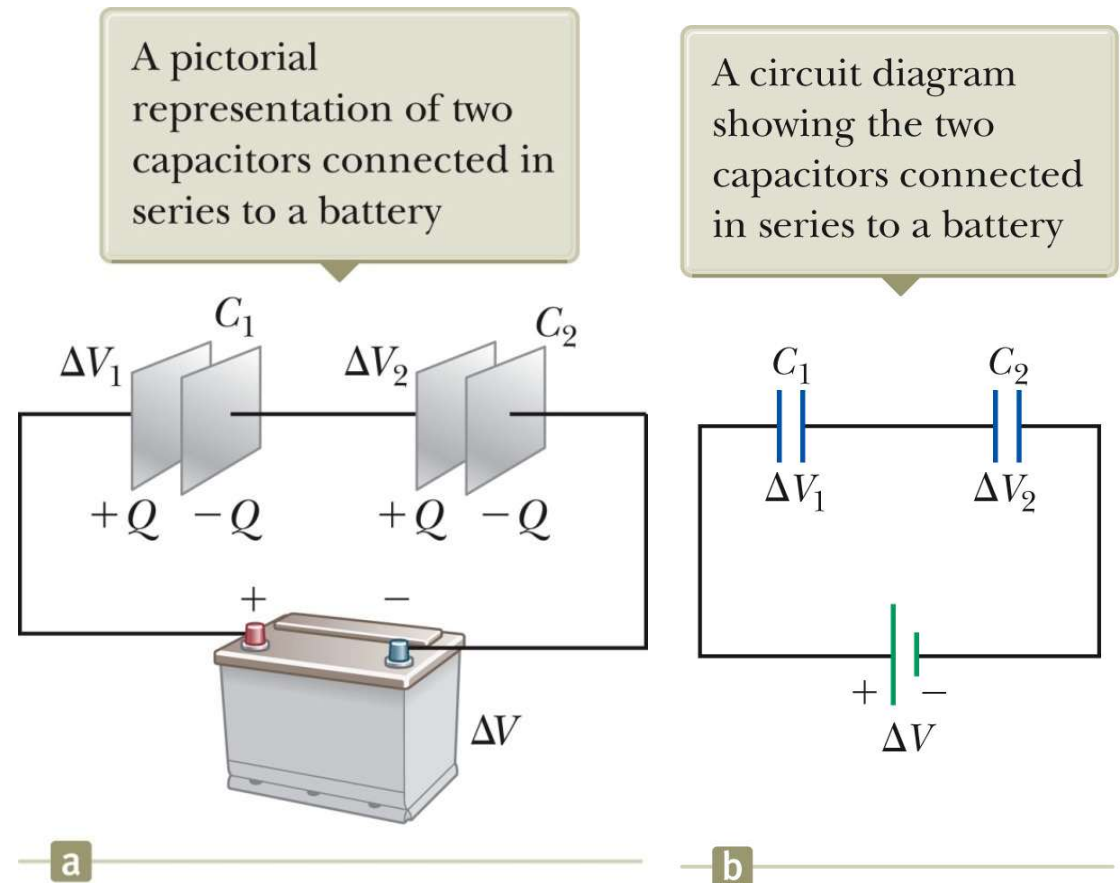
The potential differences (ΔV_1 and ΔV_2) across the capacitors are NOT the same.

The total potential is

- $\Delta V = \Delta V_1 + \Delta V_2$

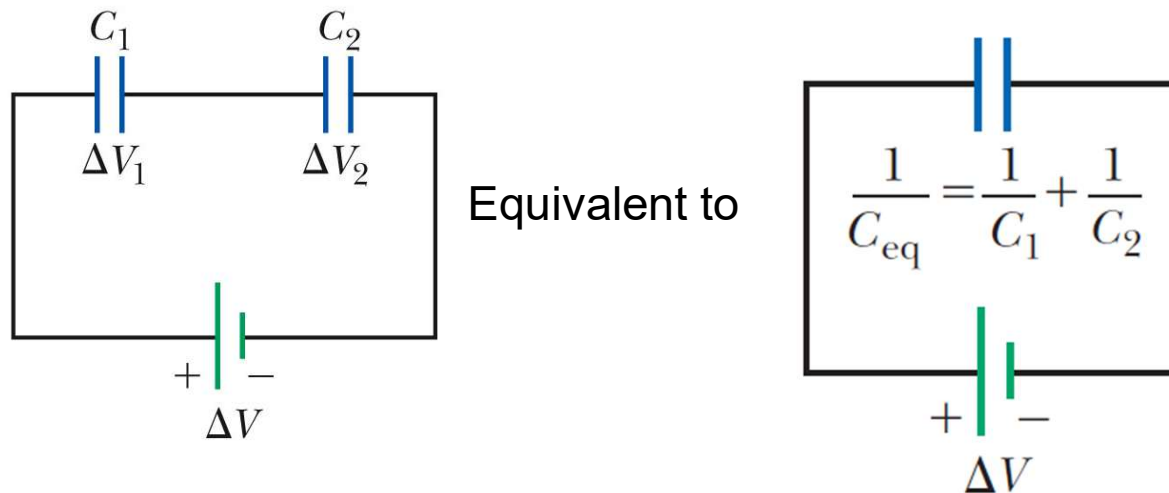
The charge on each capacitor is same.

- $Q_1 = Q_2 = Q$



Capacitors in Series (Equivalent capacitance)

The capacitors can be represented by one capacitor with a capacitance of C_{eq} .



$$(1) Q_1 = Q_2 = Q$$

$$(2) \Delta V_{tot} = \Delta V_1 + \Delta V_2$$

$$\begin{aligned} Q/C_{eq} &= \Delta V_{tot} = \Delta V_1 + \Delta V_2 \\ &= Q_1/C_1 + Q_2/C_2 \end{aligned}$$

Therefore,

$$1/C_{eq} = 1/C_1 + 1/C_2$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

(**series** combination)

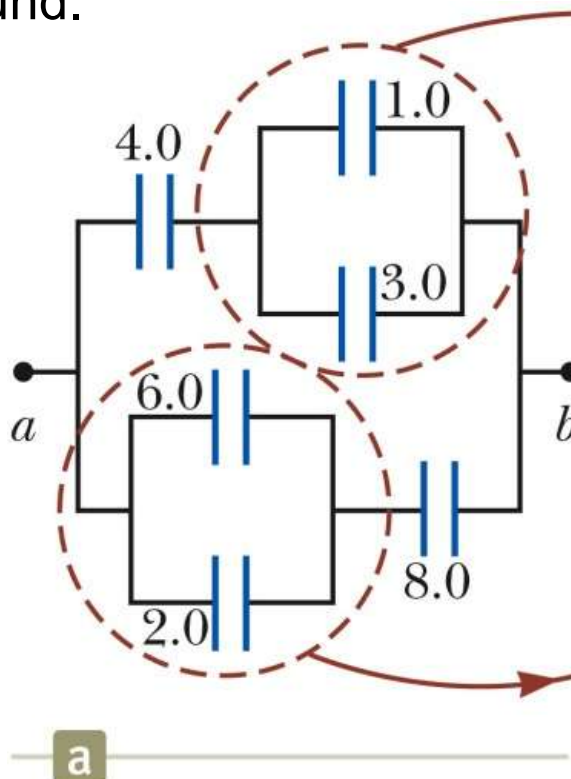
Equivalent Capacitance, Example

The 1.0-mF and 3.0-mF capacitors are in parallel.

The 6.0-mF and 2.0-mF capacitors are in parallel.

These parallel combinations are in series with the capacitors next to them.

The series combinations are in parallel and the final equivalent capacitance can be found.



Energy in a Capacitor – Overview

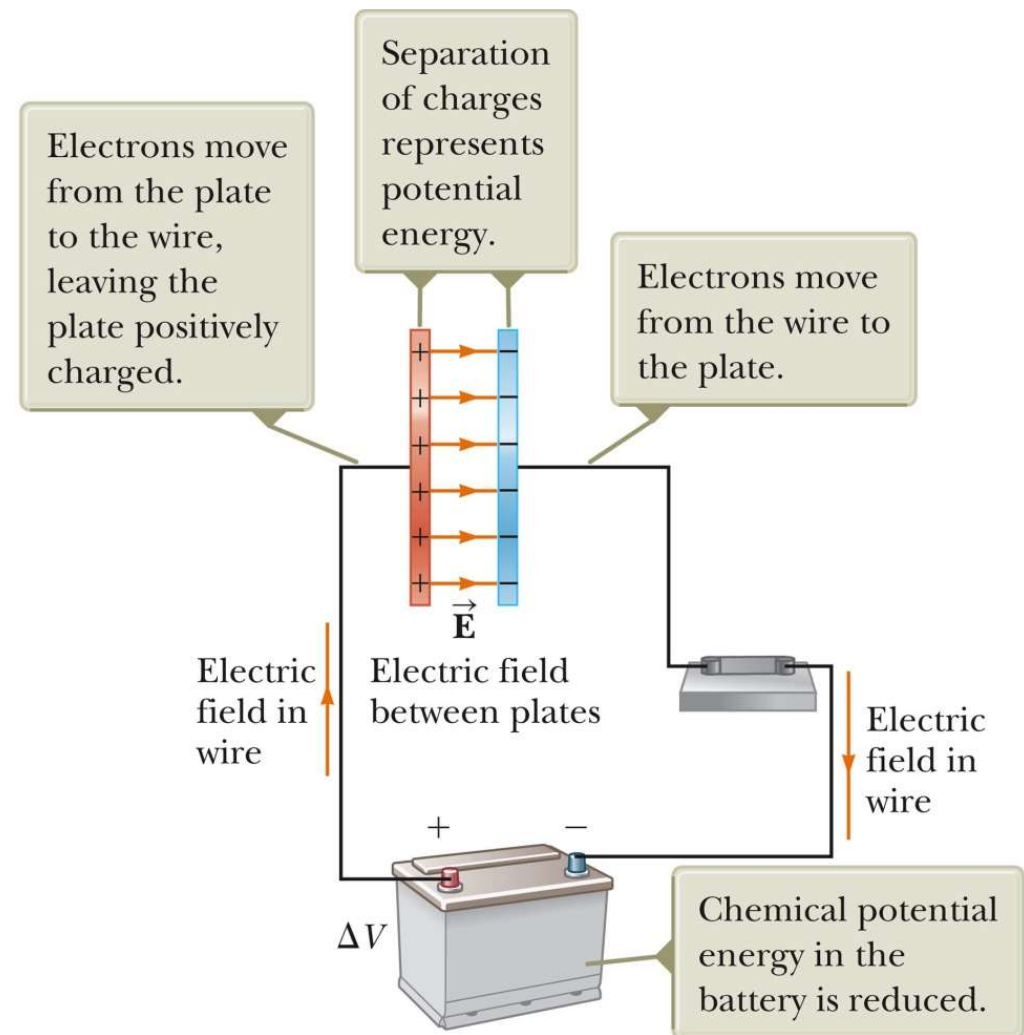
Consider the circuit to be a system.

Before the switch is closed, the energy is stored as chemical energy in the battery.

When the switch is closed, the energy is transformed from chemical potential energy to electric potential energy.

The electric potential energy is related to the separation of the positive and negative charges on the plates.

A capacitor can be described as a device that stores energy as well as charge.



Energy Stored in a Capacitor

Assume the capacitor is being charged and, at some point, has a charge q on it.

The work needed to transfer a charge (dq) from one plate to the other is

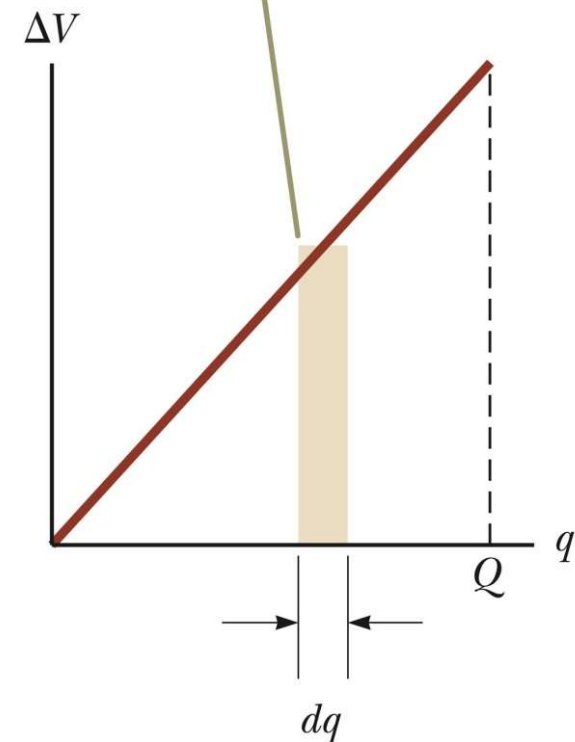
$$dW = \Delta V dq = \frac{q}{C} dq$$

The total work required is the area of the shaded rectangle.

The total work required is

$$W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$

The work required to move charge dq through the potential difference ΔV across the capacitor plates is given approximately by the area of the shaded rectangle.



Energy, cont

The work done in charging the capacitor appears as electric potential energy U :

$$U = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2$$

This applies to a capacitor of any geometry.

The energy stored increases as the charge increases and as the potential difference increases.

In practice, there is a maximum voltage before discharge occurs between the plates.

The energy can also be considered to be stored in the electric field.

For a parallel-plate capacitor, the energy can be expressed in terms of the field as

$$U = \frac{1}{2} (\epsilon_0 A/d) (E^2 d^2) = \frac{1}{2} (\epsilon_0 A d) E^2.$$

It can also be expressed in terms of the energy density (energy per unit volume)

$$u_E = U/Ad = \frac{1}{2} \epsilon_0 E^2.$$

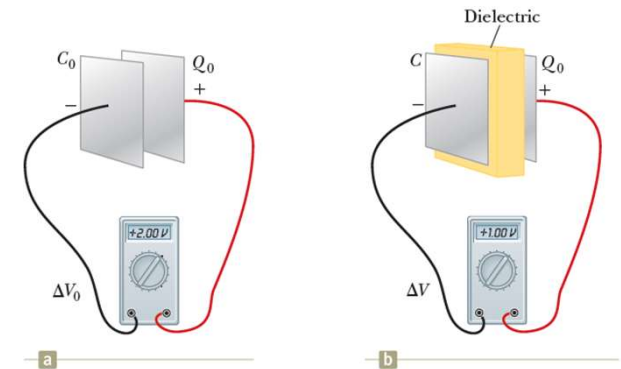
Capacitors with Dielectrics

A dielectric is a non-conducting material that, when placed between the plates of a capacitor, increases the capacitance.

- Dielectrics include rubber, glass, and waxed paper

With a dielectric, the capacitance becomes

$$C = \kappa C_0$$



If the same charge is kept $Q = Q_0$, the potential difference is reduced $\Delta V = \Delta V_0 / \kappa$.

- The capacitance increases by the factor κ when the dielectric completely fills the region between the plates.
- κ is the dielectric constant of the material.

If the capacitor remains connected to a battery, the voltage across the capacitor necessarily remains the same.

If the capacitor is disconnected from the battery, the capacitor is an isolated system and the charge remains the same.

Dielectrics, cont

For a parallel-plate capacitor,

$$C = \kappa (\epsilon_0 A) / d$$

In theory, d could be made very small to create a very large capacitance.

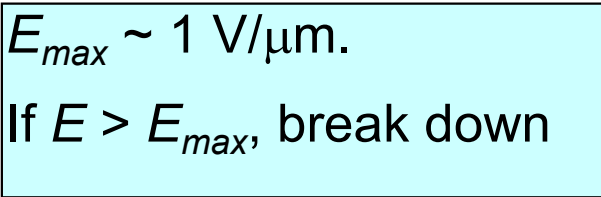
In practice, there is a limit to d .

- d is limited by the electric discharge that could occur through the dielectric medium separating the plates.

For a given d , the maximum voltage that can be applied to a capacitor without causing a discharge depends on the **dielectric strength** of the material.

Dielectrics provide the following advantages:

- Increase in capacitance
- Increase the maximum operating voltage
- Possible mechanical support between the plates
 - This allows the plates to be close together without touching.
 - This decreases d and increases C .


$$E_{max} \sim 1 \text{ V}/\mu\text{m}.$$

If $E > E_{max}$, break down

Some Dielectric Constants and Dielectric Strengths

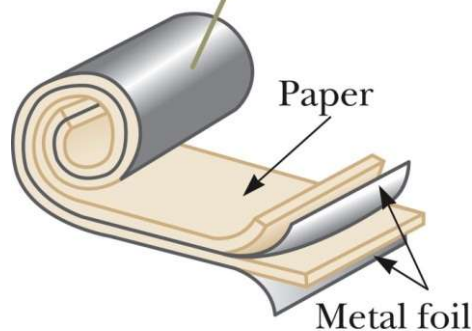
TABLE 26.1 *Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature*

Material	Dielectric Constant κ	Dielectric Strength ^a (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

^aThe dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. These values depend strongly on the presence of impurities and flaws in the materials.

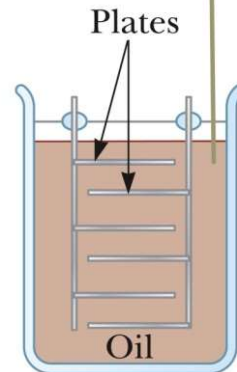
Types of Capacitors – Tubular, oil filled and electrolytic (Optional)

A tubular capacitor whose plates are separated by paper and then rolled into a cylinder

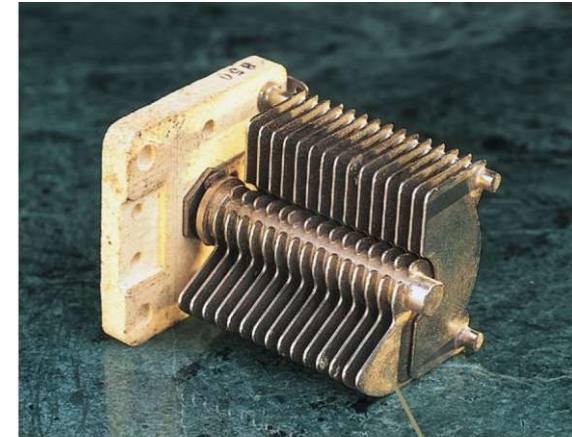


a

A high-voltage capacitor consisting of many parallel plates separated by insulating oil



b



When one set of metal plates is rotated so as to lie between a fixed set of plates, the capacitance of the device changes.

c

(a) Metallic foil interlaced with thin sheets of paraffin-impregnated paper or Mylar. The layers are rolled into a cylinder to form a small package for the capacitor.

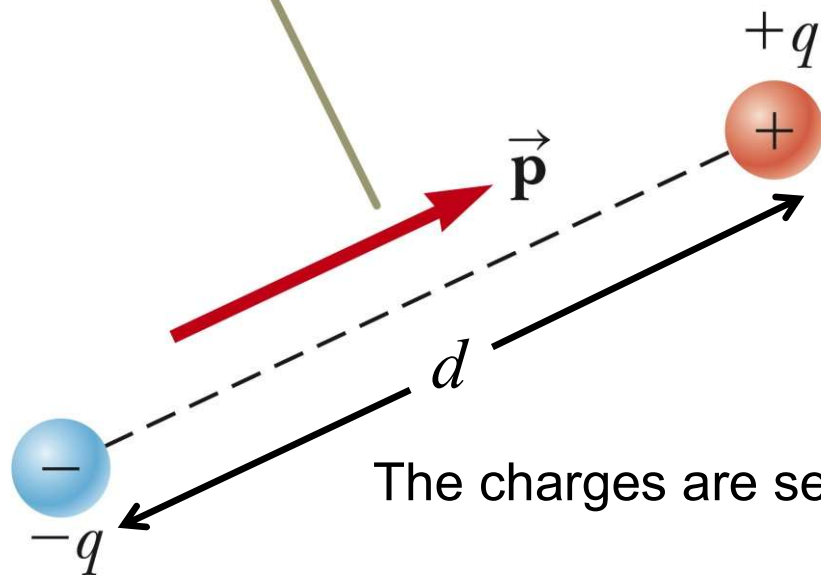
(b) High-voltage capacitors. A number of interwoven metallic plates are immersed in silicon oil.

(c) Variable capacitors consist of two interwoven sets of metallic plates. One plate is fixed and the other is movable (area changes). Contain air as the dielectric. C varying from 10 to 500 pF.

Electric Dipole (Optional)

An electric dipole consists of two charges of equal magnitude and opposite signs.

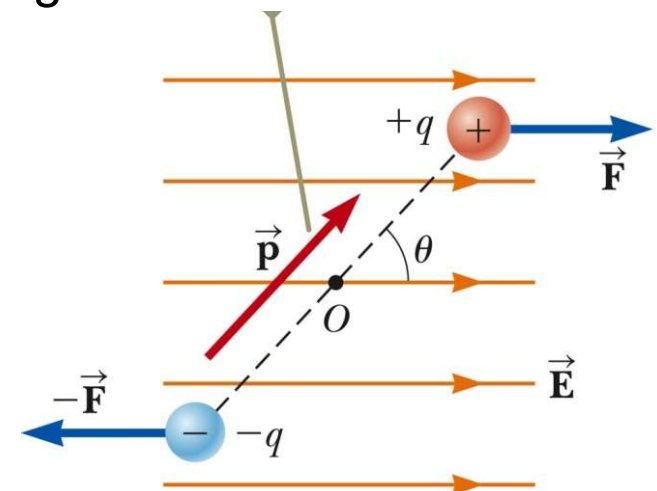
The electric dipole moment \vec{p} is directed from $-q$ toward $+q$.



The charges are separated by d .

The electric dipole moment (\vec{p}) has a magnitude of $p \equiv qd$

An electric field can cause a dipole to change its orientation.



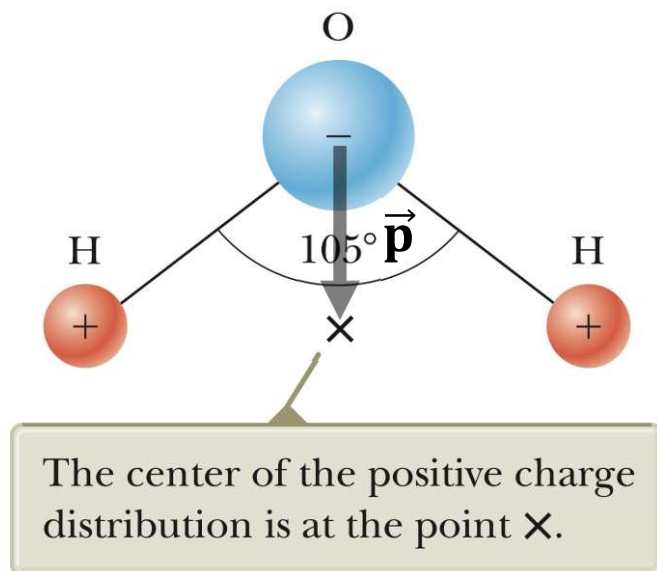
Polar vs. Nonpolar Molecules (Optional)

Before we apply electric field,

- a molecule with a net dipole moment is called a **polar molecule**.
- a molecule without a net dipole moment is called a **nonpolar molecule**.

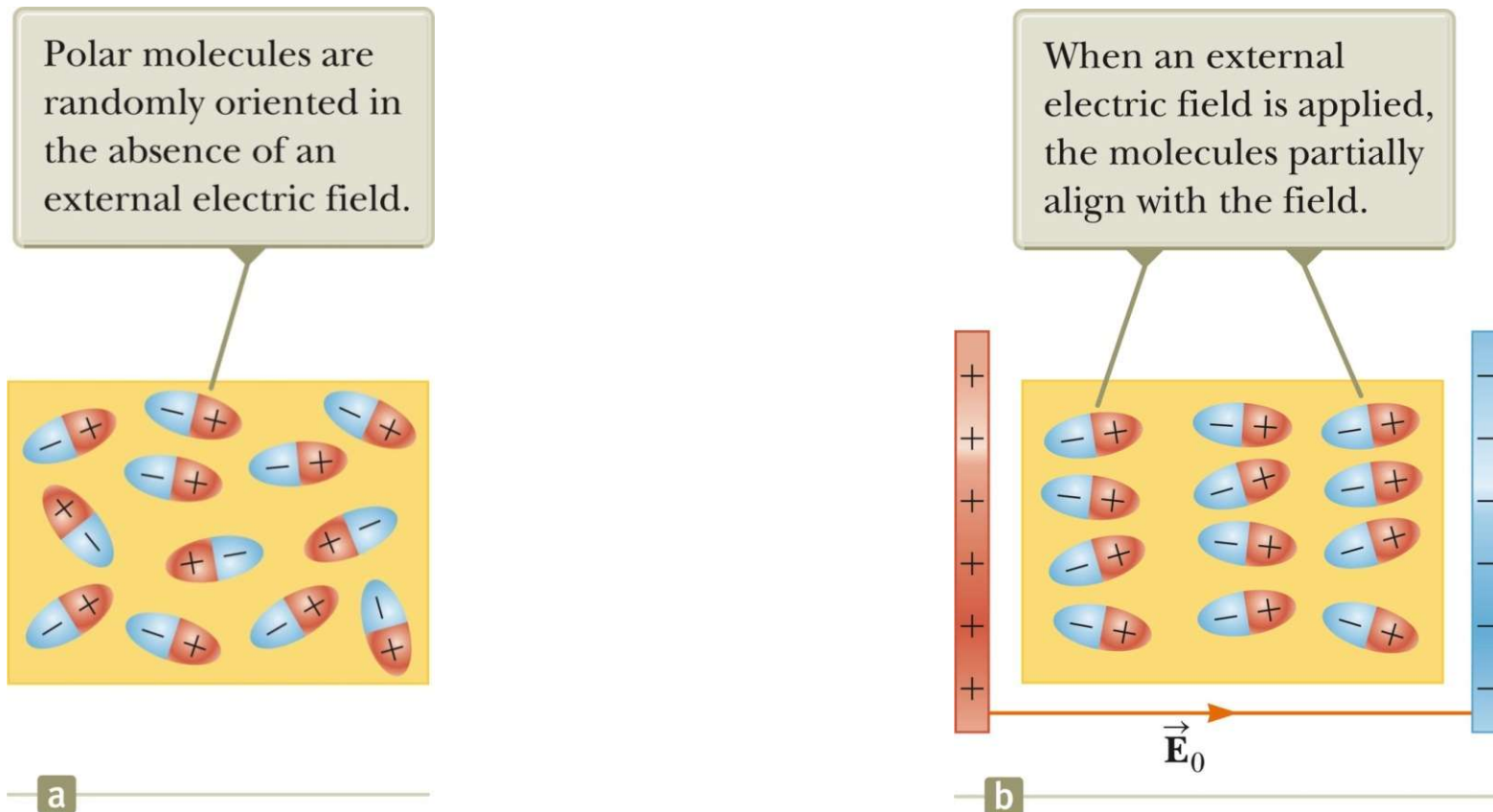
Example:

Water molecule



- A water molecule is an example of a polar molecule.
- The center of the negative charge is near the center of the oxygen atom.
- The x is the center of the positive charge distribution.
- The dipole moment is pointing downward

Dielectrics with polar molecules – An Atomic View



The polar molecules that make up the dielectric are modeled as dipoles.

The molecules are randomly oriented in the absence of an electric field.

An external electric field is applied - > torque

The molecules partially align with the field.

The degree of alignment depends on temperature and the magnitude of the field.

Dielectrics with nonpolar molecules – An Atomic View, 2

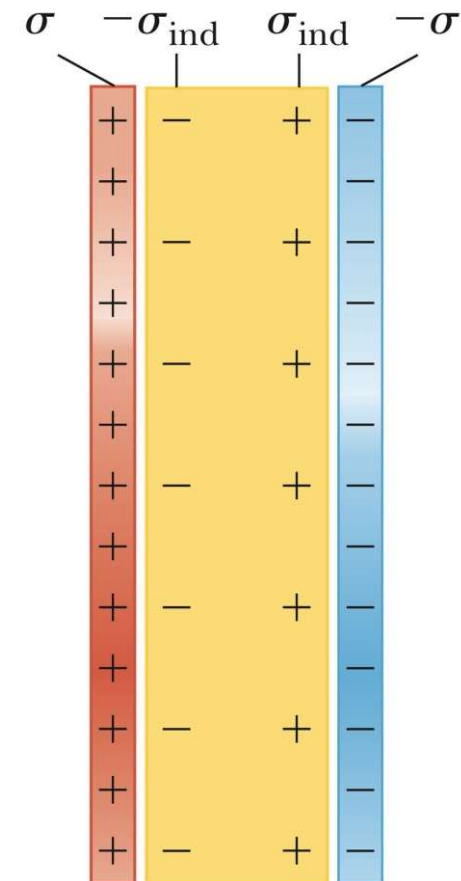
If the molecules of the dielectric are nonpolar molecules, the electric field produces some charge separation.

This produces an *induced dipole moment*.
The effect is then the same as if the molecules were polar.

As a result, an external field can polarize the dielectric whether the molecules are polar or nonpolar.

The charged edges of the dielectric act as a second pair of plates producing an induced electric field in the direction opposite the original electric field.

The induced charge density σ_{ind} on the dielectric is *less* than the charge density σ on the plates.



Induced Charge and Field

The electric field due to the plates is directed to the right and it polarizes the dielectric.

The net effect on the dielectric is an induced surface charge that results in an induced electric field.

If the dielectric were replaced with a conductor, the net field between the plates would be zero.

The charged edges of the dielectric can be modeled as an additional pair of parallel plates establishing an electric field \vec{E}_{ind} in the direction opposite that of \vec{E}_0 .

