

Policy Learning for Active Target Tracking over Continuous $SE(3)$ Trajectories

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Abstract

This paper proposes a novel *model-based policy gradient algorithm* for tracking dynamic targets using a mobile robot, equipped with an onboard sensor with limited field of view. The task is to obtain a continuous control policy for the mobile robot to collect sensor measurements that reduce uncertainty in the target states, measured by the target distribution entropy. We design a neural network control policy with the robot $SE(3)$ pose and the mean vector and information matrix of the joint target distribution as inputs and attention layers to handle variable numbers of targets. We also derive the gradient of the target entropy with respect to the network parameters explicitly, allowing efficient model-based policy gradient optimization.

Keywords: Active target tracking, model-based reinforcement learning, SLAM

1. Introduction

Active target tracking serves as a problem to reduce the uncertainty in the target state of interest by planning the trajectory of the sensing robot gathering information about the dynamic target state, which is motivated by several applications including search-and-rescue (Kumar et al., 2004), security and surveillance (Grocholsky et al., 2006), wildfire detection (Julian and Kochenderfer, 2019), pursuit-evasion (Chung et al., 2011), and so on. Under a static target state, an analogous active information acquisition problem has been applied to mapping an unknown environment and Simultaneous Localization and Mapping (SLAM) (Cadena et al., 2016; Placed et al., 2022). The difficulty of the general active target tracking problem is inherent in predicting the future target state, optimizing the sensing robot trajectory with a limited Field of View (FoV), and taking into account the stochasticity of the target motion and the observation from the sensor.

While the general active target tracking problem is posed as a stochastic optimal control problem due to the probabilistic inference of the uncertainty reduction, there has been some earlier work that relieves the complexity. Under the assumption of a linear Gaussian target motion model, active target tracking with information-theoretic cost has resulted in the deterministic optimal control problem by Le Ny and Pappas (2009). Atanasov et al. (2014) proposed a computationally efficient approach for solving the non-myopic planning with a strong performance guarantee even under a long planning horizon. Schlotfeldt et al. (2019) developed a consistent and admissible heuristic for applying A^* search to the active information acquisition with deriving maximum upper bounds in the information measure. The multi-agent formulation with tracking multiple targets for the active information acquisition and its scalable approaches have been studied by Atanasov et al. (2015);

Schlotfeldt et al. (2018); Kantaros et al. (2019); Cai et al. (2021). While those works consider planning over discrete control space, Koga et al. (2021) proposed "iterative Covariance Regulation (iCR)", which optimizes the trajectory over continuous $SE(3)$ space by deriving analytical gradient of the cost with respect to the multi-step control sequence. Extensions of the work to occlusion-aware planning and to active SLAM under uncertain robot state have been proposed by Asghari-vaskasi et al. (2022) and Koga et al. (2022), respectively. However, all the aforementioned work provides a control input computed for a given environment, which cannot be directly applicable to a new environment without replanning.

Learning a control policy from several environments and data has been studied as reinforcement learning (RL) (Sutton and Barto, 2018). RL methods associated with the deep neural networks have been developed under both discrete control space (Mnih et al., 2015) applied to games and continuous control (Lillicrap et al., 2015; Schulman et al., 2017) applied to various robotics tasks. Learning policy for active target tracking has been proposed by Jeong et al. (2019) via Q -learning to maximize the mutual information between sensor data and the target states. Hsu et al. (2021) have developed a multi-agent version of Jeong et al. (2019) by incorporating attention-block in Q -network architecture for multi-robot active information acquisition problem. As other active perception problems, Chen et al. (2020) have focused on active landmark mapping using a graph neural network representing the exploration policy, with Q -learning within a framework of Expectation Maximization (Wang and Englot, 2020). Chaplot et al. (2020) propose a modular and hierarchical approach to obtain a local policy by imitation learning from analytical path planners with a learned SLAM module and a global policy to maximize area coverage. Lodel et al. (2022) apply PPO (Schulman et al., 2017) for learning an information-theoretic active mapping policy to acquire reference viewpoints that maximize the reward with local sensing of obstacles and the robot position. Yang et al. (2022) proposes a learning method in continuous trajectory for active perception to localize multiple static landmarks by applying differentiable FoV to reward shaping and designing the attention-based network architecture. However, when actuating low-level controller (e.g., velocity, torque) which drives the robot motion over $SE(3)$ kinematic space, learning the control policy by model-free RL methods often fail or struggle with convergence of the policy, or at least require sufficiently large amount of data especially for complicated tasks.

Utilizing a known or predicted state transition model in learning algorithms can significantly reduce the required amount of samples and computation relative to the model-free RL methods. Levine and Koltun (2013) has developed a guided policy search that optimizes the system trajectory associated with the model by Differential Dynamic Programming (DDP) for direct policy learning in control. Several variants and extensions of the guided policy search have been proposed by Levine and Abbeel (2014) for policy under unknown dynamics and by Levine et al. (2016) for end-to-end policy from visual sensing to robot action. Luo et al. (2019) has incorporated a kinematic model of force/torque information into RL for enabling high-precision tasks in the robot manipulator. A recent comprehensive review of the model-based RL is presented by Janner et al. (2019).

Contributions: The contributions of the paper are summarized below:

- This paper proposes a novel model-based policy gradient algorithm for tracking multiple dynamics targets over continuous $SE(3)$ trajectories. The differentiable FoV formulation is incorporated to enable offline learning under sensor models with limited FoV.
- The network architecture is designed with the attention block to handle multiple targets, and with padding and masking to enable learning over a varying number of targets during training.

2. Problem Statement

Consider a robot with pose $T_k \in SE(3) \subset \mathbb{R}^{4 \times 4}$ at time $t_k \in \mathbb{R}_+$, where $\{t_k\}_{k=0}^K$ for some $K \in \mathbb{N}$ is an increasing sequence. The definition of pose and its discrete-time kinematic model are:

$$T_k := \begin{bmatrix} R_k & \mathbf{p}_k \\ \mathbf{0}_{3 \times 1}^\top & 1 \end{bmatrix}, \quad T_{k+1} = T_k \exp(\tau_k \hat{\mathbf{u}}_k), \quad (1)$$

where $\mathbf{p}_k \in \mathbb{R}^3$ is position, $R_k \in SO(3) \subset \mathbb{R}^{3 \times 3}$ is orientation, $\tau_k := t_{k+1} - t_k > 0$ is the sampling-time interval, and $\mathbf{u}_k = [\mathbf{v}_k^\top, \boldsymbol{\omega}_k^\top]^\top \in \mathbb{R}^6$ is a control input, consisting of linear velocity $\mathbf{v}_k \in \mathbb{R}^3$ and angular velocity $\boldsymbol{\omega}_k \in \mathbb{R}^3$. The hat operator $\hat{\cdot} : \mathbb{R}^6 \rightarrow se(3)$ maps vectors in \mathbb{R}^6 to the Lie algebra $se(3)$ associated with the $SE(3)$ Lie group (Barfoot, 2017).

We consider a finite number of moving targets $\mathbf{y}_k = [\mathbf{y}_k^{(1)}, \dots, \mathbf{y}_k^{(n_l)}]$, where $\mathbf{y}_k^{(j)} \in \mathbb{R}^{n_y}$ for $j \in \{1, \dots, n_l\}$ denotes the n_y -dimensional state of j -th target at time k and n_l is the total number of targets. Assuming that the target has a homogeneous dynamics governed by a linear Gaussian process, each target dynamics is given as

$$\mathbf{y}_{k+1}^{(j)} = A\mathbf{y}_k^{(j)} + B\xi_k^{(j)} + \mathbf{w}_k^{(j)}, \quad (2)$$

where $A : \mathbb{R}^{n_y \times n_y}$ and $B : \mathbb{R}^{n_y \times m_y}$ are the system matrices, $\xi_k^{(j)} \in \mathbb{R}^{m_y}$ is a known target input, and $\mathbf{w}_k^{(j)} \sim \mathcal{N}(0, W_k)$ is an unknown process noise assumed to be Gaussian with zero mean and the covariance $W_k \in \mathbb{R}^{n_y \times n_y}$.

The onboard sensor attached to the robot has a limited FoV for sensing the targets. Let $\mathcal{F} \subset \mathbb{R}^3$ represent the FoV of the onboard sensor within the robot's body frame. The set of target indices within the FoV is:

$$\mathcal{I}_{\mathcal{F}}(T, \{\mathbf{y}^{(j)}\}) = \left\{ j \in \{1, \dots, n_l\} \mid \mathbf{q}(T, \zeta(\mathbf{y}^{(j)})) \in \mathcal{F} \right\} \quad (3)$$

where $\zeta : \mathbb{R}^{n_y} \rightarrow \mathbb{R}^3$ transforms the target state to the 3-D coordinate of the target's location, and $\mathbf{q} : SE(3) \times \mathbb{R}^3 \rightarrow \mathbb{R}^3$ returns the robot-body-frame coordinates of $\zeta \in \mathbb{R}^3$ given by

$$\mathbf{q}(T, \zeta) = QT^{-1}\zeta, \quad (4)$$

where the projection matrix Q and the homogeneous coordinates $\underline{\zeta}$ are defined as:

$$Q = \begin{bmatrix} I_3 & \mathbf{0}_{3 \times 1} \end{bmatrix} \in \mathbb{R}^{3 \times 4}, \quad \underline{\zeta} = \begin{bmatrix} \zeta \\ 1 \end{bmatrix} \in \mathbb{R}^4. \quad (5)$$

Then, a sensor measurement is denoted by $\mathbf{z}_k = [\{\mathbf{z}_k^{(j)}\}_{j \in \mathcal{I}_{\mathcal{F}}(\mathbf{x}_k, \{\mathbf{y}^{(j)}\})}] \in \mathbb{R}^{n_z |\mathcal{I}_{\mathcal{F}}(T_k, \{\mathbf{y}_k^{(j)}\})|}$ where $\mathbf{z}_k^{(j)} \in \mathbb{R}^{n_z}$ is an observation of j -th target with model:

$$\mathbf{z}_k^{(j)} = H\mathbf{y}_k^{(j)} + \boldsymbol{\eta}_k, \quad \boldsymbol{\eta}_k \sim \mathcal{N}(0, V), \quad (6)$$

for all $j \in \mathcal{I}_{\mathcal{F}}(T_k, \{\mathbf{y}_k^{(j)}\})$, where the matrix $H \in \mathbb{R}^{n_z \times n_y}$ is the sensor matrix and $V \in \mathbb{R}^{n_z \times n_z}$ is the sensing noise covariance.

Our task is to develop a control policy for the robot to maximize the information about the multiple targets acquired through the onboard sensor. In particular, we consider minimizing $\mathbb{H}(\mathbf{y}_K | \mathbf{z}_{0:K}, T_{0:K})$, which is the differential entropy of the terminal target state given the history of the sensor states and the robot trajectory. Since each target state is independent of all other target states due to the independent motion model (2), the problem is equivalent to ¹

$$\min \sum_{j=1}^{n_l} \mathbb{H}(\mathbf{y}_K^{(j)} | \mathbf{z}_{0:K}, T_{0:K}). \quad (7)$$

Under the Gaussian target state obeying (2) with the linear Gaussian sensor model (6), the problem (7) is equivalent to

$$\max \sum_{j=1}^{n_l} \log \det \left(Y_K^{(j)} \right), \quad (8)$$

where Y_K is the terminal information matrix of the posterior distribution of the target state. More precisely, we provide the belief of the target state as both the prior and the posterior distributions of the target given a history of measurements as

$$\mathbf{y}_k^{(j)} | \mathbf{z}_{0:k-1} \sim \mathcal{N}(\mathbf{p}_k^{(j)}, (P_k^{(j)})^{-1}), \quad \mathbf{y}_k^{(j)} | \mathbf{z}_{0:k} \sim \mathcal{N}(\boldsymbol{\mu}_k^{(j)}, (Y_k^{(j)})^{-1}), \quad (9)$$

for all $j \in \{1, \dots, n_l\}$ and $k \in \{1, \dots, K\}$. The mean and covariance (or information) matrix are updated based on the Kalman Filter, which is given by the following prediction and update steps (here we omit the superscripts ^(j) to ease the notations but actually the variables are for each j -th target):

$$\textbf{Prediction (for all } j \in \{1, \dots, n_l\}): \quad \mathbf{p}_{k+1} = A\boldsymbol{\mu}_k + B\boldsymbol{\xi}_k, \quad (10)$$

$$P_{k+1} = (AY_k^{-1}A^\top + W_k)^{-1}, \quad (11)$$

$$\textbf{Update (for } j \in \mathcal{I}_{\mathcal{F}}(T_{k+1}, \{\mathbf{y}_{k+1}^{(j)}\}): \quad \boldsymbol{\mu}_{k+1} = \mathbf{p}_{k+1} + K_{k+1}(\mathbf{z}_{k+1} - H\mathbf{p}_{k+1}), \quad (12)$$

$$Y_{k+1} = P_{k+1} + H^\top V^{-1}H, \quad (13)$$

$$K_{k+1} = P_{k+1}^{-1}H_{k+1}^\top (H_{k+1}P_{k+1}^{-1}H_{k+1}^\top + V_{k+1})^{-1}, \quad (14)$$

However, due to the index condition in the update step, the implementation above can be done only after obtaining the sensor value, which is not possible in the planning stage. To enable the planning before obtaining the measurement, following Koga et al. (2021), we introduce the differentiable FoV formulation to relax the index condition with enabling the gradient computation. Moreover, during training, we suppose that the sensor noise is negligible to enable the offline non-myopic planning without acquiring measurements, thereby the prior and posterior means are identical. We then design the control policy $\mathbf{u}_k = \boldsymbol{\pi}_\theta(\mathbf{s}_k)$ as a deep neural network, where \mathbf{s}_k is the input of the network. Since the differentiable FoV renders the posterior information matrix at next time step dependent on the prior mean and information matrix of the target state, the input of the network is designed to include them. Finally, we consider the following problem for policy optimization.

1. The differential entropy of a continuous random variable Y with probability density function p is defined as $\mathbb{H}(Y) := -\int p(y) \log p(y) dy$.

Problem Given a prior Gaussian distribution over the moving target $\mathbf{y}^{(j)} \sim \mathcal{N}(\mathbf{p}_0^{(j)}, (P_0^{(j)})^{-1})$ with a mean $\boldsymbol{\mu}_0^{(j)} \in \mathbb{R}^{n_y}$ and the information matrix $Y_0^{(j)} \in \mathbb{S}_{\succ 0}^{n_y \times n_y}$ for all $j \in \{1, \dots, n_l\}$, obtain a learned parameter $\boldsymbol{\theta} \in \mathbb{R}^{n_p}$ in a control policy $\mathbf{u}_k = \boldsymbol{\pi}_{\boldsymbol{\theta}}(\mathbf{s}_k)$ where $\mathbf{s}_k = [\log(T_k)^\top, \{\mathbf{p}_{k+1}^{(j)}, \text{vech}(P_{k+1}^{(j)})\}_{j=1}^{n_l}]$, to solve the following policy optimization problem:

$$\max_{\boldsymbol{\theta} \in \mathbb{R}^{n_p}} \sum_{j=1}^{n_l} \log \det(Y_K^{(j)}), \quad (15)$$

subject to

$$T_{k+1} = T_k \exp(\tau \boldsymbol{\pi}_{\boldsymbol{\theta}}(\mathbf{s}_k)) \quad (16)$$

$$\mathbf{p}_{k+1}^{(j)} = A \mathbf{p}_k^{(j)} + B \boldsymbol{\xi}_k^{(j)}, \quad (17)$$

$$P_{k+1}^{(j)} = (A(Y_k^{(j)})^{-1} A^\top + W_k)^{-1}, \quad (18)$$

$$Y_{k+1}^{(j)} = P_{k+1}^{(j)} + M(T_{k+1}, \mathbf{p}_{k+1}^{(j)}), \quad (19)$$

$$M(T, \mathbf{p}^{(j)}) = \left(1 - \Phi(d(\mathbf{q}(T, \mathbf{p}^{(j)}), \mathcal{F}))\right) H^\top V^{-1} H, \quad (20)$$

for all $j \in \{1, \dots, n_l\}$ and $k \in \{0, \dots, K-1\}$, where Φ is a probit function (Bishop, 2006), defined by the Gaussian CDF $\Phi : \mathbb{R} \rightarrow [0, 1]$, $\Phi(x) = \frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{x}{\sqrt{2\kappa}} - 2\right)\right]$, and d is a signed distance function associated with the FoV \mathcal{F} defined below.

Definition 1 The signed distance function $d : \mathbb{R}^3 \rightarrow \mathbb{R}$ associated with a set $\mathcal{F} \subset \mathbb{R}^3$ is:

$$d(\mathbf{q}, \mathcal{F}) = \begin{cases} -\min_{\mathbf{q}^* \in \partial \mathcal{F}} \|\mathbf{q} - \mathbf{q}^*\|, & \text{if } \mathbf{q} \in \mathcal{F}, \\ \min_{\mathbf{q}^* \in \partial \mathcal{F}} \|\mathbf{q} - \mathbf{q}^*\|, & \text{if } \mathbf{q} \notin \mathcal{F}, \end{cases} \quad (21)$$

where $\partial \mathcal{F}$ is the boundary of \mathcal{F} .

3. Model-Based RL over Continuous $SE(3)$ Trajectory

We approach the problem in the previous section by the following steps. First, we derive the gradient of the total reward with respect to the policy parameters analytically utilizing the $SE(3)$ pose kinematics and the mean and information update, similarly to iCR (Koga et al., 2021). Then, we design a network architecture to handle multiple targets and to enable learning over a varying number of targets.

3.1. Analytical Policy Gradient

The following proposition provides an update rule for the policy function parameters $\boldsymbol{\theta}$ using the gradient of the reward function in (15) with respect to $\boldsymbol{\theta}$.

Proposition 1 The gradient-descent update for solving active exploration (15)–(19) with differentiable field of view (20) is given by

$$\theta^{(i)} \leftarrow \theta^{(i)} + \gamma^{(i)} \frac{\partial r^{\boldsymbol{\pi}_{\boldsymbol{\theta}}}}{\partial \theta^{(i)}}, \quad (22)$$

where $\gamma^{(i)} \in \mathbb{R}_+$ for all $i \in \{1, \dots, n_p\}$ is a step size, and the gradient is given by

$$\frac{\partial r^{\pi_\theta}}{\partial \theta^{(i)}} = \sum_{j=1}^{n_l} \text{tr} \left((Y_K^{(j)})^{-1} \Omega_K^{(j,i)} \right), \quad (23)$$

where

$$\Lambda_k^{(i)} := \frac{\partial T_k}{\partial \theta^{(i)}} \in \mathbb{R}^{4 \times 4}, \quad \Omega_k^{(j,i)} := \frac{\partial Y_k^{(j)}}{\partial \theta^{(i)}} \in \mathbb{R}^{n_y \times n_y}. \quad (24)$$

are obtained via:

$$\Lambda_0^{(i)} = 0, \quad \Omega_0^{(j,i)} = 0, \quad \forall i \in \{1, \dots, n_p\}, \quad \forall j \in \{1, \dots, n_l\} \quad (25)$$

$$\begin{aligned} \Omega_{k+1}^{(j,i)} = & (A^\top + Y_k^{(j)} A^{-1} W_k)^{-1} \Omega_k^{(j,i)} (A + W_k A^{-\top} Y_k^{(j)})^{-1} \\ & + \left(\Phi'(d(\mathbf{q}, \mathcal{F})) \frac{\partial d}{\partial \mathbf{q}} \right) \Big|_{\mathbf{q}=\mathbf{q}(T_{k+1}, \mathbf{p}_{k+1}^{(j)})} Q T_{k+1}^{-1} \Lambda_{k+1}^{(i)} T_{k+1}^{-1} \underline{\mathbf{p}_{k+1}^{(j)}} H^\top V^{-1} H, \end{aligned} \quad (26)$$

$$\Lambda_{k+1}^{(i)} = \Lambda_k^{(i)} \exp(\tau \pi_\theta(\mathbf{s}_k)) + T_k \sum_{j=1}^6 \mathbf{e}_{6,j}^\top \frac{\partial \pi_\theta(\mathbf{s}_k)}{\partial \theta} \mathbf{e}_{n_p,i} \frac{\partial \exp(\tau \hat{\mathbf{u}})}{\partial \mathbf{u}^{(j)}} \Big|_{\mathbf{u}=\pi_\theta(\mathbf{s}_k)}, \quad (27)$$

where $\mathbf{e}_{n,m} \in \mathbb{R}^n$ is a n -dimensional unit vector whose m -th element is 1 and all others are 0.

Proof Taking the gradient of the reward (15) directly leads to (23) by defining $\Omega_k^{(j,i)}$ as (24), which is the perturbation of the information matrix with respect to the policy parameter. Then, the update equation (26) is derived by taking the gradient of both sides in (19) and in (18). The same manner can be performed for the $SE(3)$ pose state to derive (27) from the gradient of (16). Note that the prior mean is not affected by the control policy due to the update equation (17), and hence we do not need to define the perturbation of the prior mean. ■

3.2. Network Architecture

In order to generalize training and testing to varying number of targets, we utilize a padding and masking scheme that allows tracking an arbitrary number of targets (up to a defined maximum n_l^{\max}) while keeping the network architecture unchanged. In particular, we consider a fixed length input vector with $n_l^{\max} \times n_y$ elements for both target state and target information, where only the first $n_l \times n_y$ elements contain non-zero values. Additionally, a binary *Mask* vector contains instruction on which elements of the subsequent computations should be ignored to cancel out the effect of padding values in the output of the network. Fig. 1 illustrates the policy network architecture using the padding and masking scheme, where similarly to Yang et al. (2022) we employ an attention mechanism (Long et al., 2020) so that the agent takes into account the relationship between its current pose state and the moving target states in order to prioritize observing uncertain targets. The fully-connected layers *AP_FC* and *LI_FC* alongside the *ReLU* nonlinearities compute embeddings for agent pose and target information, denoted as Emb_a and Emb_l respectively. *Masked Attention* block blends information from agent, targets, and the masking as follows:

$$\text{Masked Attention}(\text{Emb}_a, \text{Emb}_l, \text{Mask}) = \text{softmax} \left((1 + \log(\text{Mask})) \odot \left(\frac{\text{Emb}_a \text{Emb}_l^\top}{\alpha} \right) \right), \quad (28)$$

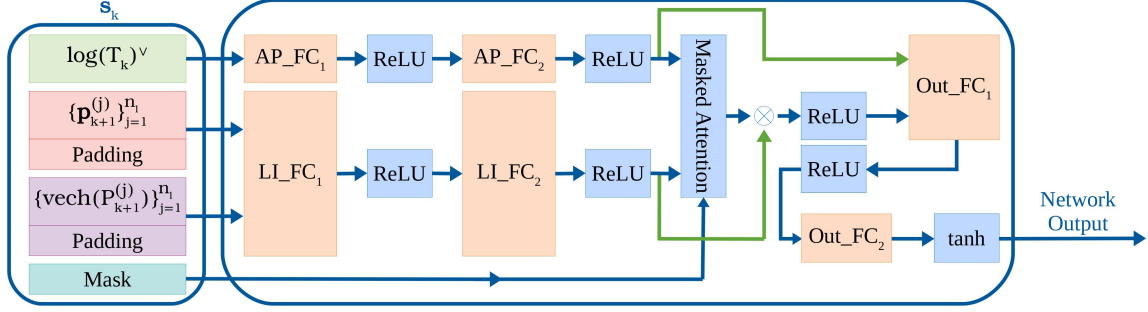


Figure 1: Deep neural network architecture used for the parameterized policy $\pi_{\theta}(s_k)$. The input s_k contains the current robot pose in log representation $\log(T_k)^v$, as well as zero-padded predicted target states $\{p_{k+1}^{(j)}\}_{j=1}^{n_l}$ and target information $\{P_{k+1}^{(j)}\}_{j=1}^{n_l}$. The *Mask* vector indicates which elements of the padded target input contains relevant values, allowing to remove the influence of zero-padding in the final output. For each input, the network computes continuous controls $u_k = [v_k^T, \omega_k^T]^T$.

where the operator \odot denotes to element-wise vector multiplication and α is a network hyper-parameter. As (28) shows, the components affected by padding can be nullified via the element-wise multiplication since the softmax operator eliminates the influence of the components corresponding to the zero elements of *Mask* in the subsequent matrix multiplication with Emb_l . Therefore, for every input vector s_k with an arbitrary choice of targets, the policy network $\pi_{\theta}(s_k)$ computes continuous controls $u_k = [v_k^T, \omega_k^T]^T$.

4. Experiments

In this section, we examine the performance of both our proposed model-based RL and a benchmark model-free RL for active target tracking in experiments. We provide simulation results to visualize the tracking trajectories and quantitative comparison results of the reward value to demonstrate the proposed method’s robust performance.

4.1. Experimental Settings

In the evaluation, we consider a 2-D target tracking by considering the ground vehicle governed by $SE(2)$ differential-drive kinematics and 2-D position as the target state ($n_y = 2$). As in practice, we consider controlling only the agent’s forward and angular speeds within a limited range for linear velocity $v_x \in [0, 4]$ m/s and angular velocity $\omega \in [-\pi/3, \pi/3]$ rad/s. The agent is equipped with a sensor detecting a relative position from the agent to the targets within the triangular FoV which has a depth of 2 m and angular range of $2\pi/3$ rad/s. When the targets are inside the agent’s FoV, their estimated position is updated based on (12).

We set the matrices in the target model as $A = I$ and $B = I$. Regarding the known input $\xi \in \mathbb{R}^2$, we tested two cases:

1) **Unbiased motion:** The known input ξ is sampled from a uniform distribution as $\xi \sim \text{Uniform}[-\bar{\xi}, \bar{\xi}]$, i.e., the mean velocities are 0 and hence the target motion is unbiased. The targets move within small areas based on their current position at every episodes.

2) **Biased motion:** On the other hand, the absolute mean of the uniform distribution is set to larger than 0, i.e., the targets have a base linear velocity heading to the same direction with small randomness at each time step. With this setting, the targets do not move too far from each other during tracking.

We trained the neural network policy only from the biased target motion, because the mean of the uniform distribution is also sampled from the uniform distribution with zero mean, which includes the case of unbiased motion. Besides, the same hyper-parameters, environment settings, and network architecture have been applied for model-free and model-based training. Specifically, regarding the environment, the smoothing factor κ , the magnitude of the Gaussian sensor noise σ_1 , and the magnitude of the Gaussian motion noise σ_2 were set with values of 0.4, 0.2, and 0.05 respectively. Besides, the time horizon and targets' initialization position boundaries are set to same constants dependent on the number of targets which is varied between [3, 8] at each episode during training. For the policy network, the fully-connected layers AP_FC_1 , AP_FC_2 , LI_FC_1 , LI_FC_2 , Out_FC_1 , and Out_FC_2 have 32, 32, 64, 32, 64 and 2 units, respectively. We choose the hyper-parameter $\alpha = 4$ throughout our experiments. For the model-free reinforcement learning, we apply PPO (Schulman et al., 2017) for training, while the model-based policy was directly trained using gradient descent over a batch of the last 20 episodes of an epoch, without a replay buffer.

Table 1: Comparison of the proposed model-based RL with the model-free RL. The table shows the average and standard derivation for normalized rewards.

Method	Target Motion Model	3 Targets	5 Targets	7 Targets
		Episodic Reward	Episodic Reward	Episodic Reward
<i>Model-free RL</i>	Unbiased Motion	4.57 ± 2.13	3.65 ± 1.54	1.94 ± 1.70
	Biased Motion	4.23 ± 1.77	3.27 ± 1.30	2.01 ± 1.67
<i>Model-based RL</i>	Unbiased Motion	6.71 ± 1.47	6.56 ± 0.55	5.41 ± 0.85
	Biased Motion	6.87 ± 1.21	5.96 ± 0.96	4.92 ± 1.07

4.2. Comparison Results and Analysis

As shown in Fig. 2, we compare the trajectories generated by model-free and model-based trained networks in three scenarios of 3, 5, and 7 targets. In each scenario, the targets' initial position, moving velocity, and agent's initial pose are identical for fair comparisons. We can clearly see that the network trained with model-based algorithm is capable of controlling the agent in a better manner for target tracking, while the network trained with model-free algorithm renders the agent to be prone to move in a small area.

Quantitative comparisons are shown in Table 1. The metric of the episodic reward is computed based on (15). We normalized the value by dividing the number of targets at the end of each episode. We chose three random seeds 0, 10, and 100 for both algorithms and tested all the models with two target motions in each scenario. With each setting in one scenario, both methods were tested for 30 runs. According to Table 1, model-based RL has an overall better performance in terms of the larger episodic reward with smaller variance for both unbiased and biased motion.

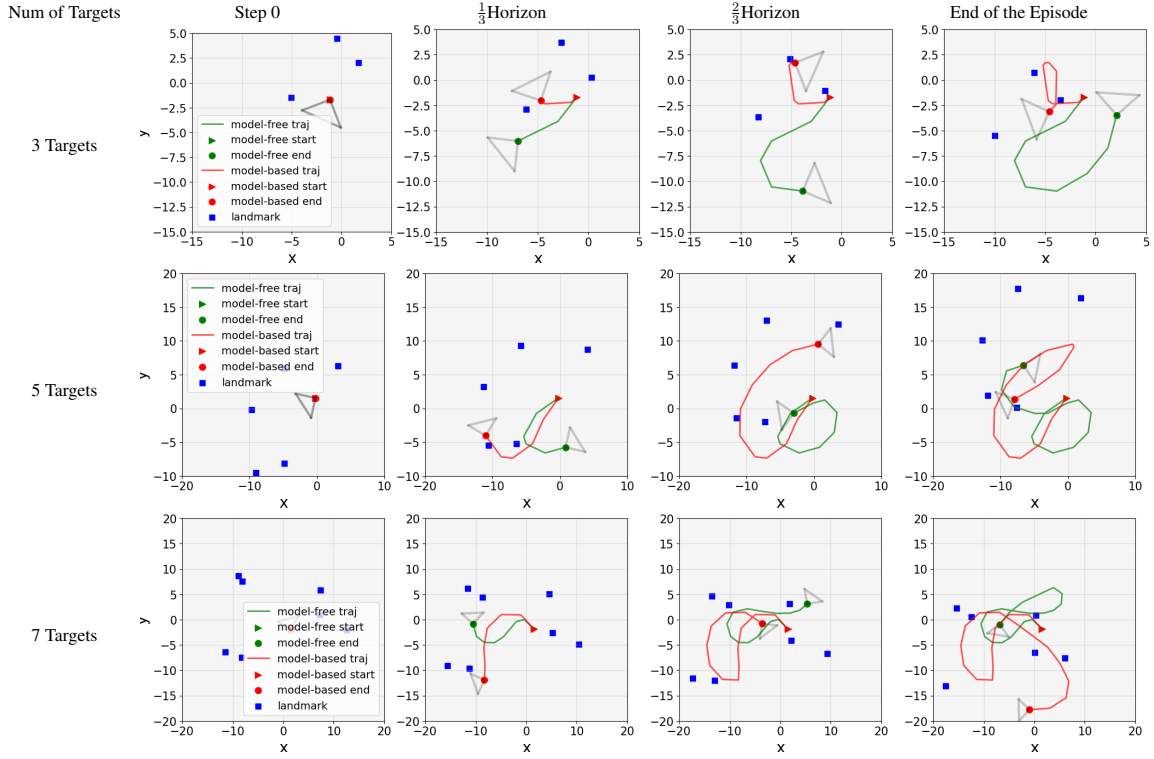


Figure 2: Trajectories of the target tracking. Two methods are compared in three scenarios with 3, 5, and 7 targets. The blue squares represent the moving targets. Green and red curves show the tracking trajectories generated by model-free and model-based trained networks. The grey triangles illustrate the agents’ forward triangular Field of View.

It is also observed that the average reward is inversely proportional to the number of targets for both methods. We conjecture that when the number of targets increases while the size of the map is also larger, an efficient planning for the target tracking becomes more challenging accordingly.

5. Conclusion

This paper proposed a model-based reinforcement learning algorithm for tracking multiple dynamic targets using a mobile robot with limited FoV. The prior and posterior mean and information matrix of each target state are provided by Kalman filtering for a given sensor state within FoV. We derive an analytical gradient of the cost function of the target entropy with respect to the network policy parameter by introducing a differentiable FoV and using perturbation of the $SE(3)$ state and the information matrix to obtain a continuous control policy. We compare the performance of our model-based RL policy with a model-free RL policy by PPO in simulated environment of tracking multiple targets, and observe better tracking. In future research, we will consider learning the policy under an unknown number of targets, in the presence of obstacles in the environment, and for target tracking with a team of robots with a limited communication.

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