Functional form of the total derivative of an option price $C(K, \sigma(K))$ in the presence of a skew $\sigma(K)$

$$\begin{aligned}\frac{d^2}{dK^2}C(K,\sigma(K)) &= \frac{d}{dK}\left(\sigma'(K)\frac{\partial C(K,\sigma(K))}{\partial \sigma} + \frac{\partial C(K,\sigma(K))}{\partial K}\right) \\ &= \frac{\partial^2 C}{\partial \sigma^2}\sigma'(K)^2 + 2\frac{\partial^2 C}{\partial K \partial \sigma}\sigma'(K) + \sigma''(K)\frac{\partial C}{\partial \sigma} + \frac{\partial^2 C}{\partial K^2}\end{aligned}$$