1 Simplify Expressions

Simplify the following expressions as much as possible

1. $(-x^4 y^2)^2 = (-1)^2 x^4 y^2 = x^8 y^4$

2. $9(3^0) = 9(1) = 9$

3. $(2a^2)(4a^4) = 8a^6$

4. $\frac{x^{4\,3}}{x^3} = \left(\frac{x^4}{x^3}\right)^3 = (x)^3 = x^3$

5. $(-2)^{4-7} = (-2)^{-3} = \frac{1}{-2^3} = -\frac{1}{8}$

6. $\left(\frac{1}{27b^3}\right)^{1/3} = \frac{1}{(27b^3)^{1/3}} = \frac{1}{3b}$

7. $y^7 y^6 y^5 y^4 = y^{7+6+5+4} = y^{22}$

8. $\frac{2a/7b}{11b/5a} = \frac{2a}{7b} \cdot \frac{5a}{11b} = \frac{10a^2}{77b^2}$

9. $(z^2)^4 = z^8$

2 Simplify a (more complex) expression

Simplify the following expression

$$(a+b)^2 + (a-b)^2 + 2(a+b)(a-b) - 3a^2$$
$$= (a^2 + 2ab + b^2) + (a^2 - 2ab + b^2) + (2a^2 - 2b^2) - 3a^2$$
$$= a^2$$

3 Graph sketching

Let the functions $f(x)$ and $g(x)$ be defined for all $x \in \mathbb{R}$ by

$$f(x) = \begin{cases} |x| & \text{if } x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}, \qquad g(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 4 & \text{if } x \geq 2 \end{cases}$$
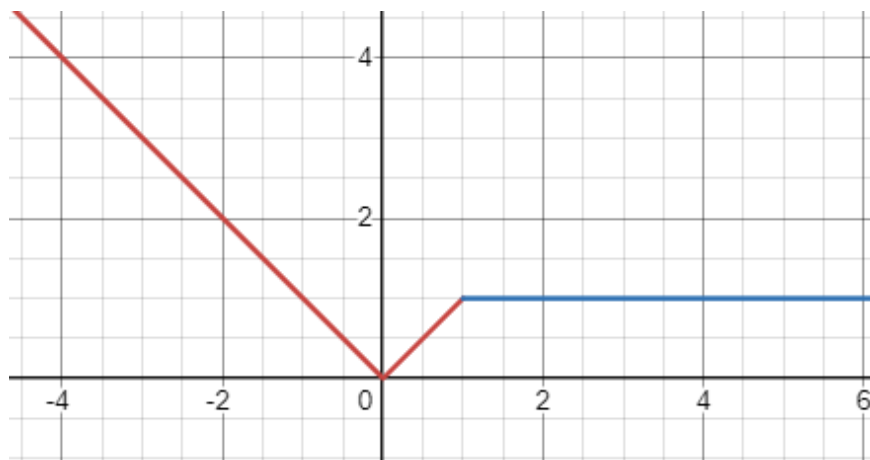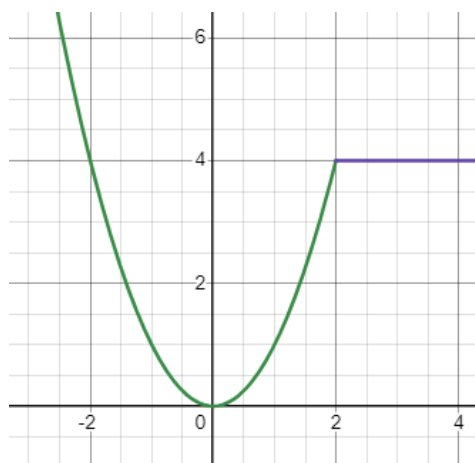
1. Sketch the graphs of:

2. $y = f(x)$

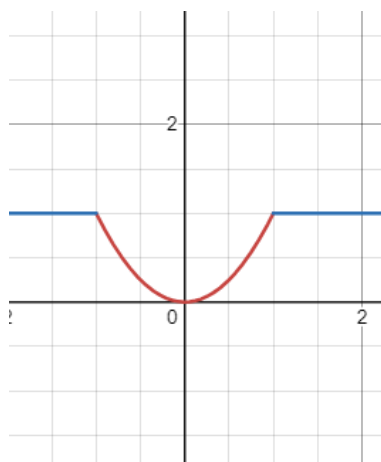3. $y = g(x)$
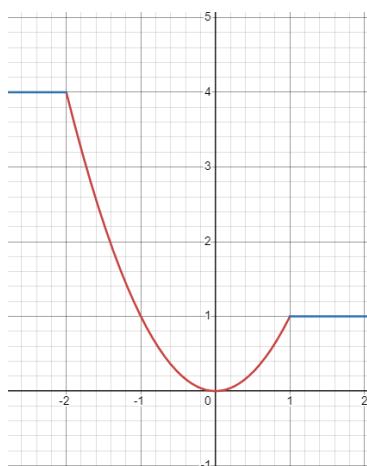
4. $y = f(g(x))$

5. $y = g(f(x))$

2. $y = f(x)$

3. $y = g(x)$

4   Root finding

Find the roots (solutions) to the following quadratic equations.

4. $y = f(g(x))$

5. $y = g(f(x))$

1. $9x^2 - 3x - 12 = 0$

$$9x^2 - 3x - 12 = 0$$
$$3(3x^2 - x - 4) = 0$$
$$3x^2 - x - 4 = 0$$
$$3x^2 - 4x + 3x - 4 = 0$$
$$3x(x + 1) - 4(x + 1) = 0$$
$$(3x - 4)(x + 1) = 0$$
$$x = \frac{4}{3}, -1$$

2. $x^2 - 2x - 16 = 0$

$$x^2 - 2x - 16 = 0$$
$$x = \frac{2 \pm \sqrt{4 - (4)(1)(-16)}}{2(1)}$$
$$x = \frac{2 \pm \sqrt{68}}{2}$$
$$x = \frac{2 \pm 2\sqrt{17}}{2}$$
$$x = 1 + \sqrt{17}, 1 - \sqrt{17}$$

3. $6x^2 - 6x - 6 = 0$

$$6x^2 - 6x - 6 = 0$$
$$x^2 - x - 1 = 0$$
$$x = \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2}$$
$$x = \frac{1 \pm \sqrt{5}}{2}$$

5 Systems of linear equations

Solve the following systems of equations for their unknown values. If there is no solution, indicate as such.

1. Two unknowns

$$3x - 2y = 18$$
$$5x + 10y = -10$$

Solution:

$$5(x + 2y) = -10$$
$$x + 2y = -2$$
$$x = -2y - 2$$
$$3(-2y - 2) - 2y = 18$$
$$-6y - 6 - 2y = 18$$
$$-8y = 24$$
$$y = -3$$
$$x = -2(-3) - 2 = 4$$
$$(x, y) = (4, -3)$$

2. Three unknowns

$$5x - 2y + 3z = 20$$
$$2x - 4y - 3z = -9$$
$$x + 6y - 8z = 21$$

Solution:

$$x + 6y - 8z = 21$$
$$x = -6y + 8z + 21$$
$$2x - 4y - 3z = -9$$
$$2(-6y + 8z + 21) - 4y - 3z = -9$$
$$-12y + 16z + 42 - 4y - 3z = -9$$
$$-16y + 13z = -51$$
$$y = \frac{13z + 51}{16}$$
$$5x - 2y + 3z = 20$$
$$5\left(-6\left(\frac{13z + 51}{16}\right) + 8z + 21\right) - 2\left(\frac{13z + 51}{16}\right) + 3z = 20$$
$$-30\left(\frac{13z + 51}{16}\right) + 40z + 105 - 2\left(\frac{13z + 51}{16}\right) + 3z = 20$$
$$-32\left(\frac{13z + 51}{16}\right) + 43z = -85$$
$$\frac{(-32)(13)z + (51)(-32)}{16} + \frac{(16)(43)z}{16} = -85$$
$$\frac{272z + (51)(-32)}{16} = -85$$
$$17z + (51)(-2) = -85$$
$$17z = 17$$
$$z = 1$$
$$y = \frac{13(1) + 51}{16} = 4$$
$$x = -6(4) + 8(1) + 21 = 5$$
$$(xyz) = (5, 4, 1)$$

3. An animal shelter has a total of 350 animals comprised of cats, dogs, and rabbits. If the number of rabbits is 5 less than one-half the number of cats, and there are 20 more cats than dogs, how many of each animal are at the shelter?

Equation set:

$$c + d + r = 350$$
$$r = \frac{1}{2}c - 5$$
$$c = d + 20$$

Solution:

$$(d + 20) + d + \left(\frac{1}{2}(d + 20) - 5\right) = 350$$

$$\frac{5}{2}d + 25 = 350$$

$$d = 130$$

$$c = (130) + 20 = 150$$

$$r = \frac{1}{2}(150) - 5 = 70$$

$$(c, d, r) = (150, 130, 70)$$

6 Work with sets

Using the setes

$$A = \{2, 3, 7, 9, 13, 16\}$$
$$B = \{x : 4 \leq x \leq 8 \text{ and } x \text{ is an integer}\}$$
$$C = \{x : 2 < x < 25 \text{ and } x \text{ is prime}\}$$
$$D = \{1, 4, 9, 16, 25, ...\}$$

Identify the following:

1. $A \cup B$

2. $(A \cup B) \cap C$

3. $C \cap D$

Solutions:

1. $A \cup B = \{2, 3, 9, 13, 16 \text{ and } x \in \mathbb{Z} | 4 \leq x \leq 8\} = \{2, 3, 4, 5, 6, 7, 8, 9, 13, 16\}$

2. $(A \cup B) \cap C = \{2, 3, 4, 5, 6, 7, 8, 9, 13, 16\} \cap \{x : 2 < x < 25 \text{ and } x \text{ is prime}\} = \{3, 5, 7, 13\}$

3. $C \cap D = \emptyset$