d-dimensional

$\perp_1 \quad \perp_2 \quad \perp_3 \quad \cdots \quad \perp_{d-1} \quad \perp_d$

index i

$(x_1, x_2, x_3, \cdots, x_{d-1}, x_d)$

(1) index → coordinate

$x_d = i \,//\, (\perp_1 * \perp_2 * \perp_3 * \cdots * \perp_{d-1}) = i \,//\, (\prod_1^{d-1} \perp_i)$

$x_{d-1} = \left(i \bmod (\perp_1 * \perp_2 * \perp_3 * \cdots * \perp_{d-1}) \right) // (\perp_1 * \perp_2 * \cdots * \perp_{d-2}) = \left(i \bmod (\prod_0^{d-1} \perp_i) \right) // \prod_0^{d-2} \perp_i$

$x_{d-2} = (\sout{i \bmod (\perp_1 * \perp_2 * \cdots} = \left(\left(i \bmod (\prod_0^{d-1} \perp_i) \right) \bmod (\prod_0^{d-2} \perp_i) \right) // \prod_0^{d-3} \perp_i$

...

$x_2 = (\cdots (i \bmod (\prod_0^{d-1} \perp_i)) \bmod (\prod_0^{d-2} \perp_i)) \bmod \cdots) // \perp_1$

$x_1 = (\cdots (i \bmod (\prod_0^{d-1} \perp_i)) \bmod (\prod_0^{d-2} \perp_i)) \bmod \cdots) \bmod \perp_1$

(2) coordinate → index

$(x_1, x_2, \cdots, x_d)$

$i = x_1 + x_2 * \perp_1 + x_3 * (\prod_0^{2} \perp_i) + x_4 * (\prod_0^{3} \perp_i) + \cdots + x_d * (\prod_0^{d-1} \perp_i)$