

Epic Morphisms

Category Theory > Universal Constructions

test A morphism $\alpha : x \rightarrow y$ is *epic* iff in the situation

$$x \xrightarrow{\alpha} y \xrightarrow{\beta_1} z , \\ \xrightarrow{\beta_2}$$

$$\beta_1\alpha = \beta_2\alpha \rightarrow \beta_1 = \beta_2,$$

i.e. it can be cancelled on the right.

Remark. This definition is [dual](#) to that of [monic morphisms](#).

Theorem 1. In a concrete category, every surjective morphism is monic.

Proof. Let $\alpha : X \rightarrow Y$ be an injective morphism in a concrete category \mathcal{C} . Let $Z \in \mathcal{C}$ and $\beta_1, \beta_2 : Z \rightarrow X$ such that $\alpha\beta_1 = \alpha\beta_2$. For each $z \in Z$,

$$\alpha(\beta_1(z)) = \alpha\beta_1(z) = \alpha\beta_2(z) = \alpha(\beta_2(z)),$$

meaning $\beta_1(z) = \beta_2(z)$ by the injectivity of α . Since this applies for all $z \in Z$, this gives $\beta_1 = \beta_2$.

Theorem 2. In the [category](#) of sets, topologies, groups, and rings, a morphism is injective iff it is monic.