

Bidirectional Typing Rules for Mini

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Basically the same as [1], except bidirectional. The reason we still have rigidity on checking is that certain operations (like APP) check non-rigidly.

$$\boxed{\Gamma \vdash t \Rightarrow^m \tau}$$

$$\frac{\text{VAR} \quad v : \forall \bar{a}. \tau \in \Gamma}{\Gamma \vdash v \Rightarrow^m [\bar{a} \mapsto v] \tau}$$

$$\frac{\text{APP} \quad \Gamma \vdash f \Rightarrow^w \tau_1 \rightarrow \tau_2 \quad \Gamma \vdash x \Leftarrow^w \tau_1}{\Gamma \vdash (f \ x) \Rightarrow^m \tau_2}$$

$$\frac{\text{ABS-INFER} \quad \Gamma, x :^m \tau_1 \vdash t \Leftarrow^m \tau_2}{\Gamma \vdash (\text{lam } x \ t) \Rightarrow^m \tau_1 \rightarrow \tau_2}$$

$$\frac{\text{LET-INFER} \quad \Gamma \vdash u \Rightarrow^w \tau_1 \quad \bar{a} = \text{ftv}(\tau_1) - \text{ftv}(\Gamma) \quad \Gamma \vdash u \uparrow^{m_1} \quad \Gamma, x :^{m_1} \forall \bar{a}. \tau_1 \vdash t \Rightarrow^{m_2} \tau_2}{\Gamma \vdash (\text{let } (x \ u) \ t) \Rightarrow^{m_2} \tau_2}$$

$$\frac{\text{FIX-INFER} \quad \Gamma \vdash (x \ u) \mapsto \Delta \quad \Gamma, \Delta \vdash t \Rightarrow^{m_2} \tau_2}{\Gamma \vdash (\text{fix } \overline{(x \ u)} \ t) \Rightarrow^{m_2} \tau_2}$$

$$\frac{\text{ANN} \quad \bar{a} = \text{ftv}(\tau) - \text{ftv}(\Gamma) \quad \Gamma, \bar{a} \vdash x \Leftarrow^r \tau}{\Gamma \vdash (x :: \tau) \Rightarrow^m \tau}$$

$$\frac{\text{MATCH-INFER} \quad \Gamma \vdash u \Rightarrow^w \tau_p \quad \Gamma \vdash \overline{p \Leftarrow^w \bar{\tau}_v \rightarrow \tau_p} \quad \overline{\Gamma, p_v :^w \bar{\tau}_v \vdash t \Rightarrow \tau_t}}{\Gamma \vdash (\text{match } u \ \overline{(p \rightarrow t)}) \Rightarrow^{m_t} \tau_t}$$

$$\frac{\text{LIT} \quad L : \tau \in \Gamma}{\Gamma \vdash L \Rightarrow^m \tau}$$

$$\frac{\text{CONS} \quad C : \forall \bar{a}. \bar{\tau}_1 \rightarrow \bar{\tau}_2 \in \Gamma \quad \theta = [\bar{a} \mapsto v] \text{ s.t. } \overline{\Gamma \vdash \theta(v) \Leftarrow^r \tau_1}}{\Gamma \vdash C\bar{v} \Rightarrow^m \theta(\tau_2)}$$

$$\frac{\text{PRIMOP} \quad O : \bar{\tau}_1 \rightarrow \bar{\tau}_2 \quad \theta = [\bar{a} \mapsto v] \text{ s.t. } \overline{\Gamma \vdash \theta(v) \Leftarrow^r \tau_1}}{\Gamma \vdash O\bar{v} \Rightarrow^m \tau_2}$$

$$\boxed{\Gamma \vdash t \Leftarrow^m \tau}$$

$$\frac{\text{ABS-CHECK} \quad \Gamma, x :^m \tau_1 \vdash t \Leftarrow^m \tau_2}{\Gamma \vdash (\mathbf{lam} \ x \ t) \Leftarrow^m \tau_1 \rightarrow \tau_2}$$

$$\frac{\text{LET-CHECK} \quad \Gamma \vdash u \uparrow^{m_1} \quad \bar{a} = \text{ftv}(\tau_1) - \text{ftv}(\Gamma) \quad \Gamma, x :^{m_1} \forall \bar{a}. \tau_1 \vdash t \Leftarrow^{m_2} \tau_2}{\Gamma \vdash (\mathbf{let} \ (x \ u) \ t) \Leftarrow^{m_2} \tau_2}$$

$$\frac{\text{FIX-CHECK} \quad \Gamma \vdash \overline{(x \ u)} \mapsto \Delta \quad \Gamma, \Delta \vdash t \Leftarrow^{m_2} \tau_2}{\Gamma \vdash (\mathbf{fix} \ \overline{(x \ u)} \ t) \Leftarrow^{m_2} \tau_2}$$

$$\frac{\text{MATCH-CHECK} \quad \Gamma \vdash u \uparrow^{m_p} \quad \Gamma \vdash u \Rightarrow^w \tau_p \quad \Gamma \vdash \overline{p, t} \Leftarrow^{\langle m_p, m_t \rangle} \tau_p, \tau_t}{\Gamma \vdash (\mathbf{match} \ u \ (\overline{p \rightarrow t})) \Leftarrow^{m_t} \tau_t} \quad \frac{\text{CHECK-INFER} \quad \Gamma \vdash t \Rightarrow^m \tau}{\Gamma \vdash t \Leftarrow^m \tau}$$

$$\boxed{\Gamma \vdash p, t \Rightarrow^{\langle m_t, \tau_p \rangle} \tau_t}$$

$$\frac{\text{PCON-INFER} \quad C : \forall \bar{a}. \bar{\tau}_1 \rightarrow P \bar{\tau}_v \in \Gamma \quad \Gamma, v :^w \tau_v \vdash t \Rightarrow^{m_t} \tau_t}{\Gamma \vdash C \bar{v}, t \Rightarrow^{\langle m_t, P \bar{\tau}_v \rangle} \tau_t} \quad \frac{\text{PLIT-INFER} \quad L : \tau_p \in \Gamma \quad \Gamma \vdash t \Rightarrow^{m_t} \tau_t}{\Gamma \vdash L, t \Rightarrow^{\langle m_t, \tau_p \rangle} \tau_t}$$

$$\frac{\text{PWILD-INFER} \quad \Gamma \vdash t \Rightarrow^{m_t} \tau_t}{\Gamma \vdash, t \Rightarrow^{\langle m_t, \tau_p \rangle} \tau_t}$$

$$\boxed{\Gamma \vdash p, t \Leftarrow^{\langle m_p, m_t \rangle} \tau_p, \tau_t}$$

$$\frac{\text{PCON-W} \quad C : \forall \bar{a}. \bar{\tau}_1 \rightarrow P \bar{\tau}_2 \in \Gamma \quad \Gamma, v :^w \tau_v \vdash t \Leftarrow^m \tau_t}{\Gamma \vdash C \bar{v}, t \Leftarrow^{\langle w, m \rangle} P \bar{\tau}_v, \tau_t}$$

$$\frac{\text{PCON-R} \quad C : \forall \bar{a}. \bar{\tau}_1 \rightarrow P \bar{\tau}_2 \in \Gamma \quad \theta = \text{mgu}(\bar{\tau}_p, \bar{\tau}_2) \quad \Gamma, v :^r \theta(\tau_v) \vdash \theta(t) \Leftarrow^m \theta(\tau_t)}{\Gamma \vdash C \bar{v}, t \Leftarrow^{\langle r, m \rangle} P \bar{\tau}_v, \tau_t}$$

$$\frac{\text{PLIT-CHECK} \quad L : \tau_p \in \Gamma \quad \Gamma \vdash t \Leftarrow^{m_t} \tau_t}{\Gamma \vdash L, t \Leftarrow^{\langle m_p, m_t \rangle} \tau_p, \tau_t} \quad \frac{\text{PWILD-CHECK} \quad \Gamma \vdash t \Leftarrow^{m_t} \tau_t}{\Gamma \vdash -, t \Leftarrow^{\langle m_p, m_t \rangle} \tau_p, \tau_t}$$

$$\boxed{\Gamma \vdash \overline{(x \ u)} \mapsto \Delta}$$

$$\frac{\text{FIXDEFS} \quad \overline{a_1 = \text{ftv}(\tau_3) - \text{ftv}(\Gamma)} \quad \overline{\Gamma, x_1 :^{m_1} \forall a_1. \tau_3, x_2 :^{m_2} \tau_1 \vdash u_1 \Leftarrow^w \forall a_1. \tau_3, u_2 \Leftarrow^w \tau_1} \quad \overline{a_2 = \text{ftv}(\tau_1) - \text{ftv}(\Gamma)} \quad \overline{\Delta = x_1 :^{m_1} \forall a_1. \tau_3, x_2 :^{m_2} \forall a_2. \tau_1}}{\Gamma \vdash \overline{(x_1 \ (u_1 : \tau_3))}, \overline{(x_2 \ u_2)} \mapsto \Delta}$$

$$\boxed{\Gamma \vdash t \uparrow^m}$$

$$\begin{array}{c}
\text{SCR-VAR} \\
\frac{v \uparrow^m \in \Gamma}{\Gamma \vdash v \uparrow^m}
\end{array}
\quad
\begin{array}{c}
\text{SCR-APP} \\
\frac{\Gamma \vdash f \uparrow^{m_1} \quad \Gamma \vdash x \uparrow^{m_2}}{\Gamma \vdash (f \ x) \uparrow^{[m_1, m_2]}}
\end{array}
\quad
\begin{array}{c}
\text{SCR-LET} \\
\frac{\Gamma \vdash u \uparrow^{m_1} \quad \Gamma, x \uparrow^{m_1} \vdash t \uparrow^{m_2}}{\Gamma \vdash (\text{let } (x \ u) \ t) \uparrow^{m_2}}
\end{array}$$

$$\begin{array}{c}
\text{SCR-ABS} \\
\frac{\Gamma, x \uparrow^w \vdash t \uparrow^{m_1}}{\Gamma \vdash (\text{lam } x \ t) \uparrow^{m_1}}
\end{array}
\quad
\begin{array}{c}
\text{SCR-ANN} \\
\frac{}{\Gamma \vdash (x : \tau) \uparrow^r}
\end{array}
\quad
\begin{array}{c}
\text{SCR-OTHER} \\
\frac{}{\Gamma \vdash t \uparrow^w}
\end{array}$$

References

- [1] Simon Peyton Jones, Dimitrios Vytiniotis, and Stephanie Weirich. *Simple unification-based type inference for GADTs*. Jan. 2005.