

# Categories

Category Theory

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A *category* consists of:

1. a class of  $\mathcal{C}$  of **objects**,
2. for each pair  $x, y \in \mathcal{C}$ , a set  $\mathcal{C}(x, y)$  of pairwise disjoint **morphisms**, and
3. for each triple  $x, y, z \in \mathcal{C}$ , a map  $\mathcal{C}(x, y) \times \mathcal{C}(y, z) \rightarrow \mathcal{C}(x, z)$ , called a **composition** and denoted  $(\alpha, \beta) \rightarrow \beta\alpha$ , such that
  - (*Associativity*)  $\gamma(\beta\alpha) = (\gamma\beta)\alpha$  for all **morphisms**  $\alpha, \beta, \gamma$ , and
  - (*Identity*) for all  $x \in \mathcal{C}$ , there exists an **identity morphism**  $1_x \in \mathcal{C}(x, x)$  such that  $1_x\alpha = \alpha$  and  $\beta 1_x = \beta$  for any **morphisms**  $\alpha, \beta$  where the indicated **composition** is defined.