$\cancel{\vec{a_0} = [a^0_0, a^1_0, a^2_0, \quad a^n_0]^T}$

$\cancel{\vec{b_{01}} = [b}$

$$\vec{a_0} = [a^0_0, a^0_1, a^0_2, \cdots, a^0_n]^T$$

$$\vec{b_1} = [b'_0, b'_1, b'_2, \cdots, b'_n]^T$$

$$W' = \begin{bmatrix} w'_{0,0} & w'_{0,1} & w'_{0,2} \cdots & w'_{0,n} \\ w'_{1,0} & w'_{1,1} & w'_{1,2} \cdots & w'_{1,n} \\ \vdots & & & \\ w'_{n,0} & w'_{n,1} & w'_{n,2} \cdots & w'_{n,n} \end{bmatrix}$$

$$\vec{a_1} = W'\vec{a_0} + \vec{b_1}$$

so, $a'_0 = a^0_0 w'_{0,0} + a^0_1 w'_{0,1} + a^0_2 w'_{0,2} + \cdots + a^0_n w'_{0,n} + b'_0 = \sum_0^n a^0_i w'_{0,i} + b'_0$

$a'_{\cancel{0}1} = a^0_0 w'_{1,0} + a^0_1 w'_{1,1} + a^0_2 w'_{1,2} + \cdots + a^0_n w'_{1,n} + b'_1 = \sum_0^n a^0_i w'_{1,i} + b'_1$

$\vdots$

$a'_n = a^0_0 w'_{n,0} + a^0_1 w'_{n,n} + a^0_2 w'_{n,2} + \cdots + a^0_n w'_{n,n} + b'_n = \sum_0^n a^0_i w'_{n,i} + b'_n$

$$\vec{a_2} = W^2\vec{a_1} + \vec{b_2}$$

so, $a^2_0 = a'_0 w^2_{0,0} + a'_1 w^2_{0,1} + a'_2 w^2_{0,2} + \cdots + a'_n w^2_{0,n} + b^2_0$

$= (a^0_0 w'_{0,0} + a^0_1 w'_{0,1} + a^0_2 w'_{0,2} + \cdots + a^0_n w'_{0,n} + b'_0) w^2_{0,0}$

$+ (a^0_0 w'_{1,0} + a^0_1 w'_{1,1} + a^0_2 w'_{1,2} + \cdots + a^0_n w'_{1,n} + b'_1) w^2_{0,1}$

$+ (a^0_0 w'_{2,0} + a^0_1 w'_{2,1} + a^0_2 w'_{2,2} + \cdots + a^0_n w'_{2,n} + b'_2) w^2_{0,2}$

$+ \cdots + (a^0_0 w'_{n,0} + a^0_1 w'_{n,1} + a^0_2 w'_{n,2} + \cdots + a^0_n w'_{n,n} + b'_n) w^2_{0,n} + b^2_0$

$= a^0_0 (w'_{0,0} w^2_{0,0} + w'_{1,0} w^2_{0,1} + w'_{2,0} w^2_{0,2} + \cdots + w'_{n,0} w^2_{0,n})$

$+ a^0_1 (w'_{0,1} w^2_{0,0} + w'_{1,1} w^2_{0,1} + w'_{2,1} w^2_{0,2} + \cdots + w'_{n,1} w^2_{0,n})$

$+ a^0_2 (w'_{0,2} w^2_{0,0} + w'_{1,2} w^2_{0,1} + w'_{2,2} w^2_{0,2} + \cdots + w'_{n,2} w^2_{0,n})$

$+ \cdots$

$$+ a^0_n (w^1_{0,n} w^2_{0,0} + w^1_{1,n} w^2_{0,1} + w^1_{2,n} w^2_{0,2} + \cdots + w^1_{n,n} w^2_{0,n})$$

$$+ (b^1_0 w^2_{0,0} + b^1_1 w^2_{0,1} + b^1_2 w^2_{0,2} + \cdots + b^1_n w^2_{0,n}) + b^2_0$$

if $\vec{a}_2 = W^{12} \vec{a}_0 + \vec{b}_{12}$

then $w^{12}_{0,0} = w^1_{0,0} w^2_{0,0} + w^1_{1,0} w^2_{0,1} + w^1_{2,0} w^2_{0,2} + \cdots + w^1_{n,0} w^2_{0,n} = \sum_0^n w^1_{i,0} w^2_{0,i}$

$w^{12}_{0,1} = w^1_{0,1} w^2_{0,0} + w^1_{1,1} w^2_{0,1} + w^1_{2,1} w^2_{0,2} + \cdots + w^1_{n,1} w^2_{0,n} = \sum_0^n w^1_{i,1} w^2_{0,i}$

...

$w^{12}_{0,n} = w^1_{0,n} w^2_{0,0} + w^1_{1,n} w^2_{0,1} + w^1_{2,n} w^2_{0,2} + \cdots + w^1_{n,n} w^2_{0,n} = \sum_0^n w^1_{i,n} w^2_{0,i}$

$$W^{12} = \Big[\sum_0^n w^1_{i,0} w^2_{0,i},\ \sum_0^n w^1_{i,1} w^2_{0,i},\ \sum_0^n w^1_{i,2} w^2_{0,i}, \ldots, \sum_0^n w^1_{i,n} w^2_{0,i};$$

$$\sum_0^n w^1_{i,0} w^2_{1,i},\ \sum_0^n w^1_{i,1} w^2_{1,i},\ \sum_0^n w^1_{i,2} w^2_{1,i}, \ldots, \sum_0^n w^1_{i,n} w^2_{1,i};$$

...

$$\sum_0^n w^1_{i,0} w^2_{n,i},\ \sum_0^n w^1_{i,1} w^2_{n,i},\ \sum_0^n w^1_{i,2} w^2_{n,i}, \ldots, \sum_0^n w^1_{i,n} w^2_{n,i} \Big]$$

that is $W^{12}_{x,y} = \sum_0^n w^1_{i,y} w^2_{x,i}$

$\cancel{b_{12}}\ b^{12}_x = (b^1_0 w^2_{x,0} + b^1_1 w^2_{x,1} + b^1_2 w^2_{x,2} + \cdots + b^1_n w^2_{x,n}) + b^2_x = \sum_0^n b^1_i w^2_{x,i} + b^2_x$

$$\cancel{W^{123}_{x,y} = \left(\sum_0^n W^{12}_{i,y}\right) w^3_{x,i}}$$

"RECURSION"

$$\cancel{W^{123}_{x,y} = \sum_0^n (} \qquad W^{123}_{x,y} = \sum_0^n \left(W^{12}_{i,y} w^3_{x,i} \right)$$

$$= W^{12}_{0,y} w^3_{x,0} + W^{12}_{1,y} w^3_{x,1} + W^{12}_{2,y} w^3_{x,2} + \cdots + W^{12}_{n,y} w^3_{x,n}$$

$$= \cancel{\sum_0^n w^1_{j,y} w^2}$$

$$\sum_0^n w^1_{j,y} w^2_{0,j} w^3_{x,0} + \sum_0^n w^1_{j,y} w^2_{1,j} w^3_{x,1} + \sum_0^n w^1_{j,y} w^2_{2,j} w^3_{x,2} + \cdots + \sum_0^n w^1_{j,y} w^2_{n,j} w^3_{x,n}$$

$$= \sum_0^n \sum_0^n w^1_{j,y} w^2_{0,j} w^3_{x,i}$$

$$\vec{b_3} =$$

$$b^{123}_x = \left(b^{12}_0 w^3_{x,0} + b^{12}_1 w^3_{x,1} + b^{12}_2 w^3_{x,2} + \cdots + b^{12}_n w^3_{x,n}\right) + b^3_x$$

$$= (b^1_0 w^2_{0,0} + b^1_1 w^2_{0,1} + b^1_2 w^2_{0,2} + \cdots + b^1_n w^2_{0,n} + b^2_0) w^3_{x,0}$$

$$+ (b^1_0 w^2_{1,0} + b^1_1 w^2_{1,1} + b^1_2 w^2_{1,2} + \cdots + b^1_n w^2_{1,n} + b^2_1) w^3_{x,1}$$

$$+ (b^1_0 w^2_{2,0} + b^1_1 w^2_{2,1} + b^1_2 w^2_{2,2} + \cdots + b^1_n w^2_{2,n} + b^2_2) w^3_{x,2}$$

$$+ \cdots + (b^1_0 w^2_{n,0} + b^1_1 w^2_{n,1} + b^1_2 w^2_{n,2} + \cdots + b^1_n w^2_{n,n} + b^2_n) w^3_{x,n} + b^3_x$$

$$= b^1_0 (w^2_{0,0} w^3_{x,0} + w^2_{1,0} w^3_{x,1} + w^2_{2,0} w^3_{x,2} + \cdots + w^2_{n,0} w^3_{x,n})$$

$$+ b^1_1 (w^2_{0,1} w^3_{x,0} + w^2_{1,1} w^3_{x,1} + w^2_{2,1} w^3_{x,2} + \cdots + w^2_{n,1} w^3_{x,n})$$

$$+ b^1_2 (w^2_{0,2} w^3_{x,0} + w^2_{1,2} w^3_{x,1} + w^2_{2,2} w^3_{x,2} + \cdots + w^2_{n,2} w^3_{x,n})$$

$$+ \cdots + b^1_n (w^2_{0,n} w^3_{x,0} + w^2_{1,n} w^3_{x,1} + w^2_{2,n} w^3_{x,2} + \cdots + w^2_{n,n} w^3_{x,n}) + b^3_x$$

$$\cancel{= \sum b^1_i w^2} = \sum_0^n \sum_0^n b^1_j w^2_{i,j} w^3_{x,i} + b^3_x$$