

3.14 [15 分]

$$(1) f(t) = \frac{\sin(2\pi(t-2))}{\pi(t-2)} = 2Sa(2\pi(t-2))$$

$$\therefore \varepsilon\left(t + \frac{\tau}{2}\right) - \varepsilon\left(t - \frac{\tau}{2}\right) \leftrightarrow \tau Sa\left(\frac{\tau\omega}{2}\right)$$

$$\therefore \text{当 } \tau = 4\pi, \varepsilon(t + 2\pi) - \varepsilon(t - 2\pi) \leftrightarrow 4\pi Sa(2\pi\omega)$$

$$\text{由对称特性 } F(t) \leftrightarrow 2\pi f(-\omega), \text{ 且为实偶函数 } \therefore F(t) \leftrightarrow 2\pi f(\omega)$$

$$\therefore 4\pi Sa(2\pi t) \leftrightarrow 2\pi[\varepsilon(\omega + 2\pi) - \varepsilon(\omega - 2\pi)]$$

$$2Sa(2\pi t) \leftrightarrow \varepsilon(\omega + 2\pi) - \varepsilon(\omega - 2\pi)$$

$$\text{由延时特性 } f(t - t_0) \leftrightarrow F(j\omega)e^{-j\omega t_0}$$

$$\therefore 2Sa(2\pi(t-2)) \leftrightarrow [\varepsilon(\omega + 2\pi) - \varepsilon(\omega - 2\pi)] e^{-2j\omega}$$

$$(2) f(t) = \frac{2a}{a^2 + \omega^2}$$

$$\therefore e^{-a|t|} \leftrightarrow \frac{2a}{a^2 + \omega^2}, \text{ 由对称特性 } F(t) \leftrightarrow 2\pi f(-\omega), \text{ 且为实偶函数 } \therefore F(t) \leftrightarrow 2\pi f(\omega)$$

$$\therefore \frac{2a}{a^2 + t^2} \leftrightarrow 2\pi e^{-a|\omega|}$$

$$(3) f(t) = \left[\frac{\sin(2\pi t)}{2\pi t}\right]^2 = Sa^2(2\pi t)$$

法一:

$$\varepsilon\left(t + \frac{\tau}{2}\right) - \varepsilon\left(t - \frac{\tau}{2}\right) \leftrightarrow \tau Sa\left(\frac{\tau\omega}{2}\right)$$

$$\text{当 } \tau = 4\pi \text{ 时, } \varepsilon(t + 2\pi) - \varepsilon(t - 2\pi) \leftrightarrow 4\pi Sa(2\pi\omega)$$

$$\text{由对称特性 } F(t) \leftrightarrow 2\pi f(-\omega), \text{ 且为实偶函数 } \therefore F(t) \leftrightarrow 2\pi f(\omega)$$

$$\text{可得 } 4\pi Sa(2\pi t) \leftrightarrow 2\pi[\varepsilon(\omega + 2\pi) - \varepsilon(\omega - 2\pi)] \therefore Sa(2\pi t) \leftrightarrow \frac{1}{2}[\varepsilon(\omega + 2\pi) - \varepsilon(\omega - 2\pi)]$$

$$\text{由频域卷积定理 } f_1(t)f_2(t) \leftrightarrow \frac{1}{2\pi}[F_1(j\omega) * F_2(j\omega)]$$

$$Sa^2(2\pi t) \leftrightarrow \frac{1}{8\pi}[\varepsilon(\omega + 2\pi) - \varepsilon(\omega - 2\pi)] * [\varepsilon(\omega + 2\pi) - \varepsilon(\omega - 2\pi)]$$

$$= \frac{1}{8\pi}[\varepsilon(\omega + 2\pi) * \varepsilon(\omega + 2\pi) - 2\varepsilon(\omega + 2\pi) * \varepsilon(\omega - 2\pi) + \varepsilon(\omega - 2\pi) * \varepsilon(\omega - 2\pi)]$$

$$= \frac{1}{8\pi}[(\omega - 4\pi)\varepsilon(\omega - 4\pi) - 2\omega\varepsilon(\omega) + (\omega + 4\pi)\varepsilon(\omega + 4\pi)]$$

$$\text{当 } \omega \leq -4\pi \text{ 时, 上式} = 0$$

$$\text{当 } -4\pi < \omega \leq 0 \text{ 时, 上式} = \frac{1}{8\pi}(\omega + 4\pi) = \frac{1}{2} - \frac{|\omega|}{8\pi}$$

$$\text{当 } 0 < \omega \leq 4\pi \text{ 时, 上式} = \frac{1}{8\pi}(\omega + 4\pi - 2\omega) = \frac{1}{2} - \frac{|\omega|}{8\pi}$$

$$\text{当 } 4\pi < \omega \text{ 时, 上式} = \frac{1}{8\pi}(\omega - 4\pi - 2\omega + \omega + 4\pi) = 0$$

$$\text{综上 } [Sa(2\pi t)]^2 \leftrightarrow \begin{cases} \frac{1}{2} - \frac{|\omega|}{8\pi} & |\omega| \leq 4\pi \\ 0 & |\omega| > 4\pi \end{cases}$$

法二:

$$\therefore g(t) = \begin{cases} 1 - \frac{|t|}{\tau} & |t| \leq \tau \\ 0 & |t| > \tau \end{cases} \leftrightarrow \tau \left[ \text{Sa}\left(\frac{\omega\tau}{2}\right) \right]^2$$

$$\text{当 } \tau = 4\pi \text{ 时, } g(t) = \begin{cases} 1 - \frac{|t|}{4\pi} & |t| \leq 4\pi \\ 0 & |t| > 4\pi \end{cases} \leftrightarrow 4\pi [\text{Sa}(2\pi\omega)]^2$$

由对称特性  $F(t) \leftrightarrow 2\pi f(-\omega)$ , 且为实偶函数  $\therefore F(t) \leftrightarrow 2\pi f(\omega)$

$$\therefore 4\pi [\text{Sa}(2\pi t)]^2 \leftrightarrow 2\pi g(\omega) = \begin{cases} 2\pi(1 - \frac{|\omega|}{4\pi}) & |\omega| \leq 4\pi \\ 0 & |\omega| > 4\pi \end{cases}$$

$$\therefore [\text{Sa}(2\pi t)]^2 \leftrightarrow \frac{1}{2}g(\omega) = \begin{cases} \frac{1}{2} - \frac{|\omega|}{8\pi} & |\omega| \leq 4\pi \\ 0 & |\omega| > 4\pi \end{cases}$$

### 3.16a [15 分]

解: (a) 图

$$G_2(t) = \varepsilon(t+1) - \varepsilon(t-1) \quad G_2(t) \leftrightarrow 2\text{Sa}(\omega)$$

$$\therefore f(t) = G_2(t-2) + G_2(t+2)$$

(1) 由延时特性  $f(t-t_0) \leftrightarrow F(j\omega)e^{-j\omega t_0}$

$$G_2(t-2) \leftrightarrow 2\text{Sa}(\omega)e^{-2j\omega} \quad G_2(t+2) \leftrightarrow 2\text{Sa}(\omega)e^{2j\omega}$$

$$\therefore f(t) \leftrightarrow 2\text{Sa}(\omega)e^{-2j\omega} + 2\text{Sa}(\omega)e^{2j\omega}$$

根据欧拉公式  $e^{jx} = \cos x + j\sin x$

$$\text{可得 } f(t) \leftrightarrow 4\text{Sa}(\omega)\cos 2\omega$$

(2)  $f(t) = G_2(t-2) + G_2(t+2) = \varepsilon(t-1) - \varepsilon(t-3) + \varepsilon(t+3) - \varepsilon(t+1)$

$$f'(t) = \delta(t-1) - \delta(t-3) + \delta(t+3) - \delta(t+1)$$

$$\delta(t) \leftrightarrow 1, \text{ 由延时特性 } f(t-t_0) \leftrightarrow F(j\omega)e^{-j\omega t_0}, \delta(t-1) \leftrightarrow e^{-j\omega}$$

$$\therefore f'(t) \leftrightarrow e^{-j\omega} - e^{-3j\omega} + e^{3j\omega} - e^{j\omega}$$

根据欧拉公式  $e^{jx} = \cos x + j\sin x$ ,  $f'(t) \leftrightarrow 2j(\sin 3\omega - \sin \omega)$

$$\text{由积分特性, } f(t) = \int_{-\infty}^t f'(\tau) d\tau \leftrightarrow \frac{2}{\omega}(\sin 3\omega - \sin \omega)$$

### 3.24 [20 分]

解: 根据图像, 可得当  $0 \leq t < \frac{T}{2}$  时,  $i(t) = \frac{2I}{T}t$ , 当  $\frac{T}{2} \leq t < T$  时,  $i(t) = -\frac{2I}{T}t + 2I$

$$(1) \text{ 平均电流 } \bar{i} = \frac{1}{T} \int_0^T i(t) dt = \frac{1}{T} \int_0^{\frac{T}{2}} i(t) dt + \frac{1}{T} \int_{\frac{T}{2}}^T i(t) dt = \frac{I}{2}$$

$$\text{方均值 } \overline{i^2} = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} i^2(t) dt = \frac{2}{T} \int_0^{\frac{T}{2}} i^2(t) dt = \frac{1}{4} \int_0^4 i^2(t) dt = \frac{1}{4} \int_0^4 \left(\frac{I}{4}t\right)^2 dt = \frac{I^2}{3} \quad \therefore \bar{i} = \frac{\sqrt{3}}{3}I$$

$$(2) \text{ 平均功率 } \overline{P} = \overline{i^2} = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} i^2(t) dt = \frac{I^2}{3}$$

$$\text{直流功率 } P_1 = \left(\frac{a_0}{2}\right)^2 = \left(\frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} i(t) dt\right)^2 = \frac{I^2}{4}$$

$$\text{交流功率 } P_2 = \frac{1}{2} \sum_{n=1}^{+\infty} A_n^2 = \frac{1}{2} \sum_{n=1}^{+\infty} (a_n^2 + b_n^2)$$

由于该电流为偶函数, 所以  $b_n = 0$

$$P_2 = \frac{1}{2} \sum_{n=1}^{+\infty} \left( \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} i(t) \cos n\omega t dt \right)^2 = \frac{1}{2} \sum_{n=1}^{+\infty} \left( \frac{4}{T} \int_0^{\frac{T}{2}} \frac{2I}{T} t \cos n\omega t dt \right)^2$$

$$= \frac{32I^2}{T^4} \sum_{n=1}^{+\infty} \left( \int_0^{\frac{T}{2}} t \cos \frac{2n\pi}{T} t dt \right)^2 = \frac{1}{12} I^2$$

$\therefore \overline{P} = P_1 + P_2$   $\therefore$  符合帕塞瓦尔定理

#### 4.1 [30 分]

解：当  $0 \leq t \leq 2$  时， $e(t) = \begin{cases} A \sin(\pi t), & 0 \leq t < 1 \\ -A \sin(\pi t), & 1 \leq t \leq 2 \end{cases}, T = 2, \quad \Omega = \frac{2\pi}{T} = \pi$

$$e(t) = \frac{a_0}{2} + \sum_{n=1}^{+\infty} (a_n \cos n\pi t + b_n \sin n\pi t)$$

$\because e(t)$  为偶函数， $\therefore b_n = 0$

$$a_0 = \frac{2}{T} \int_0^T e(t) dt = \int_0^1 A \sin(\pi t) dt + \int_1^2 -A \sin(\pi t) dt = \frac{4A}{\pi}$$

$$a_n = \frac{2}{T} \int_0^T e(t) \cos n\pi t dt = 2 \int_0^1 A \sin \pi t \cos n\pi t dt$$

使用积化和差公式，可算出  $a_n = \begin{cases} \frac{4A}{(1-n^2)\pi}, & n \text{ 为偶数} \\ 0, & n \text{ 为奇数} \end{cases}$

$$\therefore e(t) = \frac{2A}{\pi} - \frac{4A}{3\pi} \cos 2\pi t - \frac{4A}{15\pi} \cos 4\pi t \dots$$

$$H(n\Omega) = H(j\omega) = \frac{\frac{1}{jn\pi C}}{\frac{1}{jn\pi C} + R} = \frac{\frac{1}{jn\pi}}{\frac{1}{jn\pi} + 1} = \frac{1}{1 + n\pi j} = \frac{1 - n\pi j}{1 + n^2\pi^2} = \frac{1}{\sqrt{1 + n^2\pi^2}} \angle \arctan(-n\pi)$$

即幅度为  $\frac{1}{\sqrt{1 + n^2\pi^2}}$ ，相位为  $\arctan(-n\pi)$

$$\text{输出直流分量 } R_0 = \frac{2A}{\pi} H(0) = \frac{2A}{\pi}$$

$$\text{输出 } n \text{ 次谐波分量 } R_n = a_n H(n\Omega) = \frac{1}{\sqrt{1 + n^2\pi^2}} \frac{4A}{(1 - n^2)\pi} \angle n\pi t - \arctan(n\pi)。$$

$$r(t) = \frac{2A}{\pi} - \sum_{n=1}^{+\infty} \frac{1}{\sqrt{1 + n^2\pi^2}} \frac{4A}{(1 - n^2)\pi} \cos(n\pi t - \arctan(n\pi))$$

不为零的前三个分量：

$$\text{直流分量 } \frac{2A}{\pi}$$

$$\text{基波分量 } \frac{-4A}{3\pi\sqrt{1 + 4\pi^2}} \cos(2\pi t - \arctan(2\pi))$$

$$\text{二次谐波分量 } \frac{-4A}{15\pi\sqrt{1 + 16\pi^2}} \cos(4\pi t - \arctan(4\pi))$$

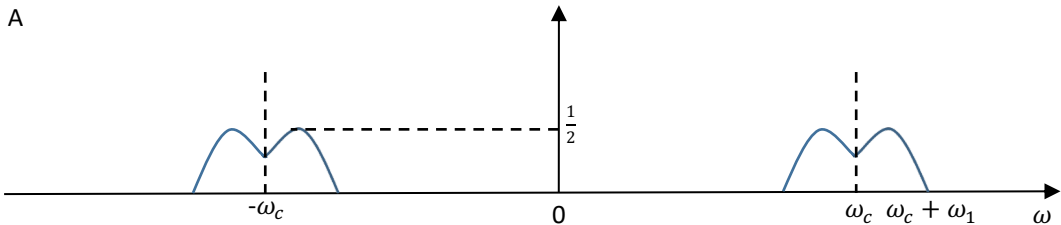
#### 4.6 [20 分]

解：

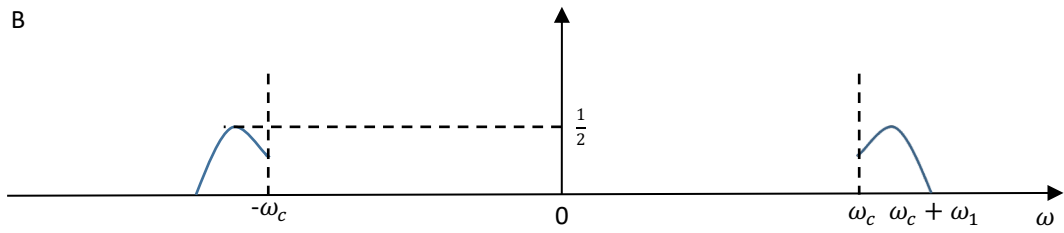
1、 $e(t) \times \cos \omega_c t$

根据频域卷积定理  $f_1(t)f_2(t) \leftrightarrow \frac{1}{2\pi} [F_1(j\omega) * F_2(j\omega)]$  可知，

$$e(t) \times \cos \omega_c t \leftrightarrow \frac{1}{2\pi} \{E(j\omega) * \pi[\delta(\omega + \omega_c) + \delta(\omega - \omega_c)]\} = \frac{1}{2} [E(j(\omega + \omega_c)) + E(j(\omega - \omega_c))]$$



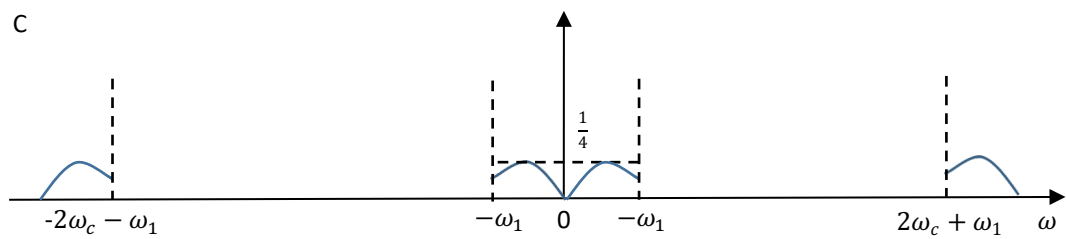
2、理想高通滤波器的截止频率为 $\omega_c$ ，因此只有 $|\omega| > \omega_c$ 才能通过



3、 $e_b(t) \times \cos [(\omega_c + \omega_1)t]$

根据频域卷积定理 $f_1(t)f_2(t) \leftrightarrow \frac{1}{2\pi} [F_1(j\omega) * F_2(j\omega)]$ 可知，

$$\begin{aligned} e_b(t) \times \cos [(\omega_c + \omega_1)t] &\leftrightarrow \frac{1}{2\pi} \{E_b(j\omega) * \pi[\delta(\omega + \omega_c + \omega_1) + \delta(\omega - \omega_c - \omega_1)]\} \\ &= \frac{1}{2} [E_b(j(\omega + \omega_c + \omega_1)) + E_b(j(\omega - \omega_c - \omega_1))] \end{aligned}$$



4、理想低通滤波器的截止频率为 $\omega_c$ ，因此只有 $|\omega| < \omega_c$ 才能通过

