Linear equations, inequalities, sets and functions, quadratics
MACSS 33000 1
Due Tuesday, August 27
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1 Simplify Expressions

Simplify the following expressions as much as possible

1. $(-x^4y^2)^2 = (-1)^2x^4y^2 = x^8y^4$
2. $9(3^0) = 9(1) = 9$
3. $(2a^2)(4a^4) = 8a^6$
4. $\frac{x^4}{x^3}^3 = \left(\frac{x^4}{x^3}\right)^3 = (x)^3 = x^3$
5. $(-2)^{4-7} = (-2)^{-3} = \frac{1}{-2^3} = -\frac{1}{8}$
6. $\left(\frac{1}{27b^3}\right)^{1/3} = \frac{1}{(27b^3)^{1/3}} = \frac{1}{3b}$
7. $y^7y^6y^5y^4 = y^{7+6+5+4} = y^{22}$
8. $\frac{2a/7b}{11b/5a} = \frac{2a}{7b} \cdot \frac{5a}{11b} = \frac{10a^2}{77b^2}$
9. $(z^2)^4 = z^8$

2 Simplify a (more complex) expression

Simplify the following expression

$$\begin{aligned}&(a+b)^2 + (a-b)^2 + 2(a+b)(a-b) - 3a^2\\ &= (a^2 + 2ab + b^2) + (a^2 - 2ab + b^2) + (2a^2 - 2b^2) - 3a^2\\ &= a^2\end{aligned}$$

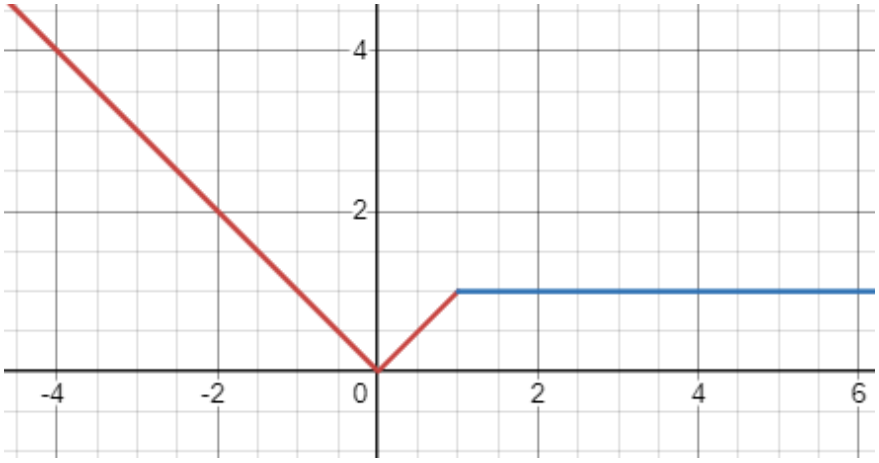
3 Graph sketching

Let the functions $f(x)$ and $g(x)$ be defined for all $x \in \mathbb{R}$ by

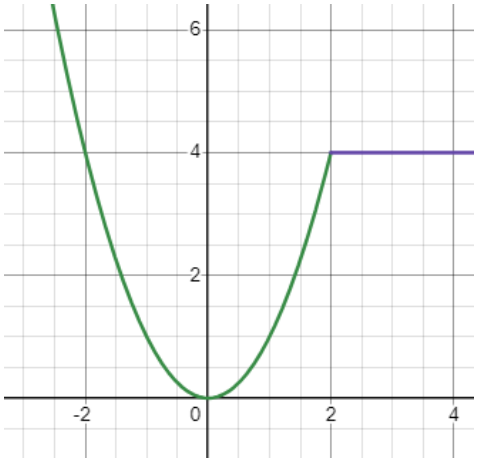
$$f(x) = \begin{cases} |x| & \text{if } x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}, \qquad g(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 4 & \text{if } x \geq 2 \end{cases}$$

1. Sketch the graphs of:
2. $y = f(x)$
3. $y = g(x)$

4. $y = f(g(x))$
5. $y = g(f(x))$



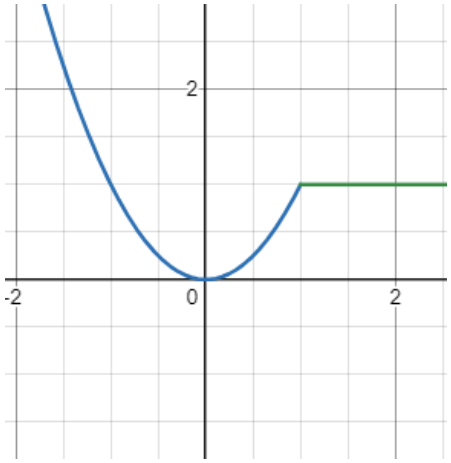
2. $y = f(x)$



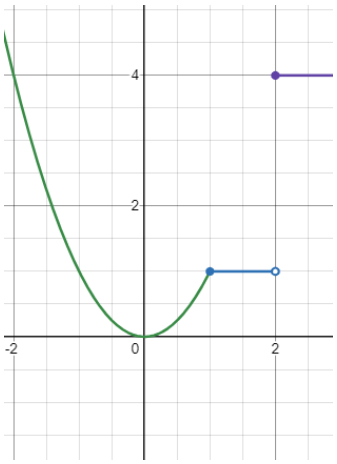
3. $y = g(x)$

4 Root finding

Find the roots (solutions) to the following quadratic equations.



4. $y = f(g(x))$



5. $y = g(f(x))$

1. $9x^2 - 3x - 12 = 0$

$$
\begin{aligned}
9x^2 - 3x - 12 &= 0 \\
3(3x^2 - x - 4) &= 0 \\
3x^2 - x - 4 &= 0 \\
3x^2 - 4x + 3x - 4 &= 0 \\
3x(x+1) - 4(x+1) &= 0 \\
(3x-4)(x+1) &= 0 \\
x &= \frac{4}{3}, -1
\end{aligned}
$$

2. $x^2 - 2x - 16 = 0$

$$\begin{aligned} x^2 - 2x - 16 &= 0 \\ x &= \frac{2 \pm \sqrt{4 - (4)(1)(-16)}}{2(1)} \\ x &= \frac{2 \pm \sqrt{68}}{2} \\ x &= \frac{2 \pm 2\sqrt{17}}{2} \\ x &= \sqrt{17}, -\sqrt{17} \end{aligned}$$

3. $6x^2 - 6x - 6 = 0$

$$\begin{aligned} 6x^2 - 6x - 6 &= 0 \\ x^2 - x - 1 &= 0 \\ x &= \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2} \\ x &= \frac{1 \pm \sqrt{5}}{2} \end{aligned}$$

5 Systems of linear equations

Solve the following systems of equations for their unknown values. If there is no solution, indicate as such.

1. Two unknowns

$$\begin{aligned} 3x - 2y &= 18 \\ 5x + 10y &= -10 \end{aligned}$$

Solution:

$$\begin{aligned} 5(x + 2y) &= -10 \\ x + 2y &= -2 \\ x &= -2y - 2 \\ 3(-2y - 2) - 2y &= 18 \\ -6y - 6 - 2y &= 18 \\ -8y &= 24 \\ y &= -3 \\ x &= -2(-3) - 2 = 4 \\ (x, y) &= (4, -3) \end{aligned}$$

2. Three unknowns

$$\begin{aligned} 5x - 2y + 3z &= 20 \\ 2x - 4y - 3z &= -9 \\ x + 6y - 8z &= 21 \end{aligned}$$

Solution:

$$
\begin{aligned}
x + 6y - 8z &= 21 \\
x &= -6y + 8z + 21 \\
2x - 4y - 3z &= -9 \\
2(-6y + 8z + 21) - 4y - 3z &= -9 \\
-12y + 16z + 42 - 4y - 3z &= -9 \\
-16y + 13z &= -51 \\
y &= \frac{13z + 51}{16} \\
5x - 2y + 3z &= 20 \\
5\left(-6\left(\frac{13z + 51}{16}\right) + 8z + 21\right) - 2\left(\frac{13z + 51}{16}\right) + 3z &= 20 \\
-30\left(\frac{13z + 51}{16}\right) + 40z + 105 - 2\left(\frac{13z + 51}{16}\right) + 3z &= 20 \\
-32\left(\frac{13z + 51}{16}\right) + 43z &= -85 \\
\frac{(-32)(13)z + (51)(-32)}{16} + \frac{(16)(43)z}{16} &= -85 \\
\frac{272z + (51)(-32)}{16} &= -85 \\
17z + (51)(-2) &= -85 \\
17z &= 17 \\
z &= 1 \\
y &= \frac{13(1) + 51}{16} = 4 \\
x &= -6(4) + 8(1) + 21 = 5 \\
(xyz) &= (5, 4, 1)
\end{aligned}
$$

3. An animal shelter has a total of 350 animals comprised of cats, dogs, and rabbits. If the number of rabbits is 5 less than one-half the number of cats, and there are 20 more cats than dogs, how many of each animal are at the shelter?

   Equation set:

$$
\begin{aligned}
c + d + r &= 350 \\
r &= \frac{1}{2}c - 5 \\
c &= d + 20
\end{aligned}
$$

Solution:

$$\begin{aligned}(d+20)+d+\left(\frac{1}{2}(d+20)-5\right) &= 350\\ \frac{5}{2}d+25 &= 350\\ d &= 130\\ c &= (130)+20=150\\ r &= \frac{1}{2}(150)+20=95\\ (c,d,r) &= (150,130,95)\end{aligned}$$

6 Work with sets

Using the setes

$$\begin{aligned}A &= \{2,3,7,9,13,16\}\\ B &= \{x : 4 \leq x \leq 8 \text{ and } x \text{ is an integer}\}\\ C &= \{x : 2 < x < 25 \text{ and } x \text{ is prime}\}\\ D &= \{1,4,9,16,25,...\}\end{aligned}$$

Identify the following:

1. $A \cup B$
2. $(A \cup B) \cap C$
3. $C \cap D$

Solutions:

1. $A \cup B = \{2,3,9,13,16 \text{ and } x \in \mathbb{Z} | 4 \leq x \leq 8\} = \{2,3,4,5,6,7,8,9,13,16\}$
2. $(A \cup B) \cap C = \{2,3,4,5,6,7,8,9,13,16\} \cap \{x : 2 < x < 25 \text{ and } x \text{ is prime}\} = \{3,5,7,13\}$
3. $C \cap D = \emptyset$