Monic Morphisms

A morphism $\alpha : x \to y$ is *monic* iff in the situation

$$z \underset{\beta_2}{\overset{\beta_1}{\rightrightarrows}} x \xrightarrow{\alpha} y \ ,$$

$$\alpha\beta_1 = \alpha\beta_2 \to \beta_1 = \beta_2,$$

i.e. it can be cancelled on the left.

Remark. This definition is dual to that of epic morphisms.

Theorem 1. *In a concrete category, every injective morphism is monic.*

Proof. Let $\alpha : X \to Y$ be an injective morphism in a concrete category $\mathcal{C}$. Let $Z \in \mathcal{C}$ and $\beta_1, \beta_2 : Z \to X$ such that $\alpha\beta_1 = \alpha\beta_2$. For each $z \in Z$,

$$\alpha(\beta_1(z)) = \alpha\beta_1(z) = \alpha\beta_2(z) = \alpha(\beta_2(z)),$$

meaning $\beta_1(z) = \beta_2(z)$ by the injectivity of α. Since this applies for all $z \in Z$, this gives $\beta_1 = \beta_2$.

Theorem 2. *In the category of sets, topologies, groups, and rings, a morphism is injective iff it is monic.*