기회인 case (ii) $f(y, p, q) = 0$, $p = a$

Ex

1. Solve : $pq = y$ — ①

Soln put $p = a$ in ①

$\therefore$ $aq = y$

$q = y/a$

Now,
The exact differential equation is

$$dz = p\,dx + q\,dy$$

$$dz = a\,dx + \frac{y}{a}\,dy$$

$$z = ax + \frac{y^2}{2a} + c$$ is a

Complete Solution.

$$\Rightarrow 2az = 2a^2x + y^2 + c_1$$

2. Solve : $p+q=y$ — (1)

Put $p=a,$

$$a+q=y$$

$$q=y-a$$

Now

$\therefore$ $dz = p\,dx + q\,dy$

$dz = a\,dx + (y-a)\,dy$

$z = ax + \frac{(y-a)^2}{2} + c$ // is a

Complete Solution.

Case (iii) $f(z, p, q) = 0$, $q = ap$

Ex

2. Solve: $p^2 = zq$ — (1)

put $q = ap$ in (1)

$\therefore p^2 = z(ap)$

$p = az$

Now,

$$dz = p\,dx + q\,dy \qquad \left\{ \because q = ap \right.$$

$$dz = (az)\,dx + a(az)\,dy$$

$$\frac{dz}{az} = dx + a\,dy$$

$$\frac{1}{a}\log z = x + ay + c \qquad (\text{con}$$

$$\log z = \underline{\underline{ax + a^2 y + c_1}}$$

4) Solve: $p + q = z \quad \text{---} \; (1)$

Put $q = ap$ in (1)

$$p + ap = z$$

$$p(1+a) = z$$

$$p = \frac{z}{(1+a)}$$

Now,

$$dz = p\,dx + q\,dy$$

$$dz = \frac{z}{(1+a)}dx + a\left[\frac{z}{(1+a)}\right]dy$$

$$(1+a)\frac{dz}{z} = dx + a\,dy$$

$$(1+a)\log z = x + ay + c$$ is a C.I

5. Solve: $p^3 = qz$ —— (1)

put $q = ap$ in (1)

(1) $\Rightarrow$ $p^{\cancel{3}\,2} = a\cancel{p}z$

$$p^2 = az \Rightarrow p = \pm\sqrt{az}$$

Now,

$$dz = p\,dx + q\,dy$$

$$dz = \pm\sqrt{a}\cdot\sqrt{z}\,dx \pm a\sqrt{a}\sqrt{z}\,dy$$

$$\frac{dz}{\sqrt{z}} = \pm\sqrt{a}\,dx \pm (a)^{3/2}\,dy$$

$$2\sqrt{z} = \pm\sqrt{a}\,x \pm (a)^{3/2}\,y + c$$ //

Type 4 $\quad f(x,p) = f(y,q)$

As a trial solution,

assume that $f(x,p) = f(y,q) = k$, say

Then solving for p, we get $p = \phi(x)$

and solving for q, we get $q = \psi(y)$

$$\therefore \quad dz = p\,dx + q\,dy$$

$$dz = \phi(x)\,dx + \psi(y)\,dy$$

Integrating,

$$z = \int \phi(x)\,dx + \int \psi(y)\,dy + c$$

Ex:-

7 Solve : $p^2 + q^2 = x + y$

Soln:-

$$p^2 - x = y - q^2 = k, \quad \text{say}$$

Now

$$p^2 - x = k$$
$$p^2 = x + k \Rightarrow p = \pm\sqrt{(x+k)}$$

$$y - q^2 = k$$

$$q^2 = y - k$$

$$q = \sqrt{y - k}$$

Now,

$$dz = p\,dx + q\,dy$$

$$dz = \pm\sqrt{(x+k)}\,dx \pm \sqrt{(y-k)}\,dy$$

$$z = \pm\frac{2}{3}(x+k)^{3/2} \pm \frac{2}{3}(y-k)^{3/2} + c$$

8. $p^2 + r^2 = x^2 + y^2 \quad - (1)$

Sol :-

$$p^2 - x^2 = y^2 - r^2 = K$$

Now,

$$p^2 - x^2 = K$$

$$p^2 = x^2 + K$$

$$p = \pm\sqrt{x^2 + K}$$

$$y^2 - q^2 = k$$

$$q^2 = y^2 - k$$

$$q = \pm\sqrt{y^2 - k}.$$

Now

$$dz = p\,dx + q\,dy$$

$$dz = \pm\sqrt{x^2 + k}\,dx \pm \sqrt{y^2 - k}\,dy$$

$$z = \pm\int\sqrt{x^2 + (\sqrt{k})^2}\,dx \pm \int\sqrt{y^2 - (\sqrt{k})^2}\,dy$$

$$\therefore \int \sqrt{x^2+a^2}\, dx = \frac{a^2}{2} \sinh^{-1}(x/a) + \frac{x}{2}\sqrt{x^2+a^2} + c$$

$$\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} \cosh^{-1}(x/a) + c$$

$$\therefore \quad z = \pm\left(\frac{K}{2} \sinh^{-1}(x/\sqrt{K}) + \frac{x}{2}\sqrt{x^2+K}\right) \pm \left(\frac{y}{2}\sqrt{y^2-K} - \frac{K}{2} \cosh^{-1}(y/\sqrt{K})\right) + c$$

9) Solve : $pq + qx = y$ — (1)

Sol^n :

$$q[p+x] = y$$

$$p+x = \frac{y}{q} = K$$

$\Rightarrow p+x = K$, $\frac{y}{q} = K$

$\Rightarrow p = K-x$ $\quad q = y/K$

$\therefore\ dz = p\,dx + q\,dy$

$dz = (k - x)\,dx + y/k\,dy$

$z = kx - \frac{x^2}{2} + \frac{y^2}{2k} + c$ (on

$2kz = 2k^2x - kx^2 + y^2 + c_1$ //