Split Epic Morphisms

A morphism $\alpha : x \to y$ is *split epic* iff there exists a morphism $\beta : y \to x$ such that $\alpha\beta = 1_y$:

$$\begin{tikzcd} x \arrow[r, shift right, "\alpha"'] & y \arrow[l, dashed, shift right, "\exists\beta"'] \arrow[loop right, "1_y"] \end{tikzcd}$$

i.e. it has a right inverse.

Remark. This definition is dual to that of split epic morphisms.

Theorem 1. *Split epic → epic*

Proof. Categories are self-dual, and the statements "*Split epic →* epic" and "Split monic → monic" are dual, so 3 gives that this is implied by 1.

Theorem 2. *In a concrete category, every split epic morphism is surjective.*

Proof. Let $\alpha : X \to Y$ be a *split epic* morphism in $\mathcal{C}$, so there exists $\beta : Y \to X$ such that $\alpha\beta = 1_Y$. For any $y \in Y$,

$$y = 1_Y(y) = \alpha\beta(y) = \alpha(\beta(y)),$$

meaning α maps $\beta(y)$ to y, so α is surjective.