Duality

We define the elementary language of category theory as the two sorted first order logic with

- objects and morphisms as the distinct sorts, together with
- relations of an object being the source or target of a morphism, and
- a symbol for composition morphisms.

Let σ be a statement in this language. The *dual* of this statement σ^{op} is the same as σ, except:

- The ”source” and ”target” relations are swapped, and
- The order of composition is swapped (every $f \circ g$ becomes $g \circ f$).

Definition 1 (Self-dual)**.** *A statement/set/class is* ***self-dual*** *iff it is equal to its dual.*

Proposition 2. *σ is true in $\mathcal{C}$ iff σ^{op} is true in $\mathcal{C}^{op}$.*

Theorem 3. *If σ holds for a* ***self-dual*** *class of categories C, so does σ^{op}.*

Proof. Let C be a class of categories that is **self-dual**. Let σ be a statement in the elementary language of category theory that holds for all $\mathcal{C} \in C$. Since C is **self-dual**, σ holds for $\mathcal{C}^{\mathrm{op}}$. Thus, by proposition 2 (and the fact that the dual of the dual is itself), σ^{op} holds for $\mathcal{C}$. $\square$