Modern Portfolio Theory with Particle Swarm Optimizer

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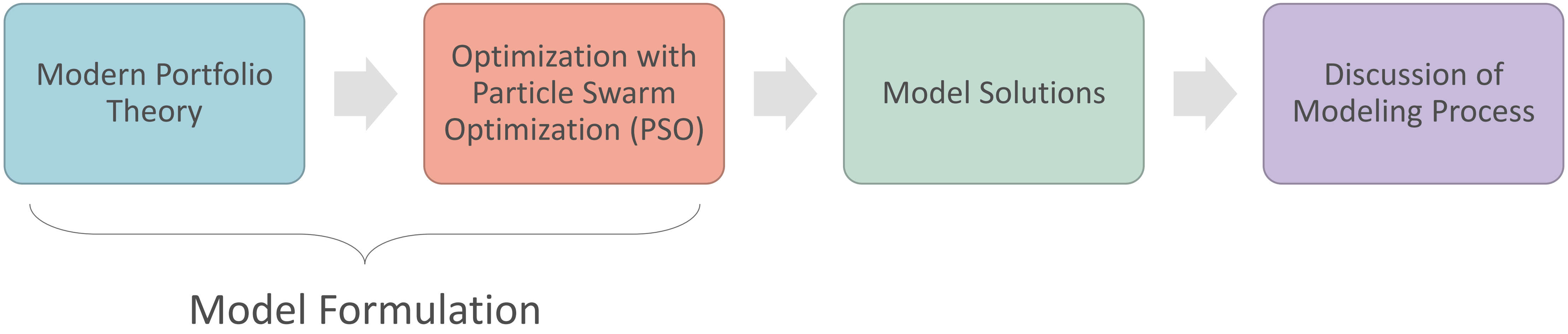
Overview


Modern Portfolio Theory
Optimization with Particle Swarm Optimization (PSO)
Model Solutions
Discussion of Modeling Process
Model Formulation

Model Formulation:
Modern Portfolio Theory

Modern Portfolio Theory (MPT) Overview

Get Stock Data (`yfinance`)

Calculations to set up MPT Modeling

Meet an Expected Return by Minimizing Risk of Portfolio

Calculate the Returns

- Calculate the percentage return of each stock's adjusted close price

$$Returns_{stock,t} = \ln\left(\frac{AdjClose_{stock,t}}{AdjClose_{stock,t-1}}\right),$$
$$\forall\ stock \in StockList,\ \forall\ t \in Periods$$

"StockList" contains the list stock tickers to consider investing in *
"Periods" are the monthly periods that associate with a stock's adjusted close price **

Calculate the Expected Returns of Each Stock

- Find the expected, or mean, return for each stock
- Pitfalls of MPT: predictive analysis

$$ExpectedReturns_{stock} = \frac{1}{numPeriods} \sum_{t \in Periods} Returns_{stock,t}, \quad \forall\ stock \in StockList$$

"StockList" contains the list stock tickers to consider investing in *
"Periods" are the monthly periods that associate with a stock's adjusted close price **

Calculate the Excess Returns

- Return in excess of the average return of the stock

$$ExcessReturns_{stock,t} = Returns_{stock,t} - ExpectedReturns_{stock,t},$$
$$\forall\ stock \in StockList,\ \forall\ t \in Periods$$

"StockList" contains the list stock tickers to consider investing in *
"Periods" are the monthly periods that associate with a stock's adjusted close price **

Account for Covariances Among Stocks

- Goal: understand how stock returns covary
- Higher risk associated with higher covariance
- Note non-linearity of variance-covariance matrix

$$VarCov = \frac{ExcessReturns^T.ExcessReturns}{N-1}$$

"N" represents the number of stocks in the "StockList" *

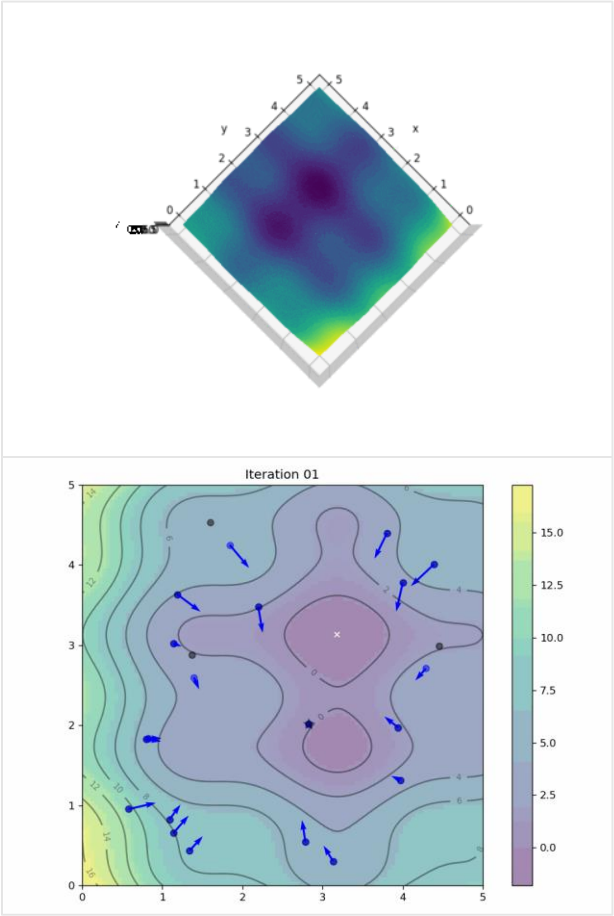
Model Formulation: Particle Swarm Optimizer

PSO Overcomes Non-Linear Objectives

1. Metaheuristic algorithm that emulates organisms, or “particles” moving in swarms

2. Helps overcome local minima for non-linear modeling

3. Basic idea:
   - Swarm of particles starting random locations
   - Each particle has no understanding of where the overall swarm is headed
   - Each particle only knows where their immediate surrounding environment is headed
   - Collectively will arrive at global optima

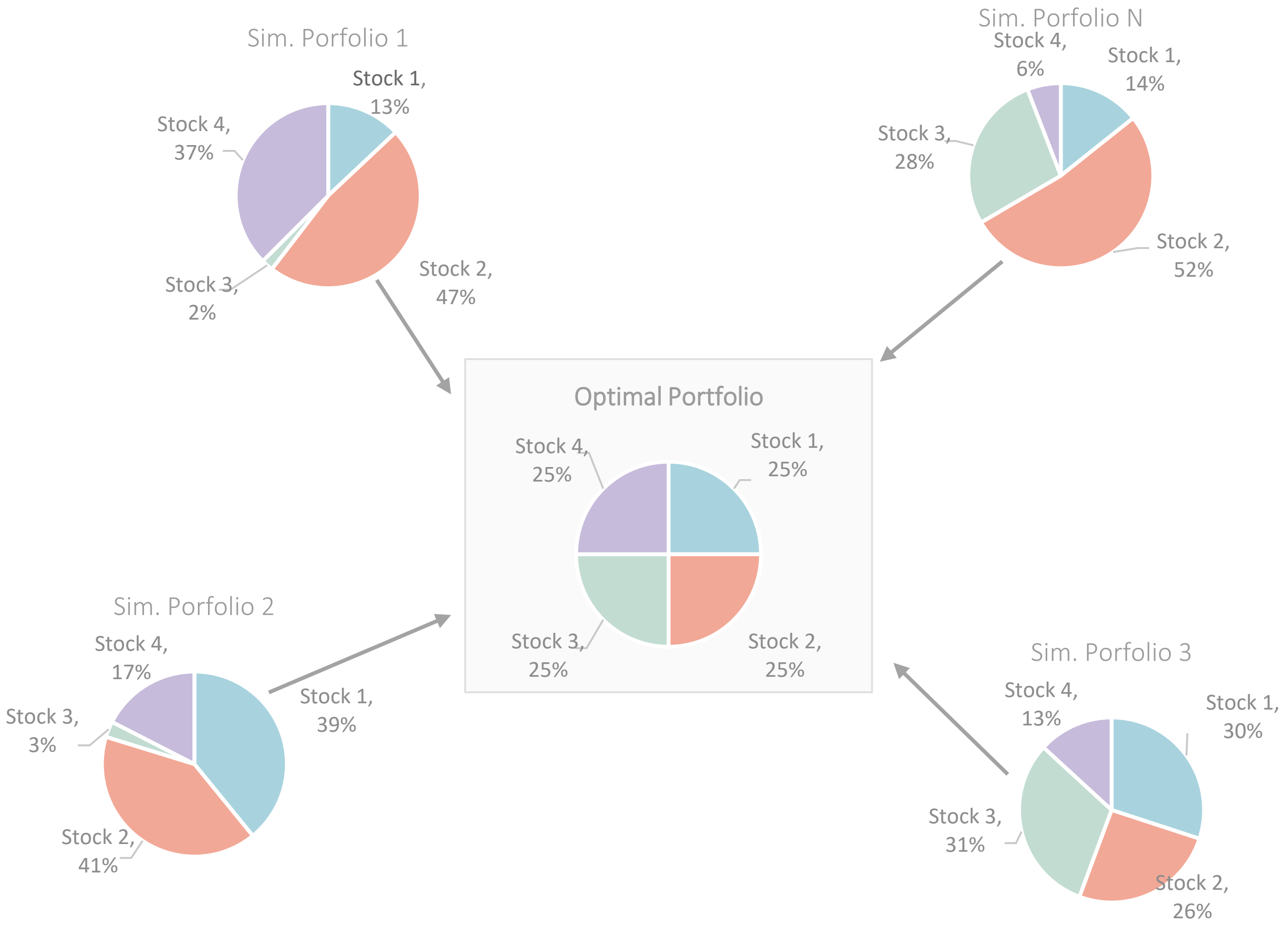


Above model animation adapted from *Machine Learning Mastery*

PSO Used to Minimize Risk of Portfolio

1. Minimizing Risk involves non-linear objective

2. Randomly use PSO algorithm to adjust swarm of stock weights to find the minimum risk portfolio*



PSO Velocity and Position update equation used for modeling *

Minimize the Risk of Portfolio

- Decision Variable: Weights to invest into each stock
- Risk Defined as

$$risk = \sqrt{12} \times \sqrt{Weights.VarCov.Weights^T}$$

- Optimization Problem:

$$\begin{array}{l} \text{Minimize: } risk \\ \text{Subject to:} \\ \sum_{stock \in StockList} Weights_{stock} = 1 \\ \sum_{stock \in StockList} (Weights_{stock} \times ExpectedReturns_{stock}) \geq minDesiredReturn \end{array}$$

Note that “12” represents the number of months in a single calendar year *

Maximize the Sharpe Ratio

- Note algorithm can solve by maximizing the Sharpe Ratio
- Sharpe Ratio defined as:

$$sharpeRatio = \frac{expectedReturn - riskFreeRate}{risk}$$

- Limitations of maximization of Sharpe Ratio with metaheuristic

"riskFreeRate" contains a proxy for the risk-free rate. *
It includes the previous period's value of the U.S. Treasury Bill **

PSO Conceptual Process Overview

Initialize

- Stochastically initialize Swarm with feasible solutions

Update

- Randomly change the weights based on PSO velocity update equation

Evaluate Risk

- Evaluate the risk (or Sharpe ratio) of all simulated portfolios

Document Best

- Document the best evaluated Weights
- Can either use global or local neighborhood

Stop Algorithm

- Algorithm stops when reached number of desired iterations, and
- Is feasible

Model Solutions

Base Solution

Parameters

- Top 50 Market Cap. Firms in S&P 500*
- 3,000 Iterations
- 30 Simulated Portfolios
- Minimizing Portfolio Risk
- Expected Return $\geq$ 7.5%
- Local Best Ring Structure
- Sample Period:
  - May 2012 – May 2022

```
Key Summary Statistics -----------------
Global Best Annualized Risk:    10.0%
Annualized Expected Return:     15.0%
Sharpe Ratio:                   1.44
Expected Return over 9.4 Years: 373.9%

Global best weights in each stock (Only includes stocks to invest in):
```

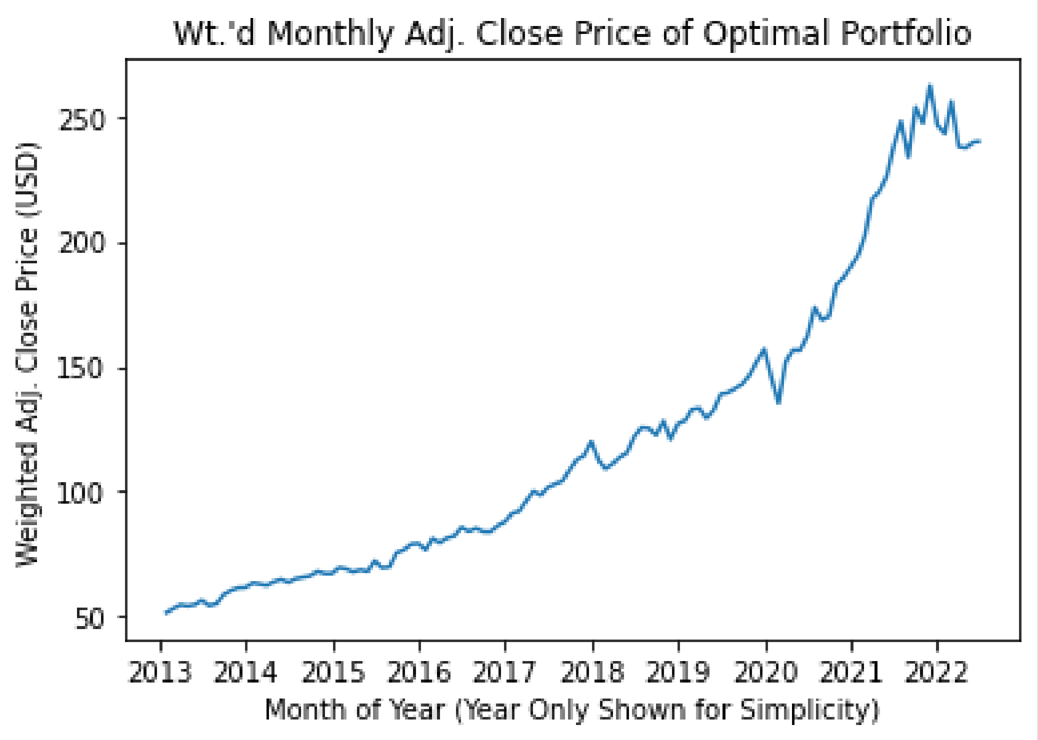
Ticker	Opt. Weight
V	0.3%
NVDA	0.7%
PG	0.3%
MA	3.4%
PFE	3.7%
KO	0.4%
ABBV	11.6%
PEP	12.2%
VZ	0.5%
COST	0.6%
AVGO	11.5%
MCD	0.1%
CSCO	9.7%
DIS	1.0%
TMUS	13.5%
UPS	5.0%
INTC	6.6%
WFC	9.0%
RTX	9.5%

S&P 500 inclusion list as of June 22, 2022 *

Base Solution Cont’d

Parameters

- Top 50 Market Cap. Firms in S&P 500*
- 3,000 Iterations
- 30 Simulated Portfolios
- Minimizing Portfolio Risk
- Expected Return ≥ 7.5%
- Local Best Ring Structure
- Sample Period:
  - May 2012 – May 2022



S&P 500 inclusion list as of June 22, 2022 *

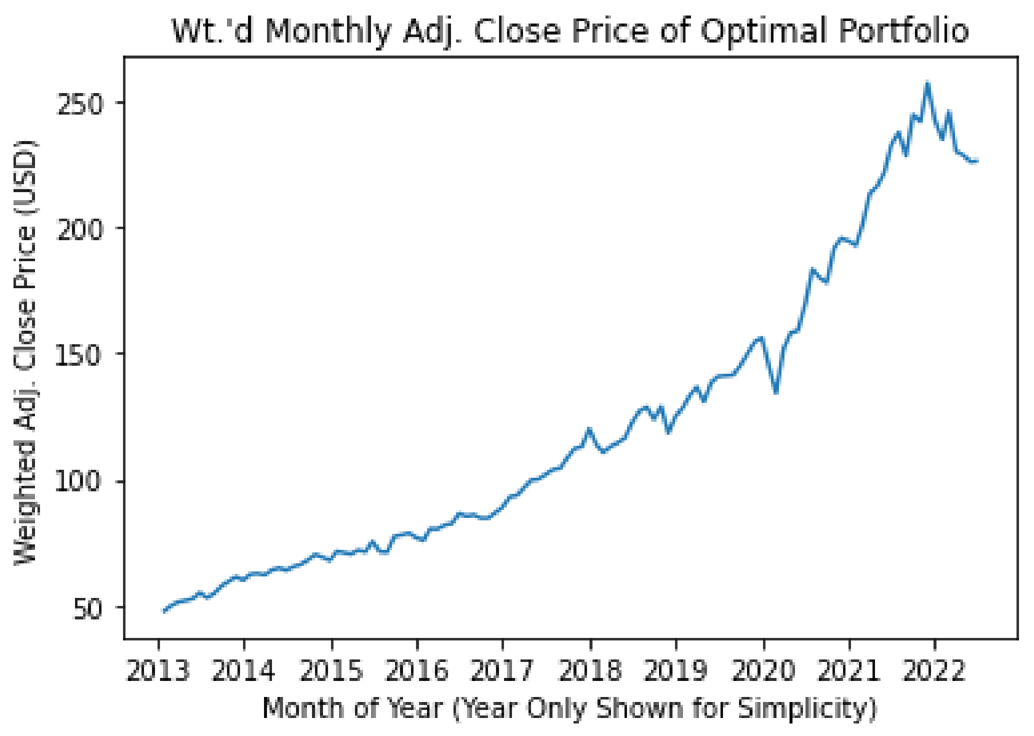
Test 1:

More Iterations and Less Simulated Portfolios: *Worse*

Parameters

- Top 50 Market Cap. Firms in S&P 500*
- 10,000 Iterations**
- 10 Simulated Portfolios***
- Minimizing Portfolio Risk
- Expected Return ≥ 7.5%
- Local Best Ring Structure
- Sample Period:
  - May 2012 – May 2022

```
Key Summary Statistics -----------------
Global Best Annualized Risk:     12.0%
Annualized Expected Return:      18.1%
Sharpe Ratio:                    1.45
Expected Return over 9.4 Years: 477.7%
```



S&P 500 inclusion list as of June 22, 2022 *
Changed from base model using 3,000 iterations **
Changed from base model using 30 portfolio simulations **
Total of 35 stocks recommended to invest in for optimal portfolio ***

Test 2:

Maximizing the Sharpe Ratio : *Worse*

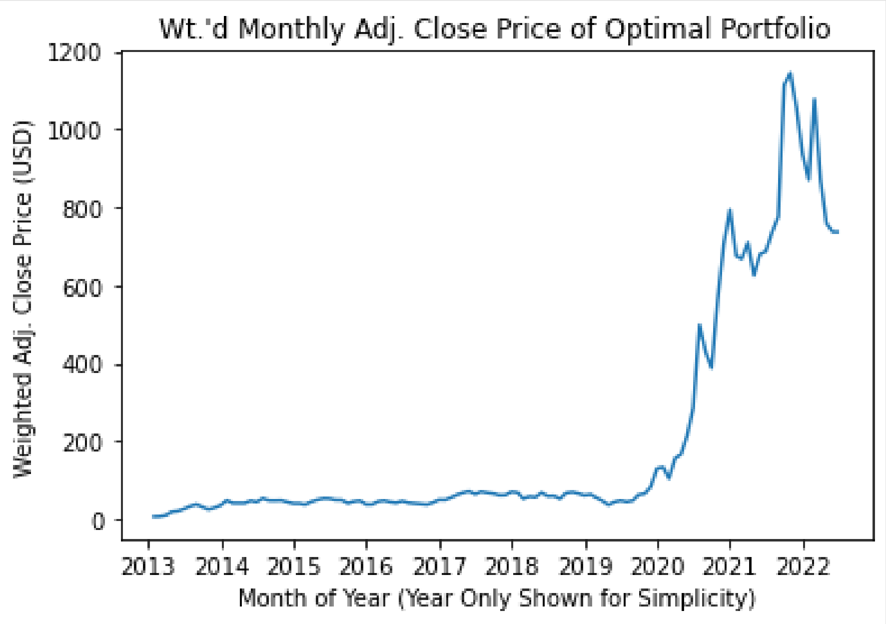
Parameters

- Top 50 Market Cap. Firms in S&P 500*
- 3,000 Iterations
- 30 Simulated Portfolios
- Maximize Portfolio Sharpe Ratio**
- Expected Return ≥ 7.5%
- Local Best Ring Structure
- Sample Period:
  - May 2012 – May 2022

```
Key Summary Statistics ----------------
Global Best Annualized Risk:     68.4%
Annualized Expected Return:      68.4%
Sharpe Ratio:                    0.99
Expected Return over 9.4 Years: 13544.6%

Global best weights in each stock (Only includes stocks to invest in):
```

Ticker	Opt. Weight
PM	100.0%



S&P 500 inclusion list as of June 22, 2022 *
Changed from base model which minimized the risk **
Note not recommended to use the Sharpe Ratio with this model **

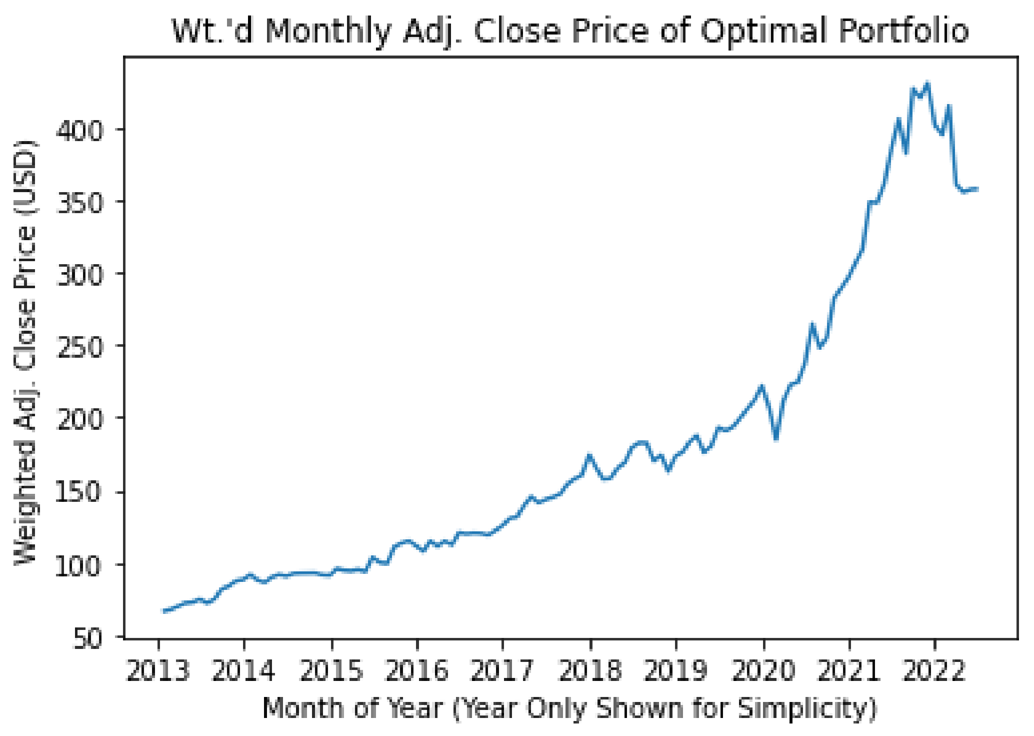
Test 3:

Global Best Method : *Worse*

Parameters

- Top 50 Market Cap. Firms in S&P 500*
- 3,000 Iterations
- 30 Simulated Portfolios
- Minimizing Portfolio Risk
- Expected Return ≥ 7.5%
- Global Best Structure**
- Sample Period:
  - May 2012 – May 2022

```
Key Summary Statistics ----------------
Global Best Annualized Risk:     12.7%
Annualized Expected Return:      19.0%
Sharpe Ratio:                    1.45
Expected Return over 9.4 Years: 513.2%
```



S&P 500 inclusion list as of June 22, 2022 *
Changed from base model using local best ring structure **
Not all stocks shown, assume weights total 100% as seen in detailed report **
Total of 47 stocks recommended to invest in for optimal portfolio ***

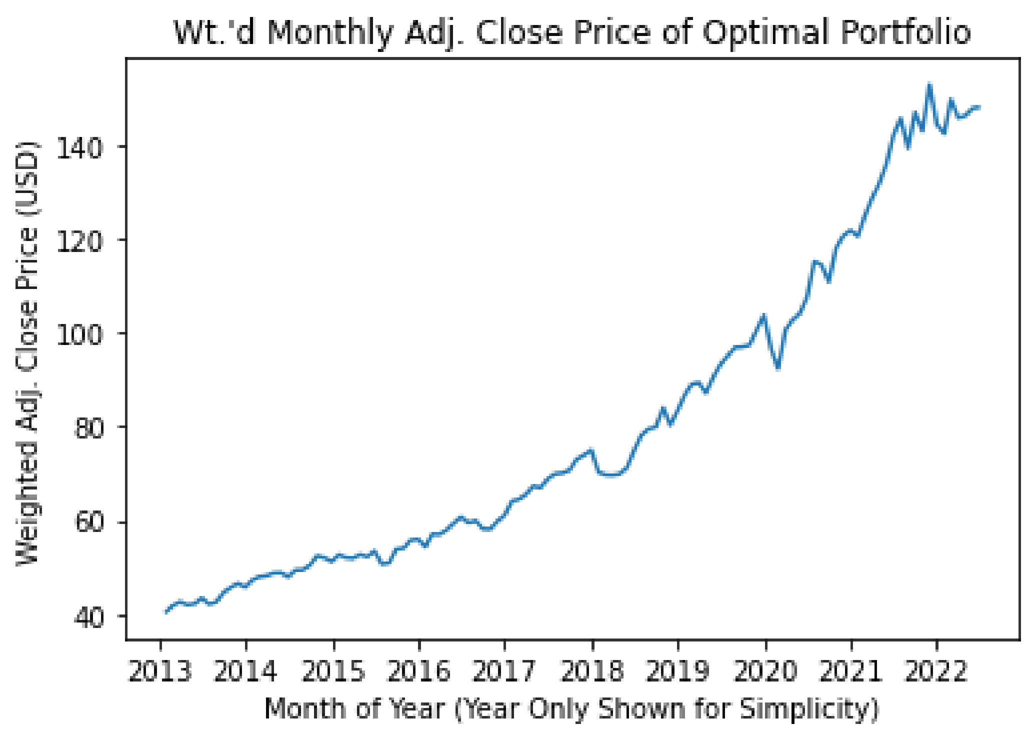
Test 4:

More Iterations and Simulated Portfolios: *Better*

Parameters

- Top 50 Market Cap. Firms in S&P 500*
- 10,000 Iterations**
- 100 Simulated Portfolios***
- Minimizing Portfolio Risk
- Expected Return ≥ 7.5%
- Local Best Ring Structure
- Sample Period:
  - May 2012 – May 2022

```
Key Summary Statistics -----------------
Global Best Annualized Risk:     9.5%
Annualized Expected Return:      14.4%
Sharpe Ratio:                    1.45
Expected Return over 9.4 Years: 355.4%
```



S&P 500 inclusion list as of June 22, 2022 *
Changed from base model using 3,000 iterations **
Changed from base model using 30 portfolio simulations **
Total of 15 stocks recommended to invest in for optimal portfolio ***

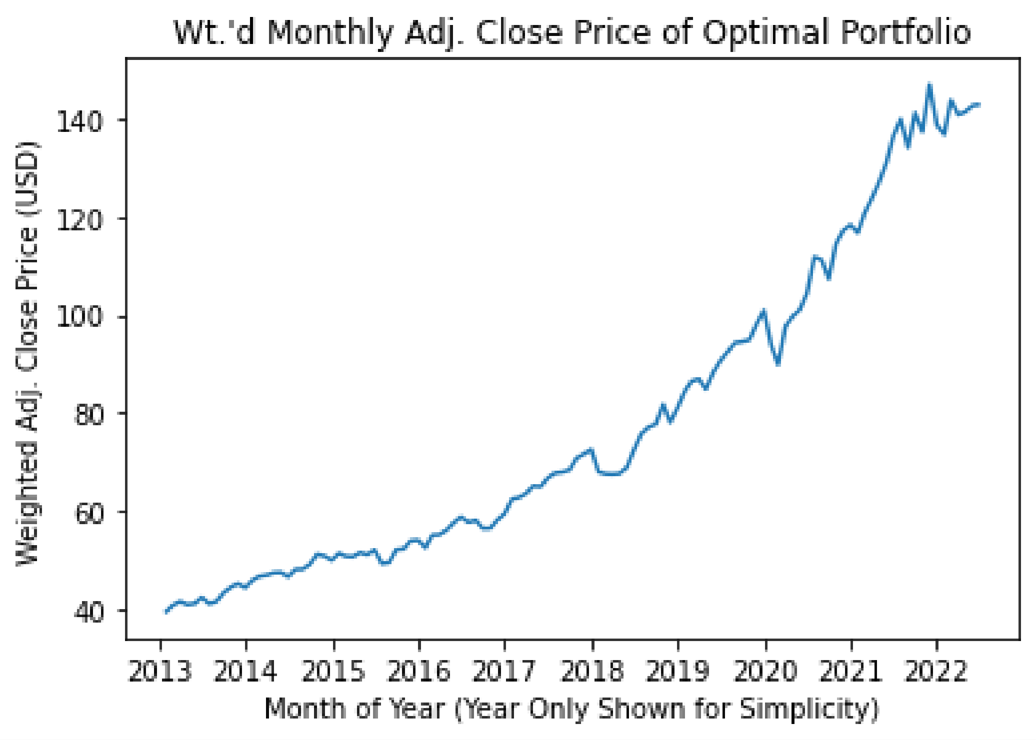
Test 5:

More Iterations and Simulated Portfolios: *Better*

Parameters

- Top 50 Market Cap. Firms in S&P 500*
- 10,000 Iterations**
- 200 Simulated Portfolios***
- Minimizing Portfolio Risk
- Expected Return ≥ 7.5%
- Local Best Ring Structure
- Sample Period:
  - May 2012 – May 2022

```
Key Summary Statistics ----------------
Global Best Annualized Risk:     9.5%
Annualized Expected Return:      14.5%
Sharpe Ratio:                    1.46
Expected Return over 9.4 Years:  357.8%
```



S&P 500 inclusion list as of June 22, 2022 *
Changed from base model using 3,000 iterations **
Changed from base model using 30 portfolio simulations **
Total of 15 stocks recommended to invest in for optimal portfolio ***

Summary of Model Solutions

- Increasing iterations and simulations is better
- Local best is better than global best
- Maximizing Sharpe ratio has future opportunity

Discussion of Modeling Process

Discussion of Modeling Process

Contribution	• MPT Applies Financial Engineering
Difficulties	• Metaheuristics Tedious To Implement
Deviation	• Fewer Metaheuristics Used

Summary of Results