Short spatial and temporal scales in the terrestrial cryosphere

Photo: Kaskawulsh Glacier, St. Elias Mountains, Yukon, Canada (Credit: Canadian Astronaut David Saint Jacques)

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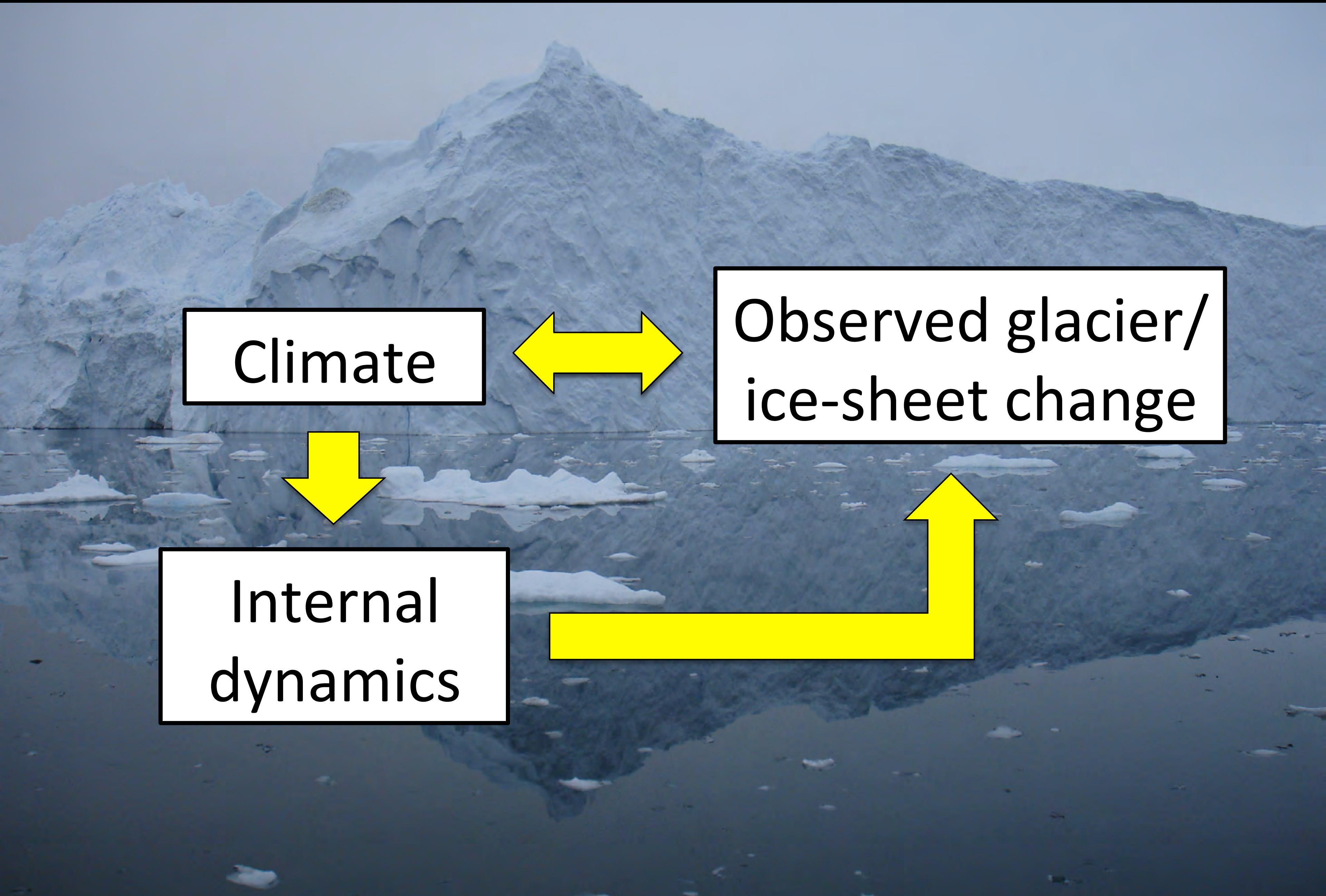
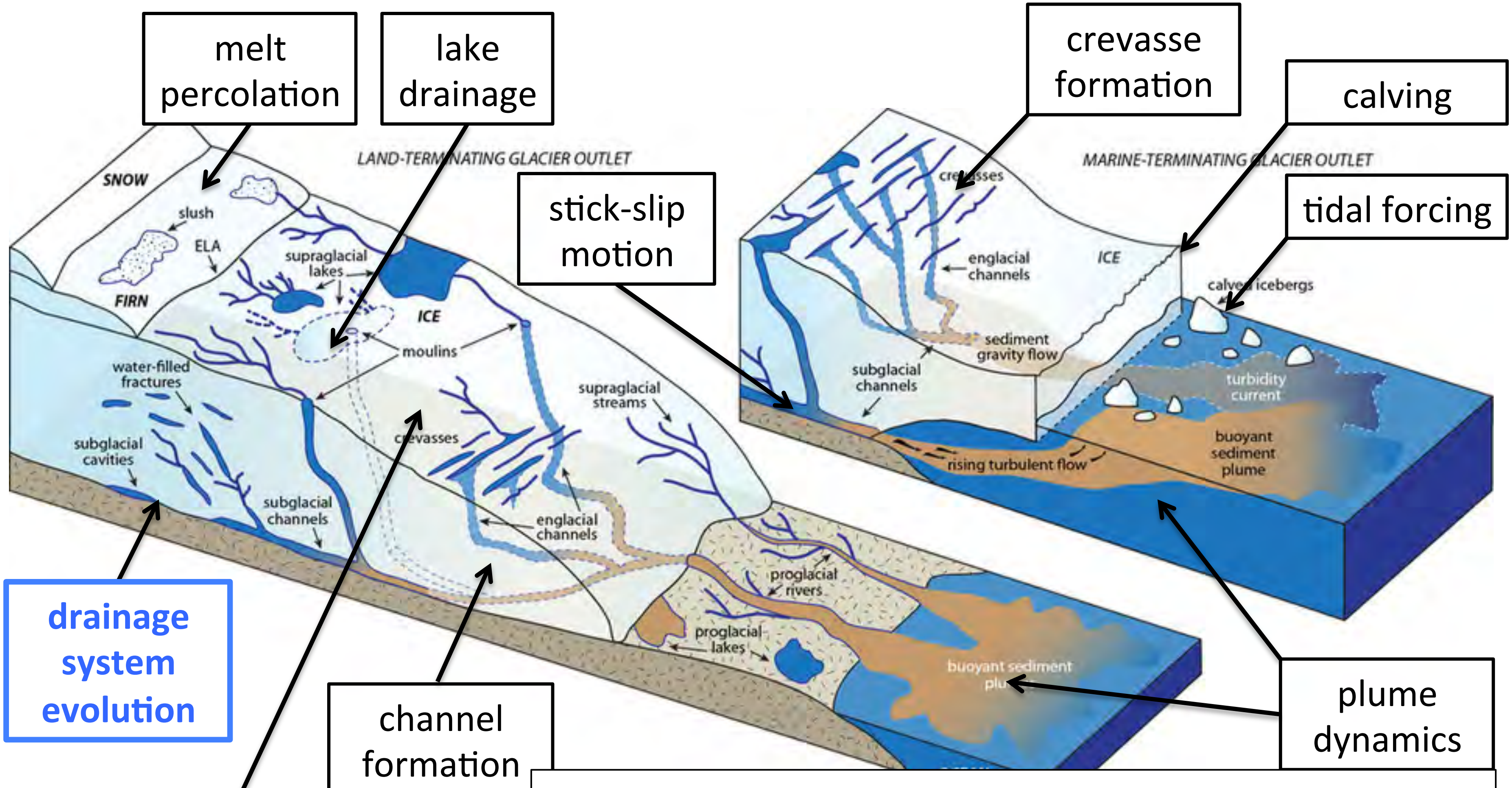
Motivation


Climate
Observed glacier/ ice-sheet change
Internal dynamics

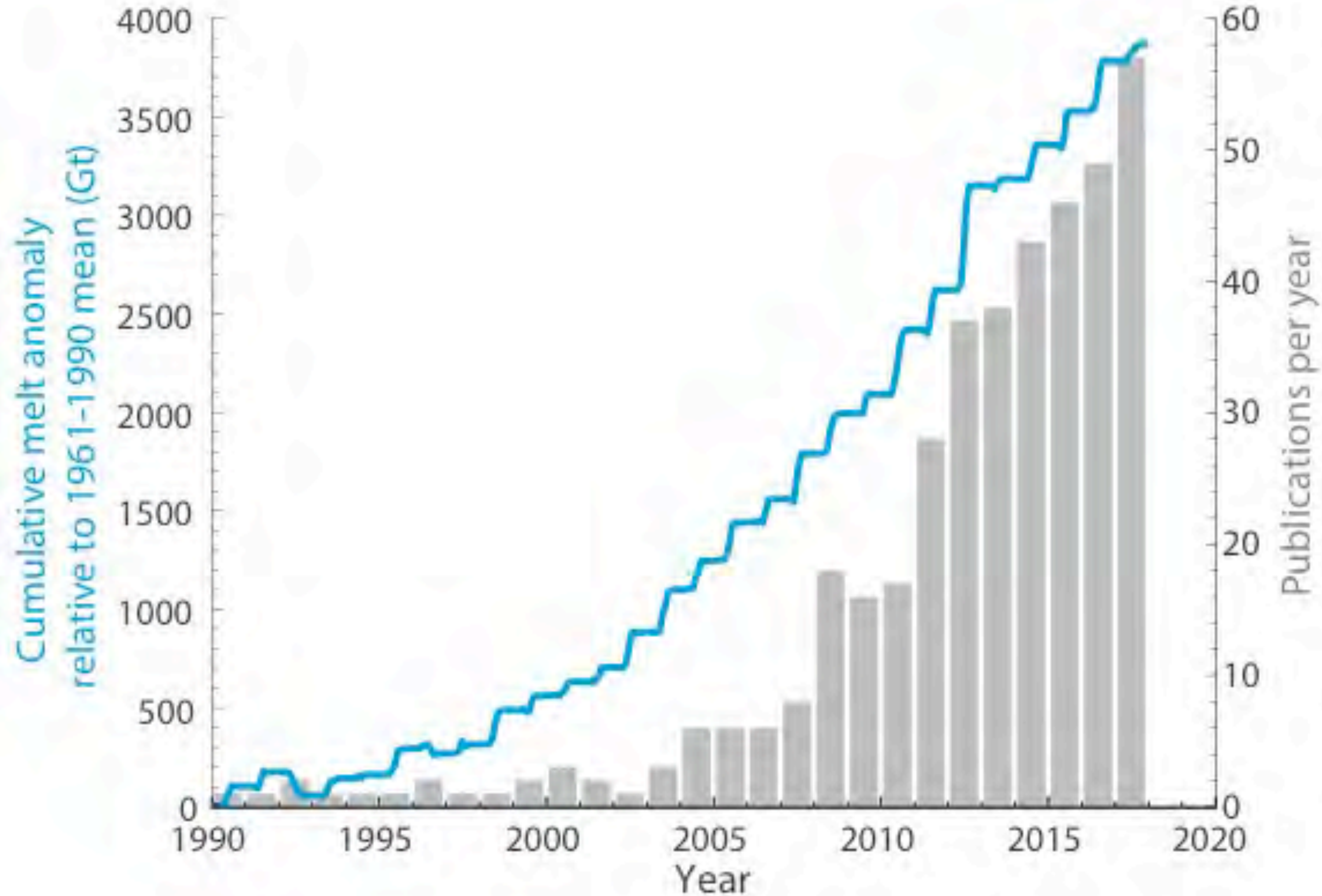
Examples: processes characterized by short spatial / temporal scales



- Current state of GrIS hydrology modeling
- Successes and challenges
- Ideas for future directions

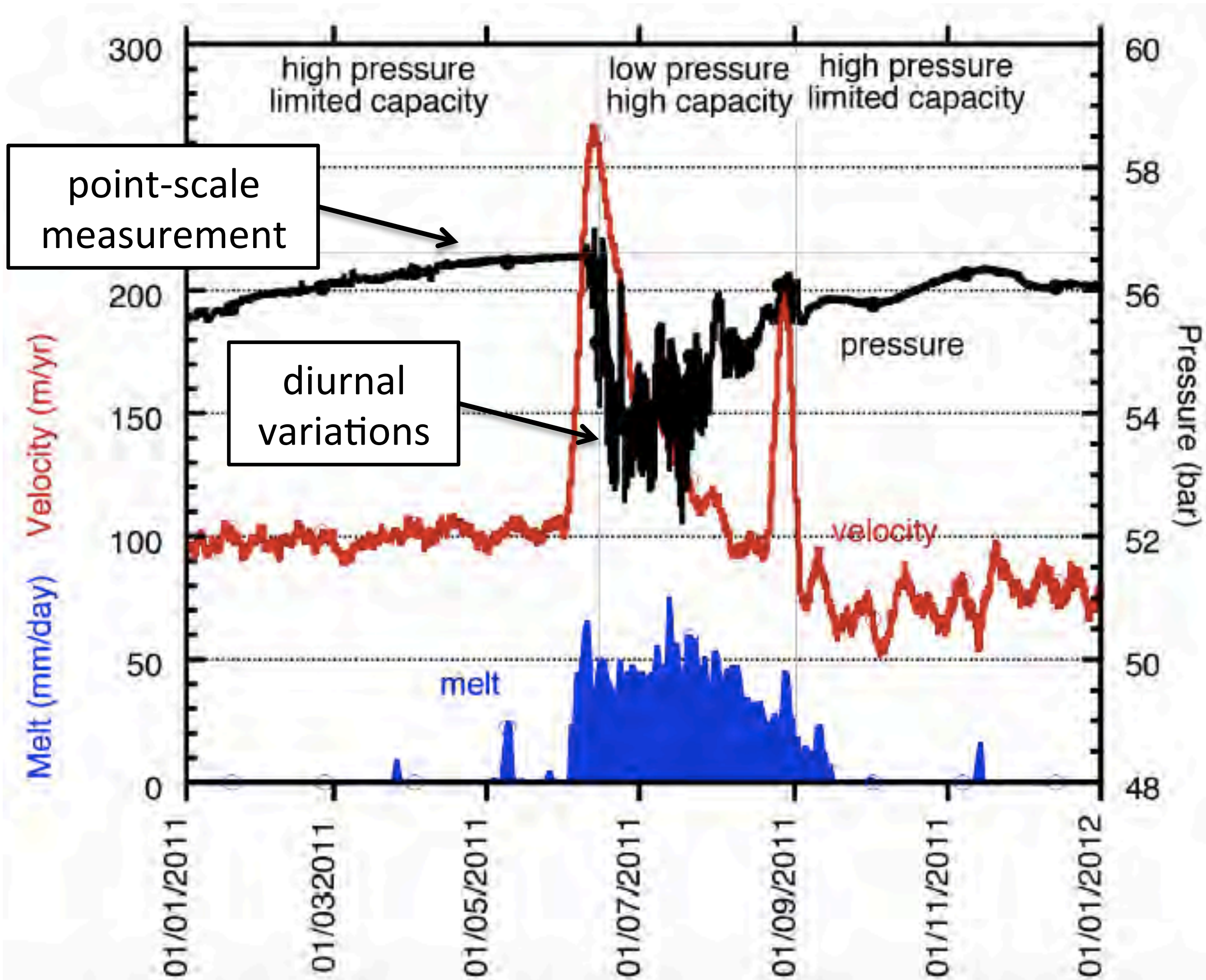
Chu, V.W. (2014). Greenland Ice Sheet hydrology: a review. *Progress in Physical Geography,* 38(1): 19-54, doi:10.1177/0309133313507075

Literature: Greenland ice-sheet hydrology



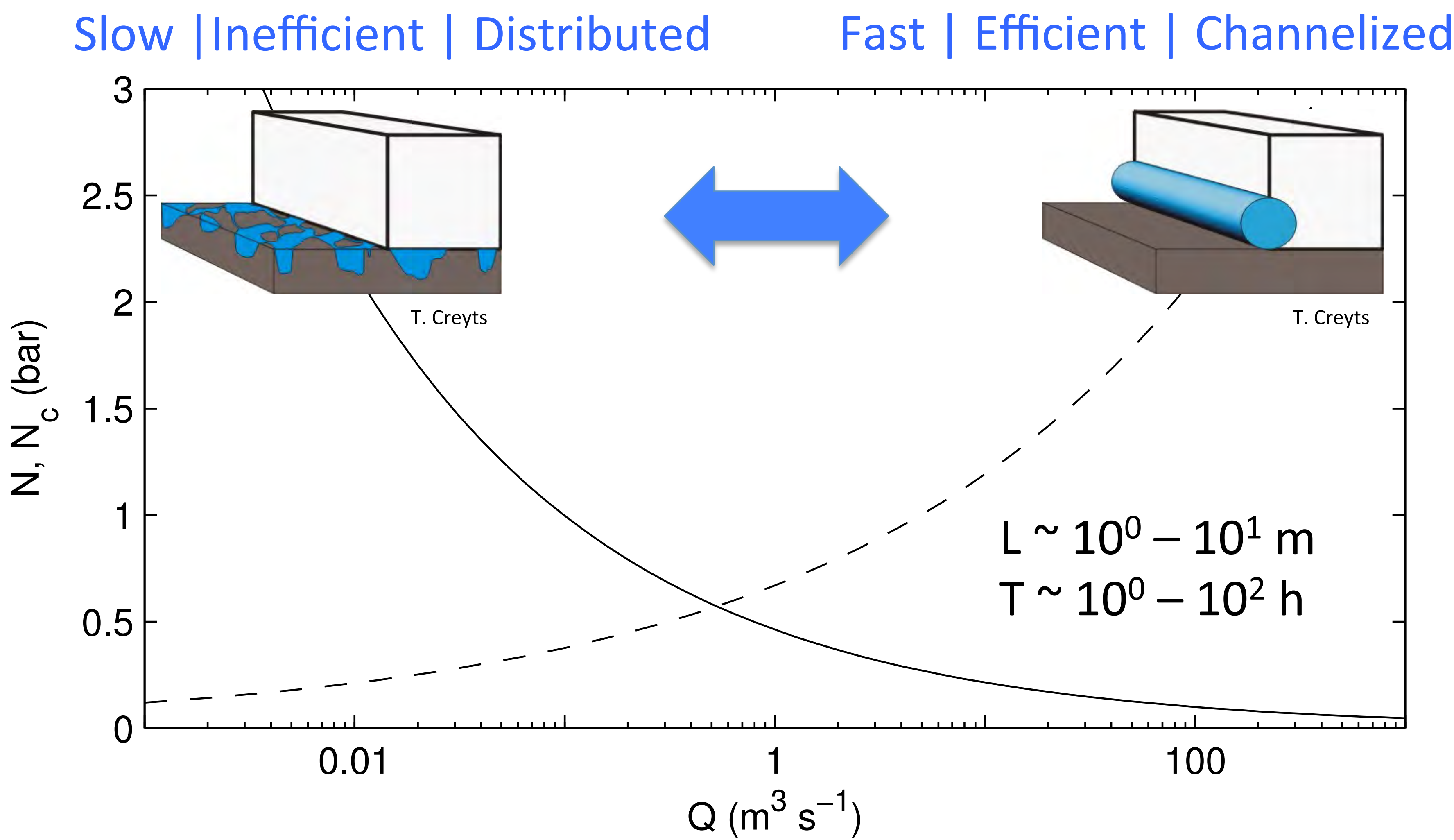
Flowers, G.E. 2018. Hydrology and the future of the Greenland Ice Sheet, *Nature Communications,* Comment.

Influence of hydrology on Greenland ice-sheet dynamics



van de Wal et al (2015). Self-regulation of ice flow varies across the ablation area in south-west Greenland, *The Cryosphere*, 9, 603–611.

Subglacial drainage system morphology and evolution



Effective pressure $N = P_{ice} - P_{water}$

Hewitt (2011). Modelling distributed and channelized subglacial drainage: the spacing of channels. J. Glaciol. 57, 302–314.

Recipe for a model of subglacial drainage

Flux

$$q = -K\, h^{\alpha}\ (\nabla \phi)^{\beta}$$

Fluid potential

$$\phi = P_{\mathrm{w}} + \rho_{\mathrm{w}}\, g\, z$$

Continuity

$$\frac{\partial h}{\partial t} + \nabla \cdot q = \dot{b}$$

Sources/sinks

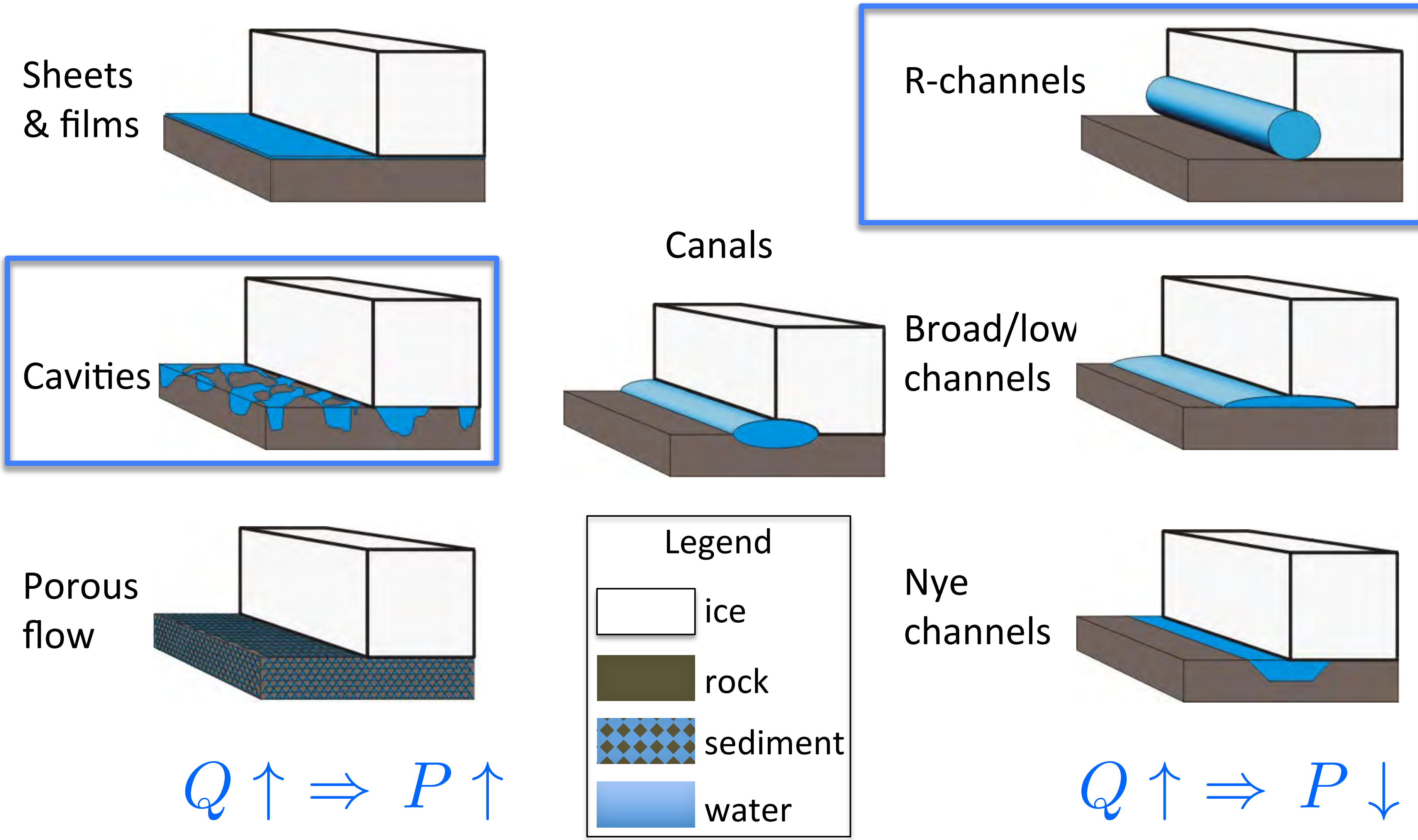
$$\dot{b} = \dot{b}_s + \dot{b}_e + \dot{b}_a + \frac{Q_G + Q_F}{\rho_{\mathrm{i}}\, L}$$

Evolution equation for element(s)

$$\frac{\partial h'}{\partial t} = \text{opening} - \text{closure}$$

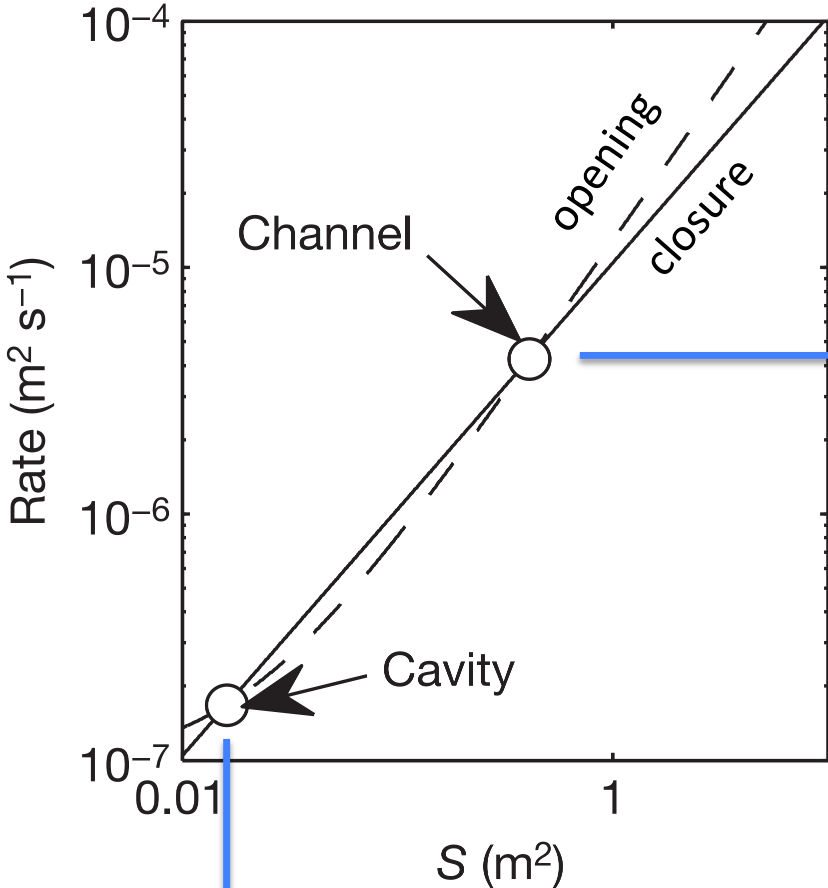
Elements of the subglacial drainage system

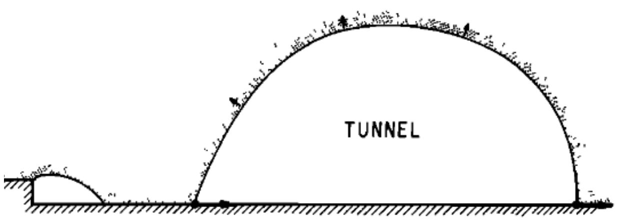
Slow | Inefficient | Distributed   Fast | Efficient | Channelized



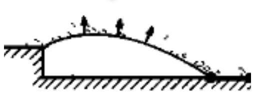
Diagrams courtesy of Tim Creyts

Coupled 2-D channel – cavity models





Kamb (1987)

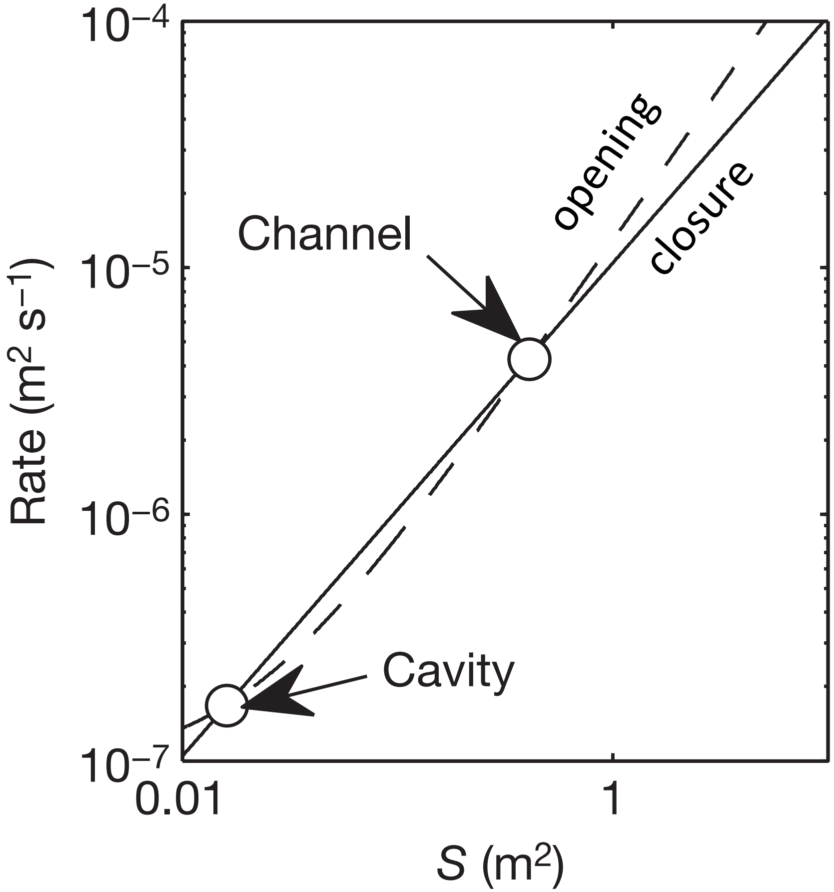

Kamb (1987)

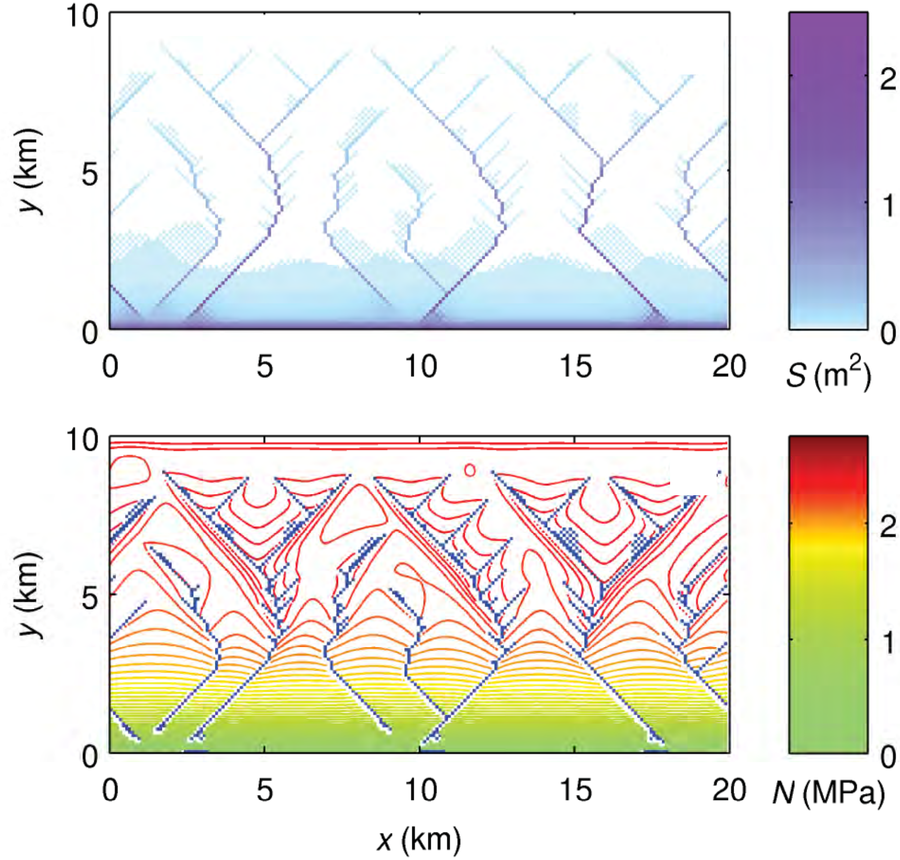
$$\frac{\mathrm{d}S}{\mathrm{d}t} = \underbrace{\boxed{c_1 Q \Psi + u_\mathrm{b} h}}_{\text{opening}} - \underbrace{\boxed{c_2 N^n S}}_{\text{closure}}$$

$$\mathrm{Q_c} = \frac{u_\mathrm{b} h}{c_1(\alpha - 1)\Psi}$$

Schoof (2010): Ice-sheet acceleration driven by melt supply variability, *Nature*

Coupled 2-D channel – cavity models



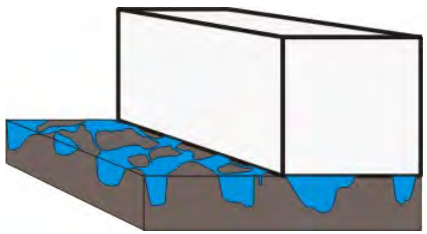


Flow element along each branch of lattice evolves into cavity or channel

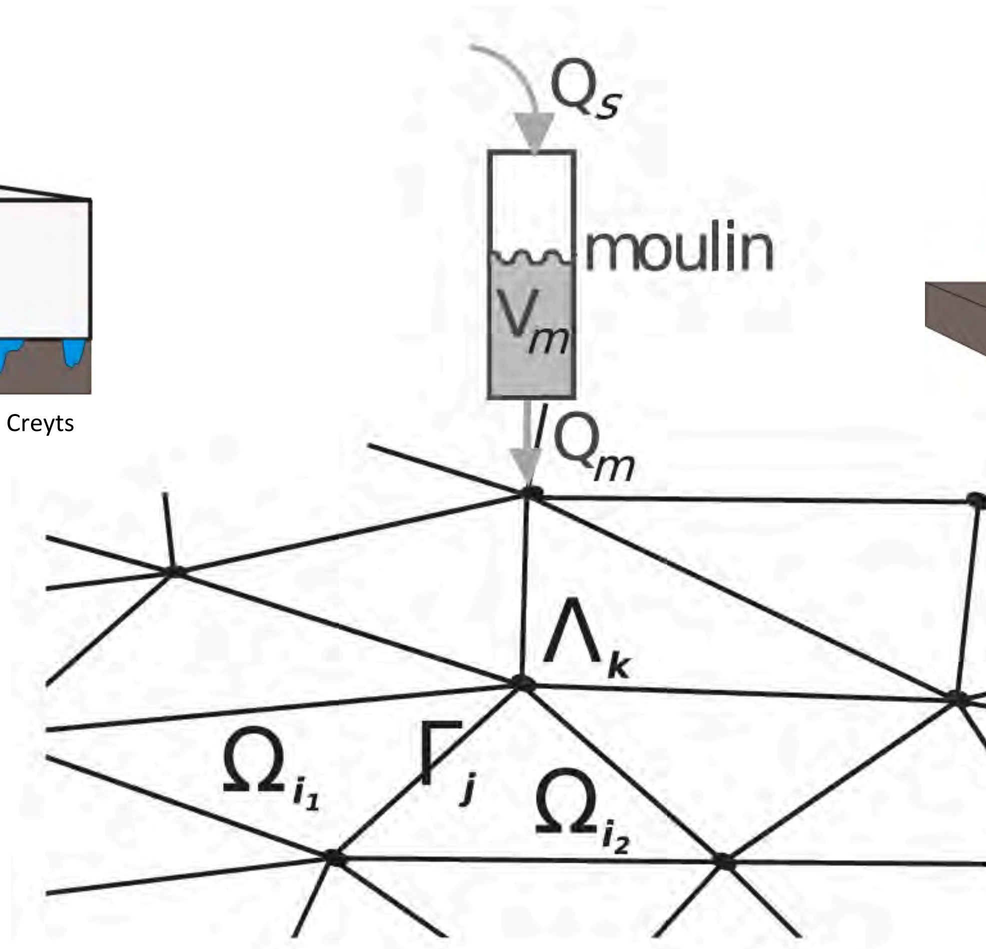
Schoof (2010): Ice-sheet acceleration driven by melt supply variability, *Nature*

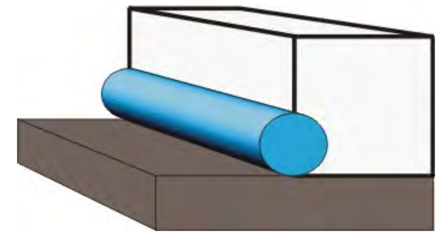
Coupled 2-D channel – cavity system models

Cavity system interacts with channel network defined along mesh edges



T. Creyts





T. Creyts

Werder et al (2013), Modeling channelized and distributed subglacial drainage in two dimensions, J. Geophys. Res. Earth Surf., 118, 2140–2158.

Governing equations for 2-D channel – cavity system model

Channel water balance

$$\frac{\partial Q}{\partial s} + \frac{\Xi - \Pi}{L}\left(\frac{1}{\rho_i} - \frac{1}{\rho_w}\right) - v_c - m_c = 0$$

Channel evolution

$$\frac{\partial S}{\partial t} = \frac{\Xi - \Pi}{\rho_i L} - v_c$$

Channel discharge

$$Q = -k_c S^{\alpha_c} \left|\frac{\partial \phi}{\partial s}\right|^{\beta_c - 2} \frac{\partial \phi}{\partial s}$$

Channel melt opening

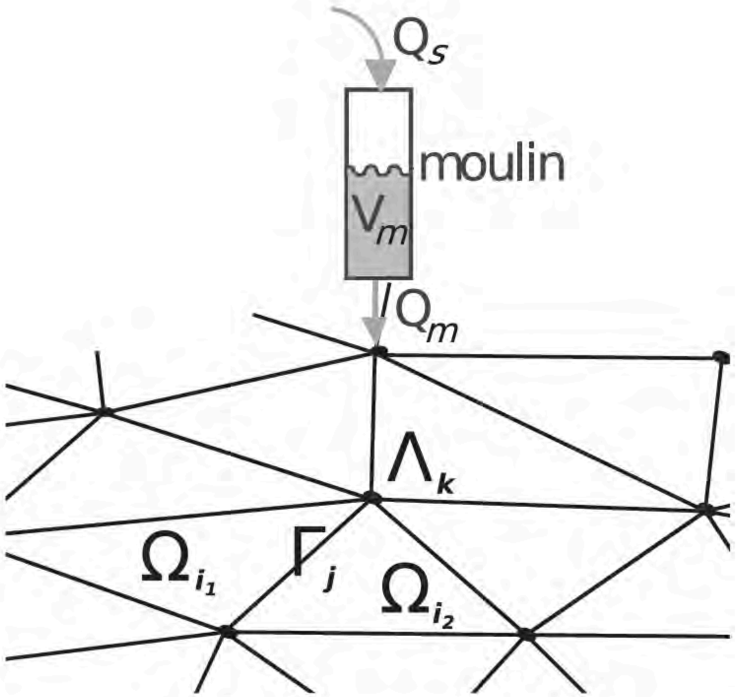
$$\Xi(S, \phi) = \left|Q \frac{\partial \phi}{\partial s}\right| + \left|l_c q_c \frac{\partial \phi}{\partial s}\right|$$

Pressure melting

$$\Pi(S, \phi) = -c_t c_w \rho_w (Q + f l_c q_c) \frac{\partial \phi - \partial \phi_m}{\partial s}$$

Channel closure by creep

$$v_c(S, \phi) = \frac{2}{n^n} A S |N|^{n-1} N$$



Werder et al (2013), Modeling channelized and distributed subglacial drainage in two dimensions, J. Geophys. Res. Earth Surf., 118, 2140–2158.

Governing equations for 2-D channel – cavity system model

Cavity system water balance

$$\frac{e_v}{\rho_w g}\frac{\partial \phi}{\partial t} - \nabla \cdot \boldsymbol{q} + w_s - v_s - m_b = 0$$

Cavity evolution

$$\frac{\partial h}{\partial t} = w_s - v_s$$

Cavity water flux

$$\boldsymbol{q} = -kh^{\alpha}|\nabla \phi|^{\beta-2}\nabla \phi$$

Cavity opening by sliding

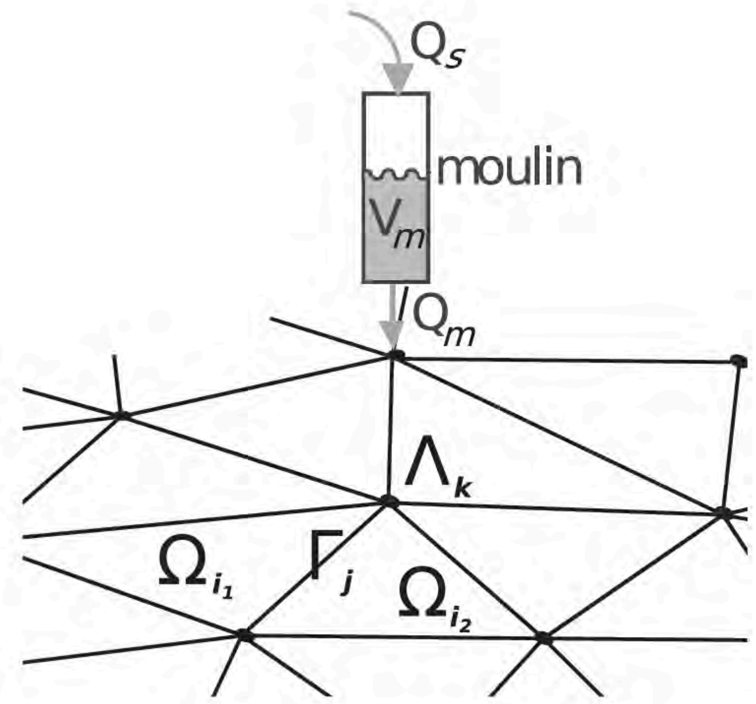
$$w_s(h) = \frac{|\boldsymbol{u}_b|}{l_r}(h_r - h)$$

Cavity closure by creep

$$v_s(h,\phi) = \frac{2}{n^n}Ah|N|^{n-1}N$$

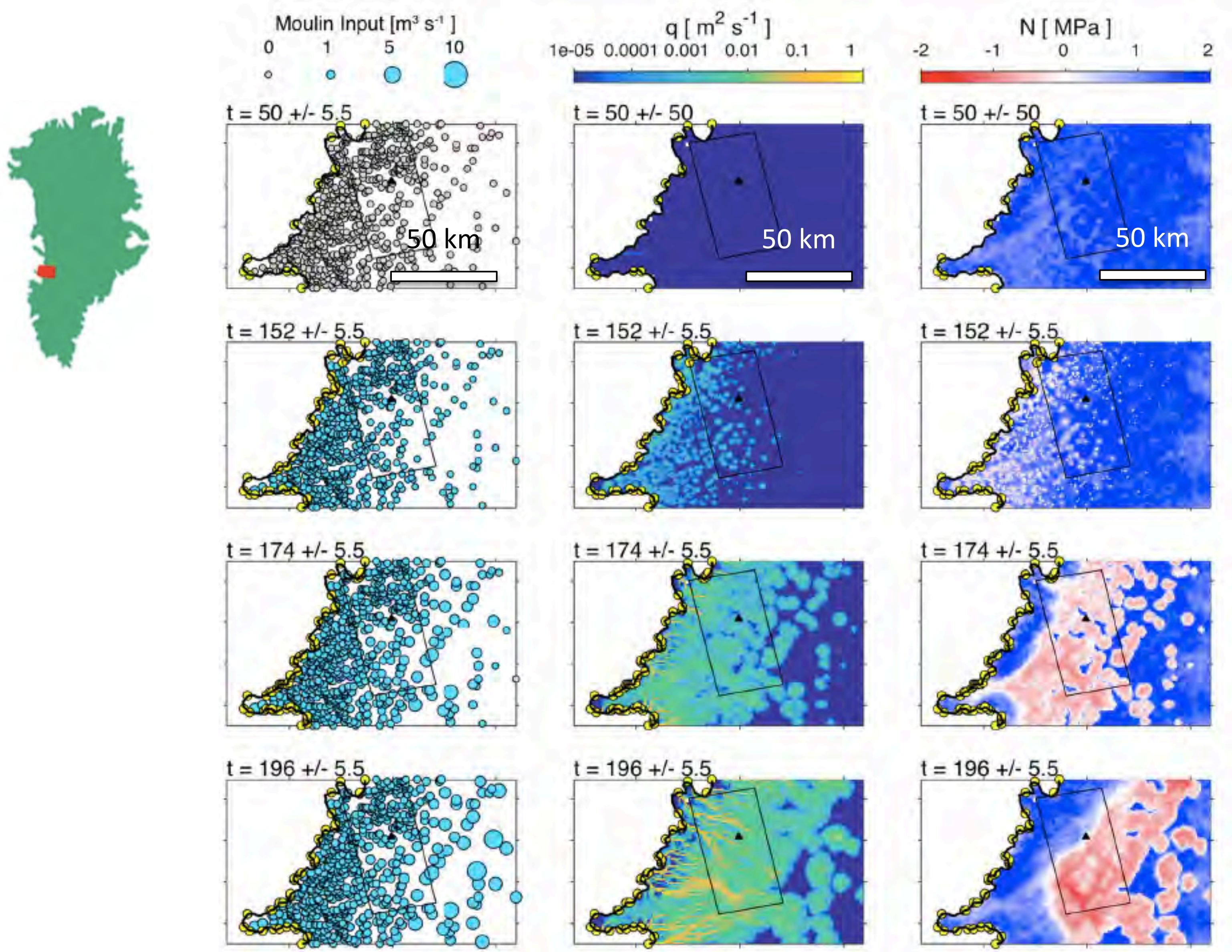
Coupling with ice dynamics

$$\boldsymbol{\tau}_b + \frac{CN_+|\boldsymbol{u}_b|^{(1/n-1)}}{(|\boldsymbol{u}_b| + C^n A_s N_+^n)^{1/n}}\boldsymbol{u}_b = 0$$



Werder et al (2013), Modeling channelized and distributed subglacial drainage in two dimensions, J. Geophys. Res. Earth Surf., 118, 2140–2158.

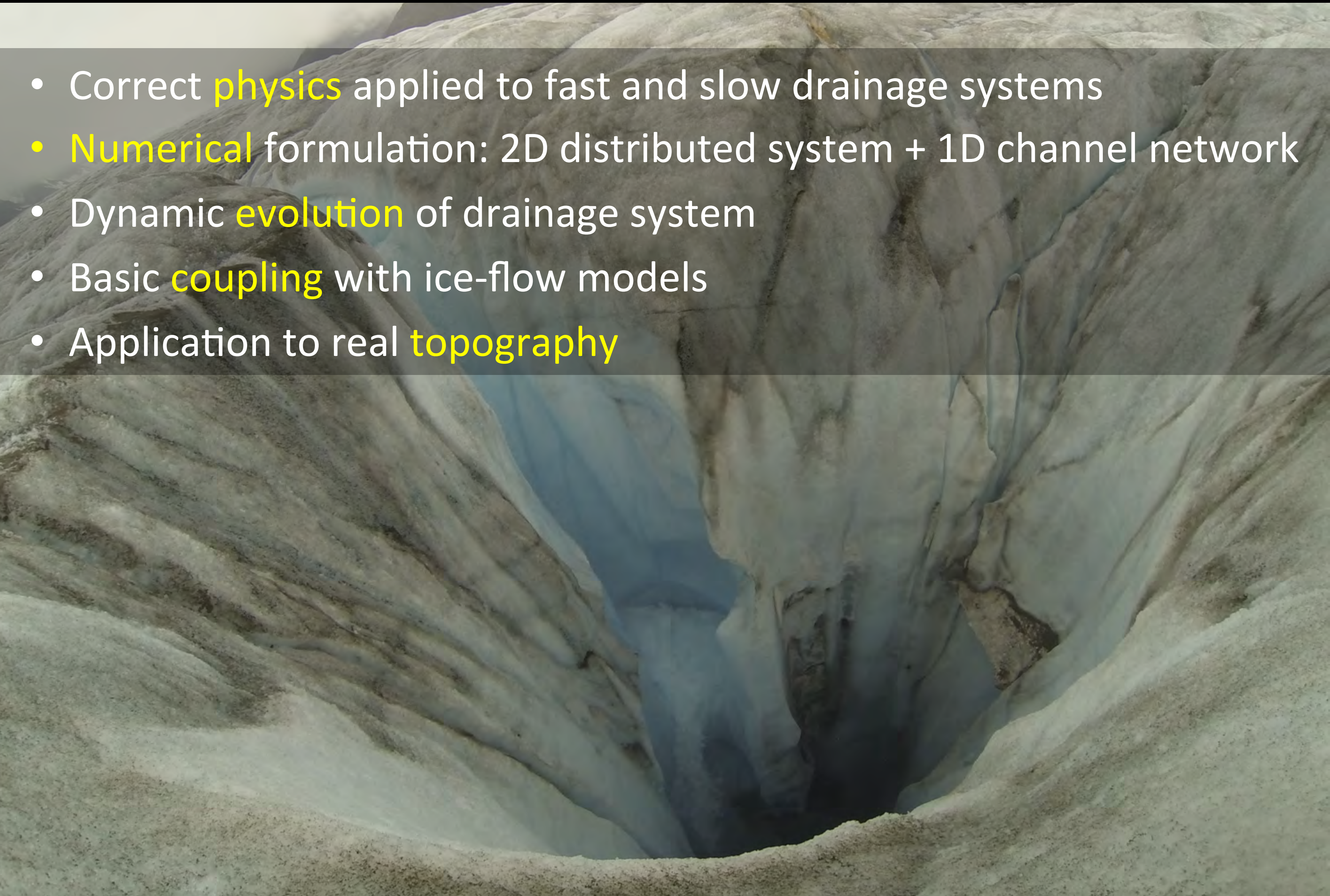
Application of coupled 2-D channel – cavity system models



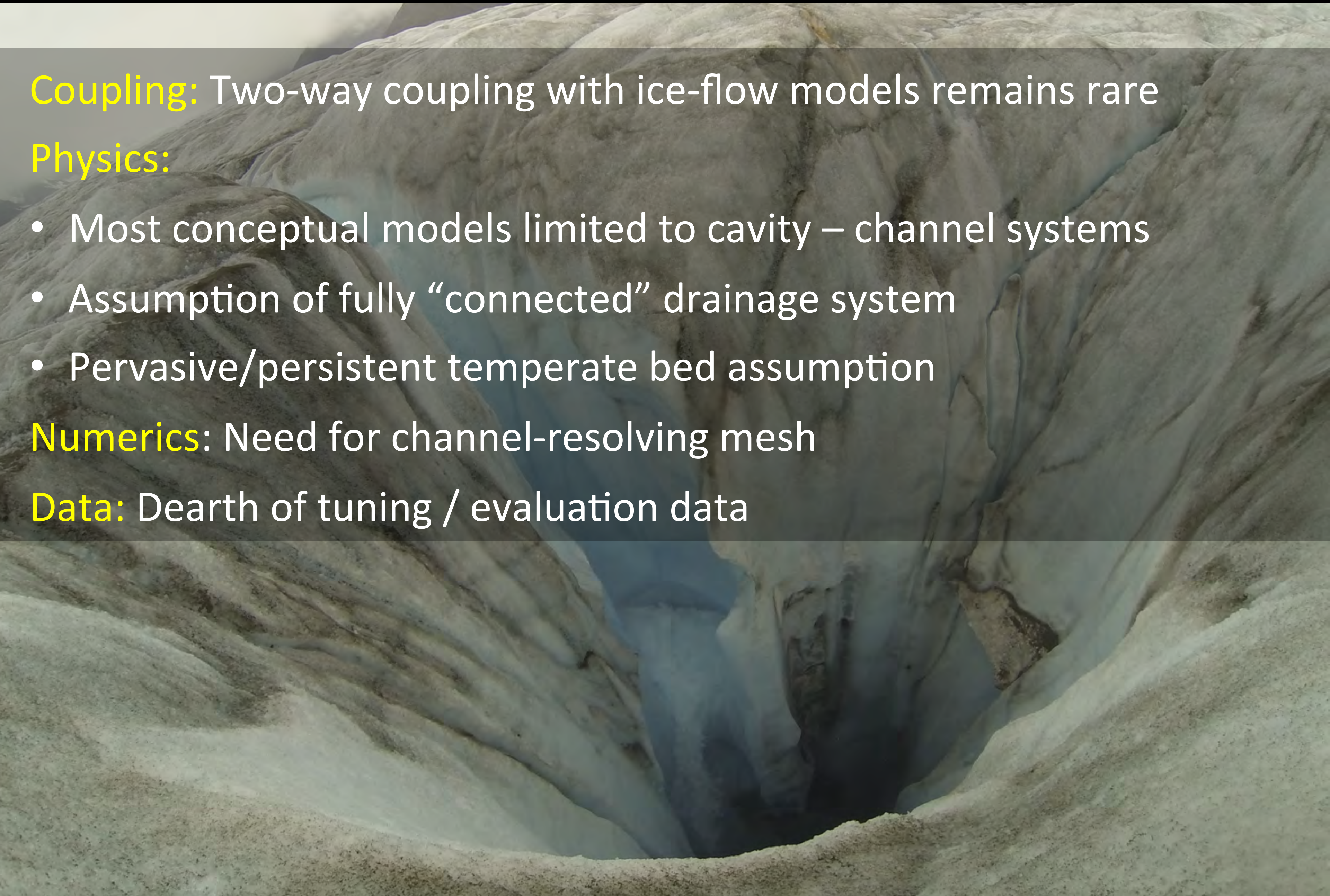
Stevens et al (2018). Relationship between Greenland ice sheet surface speed and modeled effective pressure. Journal of Geophysical Research, 123, 2258–2278.

Strengths of current models

- Correct physics applied to fast and slow drainage systems
- Numerical formulation: 2D distributed system + 1D channel network
- Dynamic evolution of drainage system
- Basic coupling with ice-flow models
- Application to real topography



Modeling challenges and limitations (short list)



Coupling: Two-way coupling with ice-flow models remains rare

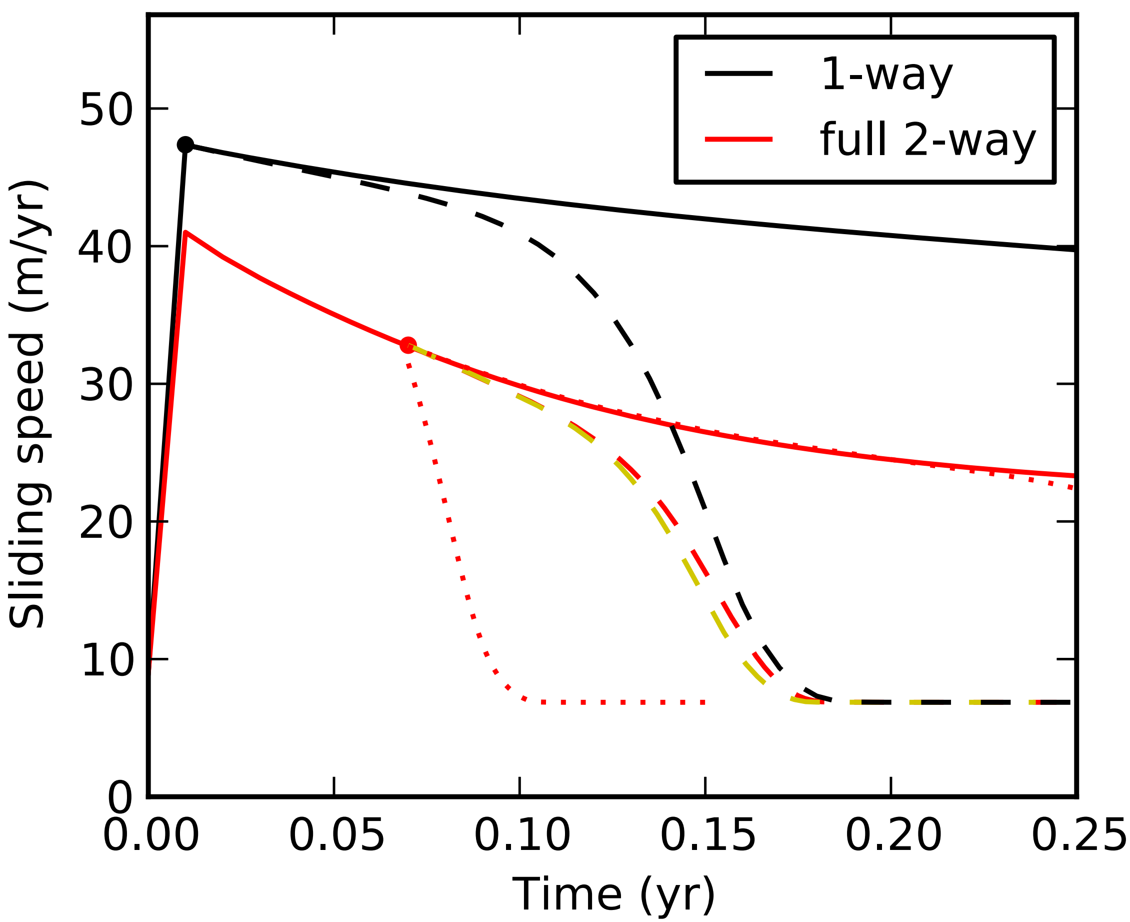
Physics:

- Most conceptual models limited to cavity – channel systems
- Assumption of fully “connected” drainage system
- Pervasive/persistent temperate bed assumption

Numerics: Need for channel-resolving mesh

Data: Dearth of tuning / evaluation data

Importance of two-way coupling between hydrology and sliding



$$\longrightarrow u_{\mathrm{b}} = u_{\mathrm{b}}(N)$$

$$\longleftarrow \frac{\partial h}{\partial t} = \frac{\partial h}{\partial t}(u_{\mathrm{b}})$$

Two-way coupling => significant negative feedback

Hoffman and Price (2014): Feedbacks between coupled subglacial hydrology and glacier dynamics, *Journal of Geophysical Research*

Unified governing equations for efficient/inefficient drainage

Basal "gap" evolution

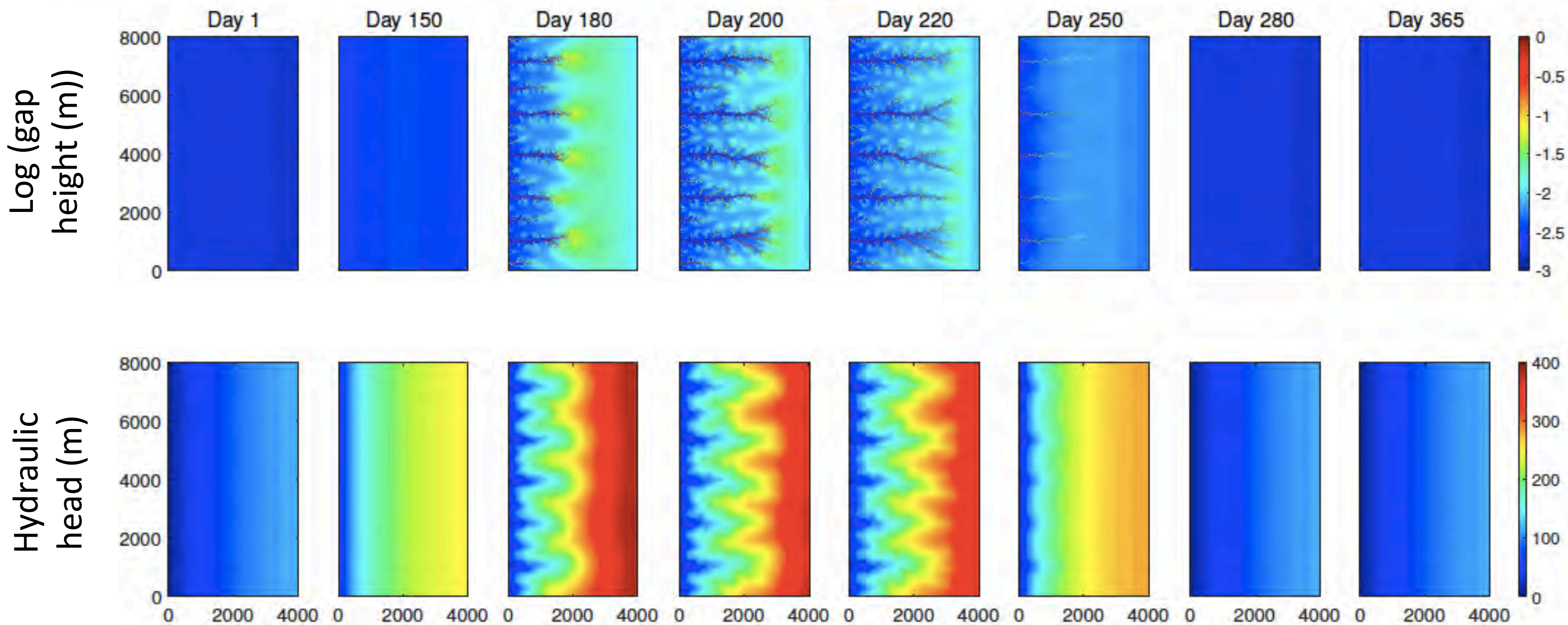
$$\frac{\partial b}{\partial t} = \frac{\dot{m}}{\rho_i} + \beta u_b - A|p_i - p_w|^{n-1}(p_i - p_w)b$$

Water flux

$$\boldsymbol{q} = \frac{-b^3 g}{12\nu(1+\omega Re)}\nabla h$$

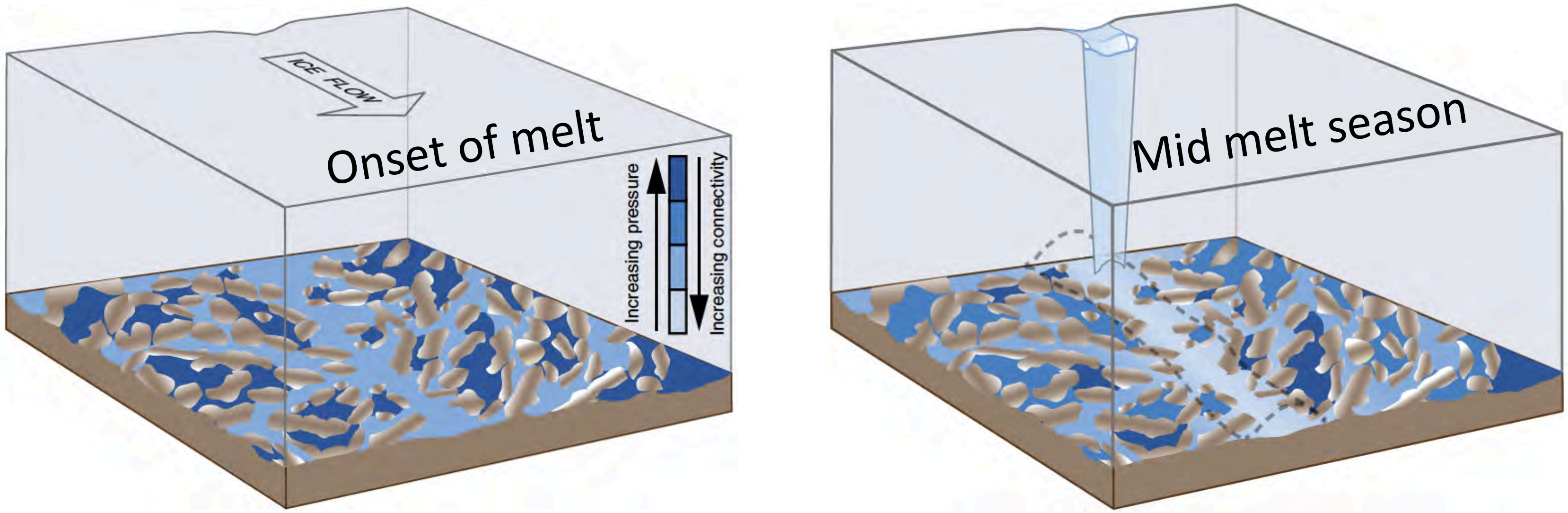
Water balance

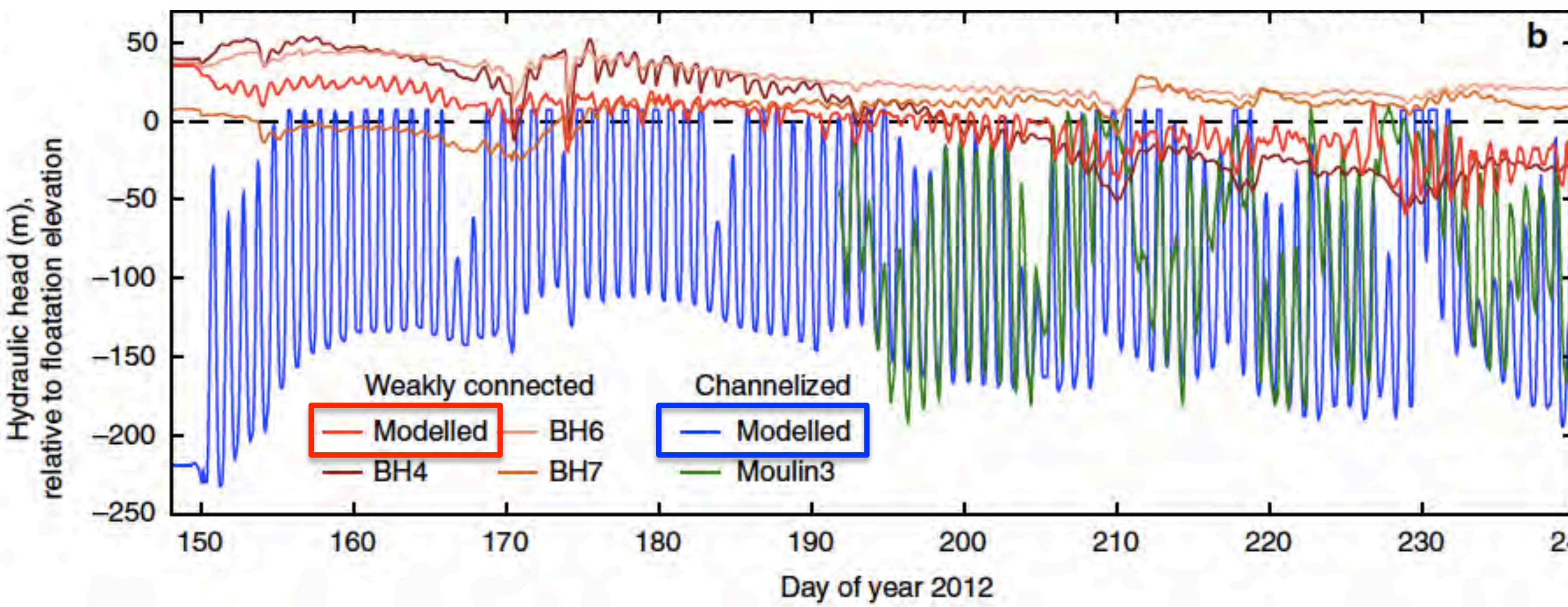
$$\nabla \cdot (-\boldsymbol{K} \cdot \nabla h) + \frac{\partial e_v(h - z_b)}{\partial t} = \dot{m}\left(\frac{1}{\rho_w} - \frac{1}{\rho_i}\right) + A|p_i - p_w|^{n-1}(p_i - p_w)b - \beta u_b + i_{e\rightarrow b}$$



Sommers et al. (2018): SHaKTI: Subglacial Hydrology and Kinetic Transient Interactions v1.0, *Geoscientific Model Development*

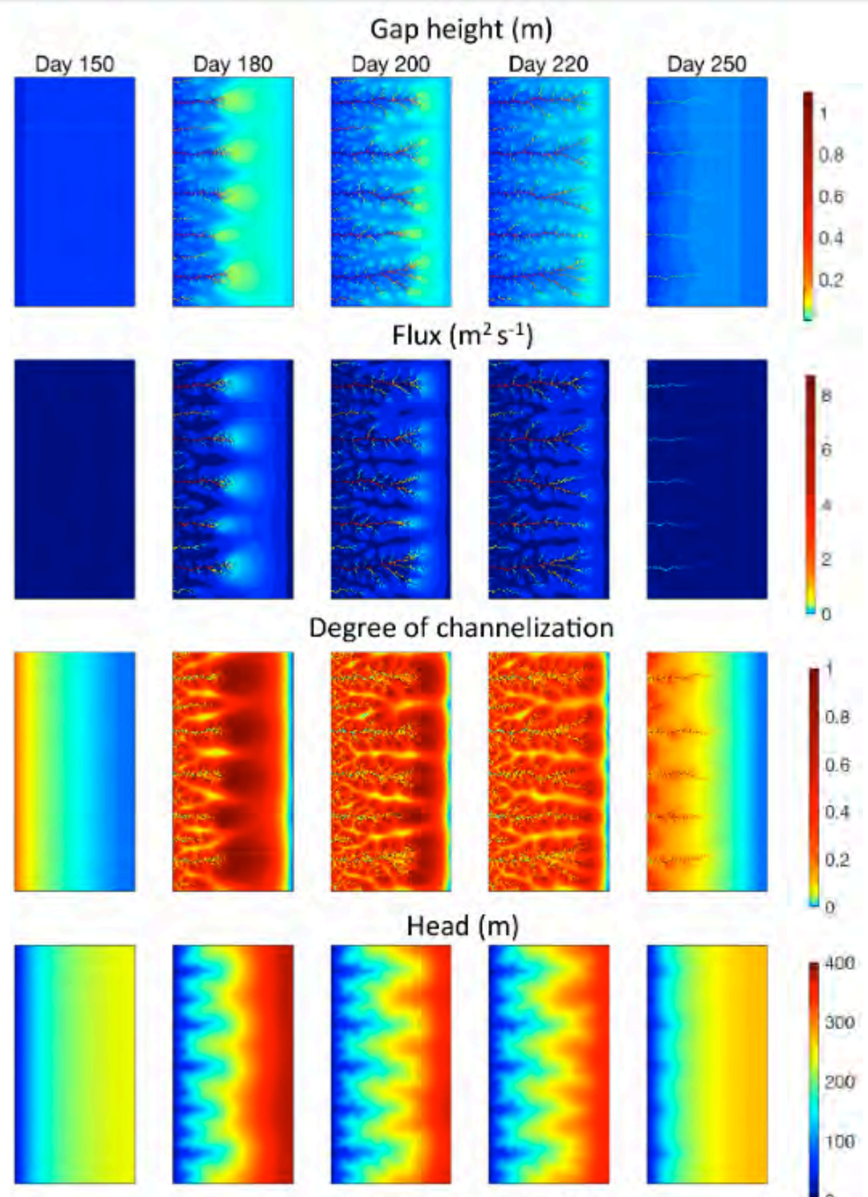
Including the "weakly connected" bed



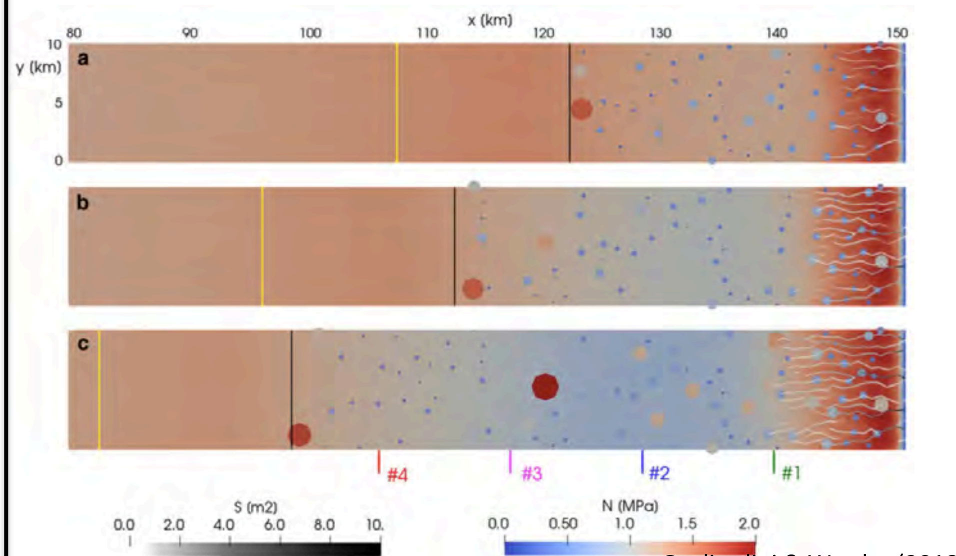


Hoffman et al. (2017): Greenland subglacial drainage evolution regulated by weakly connected regions of the bed, *Nature Communications*

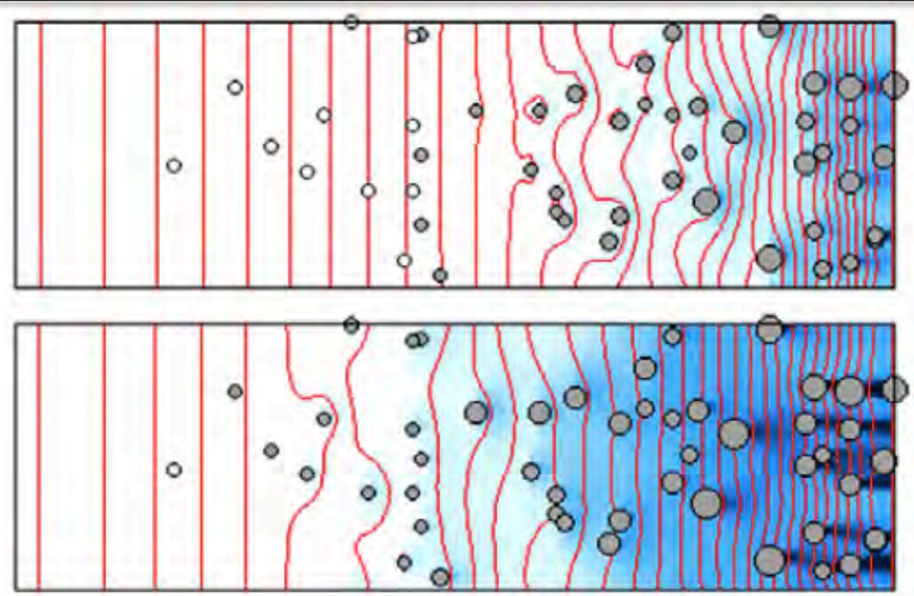
Limitations of current models: application to synthetic topography



Sommers et al (2018)

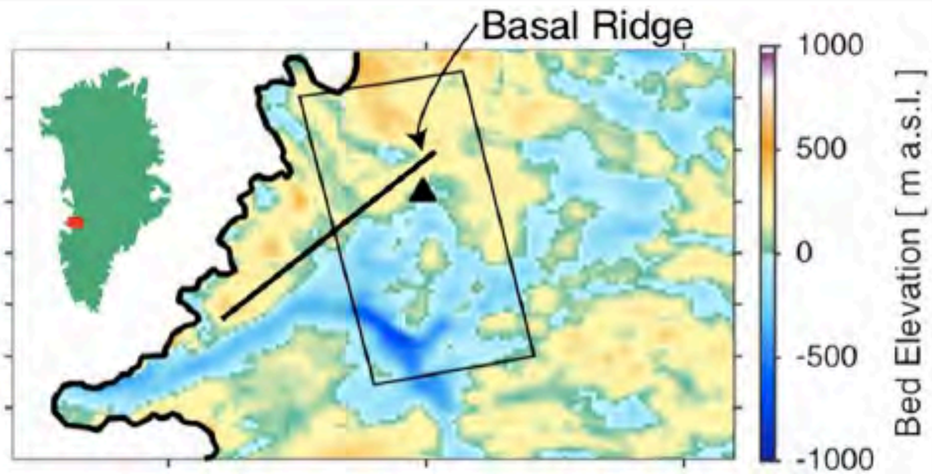


Gagliardini & Werder (2018)

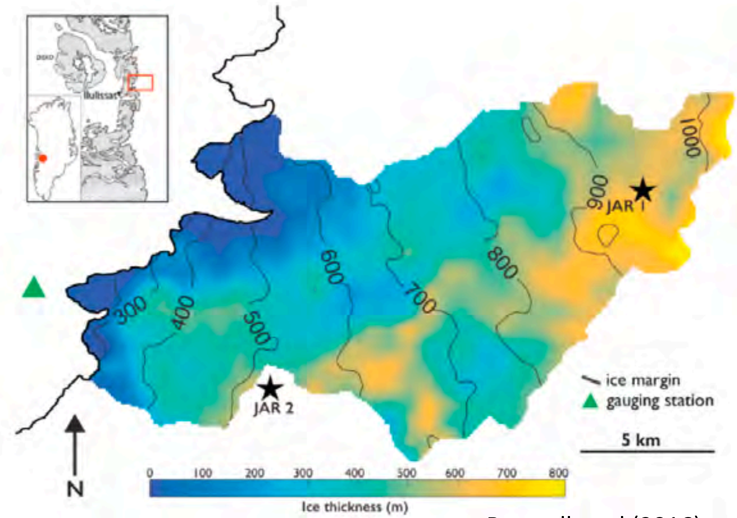


Hewit (2013)

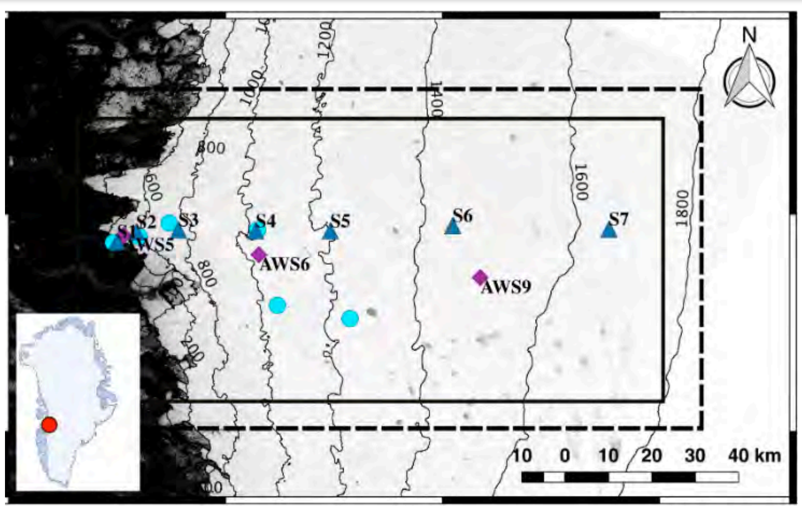
Limitations of current models: application to limited domains



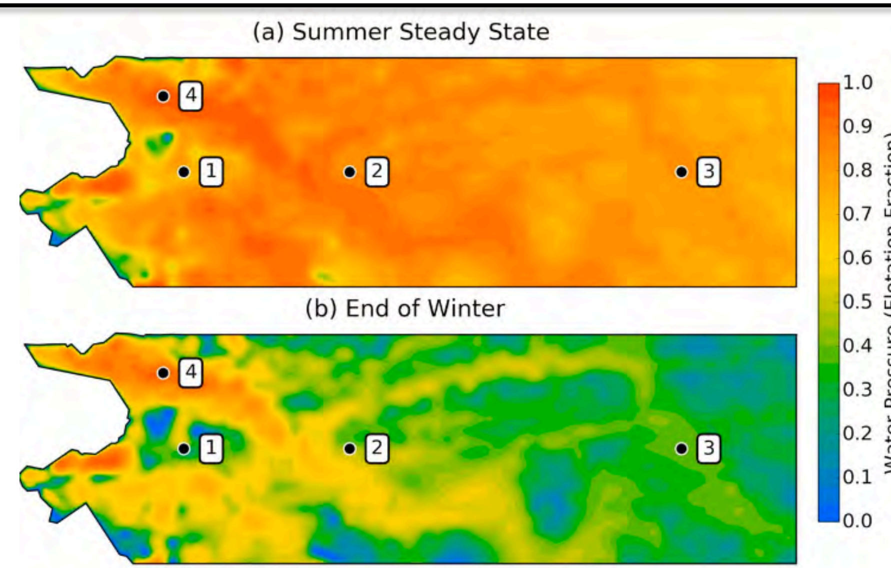
Stevens et al (2018)



Banwell et al (2016)



Koziol et al (2018)



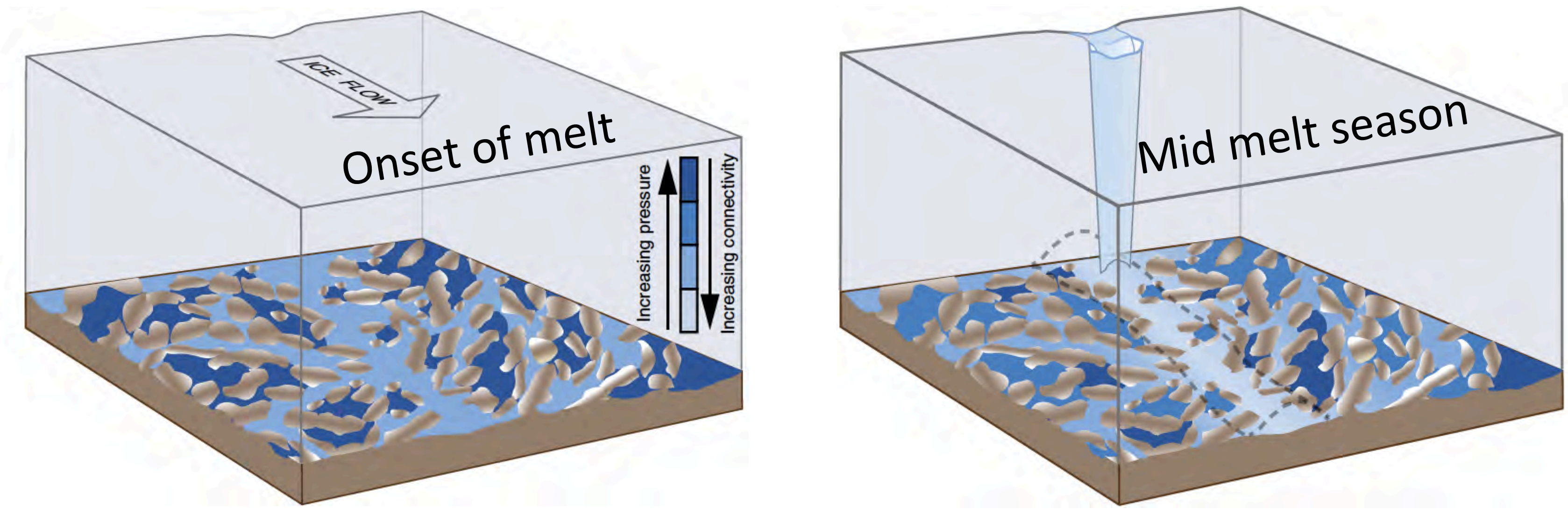
Downs et al (2018)

Modeling challenges and limitations (short list)

- Two-way coupling with ice-flow models remains rare
  - => Feedback missing
- Most conceptual models limited to cavity – channel systems
  - => Deforming bed behavior missing
- Assumption of fully “connected” drainage system
  - => Models wrong in winter
- Pervasive/persistent temperate bed assumption
  - => Mass / energy conservation
- Need for channel-resolving numerical mesh
  - ⟹ Slow
- Dearth of tuning/evaluation data, resolution of input data
  - => Calibration / validation impossible

Ways forward to address modeling challenges & limitations

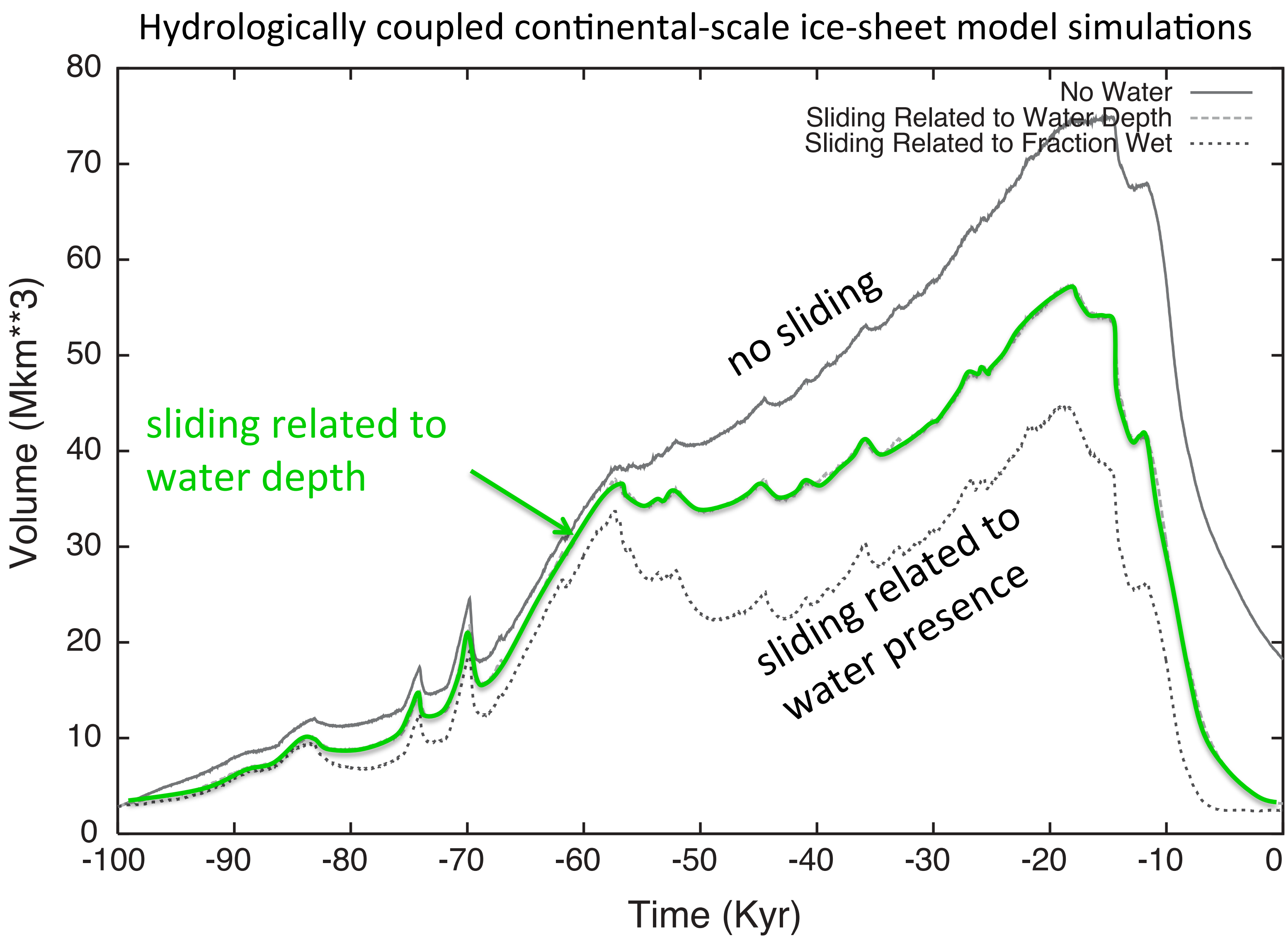
Add or better parameterize missing physics



Unified bed { Hard bed / Soft bed } { Channelized / Unchannelized { Connected / Unconnected { Temperate / Frozen } } }

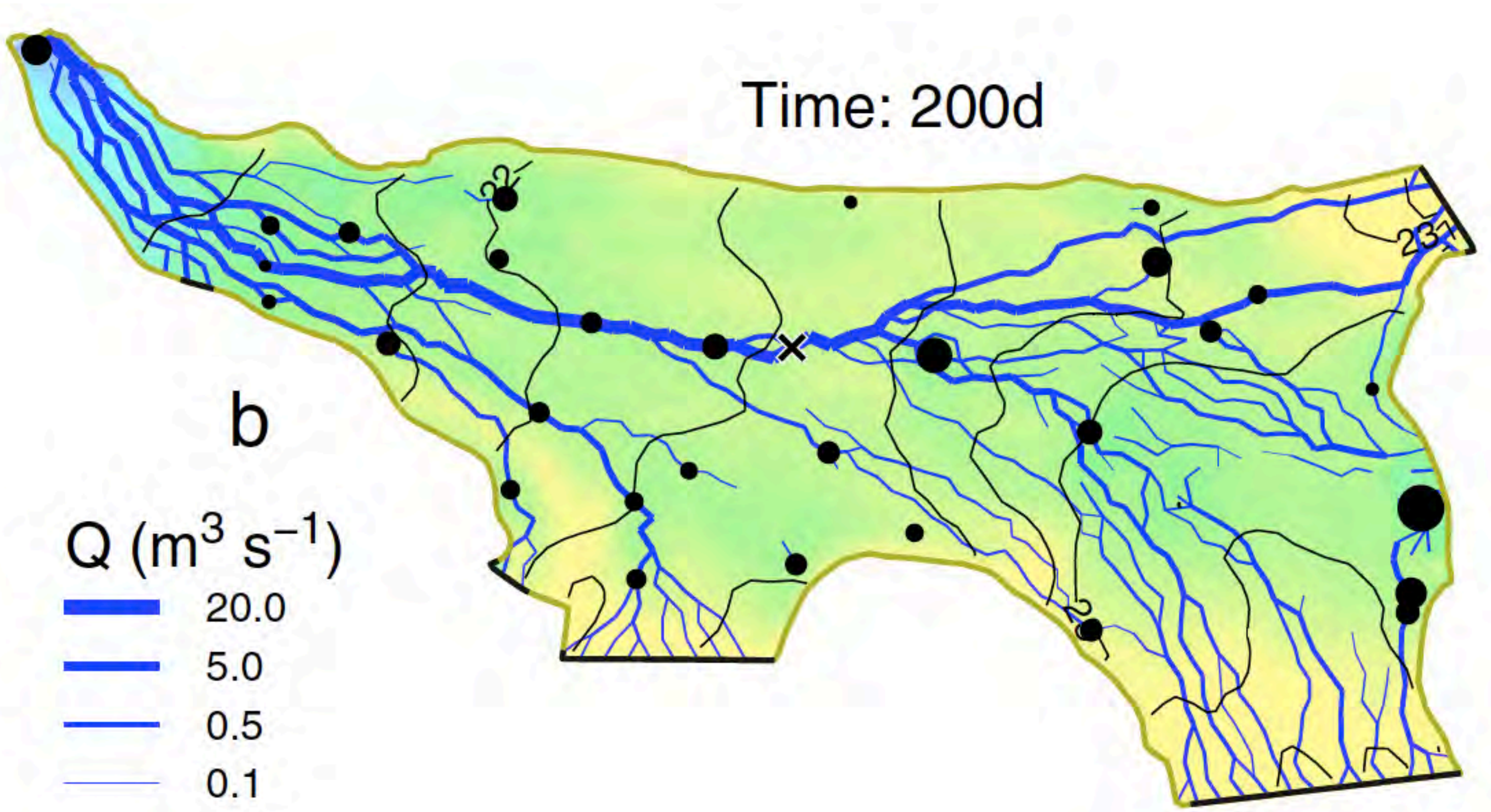
Hoffman et al. (2017): Greenland subglacial drainage evolution regulated by weakly connected regions of the bed, *Nature Communications*

How do we get back to large-scale ice-sheet hydrology models?



Johnson and Fastook (2002): Northern Hemisphere glaciation and its sensitivity to basal melt water, *Quaternary International*

Build statistical emulators of physically based models (the simulators) to reduce computational burden



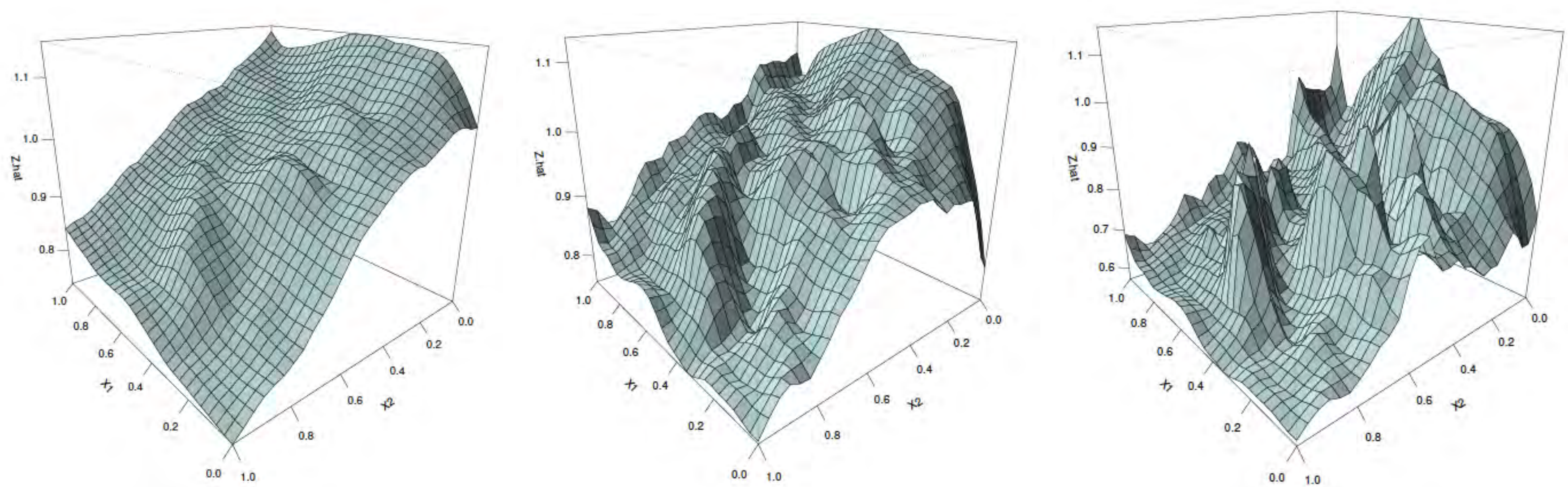
Werder et al (2013), Modeling channelized and distributed subglacial drainage in two dimensions, J. Geophys. Res. Earth Surf., 118, 2140–2158.

Ways forward to address modeling challenges & limitations

Build statistical emulators of physically based models (the simulators) to reduce computational burden

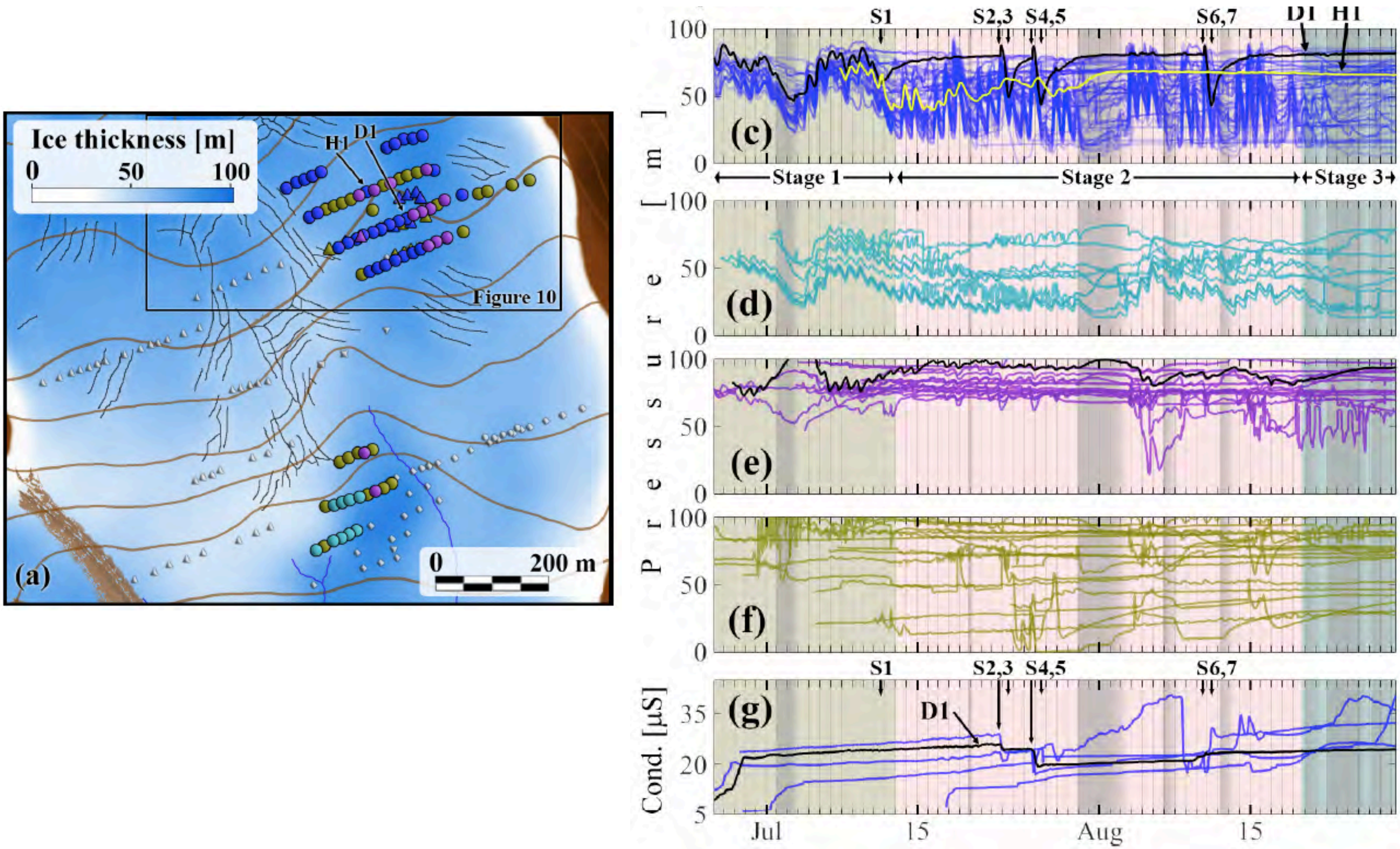
Example: Gaussian Process model to emulate glacier melt from physically based simulator model

$$\hat{Z}(\boldsymbol{x}) = \mu + \boldsymbol{r}^{\top}(\boldsymbol{x})\boldsymbol{V}^{-1}(\boldsymbol{z} - \mu\mathbf{1})$$



Surjanovic, S., D. Bingham, G.E. Flowers (In prep). Using computer model uncertainty to determine optimal design of mass-balance stake networks.

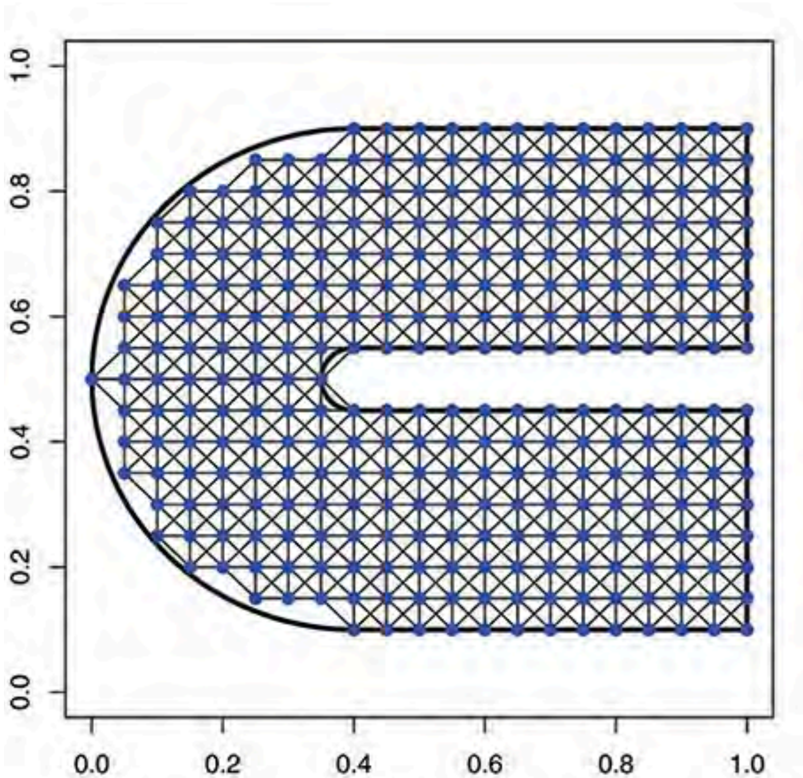
Failure of Euclidean-distance correlation: ask any borehole

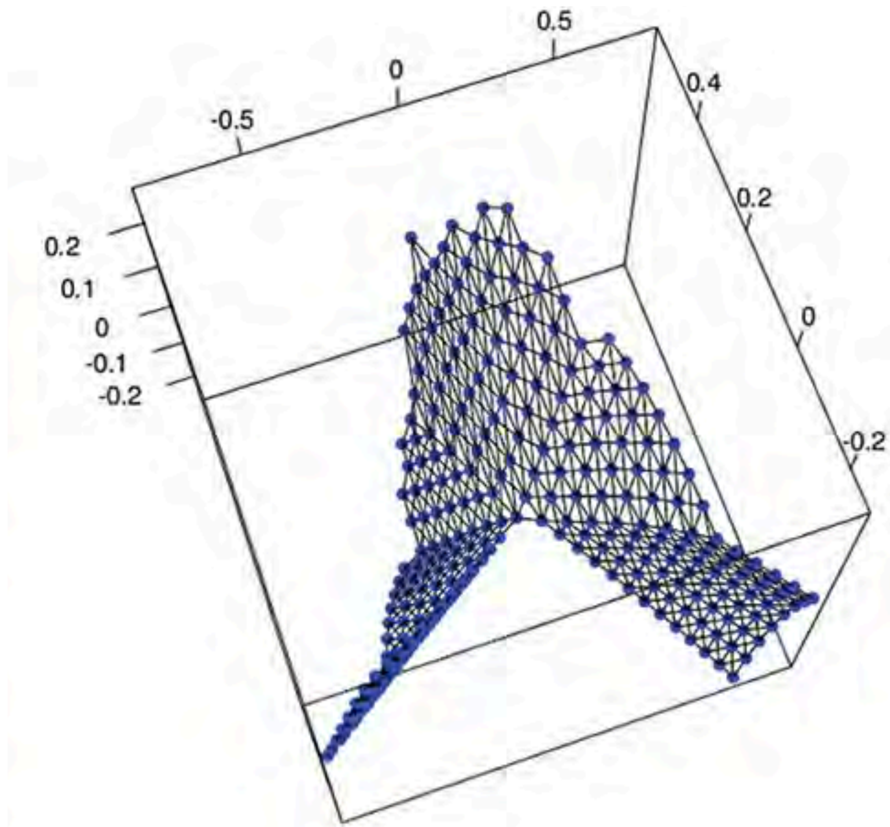


Rada, C. and C.G. Schoof (Submitted), *The Cryosphere*.

Ways forward to address modeling challenges & limitations

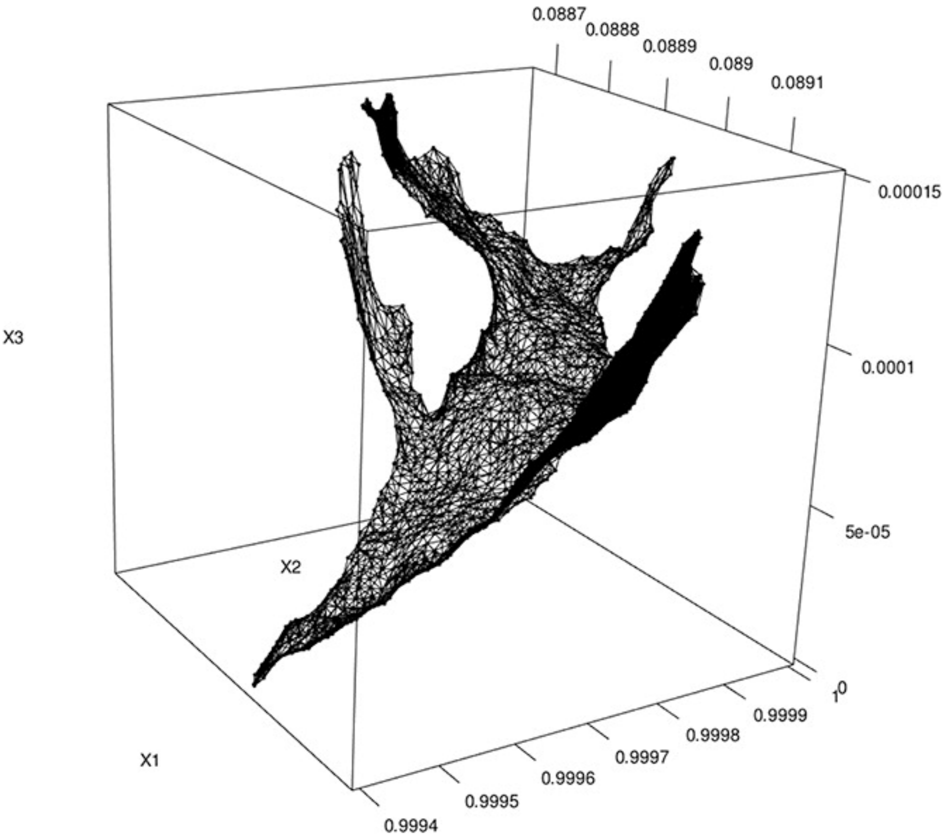
Example: Making use of geodesic distance in Gaussian Process model





Pratola, M.T., Harari, O., Bingham, D. and Flowers, G.E., 2017. Design and Analysis of Experiments on Nonconvex Regions. *Technometrics*, 59(1), pp.36-47.

Example: Making use of geodesic distance in Gaussian Process model



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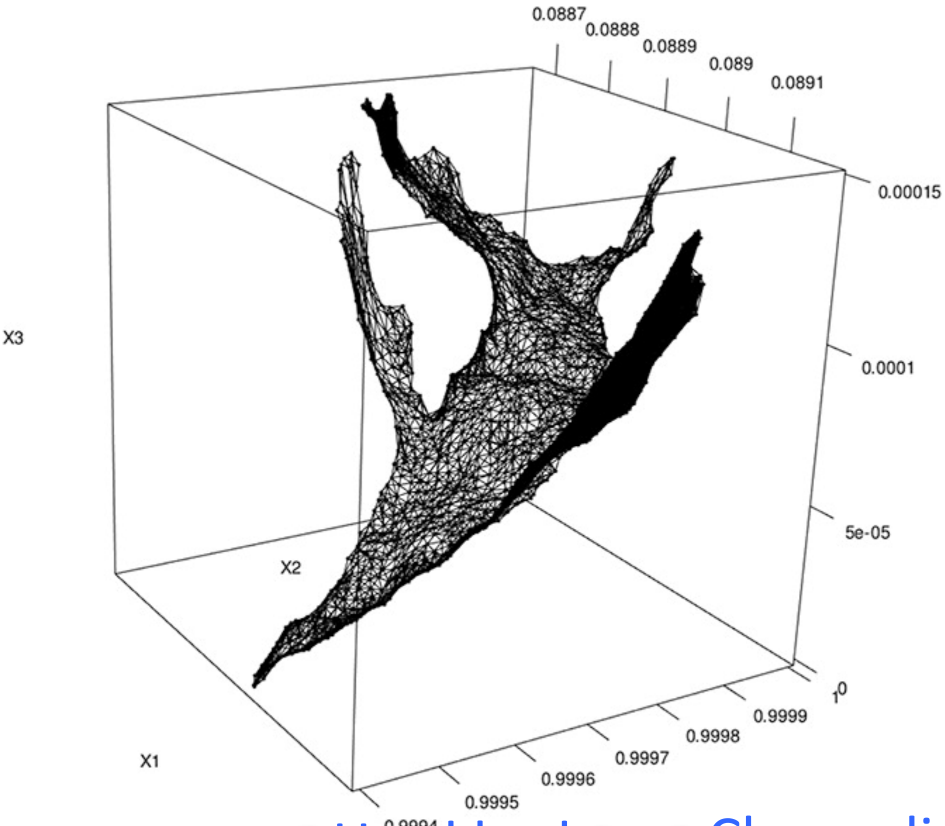
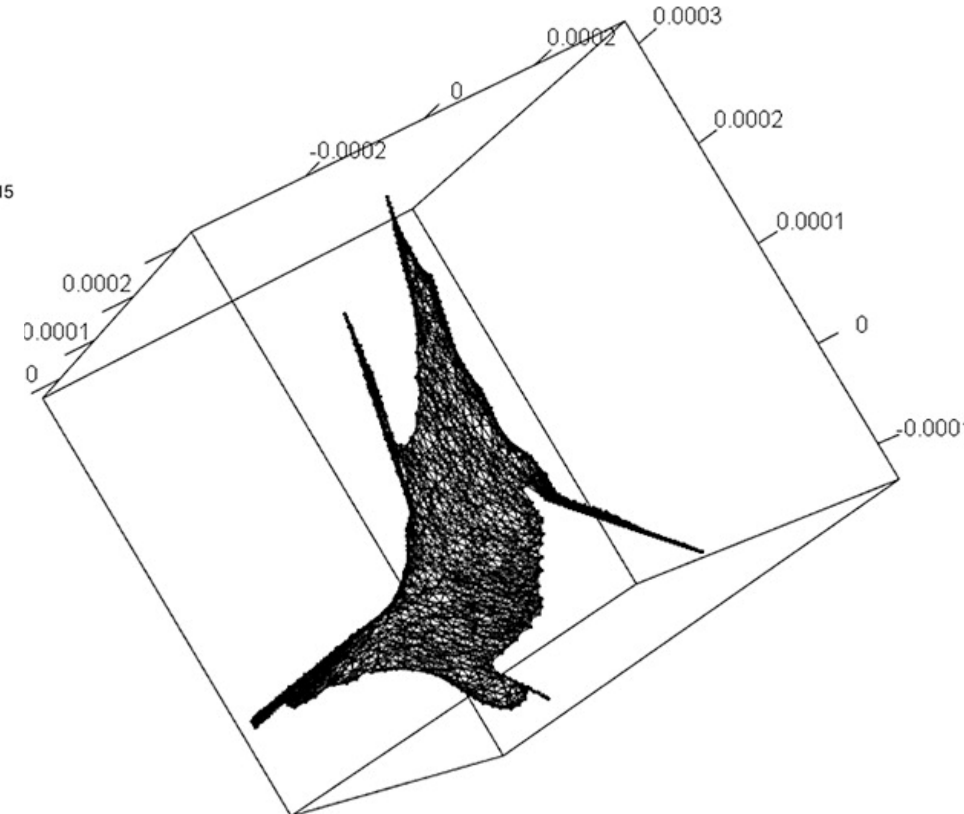
Ways forward to address modeling challenges & limitations

Example: Making use of geodesic distance in Gaussian Process model



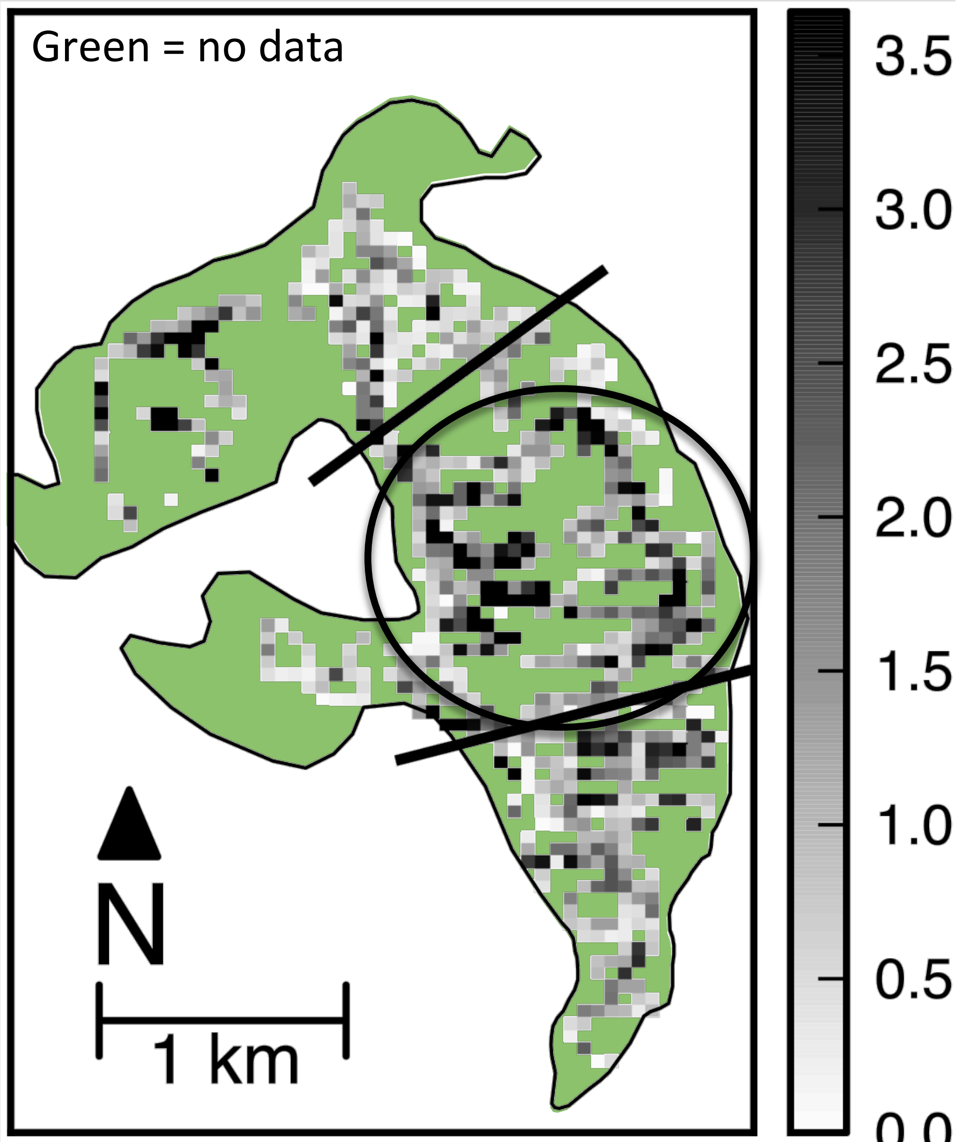


Unified bed { Hard bed / Soft bed } { Channelized / Unchannelized { Connected / Unconnected { Temperate / Frozen } } }

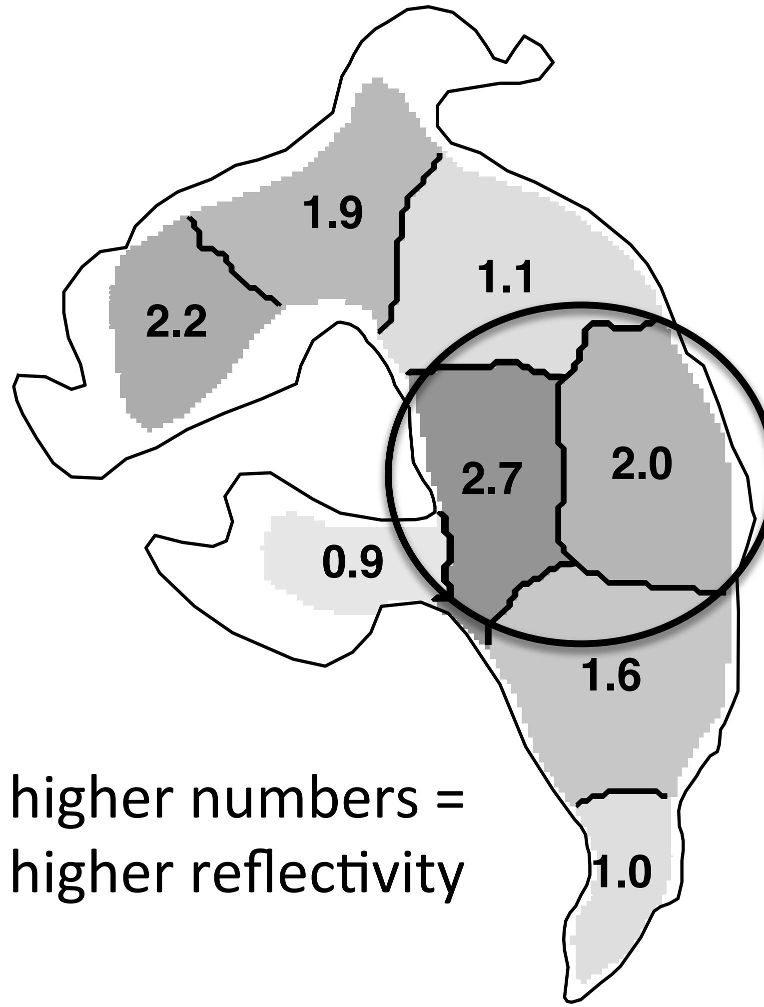
Pratola, M.T., Harari, O., Bingham, D. and Flowers, G.E., 2017. Design and Analysis of Experiments on Nonconvex Regions. *Technometrics*, 59(1), pp.36-47.

Data for model tuning / evaluation

Radar reflectivity of glacier bed



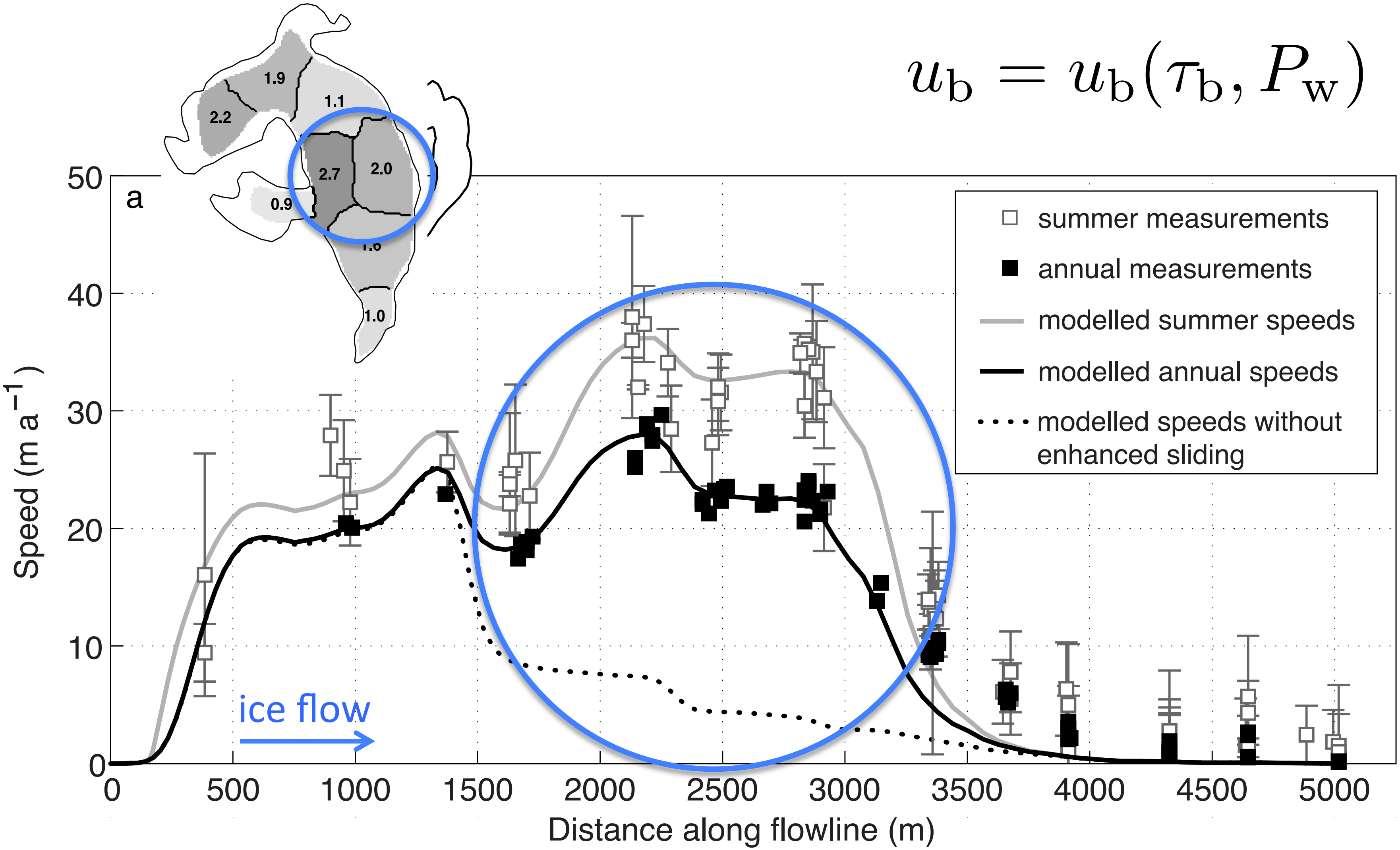
Reflectivity classified using spectral clustering



higher numbers = higher reflectivity

Wilson. N.J., G.E. Flowers, L. Mingo. 2014. Mapping and interpretation of bed reflection power from a surge-type polythermal glacier. *Annals of Glaciology*, 55(67), 1-8.

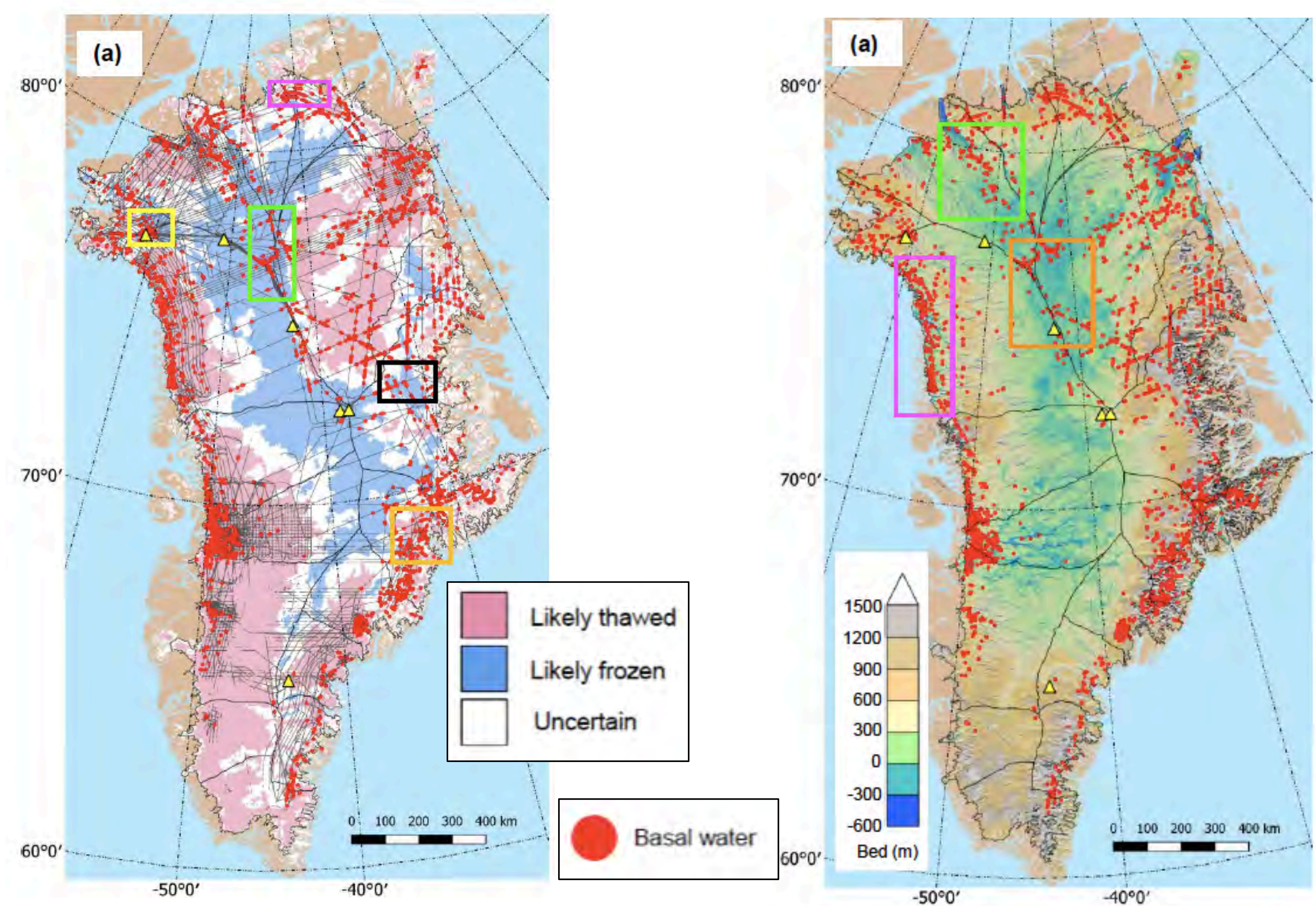
Data for model tuning / evaluation



Elevated basal water pressure can explain anomalous flow speeds

Flowers, G.E., N. Roux, S. Pimentel, C. Schoof. 2011. Present dynamics and future prognosis of a slowly surging glacier, *Cryosphere,* 5, 299-313.

Data for model tuning / evaluation



Jordan, T.M., Williams, C.N., Schroeder, D.M., Martos, Y.M., Cooper, M.A., Siegert, M.J., Paden, J.D., Huybrechts, P. and Bamber, J.L., 2018. A constraint upon the basal water distribution and thermal state of the Greenland Ice Sheet from radar bed echoes. Cryosphere, 12(9).

Synopsis: modelling Greenland ice-sheet hydrology

Successes:

- Fast/slow drainage physics, qualitatively correct behavior
- Coupling with ice-flow models, application to real topography

Challenges:

- Some missing physics (soft, unconnected, frozen beds)
- Channel-resolving models computationally prohibitive
- Lack of direct observations at appropriate scales

Possible ways forward:

- Develop emulators informed by physically based models
- Exploit concepts like geodesic distance in drainage system
- Geophysical observation of hydrological variables