

5-3 [30 分]

(1)  $2e^{-5t} \cosh 3t \varepsilon(t)$

解:  $f(t) = 2e^{-5t} \cosh 3t \varepsilon(t)$

$$= 2e^{-5t} \frac{1}{2}(e^{3t} + e^{-3t})\varepsilon(t)$$

$$= (e^{-2t} + e^{-8t})\varepsilon(t)$$

$$\therefore F(s) = \frac{1}{s+2} + \frac{1}{s+8}$$

收敛区为  $\sigma > -2$

(2)  $\frac{1}{\alpha}(1 - e^{-\alpha t})\varepsilon(t)$

解:  $f(t) = \frac{1}{\alpha}(1 - e^{-\alpha t})\varepsilon(t)$

$$= \frac{1}{\alpha}(\varepsilon(t) - e^{-\alpha t}\varepsilon(t))$$

$$\therefore F(s) = \frac{1}{\alpha}\left(\frac{1}{s} - \frac{1}{s+\alpha}\right)$$

收敛区为  $\sigma > \max[0, -\alpha]$

(3)  $(t^3 - 2t^2 + 1)\varepsilon(t)$

解:  $f(t) = (t^3 - 2t^2 + 1)\varepsilon(t)$

法一: 通过查表得:

$$F(s) = \frac{3!}{s^4} - \frac{2 \cdot 2!}{s^3} + \frac{1}{s} = \frac{s^3 - 4s + 6}{s^4}$$

收敛区为  $\sigma > \max[0, -\alpha]$

法二: 利用  $tf(t) \leftrightarrow \frac{-dF(s)}{ds}$  进行推导

则  $f(t) = \varepsilon(t)$  时,  $t\varepsilon(t) \leftrightarrow \frac{-d\frac{1}{s}}{ds} = \frac{1}{s^2}$

$$f(t) = t\varepsilon(t) \text{ 时, } t^2\varepsilon(t) \leftrightarrow -\frac{d}{ds}\frac{1}{s^2} = \frac{2}{s^3}$$

$$f(t) = t^2\varepsilon(t) \text{ 时, } t^3\varepsilon(t) \leftrightarrow -\frac{d}{ds}\frac{2}{s^3} = \frac{3!}{s^4}$$

$$\text{得到 } F(s) = \frac{3!}{s^4} - \frac{2 \cdot 2!}{s^3} + \frac{1}{s} = \frac{s^3 - 4s + 6}{s^4}$$

收敛区为  $\sigma > \max[0, -\alpha]$

$$(4) \quad \delta(t) - e^{-2t}\varepsilon(t)$$

$$\text{解: } f(t) = \delta(t) - e^{-2t}\varepsilon(t)$$

$$\therefore F(s) = 1 - \frac{1}{s+2} = \frac{s+1}{s+2}$$

收敛区为  $\sigma > -2$

5-17 [40 分]

解: 列出回路方程为:

$$\begin{cases} E(s) = [I(s) - I_2(s)]R_1 + sL_1I(s) - L_1i(0^-) \\ [I(s) - I_2(s)]R_1 = I_2(s)R_2 + sL_2I_2(s) - L_2i_2(0^-) \end{cases}$$

提示: 例如对于电感有

$$U_L t = L \frac{dI}{dt} \rightarrow U_{Ls} = LsI_s \rightarrow R_{Ls} = sL$$

以此类推, 建立 s 域元器件模型

s 域回路方程也可以通过时域回路方程进行拉氏变换得到, 本质是一样的

区别在于前者对式中各部分进行拉氏变换得到模块化表示再组合, 后者对整体直接拉氏变换

为了巩固和提高同学们对 s 域的理解, 建议利用 s 域元器件模型构建方程。

代入图中参数得到：

$$\begin{cases} E(s) = (s+2)I(s) - 2I_2(s) \\ 2I(s) - (s+5)I_2(s) = 0 \end{cases}$$

(1) 冲激响应

当  $e(t) = \delta(t)$  时,  $E(s) = 1$

$$I(s) = \frac{s+5}{(s+6)(s+1)} = \frac{1}{5} \cdot \frac{1}{s+6} + \frac{4}{5} \cdot \frac{1}{s+1}$$

$$\therefore f(t) = \left(\frac{1}{5}e^{-6t} + \frac{4}{5}e^{-t}\right)\varepsilon(t)$$

(2) 阶跃响应

当  $e(t) = \varepsilon(t)$  时,  $E(s) = \frac{1}{s}$

$$I(s) = \frac{s+5}{s(s+6)(s+1)} = \frac{k_1}{s} + \frac{k_2}{s+6} + \frac{k_3}{s+1}$$

$$k_1 = \left. \frac{s+5}{(s+6)(s+1)} \right|_{s=0} = \frac{5}{6}$$

$$k_2 = \left. \frac{s+5}{s(s+1)} \right|_{s=-6} = -\frac{1}{30}$$

$$k_3 = \left. \frac{s+5}{s(s+6)} \right|_{s=-1} = -\frac{4}{5}$$

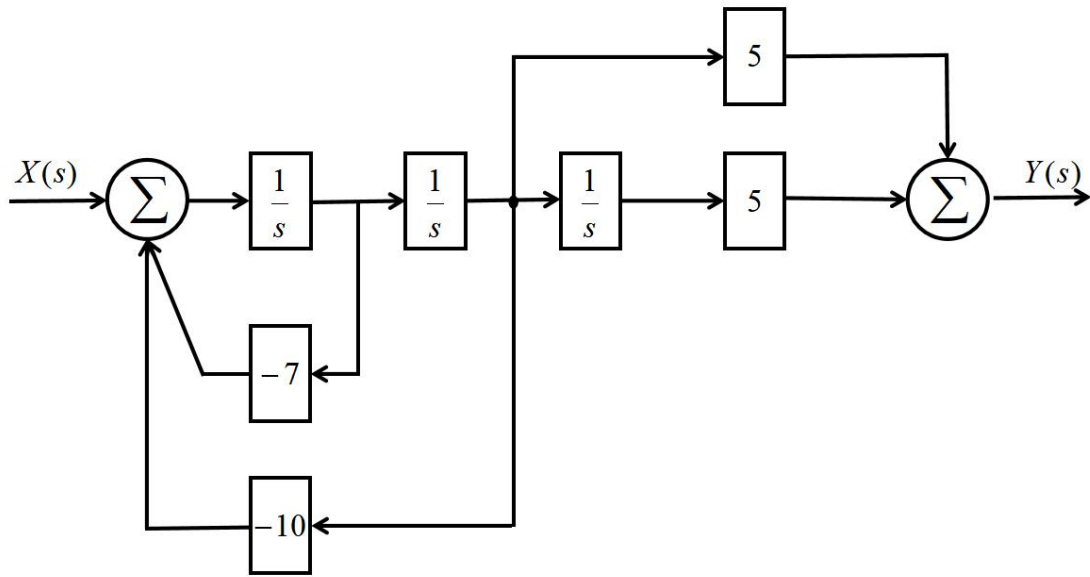
$$\therefore f(t) = \left(\frac{5}{6} - \frac{1}{30}e^{-6t} - \frac{4}{5}e^{-t}\right)\varepsilon(t)$$

5-32 [10分]

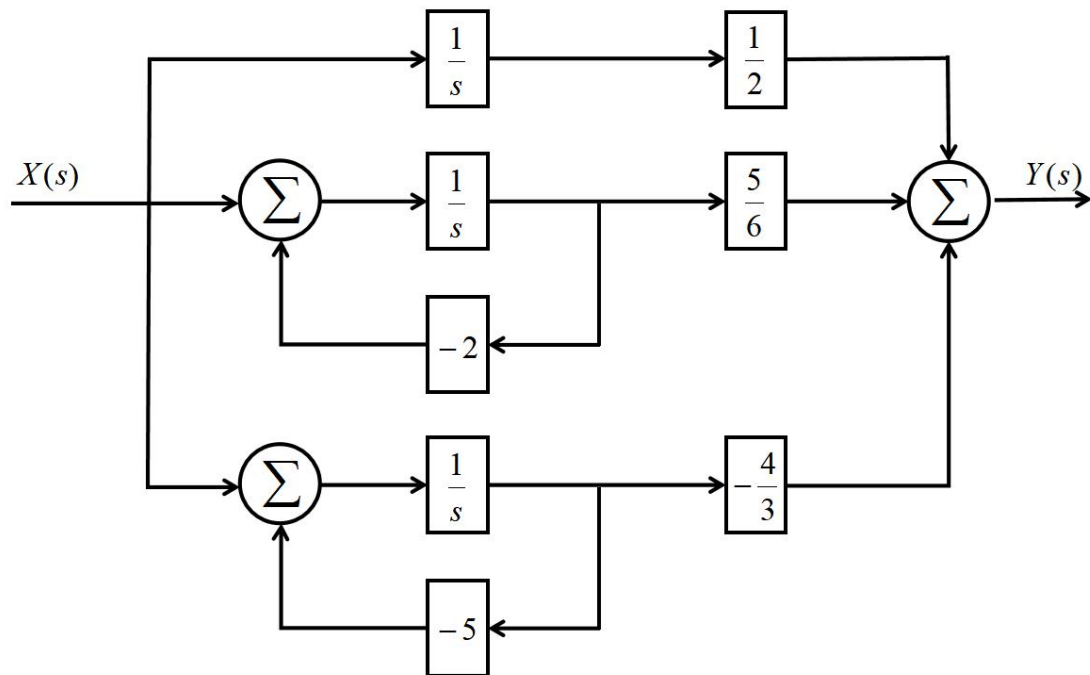
$$H(s) = \frac{5(s+1)}{s(s+2)(s+5)}$$

解：

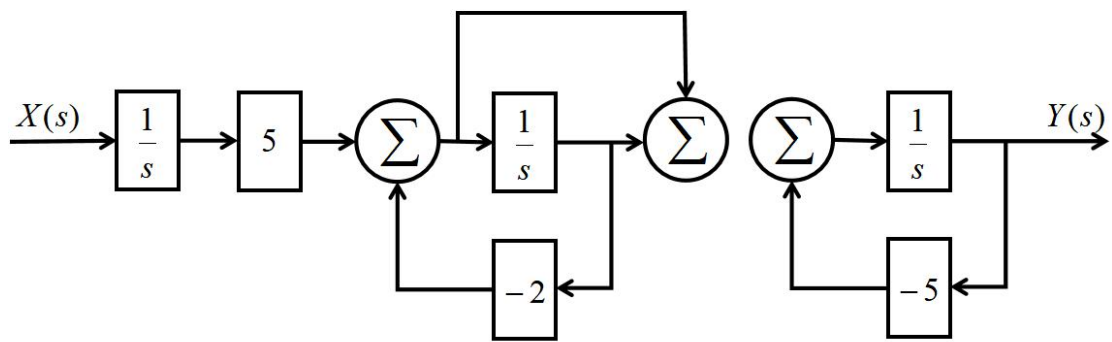
(1)  $H(s) = \frac{5s+5}{s^3+7s^2+10s}$



(2)  $H(s) = \frac{1}{2} \cdot \frac{1}{s} + \frac{5}{6} \cdot \frac{1}{s+2} - \frac{4}{3} \cdot \frac{1}{s+5}$



(3)  $H(s) = \frac{5}{s} \cdot \frac{s+1}{s+2} \cdot \frac{1}{s+5}$



6-6 [20分]

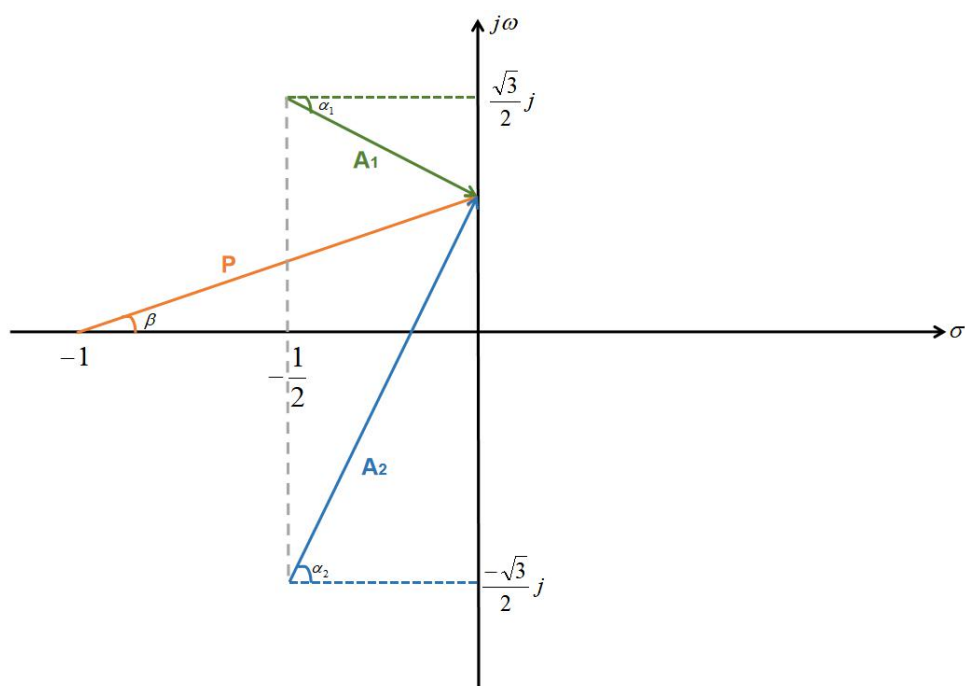
解：

$$(1) H(s) = \frac{I(s)}{E_s(s)} = \frac{1}{sL + \frac{1}{sC + \frac{1}{R}}} = \frac{1}{s + \frac{1}{s+1}} = \frac{s+1}{s^2 + s + 1}$$

$$\text{令 } s^2 + s + 1 = 0, \text{ 解得 } s = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

$$\therefore H(s) = \frac{s+1}{(s + \frac{1+j\sqrt{3}}{2})(s + \frac{1-j\sqrt{3}}{2})}$$

绘制出矢量图：



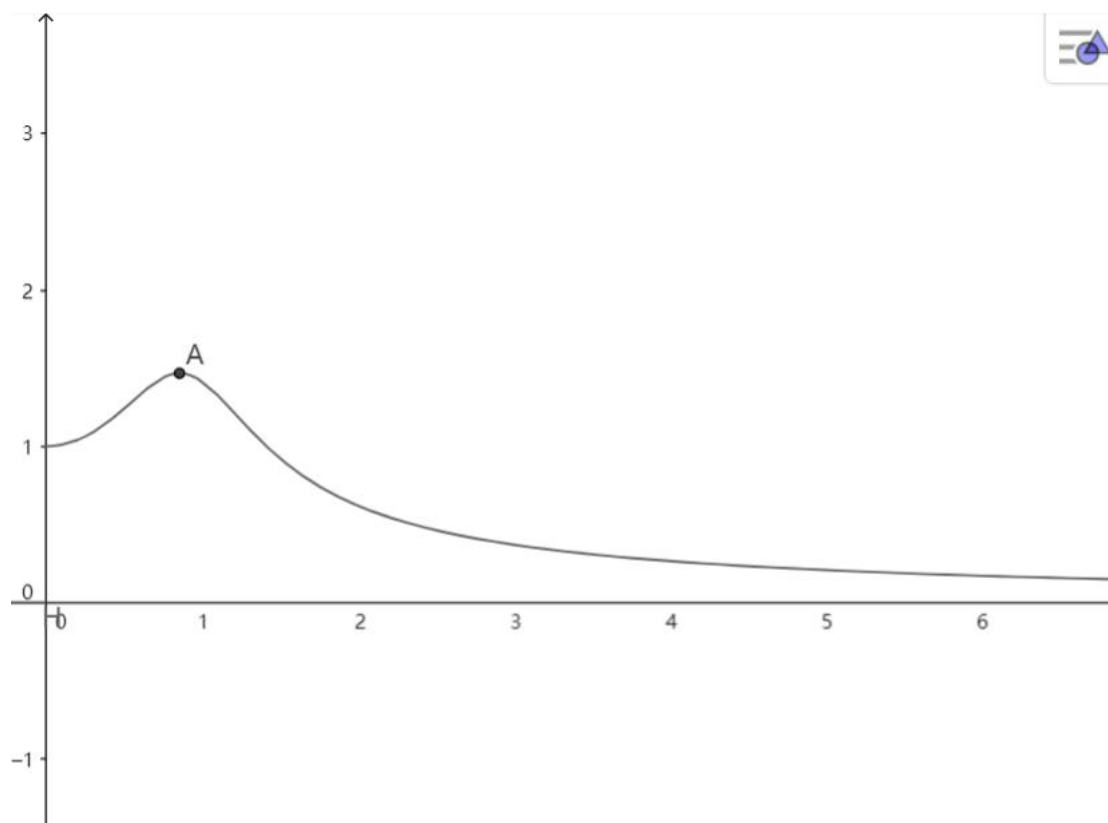
$$|H(j\omega)| = \frac{P}{A_1 \cdot A_2} = 1 \cdot \frac{|j\omega - 1|}{|j\omega + \frac{1+j\sqrt{3}}{2}| \cdot |j\omega + \frac{1-j\sqrt{3}}{2}|} = \sqrt{\frac{\omega^2 + 1}{(\omega^2 + 1)^2 - 3\omega^2}}$$

$$\text{设 } \omega^2 + 1 = \alpha \text{ 则 } |H(j\omega)| = \sqrt{\frac{\alpha}{\alpha^2 - 3(\alpha - 1)}} = \sqrt{\frac{1}{\alpha + \frac{3}{\alpha} - 3}}$$

在  $\alpha = \sqrt{3}$  时取极值点, 此时  $\omega = \sqrt{\sqrt{3} - 1} \approx 0.86$

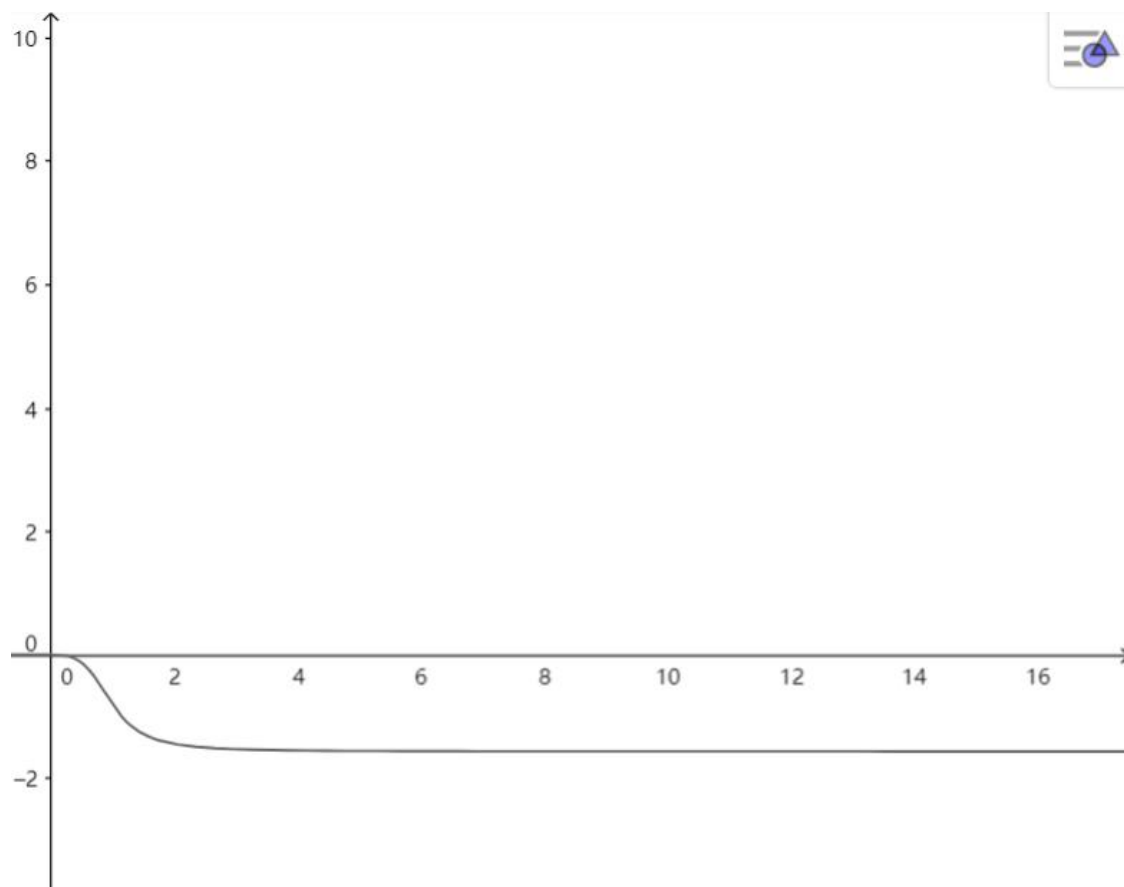
$$|H(j\omega)|_{MAX} = \sqrt{\frac{1}{2\sqrt{3} - 3}} \approx 1.47$$

幅频响应曲线如下图所示:



$$\varphi(\omega) = \sum \beta - \sum \alpha = \arctan \frac{\omega}{1} - \left( -\arctan \frac{\frac{\sqrt{3}}{2} - \omega}{\frac{1}{2}} + \arctan \frac{\frac{\sqrt{3}}{2} + \omega}{\frac{1}{2}} \right)$$

相频响应曲线如下图所示:

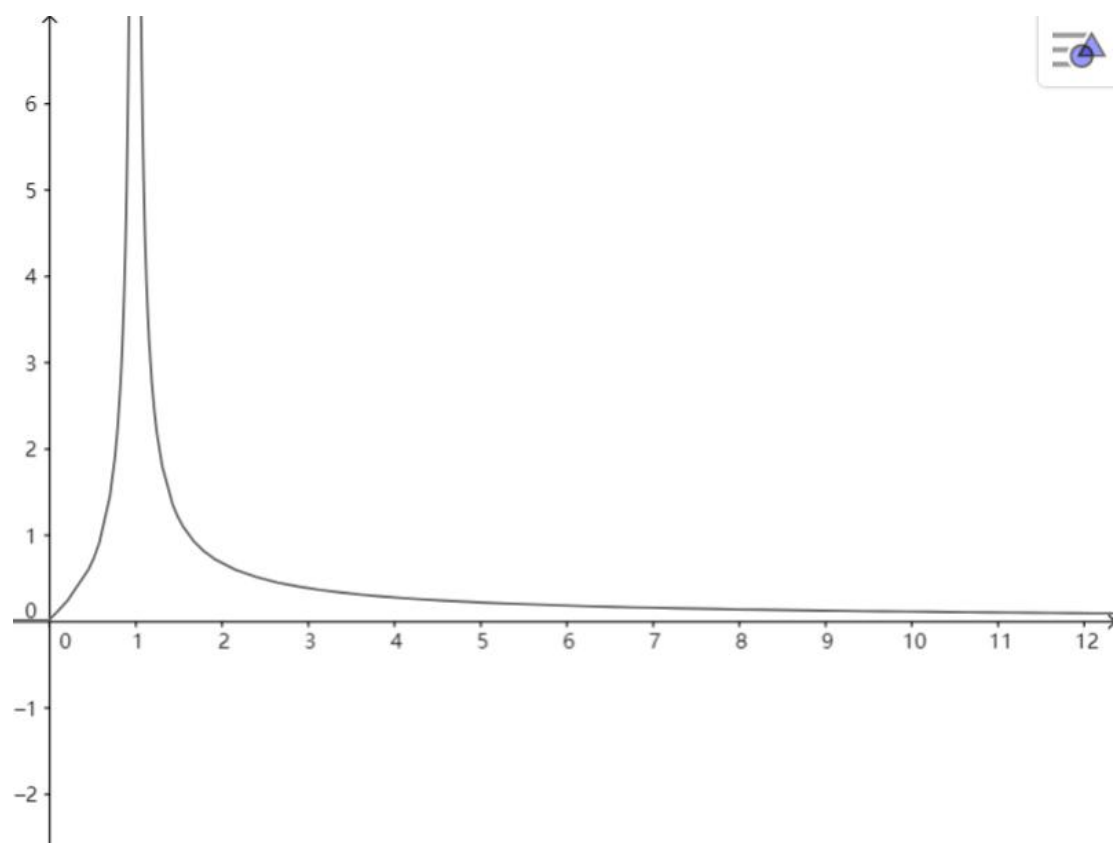


(2) 当  $R \rightarrow \infty$  时  $H(s) = \frac{s}{s^2 + 1} = \frac{s}{(s + j)(s - j)}$

$$|H(j\omega)| = \frac{|j\omega|}{|j\omega - j||j\omega + j|} = \frac{\omega}{(\omega - 1)(\omega + 1)}$$

$$\varphi(\omega) = \begin{cases} 90^\circ, 0 < \omega < 1 \\ -90^\circ, \omega > 1 \end{cases}$$

幅频响应曲线如下图所示：



相频响应曲线如下图所示:

