Part IIA Project - SF1: Data Analysis - First Interim Report

Bailey Brookes — Corpus Christi — bdb31

May 14, 2018

1 DFT vs FFT

While the DFT is fairly simple to compute, it has redundancy i it which causes it to be slow. It calculates the same exponential base (weights) numerous times. The FFT uses this recursion and redundancy to speed up the process. While the DFT requires $2N + 2N$ operations to calculate each point X_p and $X_{p+\frac{N}{2}}$, giving it complexity $O(N^2)$, the FFT has $2N + 2 + 2$ operations, meaning it has complexity $O(N \log_2 N)$. Figure 1 shows how the reduction in complexity results in a much faster computing time for longer data length.

2 Effects on the FFT

2.1 Data Length

FFT resolution increases with data length N and there are more frequency bins meaning a smaller frequency difference between bins. This increase in sharpness in shown in Figure BLANK. This is why signals are sometime s 'zero padded'. Adding zeros to the end of the signal achieves two things:

1. By adding enough zeros to make the data length a power of two, the FFT becomes much more efficient.
2. The greater the length of data, the higher the resolution.

2.2 Windowing

Windowing reduced the effect of spectral leakage caused by the sharp cut off caused by finite sampling. Figure BLANK shows this leakage for a rectangular (unwindowed) sinewave as well as a hann, hamming and triangle window. Windowing reduces the signal to zero at the end of the sampling times so there is no sharp frequency change when the sample is repeated in order to compute the FFT. Sharp charges in time result in broad frequency spectrum. This, as well as the true signal being spilt across multiple frequency bins if the frequency of the signal does not perfectly fit into one bin, is the spectral leakage.

2.3 Noise

2.4 Amplitude Modulation

3 FFT on Real Data

A Plots

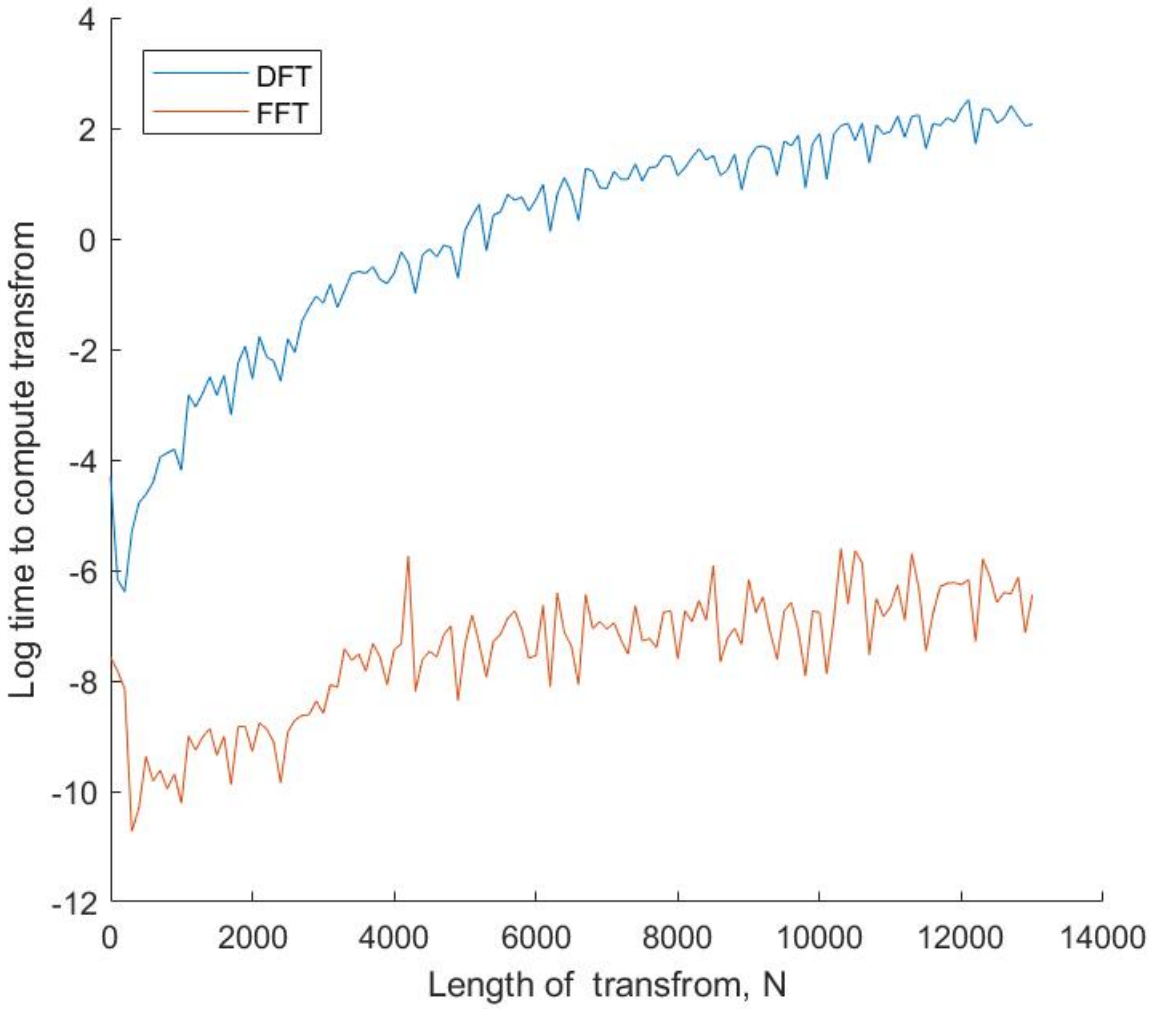

Figure 1: Comparison of the speed of the DFT and FFT, measured by the log of the time taken to compute with N samples. It is clear to see the DFT is slower and getting slower much faster than the FFT