16/3/22

$$L[f(t)] = \bar{F}(s) = \int_0^\infty e^{-st} f(t)\,dt$$

Ex

1. Find the Laplace transform of

$$e^{-3t}(2\cos 5t - 3\sin 5t).$$

$$L[e^{-3t}[2\cos 5t - 3\sin 5t]] = L[2e^{-3t}\cos 5t] - L[3e^{-3t}\sin 5t]$$

using Shifting property :

$$L[e^{at} f(t)] = \bar{F}(s-a)$$

$$= 2\,L[e^{-3t}\cos 5t] - 3\,L[e^{-3t}\sin 5t]$$

$$= 2\left[\frac{s+3}{(s+3)^2+25}\right] - 3\left[\frac{5}{(s+3)^2+25}\right]$$

$L[\cos 5t] = \dfrac{s}{s^2+25}$

$L[\sin 5t] = \dfrac{5}{s^2+25}$

$$= \frac{2s+6-15}{(s+3)^2+25} = \frac{2s-9}{(s+3)^2+25}$$

2. Find $L[e^{2t}\cos^2 t]$

$\because \quad L[\cos^2 t] = L\left[\dfrac{1+\cos 2t}{2}\right]$

$$= \frac{1}{2}L[1] + \frac{1}{2}L[\cos 2t]$$

$$= \frac{1}{2s} + \frac{1}{2}\left[\frac{s}{s^2+4}\right]$$

Now using shifting property we have,

$$L\left[e^{2t}\cos^2 t\right] = \frac{1}{2(s-2)} + \frac{1}{2}\left[\frac{s-2}{(s-2)^2+4}\right]$$

$$= \frac{1}{2}\left\{\frac{1}{s-2} + \frac{(s-2)}{(s-2)^2+4}\right\}$$

2. Find $L\left[e^{4t}\sin 2t\cos t\right]$.

Sol :-

$$L\left[\sin 2t \cos t\right] = L\left[\frac{1}{2}\left[\sin 3t + \sin t\right]\right]$$

$\because \sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$$= \frac{1}{2}\left\{L\left[\sin 3t\right] + L\left[\sin t\right]\right\}$$

$$= \frac{1}{2}\left\{\frac{3}{s^2+9} + \frac{1}{s^2+1}\right\}$$

Now,

$$L[e^{4t}\sin 2t\cos t] = \frac{1}{2}\left\{\frac{3}{(s-4)^2+9} + \frac{1}{(s-4)^2+1}\right\}$$

Note:-

If $L[f(t)] = F(s)$, then

$$L[(\sinh at) f(t)] = \frac{1}{2}\left[F(s-a) - F(s+a)\right]$$

$$L[(\cosh at) f(t)] = \frac{1}{2}\left[F(s-a) + F(s+a)\right]$$

1. Find $L[\cosh \underline{3}t \cos 2t]$

Sol

$$L[\cosh at\ f(t)] = \frac{1}{2}\left[\bar{F}(s-a) + \bar{F}(s+a)\right]$$

$$L[\cosh 3t \cos 2t] = \frac{1}{2}\left[\frac{s-3}{(s-3)^2+4} + \frac{s+3}{(s+3)^2+4}\right]$$

2 Find $L[\sinh 2t \sin 3t]$

Sol:

$$L[\sinh 2t \sin 3t] = \frac{1}{2}\left\{\frac{3}{(s-2)^2+9} - \frac{3}{(s+2)^2+9}\right]$$

4. Show that $L[t \sin at] = \dfrac{2as}{(s^2+a^2)^2}$ and

$$L[t \cos at] = \frac{s^2-a^2}{(s^2+a^2)^2}.$$

Sol:

We know that $L[t] = \dfrac{1}{s^2}$

$$L[te^{iat}] = \frac{1}{(s-ia)^2}$$

$\because e^{iat} = \cos at + i \sin at$

$$= \frac{1}{(s-ia)^2} \times \frac{[s+ia]^2}{(s+ia)^2}$$

$$= \frac{(s+ia)^2}{[(s-ia)(s+ia)]^2}$$

$$= \frac{(s+ia)^2}{(s^2+a^2)^2} \quad \left\{ \because i^2 = -1 \right.$$

$$= \frac{s^2+2ias-a^2}{(s^2+a^2)^2}$$

$$L[te^{iat}] = \frac{(s^2-a^2)+i\,2as}{(s^2+a^2)^2}$$

$$\Rightarrow L[t[\cos at + i\sin at]] = \frac{(s^2-a^2)+i\,2as}{(s^2+a^2)^2}$$

Equating the Real and imaginary parts from both sides

$$L[t\cos at] = \frac{s^2 - a^2}{(s^2+a^2)^2} \quad \text{and}$$

$$L[t\sin at] = \frac{2as}{(s^2+a^2)^2} \quad //$$

5. Find the L.T. of $f(t)$ defined as $f(t) = t/\tau$, when $0 < t < \tau$

$= 1$, $t > \tau$

Soln:

$$\mathcal{L}[f(t)] = \int_0^\tau e^{-st} \cdot \frac{t}{\tau}\,dt + \int_\tau^\infty e^{-st} \cdot 1\,dt$$

$$= \frac{1}{\tau}\left\{\left[\frac{t\,e^{-st}}{-s}\right] - \left[\frac{e^{-st}}{s^2}\right]\right\}_0^\tau + \left(\frac{e^{-st}}{-s}\right)_\tau^\infty$$

$$= \frac{1}{\tau}\left\{ \left(\frac{\tau e^{-s\tau}}{-s} - \frac{e^{-s\tau}}{s^2} \right) - \left[-\frac{1}{s^2} \right] \right\} - \frac{1}{s}\left(0 - e^{-s\tau}\right)$$

$$= \cancel{-\frac{e^{-s\tau}}{s}} - \frac{1}{\tau}\frac{e^{-s\tau}}{s^2} + \frac{1}{\tau s^2} + \cancel{\frac{e^{-s\tau}}{s}}$$

$$= \frac{\left(1 - e^{-s\tau}\right)}{\tau s^2}$$

6) Find the Laplace transform of

$$f(t) = \begin{cases} 1 & , \quad 0 < t \le 1 \\ t & , \quad 1 < t \le 2 \\ 0 & , \quad t > 2 \end{cases}$$

Soln :-

$$L[f(t)] = \int_0^1 e^{-st} \cdot 1 \, dt + \int_1^2 e^{-st} \cdot t \, dt$$

$$= \left(\frac{-e^{-st}}{s} \right)_0^1 + \left\{ \frac{t e^{-st}}{-s} - \frac{e^{-st}}{s^2} \right\}_1^2$$

$$= -\frac{1}{s}\left[e^{-s} - 1\right] + \left\{\left(\frac{-2}{s}e^{-2s} - \frac{e^{-2s}}{s^2}\right) - \left(\frac{-e^{-s}}{s} - \frac{e^{-s}}{s^2}\right)\right\}$$

$$= \frac{1}{s} - \cancel{\frac{e^{-s}}{s}} - \frac{2e^{-2s}}{s} - \frac{e^{-2s}}{s^2} + \cancel{\frac{e^{-s}}{s}} + \frac{e^{-s}}{s^2}$$

$$= \frac{1}{s} + \frac{-2e^{-2s}}{s} + \frac{1}{s^2}\left[e^{-s} - e^{-2s}\right]$$

$$= \frac{s - 2se^{-2s} + e^{-s} - e^{-2s}}{s^2}$$

H.W

$$L\left\{\left[\sqrt{t} - \frac{1}{\sqrt{t}}\right]^3\right\}$$

Ans :- $$\frac{\sqrt{\pi}}{4}\left[\frac{3}{s^{5/2}} - \frac{6}{s^{3/2}} + \frac{12}{s^{1/2}} + \frac{8}{s^{-1/2}}\right]$$