13 Large-scale Global Optimization

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Large-scale Global Optimization

Complex Optimization Problems

- One of the manifestations of complexity is that the increase in the number of variables causes the dimensionality disaster.
- Large-scale global optimization (LSGO) problems usually contains more than 1000 variables.

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Application Examples

- The large-scale power system design
- Vehicle routing problems
- Genetic identification
- Inverse chemical kinetics problem

Contents

Large-scale Global Optimization Problems-Definition

Complex Optimization Problems

- To effectively solve the large-scale global optimization problem, we normally do not search in the original search space.
- According to the correlation between variables, the problem can be separable, partially separable or non-separable.

Fully separable function

$f(\boldsymbol{x})$ is a **fully separable** function iff (if and only if)

$$\min / \max f(\boldsymbol{x}) = (f(x_1), f(x_2), \cdots, f(x_n)) \tag{1}$$

where $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$ is a D-dimensional decision vector.

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Partially separable function

$f(\boldsymbol{x})$ is a **partially separable** function with m independent subcomponents iff

$$\min / \max f(\boldsymbol{x}) = \left(f\left(\boldsymbol{x_1}, \cdots\right), \cdots, f\left(\cdots, \boldsymbol{x_m}\right)\right) \tag{2}$$

where $\boldsymbol{x_1}, \cdots, \boldsymbol{x_m}$ are disjoint sub-vectors of $\boldsymbol{x}$ and $2 \leq m < n$.

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Fully non-separable function

$f(x)$ is a **fully non-separable** function, if every pair of its decision variables interacts with each other.

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Partially additively separable function

$f(\boldsymbol{x})$ is a **partially additively separable** function iff

$$f(\boldsymbol{x}) = \sum_{i=1}^{m} f_i\left(\boldsymbol{x_i}\right) \tag{3}$$

where $\boldsymbol{x_i}$ are mutually exclusive decision vectors of f_i ; m is the number of separable subcomponents.

Large-scale Global Optimization Problems-Difficulties

Difficulties

Non-separable and **overlapping** functions are the most difficult to solve:

- The search space generally increases exponentially.
- The algorithm is easy to fall into local optima.
- The objective function generally has the characteristics of nonlinear, non-convex, multi-modal and non-differentiable.
- Variables are partially separable or completely non-separable.

It is also challenging for EAs to explore the entire search space effectively.

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Two branches of metaheuristics

- Space decomposition methods (co-evolution).
- Non-decomposition-based methods.

Co-evolution Methods

Methods Mechanism

- Decompose the large-scale global optimization problem into several low-dimensional sub-problems.
- Use the divide-and-conquer method to solve the large-scale optimization problem.

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Challenge

- The interaction between variables in indivisible problems.
- It will have a great influence on the optimization efficiency of the algorithm.

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Solution

Most decomposition methods try to identify the interacting variables and assign them to the same in the sub-question.

Co-operative co-evolutionary (CC) approach

Potter and De Jong proposed the co-operative co-evolutionary (CC) approach in 1994 to improve the performance of GAs.

Co-evolution Methods-Co-operative Co-evolution

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The framework of classic CC algorithm

- Problem decomposition.
- Subcomponent optimization.
- Cooperative combination.

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Categories

Based on different grouping strategies, the co-evolution algorithms used to solve LSGO problems are mainly divided into two categories: static grouping methods and dynamic grouping methods.

Limitations of CC algorithm

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Solutions

- Liu et al combined CC framework with FEP (fast evolutionary programming) to solve the problem of 100–1000 dimensional continuous optimization.
- Scholars tried to combine the CC idea with swarm intelligence-based algorithms, such as CPSO-SK and CPSO-HK.

Co-evolution Methods-Static Grouping

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Limitations of Static grouping

Only effective in low-dimensional problems.

Co-evolution Methods-Dynamic Grouping

Relationships between static and dynamic Grouping

- They both try to detect the interacting relationship between variables and assign the variables that interacts each other to the same sub-component.
- In static grouping, the number of sub-components (k) is fixed, while in dynamic grouping, the structure of sub-components can be dynamically adjusted.

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Categories

Dynamic grouping methods can be divided into two categories based on the substitution of variables in group components: random dynamic grouping and learning-based dynamic grouping.

DECC-G algorithm

Algorithm 1: DECC-G algorithm

while *termination criterion is not fulfilled* **do**

  Set $i = 0$;

  The n-dimensional object vector is randomly divided into m s-dimensional subcomponents;

  while $i < m$ **do**

    $i++$;

    Evolve the ith subcomponent with a certain EA;

  Assign a weight vector for each subcomponent;

  Optimize them via a certain EA;