PL Attribution And Risk

Risk Attribution

- Once we have pricing schemes available for any derivate asset/instrument we own, we can obtain *risk parameters*
- These are sensitivities of instrument values to market conditions
- Broadly speaking we think of these sensitivities as being partial derivatives of instrument value to:
  - Model parameters: underlying price, interest rate level, volatility level, volatility skew
  - Market instrument prices: ATM option prices, bond prices, CDS upfront
- The convenient thing about these is that they can be combined

Combining Partials

$$\frac{\partial \Pi}{\partial \mu} = \sum_{i=1}^{N} Q_i \frac{\partial V_i}{\partial \mu}$$

$$\Delta \Pi \approx \sum_{k=1}^{K} \sum_{i=1}^{N} \frac{\partial V_i}{\partial \mu_k} Q_i \Delta \mu_k$$

$$\Delta \Pi \approx \sum_{k=1}^{K} \frac{\partial \Pi}{\partial \mu_k} \Delta \mu_k$$

$$\Delta \Pi \approx \sum_{k=1}^{K} \sum_{i=1}^{N} \frac{V_i(\mu_i + d\mu_k) - V_i(\mu_i - d\mu_k)}{2 \cdot d\mu_k} Q_i \Delta \mu_k$$

$$\Delta \Pi \approx \sum_{k=1}^{K} \sum_{i=1}^{N} \left(\frac{\partial V_i}{\partial \mu_k} Q_i \Delta \mu_k + \frac{\partial^2 V_i}{\partial \mu_k^2} \Gamma \mu_k^2 \right)$$

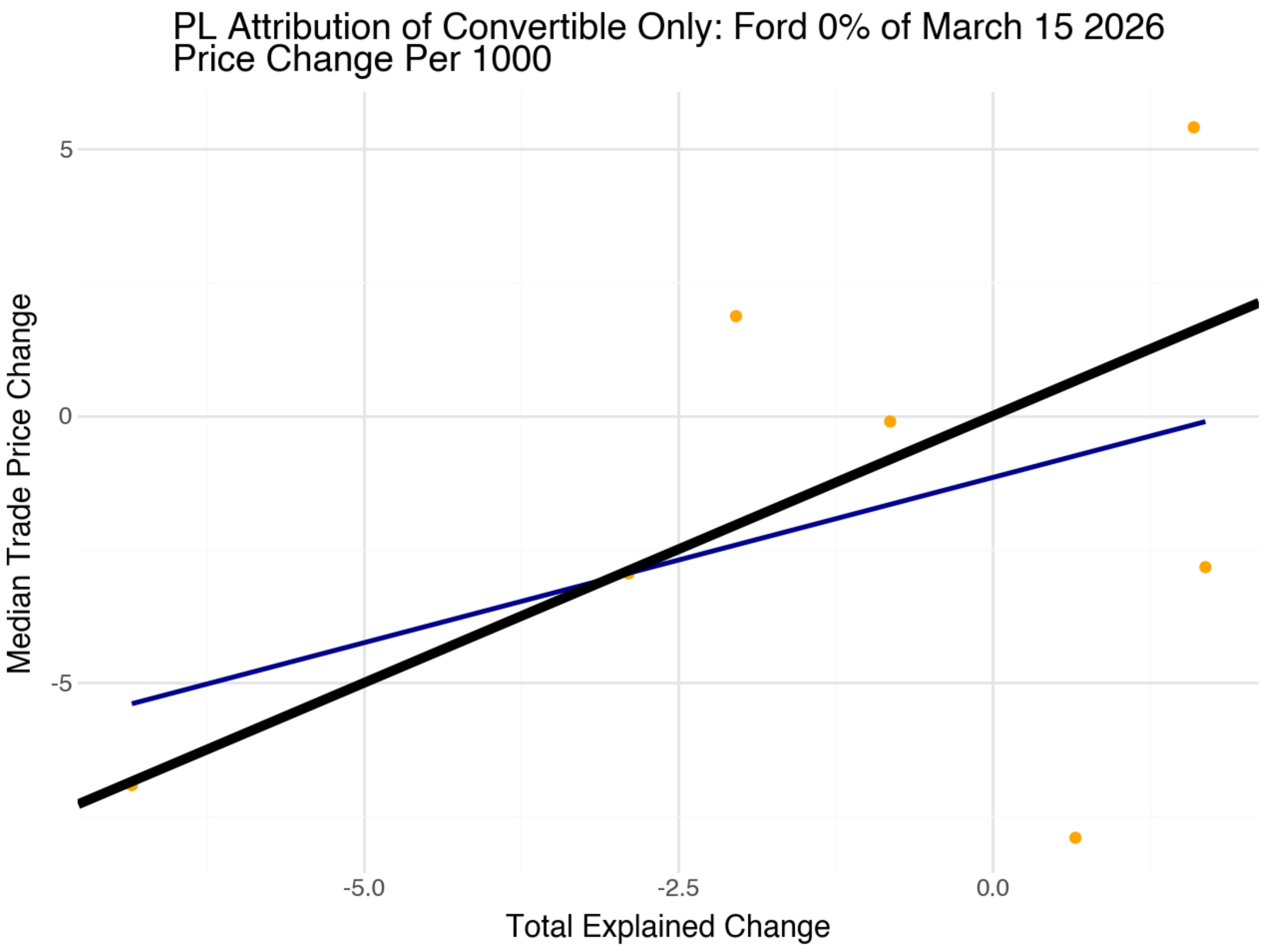
Risk Exposure And Term Structures

- We want simple expressions of risk exposure
- Instrument values depend on term structures of parameters (interest rates, volatility, credit risk)
- Decompose term structures into "level", "tilt", and "other"
  - Level is usually change in average value
  - Tilt is parameter-dependent and sometimes portfolio-dependent
    - 1 month versus 6 month
    - 2 year versus 10 year

Risk Exposure And Calibration

- We usually calibrate our models to market conditions
- Risk exposure is a partial derivative
- We ultimately care how we will judge our asset value given a new calibration
- This brings up the question of *total derivative* versus *partial derivative*
    - The total derivative for μ computes how we would perceive value change if μ changed and *other* parameters changed along with it as expected
    - The partial derivative for μ computes how value changes if *only* μ were to change
- Risk attribution typically concentrates on partial derivatives, with occasional exceptions for volatility skew

Attribution Is Imperfect



Tails Of Risk

- Let's say we have a portfolio containing derivatives
- We want to understand tails of PL, say at 1-week 95% Expected Shortfall level
    - Derivatives values are highly nonlinear with respect to market prices of base securities
    - Their value relationships are poorly represented with simple correlations
- To calculate tail risk, we use modeling instead

Volatility Skew Risk

- When the underlying moves, what happens?
  - ATM volatility moves along the skew curve, or
  - The skew curve moves its "center" to the new ATM volatility
- Neither is really captured by a partial derivative, unless the option is exactly ATM
- Same for other term structures

Expected Shortfall

$$\mathrm{ES}_\alpha(X) = \mathbb{E}\left[X \mid X \geq \mathsf{VaR}_\alpha(X)\right]$$

$$\mathrm{VaR}_\alpha(X) = \inf\{x \in \mathbb{R} : P(X \leq x) \geq \alpha\}$$

$$X = \sum_{i=1}^{n} w_i V_i(r, \sigma)$$

Derivatives Models and Tail Risk

- Rather than correlating derivatives prices with everything else, we use our pricing models
- Create a covariance matrix for model inputs along with base instruments
    - Decreases in equities are correlated with increases in volatility, dollar FX rates, and credit risk
    - They are (often) correlated with rate decreases in the long term
- Use Monte Carlo (with importance sampling) to generate scenarios
- Each scenario contains:
    - Hypothetical base instrument prices
    - Hypothetical model inputs: volatility, credit risk etc
- Each scenario gives *direct* values for base instruments, and *derived* values of derivative instruments

Computing Tail Risk

```
for sim in 1 to NumSimulations:
    σ, r, S = sample_correlated_S_sigma_r()
    for i in 1 to NumAssets:
        if is_a_derivative(Asset(i))
            V[i] <- price_derivative(i, S[i], σ, r)
        else
            V[i] <- S[i]
  X[sim] <- sum_over_i(w[i] * V[i])

sort X in descending order (losses from high to low)
VaR_index <- ceil((1 - α) * NumSimulations)
ExpectedShortfall <- mean(X[1:VaR_index])
```

Budgets Of Risk

- Every portfolio asset has a (possibly zero) exposure to every market parameter: $dX_\mu = \sum_{i=1}^{N} \frac{\partial V_i}{\partial \mu}$
- We can define a risk budget for any μ, specifying the the total dollar exposure dX_μ to μ may not exceed, say, L_μ
- Or, we can specify rules, e.g.:
  - If dX_μ is greater than $\frac{1}{2}L_\mu$ then we are required to stop increasing the contributing position sizes
  - If dX_μ is greater than $\frac{3}{4}L_\mu$ then we are required to decrease the contributing position sizes
  - If dX_μ is greater than L_μ then we are required to close all contributing positions
- Seen externally in clearing firm limits, OCC margin calculations