Charpit's Method

Consider a 1^{st} order pde with 2 independent variables

$$f[x, y, z, p, q] = 0$$ where

$p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$. Assume $f_p^2 - f_q^2 \neq 0$

A.E. of the pde,

$$\frac{dx}{F_p} = \frac{dy}{F_q} = \frac{dz}{pF_p + qF_q} = \frac{dp}{-F_x - pF_z} = \frac{dq}{-F_y - qF_z}$$

$$\Rightarrow \frac{dp}{-F_x - pF_z} = \frac{dq}{-F_y - qF_z} = 0$$

$$\Rightarrow dp = 0, \quad dq = 0$$

$$\underline{\underline{p = a}}, \quad \underline{q = b}$$

4/3/22

Non-Linear 1st order pde

Type	Descriptions	Complete Integral and singular Integral
1. $f(p,q)=0$	Equations having p and q	1. Substitute $p=a$ and $q=b$. 2. Now, $f(a,b)=0$ and solve the equation for b as, $b=\phi(a)$. 3. The total differential, $dz = a\,dx + b\,dy$ $dz = a\,dx + \phi(a)\,dy$ ---- (1)

Type	Description	C.I. & S.I.
II	"	On Integrating (1) we get C.I.

Now to find singular Integral,

$$z = ax + \phi(a)y + c \quad -(2)$$

Which is a complete Integral.

Diff (2) with respect to 'a' and 'c'

$0 = x + \phi'(a)\, y$

$\Rightarrow \phi'(a) = -x/y$

Now

'c'

$0 = 1$, which is absurd.

Type I does not exists with S. I

Ex

1. Solve : $\sqrt{p} + \sqrt{q} = 1$ — (1) (10 marks)

Soln :-

The given equation of the form

$$f(p, q) = 0$$

put $p = a$ and $q = b$ in (1)

$$\sqrt{a} + \sqrt{b} = 1$$

$$\sqrt{b} = 1 - \sqrt{a}$$

$$\Rightarrow \quad b = c(1-\sqrt{a})^2$$

The Exact differential equation is given by,

$$dz = a\,dx + b\,dy$$

$$dz = a\,dx + c(1-\sqrt{a})^2 dy \quad \text{—— (2)}$$

Integrate (2)

$$\int dz = a\int dx + c(1-\sqrt{a})^2 \int dy + c$$

$$z = ax + c(1-\sqrt{a})^2 y + c \quad \text{—— (3)}$$

is a complete Integral

To find S.I

$$z = ax + (1-\sqrt{a})^2 y + c$$

Differentiate with respect to 'a' and 'c'

'a'

$$0 = x + 2(1-\sqrt{a})\left[-\frac{1}{2}a^{-1/2}\right]y$$

$$0 = x - \frac{(1-\sqrt{a})y}{\sqrt{a}}$$

$$0 = x\sqrt{a} - y + \sqrt{a}\,y$$

$$\sqrt{a}\,[x+y] = y$$

$$\sqrt{a} = \frac{y}{x+y}$$

$$a = \frac{y^2}{(x+y)^2} \quad //$$

'c'

$0 = 1$, which is absurd

$\Rightarrow$ There is no S.I exists.

2) Solve $p+q=2$ — (1) (5 marks)

Sol :-

put $p=a$, $q=b$ in (1)

$$\Rightarrow a+b=2$$

$$b=(2-a)$$

$\therefore$ The exact differential equation,

$$dz = a\,dx + b\,dy$$

$$dz = a\,dx + (2-a)\,dy$$

The exact differential equation is given by

$$dz = a\,dx + b\,dy$$

$$dz = a\,dx + (2-a)\,dy$$

Integrate $z = ax + (2-a)y + c$ is a C.I.

The S.I does not exists for this type.

5 Solve $p + q + pq = 0$ — (1)

Sol^n :-

Put $p = a$, $q = b$ in (1)

$\therefore$ (1) $\Rightarrow$ $a + b + ab = 0$

$b(1 + a) = -a$

$b = \dfrac{-a}{1+a}$

The Exact differential equation is,

$$dz = a\,dx + b\,dy$$

$$dz = a\,dx - \frac{a}{(1+a)}\,dy$$

Integrate

$$z = ax - \frac{a}{(1+a)}\,y + c$$

is a

Complete Integral.

S.I does not exists

4) $p(p+1) = q^2$

5) $p^2 + q^2 = 4$

Sol^n

4. $z = ax + \sqrt{a(1+a)}\, y + c$ //

5. $z = ax \pm \sqrt{4-a^2}\, y + c$ //.