Bidirectional Typing Rules for Mini

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Judgements take the conventional bidirectional form of $\Gamma \vdash t \Rightarrow^m \tau$ and $\Gamma \vdash t \Leftarrow^m \tau$. Otherwise, rules operate in essentially the same manner as [1], i.e. we add a 'modifier' m, indicating whether the type τ is 'rigid'. Similarly to [1], we add a function $\Gamma \vdash t \upharpoonleft^m$, which infers the rigidity of the type of t. Note that this does not infer the type of t. We also add a commutative function $\lfloor m_1, m_2 \rfloor$, such that $\lfloor r, r \rfloor = r$, and $\forall x. \lfloor w, x \rfloor = w$, used in inference of the rigidity.

$\boxed{\Gamma \vdash t \Rightarrow^m \tau}$

$$\textsc{Var}\quad \frac{v : \forall \bar{a}.\tau \in \Gamma}{\Gamma \vdash v \Rightarrow^m [\overline{a \mapsto v}]\tau}$$

$$\textsc{App}\quad \frac{\Gamma \vdash f \Rightarrow^w \tau_1 \to \tau_2 \qquad \Gamma \vdash x \Leftarrow^w \tau_1}{\Gamma \vdash \texttt{(}f\ \ x\texttt{)} \Rightarrow^m \tau_2}$$

$$\textsc{Abs-Infer}\quad \frac{\Gamma, x :^m \tau_1 \vdash t \Leftarrow^m \tau_2}{\Gamma \vdash \texttt{(lam}\ x\ \ t\texttt{)} \Rightarrow^m \tau_1 \to \tau_2}$$

$$\textsc{Let-Infer}\quad \frac{\begin{array}{c}\Gamma \vdash u \upharpoonleft^{m_1} \\ \Gamma \vdash u \Rightarrow^w \tau_1 \qquad \bar{a} = \mathrm{ftv}(\tau_1) - \mathrm{ftv}(\Gamma) \qquad \Gamma, x :^{m_1} \forall \bar{a}.\tau_1 \vdash t \Rightarrow^{m_2} \tau_2\end{array}}{\Gamma \vdash \texttt{(let}\ \texttt{(}x\ \ u\texttt{)}\ \ t\texttt{)} \Rightarrow^{m_2} \tau_2}$$

$$\textsc{Fix-Infer}\quad \frac{\Gamma \vdash \overline{(x\ \ u)} \mapsto \Delta \qquad \Gamma, \Delta \vdash t \Rightarrow^{m_2} \tau_2}{\Gamma \vdash \texttt{(fix}\ \overline{\texttt{(}x\ \ u\texttt{)}}\ \ t\texttt{)} \Rightarrow^{m_2} \tau_2}$$

$$\textsc{Ann}\quad \frac{\bar{a} = \mathrm{ftv}(\tau) - \mathrm{ftv}(\Gamma) \qquad \Gamma, \bar{a} \vdash x \Leftarrow^r \tau}{\Gamma \vdash \texttt{(}x\texttt{::}\tau\texttt{)} \Rightarrow^m \tau}$$

$$\textsc{Match-Infer}\quad \frac{\Gamma \vdash u \Rightarrow^w \tau_p \qquad \Gamma \vdash \overline{p \Leftarrow^w \bar{\tau}_v \to \tau_p} \qquad \overline{\Gamma, \overline{p_v :^w \tau_v} \vdash t \Rightarrow \tau_t}}{\Gamma \vdash \texttt{(match}\ u\ \ \overline{\texttt{(}p\ \texttt{->}\ t\texttt{)}}\texttt{)} \Rightarrow^{m_t} \tau_t}$$

$$\textsc{Lit}\quad \frac{L : \tau \in \Gamma}{\Gamma \vdash L \Rightarrow^m \tau}$$

$$\textsc{Cons}\quad \frac{C : \forall \bar{a}.\bar{\tau_1} \to \bar{\tau_2} \in \Gamma \qquad \theta = [\overline{a \mapsto v}] \text{ s.t. } \overline{\Gamma \vdash \theta(v) \Leftarrow^r \tau_1}}{\Gamma \vdash C\bar{v} \Rightarrow^m \theta(\tau_2)}$$

$$\textsc{Primop}\quad \frac{O : \bar{\tau_1} \to \tau_2 \qquad \theta = [\overline{a \mapsto v}] \text{ s.t. } \overline{\Gamma \vdash \theta(v) \Leftarrow^r \tau_1}}{\Gamma \vdash O\bar{v} \Rightarrow^m \tau_2}$$

$\boxed{\Gamma \vdash t \Leftarrow^{m} \tau}$

Abs-Check

$$\frac{\Gamma, x :^{m} \tau_1 \vdash t \Leftarrow^{m} \tau_2}{\Gamma \vdash \texttt{(lam}\ x\ t\texttt{)} \Leftarrow^{m} \tau_1 \to \tau_2}$$

Let-Check

$$\frac{\begin{array}{c}\Gamma \vdash u \upharpoonleft^{m_1} \\ \Gamma \vdash u \Rightarrow^{w} \tau_1 \qquad \bar{a} = \mathrm{ftv}(\tau_1) - \mathrm{ftv}(\Gamma) \qquad \Gamma, x :^{m_1} \forall \bar{a}.\tau_1 \vdash t \Leftarrow^{m_2} \tau_2\end{array}}{\Gamma \vdash \texttt{(let (}x\ u\texttt{)}\ t\texttt{)} \Leftarrow^{m_2} \tau_2}$$

Fix-Check

$$\frac{\Gamma \vdash \overline{\texttt{(}x\ u\texttt{)}} \mapsto \Delta \qquad \Gamma, \Delta \vdash t \Leftarrow^{m_2} \tau_2}{\Gamma \vdash \texttt{(fix}\ \overline{\texttt{(}x\ u\texttt{)}}\ t\texttt{)} \Leftarrow^{m_2} \tau_2}$$

Match-Check

$$\frac{\Gamma \vdash u \upharpoonleft^{m_p} \qquad \Gamma \vdash u \Rightarrow^{w} \tau_p \qquad \Gamma \vdash \overline{p, t} \Leftarrow^{\langle m_p, m_t \rangle} \tau_p, \tau_t}{\Gamma \vdash \texttt{(match}\ u\ \overline{\texttt{(}p\ \texttt{->}\ t\texttt{)}}\texttt{)} \Leftarrow^{m_t} \tau_t}$$

Check-Infer

$$\frac{\Gamma \vdash t \Rightarrow^{m} \tau}{\Gamma \vdash t \Leftarrow^{m} \tau}$$

$\boxed{\Gamma \vdash p, t \Rightarrow^{\langle m_t, \tau_p \rangle} \tau_t}$

PCon-Infer

$$\frac{C : \forall \bar{a}.\bar{\tau_1} \to P\bar{\tau_v} \in \Gamma \qquad \Gamma, \overline{v :^{w} \tau_v} \vdash t \Rightarrow^{m_t} \tau_t}{\Gamma \vdash C\bar{v}, t \Rightarrow^{\langle m_t, P\bar{\tau_v} \rangle} \tau_t}$$

PLit-Infer

$$\frac{L : \tau_p \in \Gamma \qquad \Gamma \vdash t \Rightarrow^{m_t} \tau_t}{\Gamma \vdash L, t \Rightarrow^{\langle m_t, \tau_p \rangle} \tau_t}$$

PWild-Infer

$$\frac{\Gamma \vdash t \Rightarrow^{m_t} \tau_t}{\Gamma \vdash {}_{,}\ t \Rightarrow^{\langle m_t, \tau_p \rangle} \tau_t}$$

$\boxed{\Gamma \vdash p, t \Leftarrow^{\langle m_p, m_t \rangle} \tau_p, \tau_t}$

PCon-W

$$\frac{C : \forall \bar{a}.\bar{\tau_1} \to P\bar{\tau_2} \in \Gamma \qquad \Gamma, \overline{v :^{w} \tau_v} \vdash t \Leftarrow^{m} \tau_t}{\Gamma \vdash C\bar{v}, t \Leftarrow^{\langle w, m \rangle} P\bar{\tau_v}, \tau_t}$$

PCon-R

$$\frac{C : \forall \bar{a}.\bar{\tau_1} \to P\bar{\tau_2} \in \Gamma \qquad \theta = \mathrm{mgu}(\bar{\tau_p}, \bar{\tau_2}) \qquad \Gamma, \overline{v :^{r} \theta(\tau_v)} \vdash \theta(t) \Leftarrow^{m} \theta(\tau_t)}{\Gamma \vdash C\bar{v}, t \Leftarrow^{\langle r, m \rangle} P\bar{\tau_v}, \tau_t}$$

PLit-Check

$$\frac{L : \tau_p \in \Gamma \qquad \Gamma \vdash t \Leftarrow^{m_t} \tau_t}{\Gamma \vdash L, t \Leftarrow^{\langle m_p, m_t \rangle} \tau_p, \tau_t}$$

PWild-Check

$$\frac{\Gamma \vdash t \Leftarrow^{m_t} \tau_t}{\Gamma \vdash \_, t \Leftarrow^{\langle m_p, m_t \rangle} \tau_p, \tau_t}$$

$\boxed{\Gamma \vdash \overline{\texttt{(}x\ u\texttt{)}} \mapsto \Delta}$

FixDefs

$$\frac{\begin{array}{c}\Gamma \vdash \overline{u_1, \upharpoonleft^{m_1}}, \overline{u_2, \upharpoonleft^{m_2}} \\ \overline{a_1 = \mathrm{ftv}(\tau_3) - \mathrm{ftv}(\Gamma)} \qquad \Gamma, \overline{x_1 :^{m_1} \forall a_1.\tau_3}, \overline{x_2 :^{m_2} \tau_1} \vdash \overline{u_1 \Leftarrow^{w} \forall a_1.\tau_3}, \overline{u_2 \Leftarrow^{w} \tau_1} \\ \overline{a_2 = \mathrm{ftv}(\tau_1) - \mathrm{ftv}(\Gamma)} \qquad \Delta = \overline{x_1 :^{m_1} \forall a_1.\tau_3}, \overline{x_2 :^{m_2} \forall a_2.\tau_1}\end{array}}{\Gamma \vdash \overline{\texttt{(}x_1\ \texttt{(}u_1 \texttt{::} \tau_3\texttt{))}}, \overline{\texttt{(}x_2\ u_2\texttt{)}} \mapsto \Delta}$$

$\boxed{\Gamma \vdash t \upharpoonleft^{m}}$

$$\dfrac{v \upharpoonleft^{m} \in \Gamma}{\Gamma \vdash v \upharpoonleft^{m}}\ \textsc{Scr-Var} \qquad \dfrac{\Gamma \vdash f \upharpoonleft^{m_1} \qquad \Gamma \vdash x \upharpoonleft^{m_2}}{\Gamma \vdash \texttt{(}f\ \ x\texttt{)} \upharpoonleft^{\lfloor m_1, m_2 \rfloor}}\ \textsc{Scr-App} \qquad \dfrac{\Gamma \vdash u \upharpoonleft^{m_1} \qquad \Gamma, x \upharpoonleft^{m_1} \vdash t \upharpoonleft^{m_2}}{\Gamma \vdash \texttt{(let (}x\ \ u\texttt{)}\ \ t\texttt{)} \upharpoonleft^{m_2}}\ \textsc{Scr-Let}$$

$$\dfrac{\Gamma, x \upharpoonleft^{w} \vdash t \upharpoonleft^{m_1}}{\Gamma \vdash \texttt{(lam}\ x\ \ t\texttt{)} \upharpoonleft^{m_1}}\ \textsc{Scr-Abs} \qquad \dfrac{}{\Gamma \vdash \texttt{(}x\texttt{::}\tau\texttt{)} \upharpoonleft^{r}}\ \textsc{Scr-Ann} \qquad \dfrac{}{\Gamma \vdash t \upharpoonleft^{w}}\ \textsc{Scr-Other}$$

References

[1] Simon Peyton Jones, Dimitrios Vytiniotis, and Stephanie Weirich. *Simple unification-based type inference for GADTs*. Jan. 2005.