Nomura Fixed Income Research

Correlation Redux

October 17, 2005

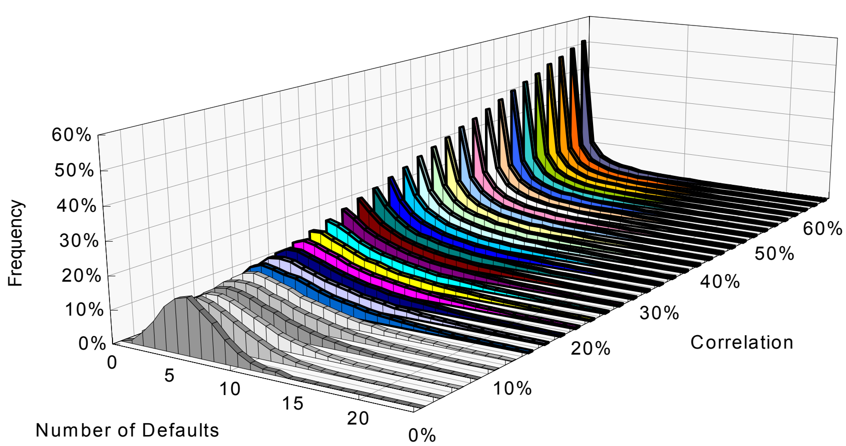
I. Introduction

Correlation has received a great amount of attention in recent months. Market participants now understand how a high degree of co-movement can increase the likelihood of extreme events or create the so-called "fat tail." Correlation, however, is extremely difficult to measure. While the idea of trading "correlation" was initially hailed as trading of a new asset class, the market is still far from mastering the nuances of the concept and some of the practical measurement issues. In this note, we illustrate how "market implied" levels of correlation are measured and how one can utilize them to make relative value decisions, with caution.

II. Correlation 101

Correlation is an important input when we analyze a portfolio of credit risk.[1] Credit risk correlation can be defined in many ways, but it generally refers to the degree of co-movement of asset values or companies' fortune. High correlation tends to increase the likelihood that assets or companies default (or survive) together, while low correlation tends to increase the likelihood of isolated defaults. The graph below shows loss distributions of a credit risk portfolio at varying levels of correlation, derived from a Monte Carlo simulation.

Graph 1: Frequency of Portfolio Defaults
(Simulation Results: 125 names, 5 years)



Based on simulations with 40,000 iterations assuming a 1% default probability. Source: Nomura

[1] For basic discussion of how correlation affects the loss distribution of a credit risk portfolio, please see; *Correlation Primer*, Nomura Fixed Income Research (6 August 2005).

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The height of the graphs indicates frequency (*i.e.*, probability) that the portfolio experiences a certain level of default losses. As we can see, correlation affects the shape of the graphs. As is widely known, high correlation makes the loss distribution "fat-tailed." In other words, high correlation increases the risk of suffering extremely high level of losses as well as the likelihood of escaping losses entirely. For example, in the graph above, the frequency of suffering 20 or more defaults is 0.0% when correlation is 0%, but it increases to 9.4% as correlation rises to 60%. At the same time, the likelihood of *escaping losses* also increases from 0% to more than 50% as correlation increases from 0% to 60%.

Correlation is only marginally important when an investor owns the *entire* portfolio, rather than a *tranche* based on it. That's because the average (*i.e.*, expected) losses in the portfolio remain unchanged regardless of the level of correlation. In contrast, the level of correlation greatly affects the value of a tranche or a CDO of a CDS index. A fat-tailed loss distribution has higher risk of suffering large losses than a thin-tailed loss distribution. Hence, the value of a senior tranche is lower in general when the loss distribution is fat-tailed. By contrast, the reverse is true for the equity (first-loss) tranche, because high correlation increases the likelihood of escaping any losses.

At present, the Gaussian copula approach is the most commonly used model in the structured credit market. The Gaussian copula model, first introduced by David Li[2], uses the normal distributions to incorporate correlation among individual credits in a portfolio. Rating agencies and dealers rely on Monte Carlo simulation based on this model when they assess risk and value of a credit risk tranche. Recently, some market participants have voiced their concerns about the heavy reliance on this model. In the absence of a better alternative, however, we recommend that investors handle it with caution and try to understand pitfalls and possible "model effects."

III. Evolution from "Compound Correlation" to "Base Correlation"

Shortly after the trading of CDS index tranches began, dealers started to quote the "market-implied" level of correlation for each tranche. Based on the observed market spread for a tranche, implied correlation can be calculated by finding the level of correlation that equates the theoretical spread and the market spread, using the Gaussian copula model. The situation is analogous to how implied volatility is calculated in the options market. This type of market-implied correlation is also called the "compound correlation."

A. The Compound Correlation

When active trading of index tranches began last year, compound correlation was hailed as a useful measure for assessing relative values among individual tranches. For example, if an investor believes that the "true" correlation is 20% but sees the market-implied correlation at 15% for the equity tranche, that tranche would be viewed as undervalued (because correlation is good for the equity tranche). On the other hand, if the senior tranche is quoted at an implied correlation of 15%, the correlation is "too low," and it would mean that the tranche is overvalued. Depending on the demand-supply condition in the market, one would observe various levels of correlation across the capital structure of the index.

The graph below plots the relation between tranche spreads and levels of correlation for the portfolio we created in the previous section. As we can see, the breakeven spread of the 0%-3% tranche ("equity tranche") goes down as correlation increases. In contrast, for the senior tranche, tranche spread increases with correlation. This makes sense because higher correlation means the increased likelihood of losses reaching even the senior tranche.

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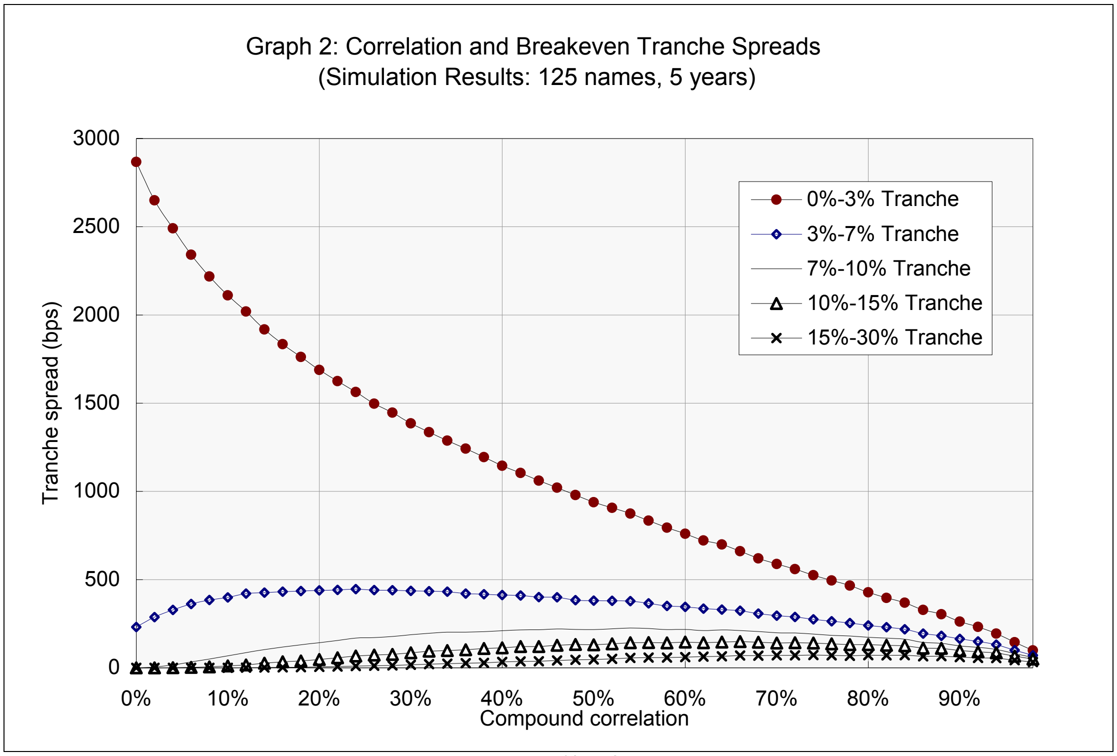
[2] Li, David, *On Default Correlation: A Copula Function Approach*, RiskMetrics Group, 2000.



Based on simulations with 40,000 iterations assuming a 1% default probability.
Source: Nomura

However, the relationship is NOT monotonic for mezzanine tranches. For these tranches, breakeven spreads initially increase as correlation goes up, but they begin to fall at high correlation. In other words, there can be two levels of correlation that yields a single market spread! Hence, one problem with the compound correlation is that, at least for some mezzanine tranches, market-implied correlation is not well defined.[3]

Moreover, the market-implied compound correlation is known to exhibit a "smile," just like the volatility smile observed in the options market. Graph 3 shows the compound correlation observed for the DJ CDX.NA.IG index last March. There are several reasons why such a smile exists, including demand-

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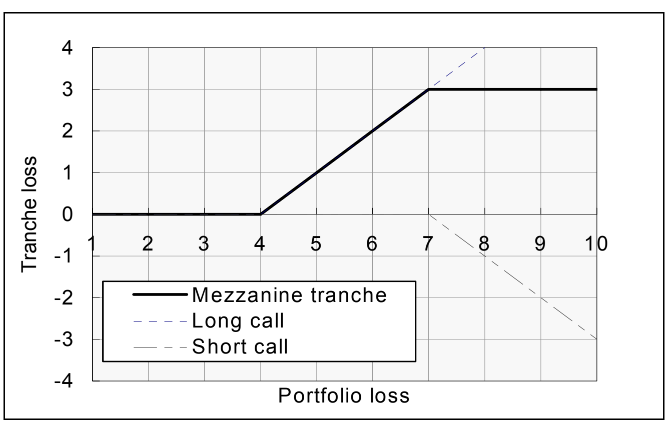
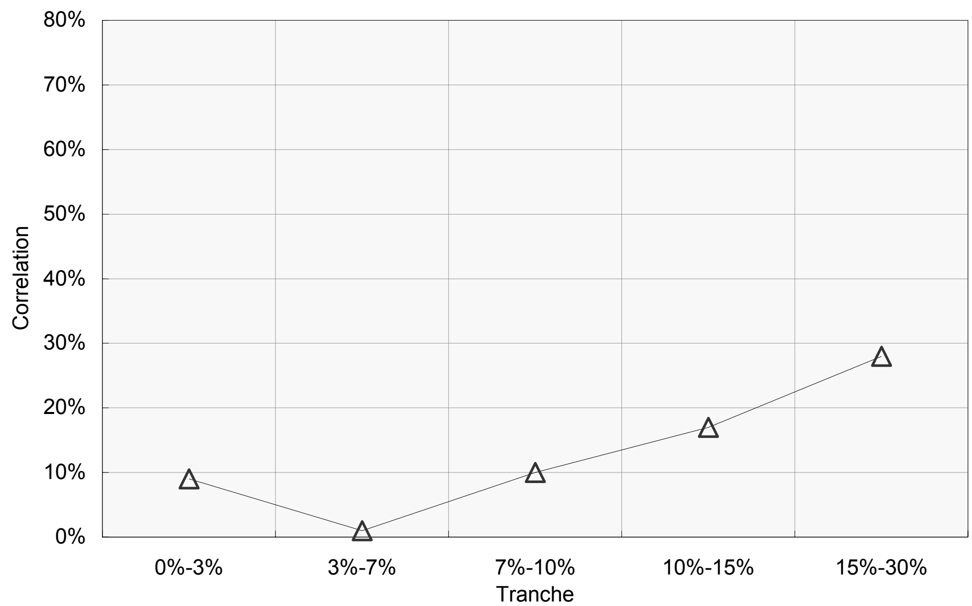
[3] Another way to look at this point is a combined option position of long call option at a low strike with short call option at a high strike. (Note that unlike an ordinary call option, "tranche loss" and "portfolio loss" replace "payoff" and "asset price," respectively.)



The payoff (*i.e.*, tranche loss) for the combined position is identical to that of a mezzanine tranche. The payoff of such an option depends on the shape of the loss distribution, which is affected by correlation. However, the effect of correlation on the combined position of long and short options is not straightforward.

supply conditions in the index tranche market and certain model effects.[4] However, this correlation smile presents another problem; the curve is difficult to interpolate for pricing a customized tranche with a non-standard attachment or detachment point.

Graph 3: Compound Correlation "Smile" of the 5-year CDX.NA.IG Index (September 27 2005)



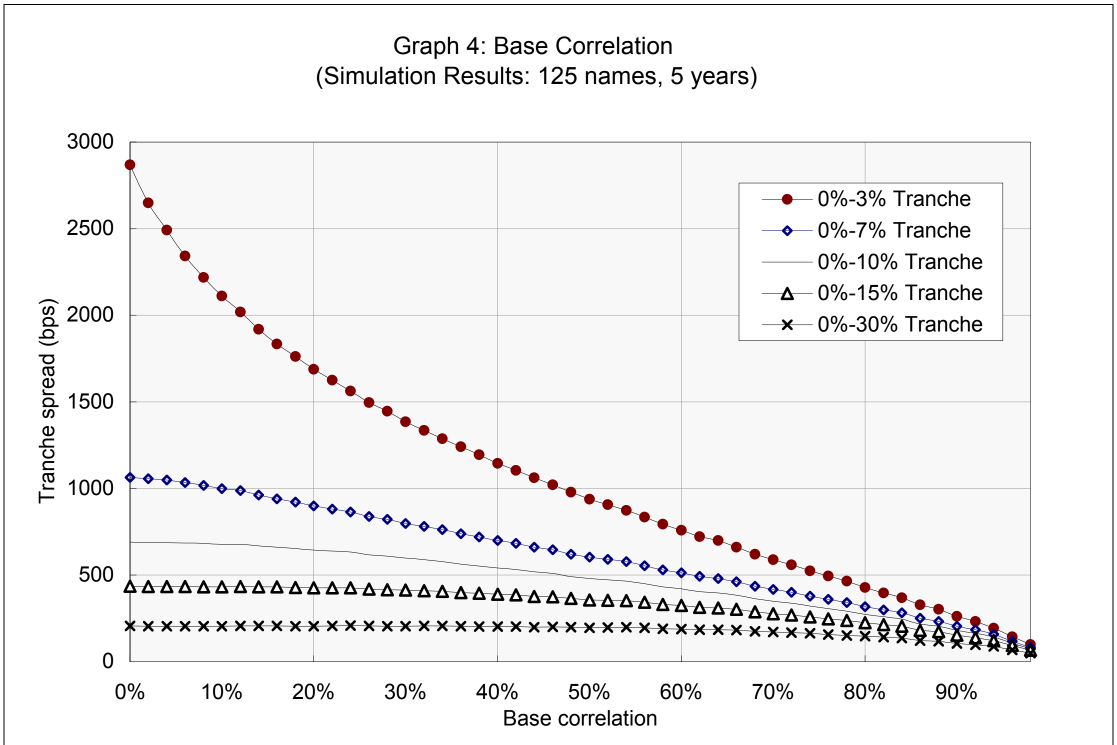
Source: Creditflux

B. The Base Correlation

The structured credit market gradually moved away from the problematic compound correlation, towards a new measure of market-implied correlation. The new measure, called "base correlation," solves some of the problems of the compound correlation.

Base correlation is a correlation number for an *equity* tranche that combines *all* tranches up to a certain detachment point. For example, the 0%-3% tranche and the 3%-7% tranche of the DJ CDX.NA.IG index are combined to create a hypothetical 0%-7% tranche. The expected loss for this hypothetical tranche is equal to the sum of expected losses for the 0%-3% tranches and the 3%-7% tranche, which can be calculated from tranche spreads. Then, we calculate the level of correlation that corresponds to the expected loss of the 0%-7% tranche, using the same method used in the calculation of compound correlation.

[4] Academic research has found that the correlation smile is partly due to the inability of the Gaussian models to capture fat tails. Using some other probability distributions and incorporating additional stochastic factors, it is possible to fit the observed tranche spreads more closely.



Based on simulations with 40,000 iterations assuming a 1% default probability.
Source: Nomura

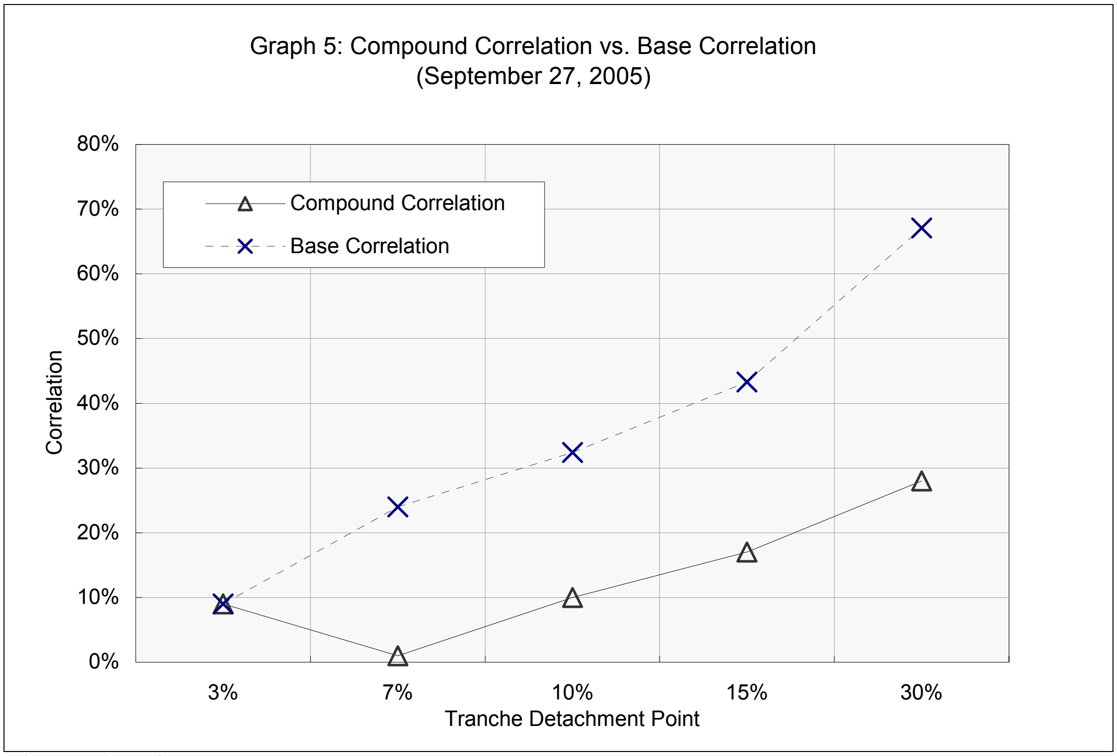
Graph 4 above plots the relation between tranche spreads and correlation. Not surprisingly, the base correlation curve for the 0%-3% tranche is identical to the compound correlation curve for that tranche, because the two measures are equivalent for this tranche. More importantly, the base correlation curves for most other tranches are now monotonic, allowing a unique level of correlation for a particular spread level.

The fact that base correlation changes monotonically with tranche spread represents an important improvement over the compound correlation. First, using this method, we can avoid the problem associated with mezzanine tranche correlation. Second, unlike in the compound correlation, it makes more sense to interpolate the base correlation curve for non-standard tranches. Graph 5 plots compound correlation and base correlation for standard tranches of the 5-year DJ CDX.NA.IG index. As we can see, the correlation "smile" disappears when we move to base correlation. Instead, the base correlation curve is upward sloping, which is often referred to as the base correlation "skew."

The Large Homogeneous Pool (LHP) Model

One of the simplest ways to model a portfolio of credit risk is called the large homogeneous pool (LHP) approach. It assumes that a portfolio is comprised of a large number of identical assets. For example, the CDS index is viewed as an equally weighted portfolio of 125 reference credits that have the same default probability and the same recovery rate. Moreover, in this setup, each pair of reference credits in the portfolio has the same correlation. Hence, a portfolio is characterized by a small number of parameters; (1) the number of reference credits, (2) the average default probability (*i.e.*, spread), and (3) the single correlation. The recovery rate is often assumed to be constant at, say, 40%.

The Gaussian copula model does not need to assume a LHP. However, combining the two approaches simplifies the analysis significantly. In fact, under the LHP, the Gaussian copula model becomes so simple that the loss distribution can be numerically calculated without running a simulation. Because of its computation ease, the LHP is commonly used in the index tranche context. All things equal, assuming a homogeneous pool would result in *lower* base correlation than a model that accounts for spread dispersion.



Source: Creditflux, Nomura

It is important to note that the base correlation curve reflects the same information as compound correlation, only in a different manner. Instead of expressing a marked-implied level of correlation for each tranche, base correlation expresses correlation for and equity tranche comprised of all tranches up to the detachment point of each tranche. Doing so simply avoids the problem in calculating mezzanine tranche correlation.

Unfortunately, base correlation is not without flaws. First, base correlation is less intuitive than compound correlation and is more difficult to interpret. One needs to know the spread levels of *all* tranches up to a certain detachment point to calculate base correlation. As a result, base correlation for a certain detachment point reflects market spreads for multiple tranches, making it difficult to separate market condition for each tranche. For example, if the 0%-3% tranche becomes undervalued ("cheap") while the 3%-7% tranche becomes overvalued ("rich"), base correlation for the 0%-7% tranche may reflect the offsetting forces and show little change. Moreover, the base correlation does not resolve the correlation smile, because the smile is simply transformed into a "skew." Researchers are still working on this unsolved mystery.

IV. Interpreting the Base Correlation

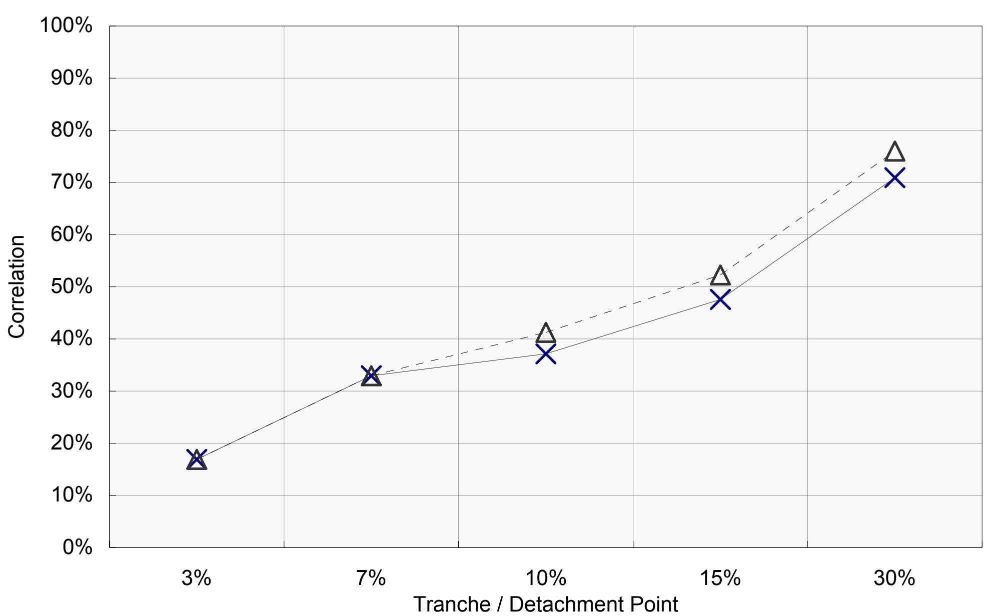
As mentioned above, base correlation is not without problems. The market is still in search for a better model to describe relative value of index tranches. However, it is possible to obtain some insight into shifting market conditions by observing the shape of the base correlation curve.

A. One Tranche Outperforming (or Underperforming)

Suppose the base correlation of the 7%-10% tranche increases while the 0%-3% and the 3%-7% tranches remain unchanged. The base correlation curve steepens between the 7% point and the 10% point. (See Graph 6 below.) The move indicates that the 7%-10% tranche outperformed *relative* to both 0%-3% and 3%-7% tranches. The fact that the base correlation for the combined 0%-10% tranche went up indicates that the combined expected losses declined. However, the expected losses for the 0%-3% and the 3%-7% tranches remained unchanged. Hence, the move indicates that the spread of the 7%-10% tranche tightened, causing it to outperform the other two tranches.

As we discussed above, the base correlation is calculated as an implied correlation of an equity tranche that combines *all* tranches up to the detachment point. Importantly, this causes the base correlation curve to move in a counter-intuitive manner when spreads and expected losses of individual tranches fluctuate. That is, when the spread of the 7%-10% tranche declines, the base correlation will *rise* while the implied correlation for the tranche *declines*. In general, a steeper base correlation curve indicates more senior tranches are outperforming, and vice versa.

Graph 6: Base Correlation Curve

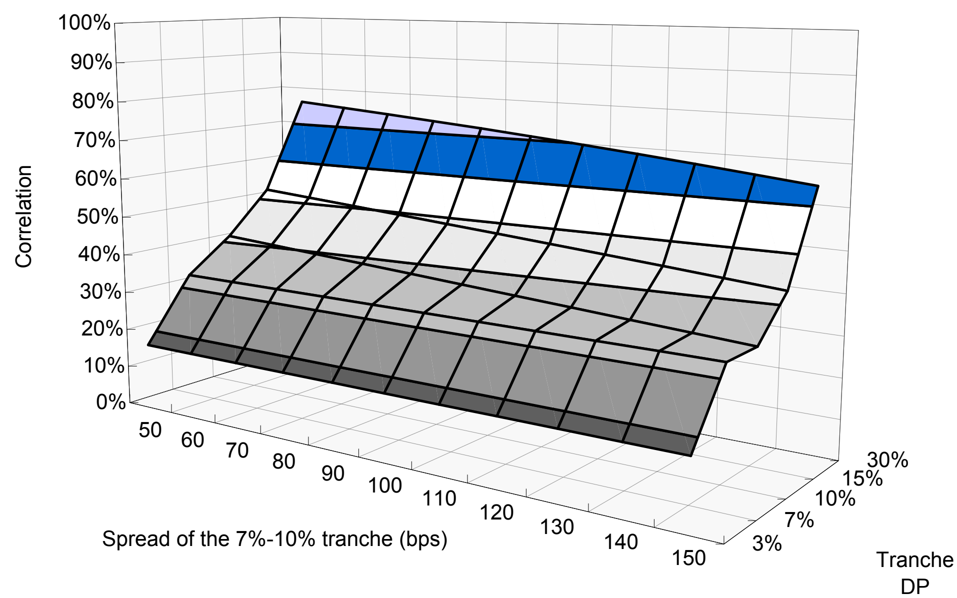


Source: Nomura

Graph 7 plots the base correlation curves when we change the spread of the 7%-10% tranche, while spreads remain unchanged for the overall index and all other tranches. As spread of the 7%-10% tranche increases, the portion of the base correlation curve up to the 7% point remains unchanged, but the *rest* of the curve (above the 10% point) moves downward.

(For Graphs 7, 8, and 9, we assumed a 5-year index spread of 50 bps, with tranche spreads of 40% upfront payment plus a 500-bp running spread for the equity tranche. Also, we assume 150 bps, 50 bps, 25 bps, and 10 bps, unless otherwise noted, for the 3%-7%, 7%-10%, 10%-15%, and 15%-30% tranches, respectively.)

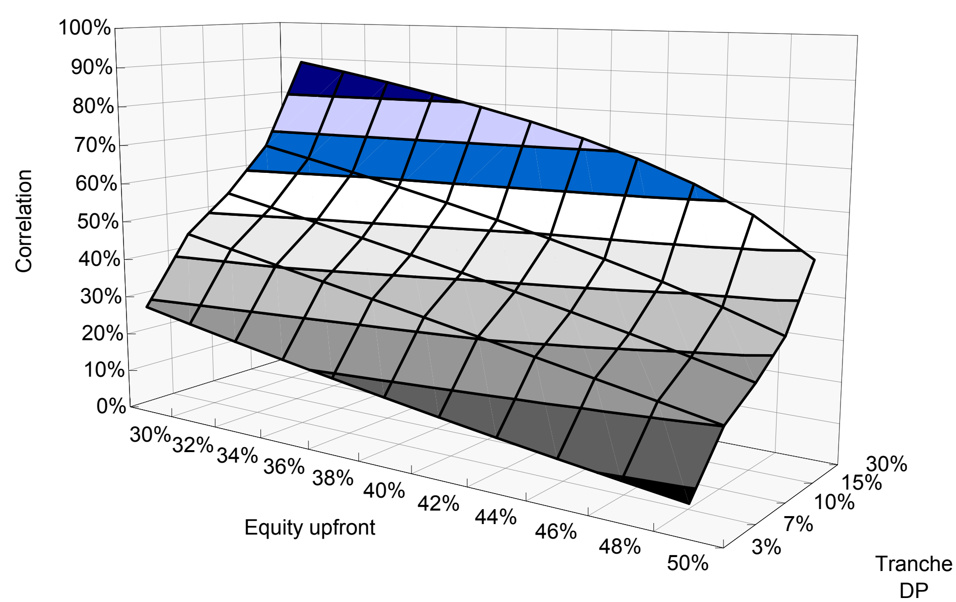
Graph 7 : Shifting Base Correlation Curve



Source: Nomura

On the other hand, *the entire curve* shifts up or down as the spread (*i.e.*, upfront fee) of the 0%-3% tranche changes, while spreads of the overall index and other tranches stay unchanged. Graph 8 shows that the entire base correlation curve shifts down, as the equity tranche underperforms.

Graph 8 : Shifting Base Correlation Curve



Source: Nomura

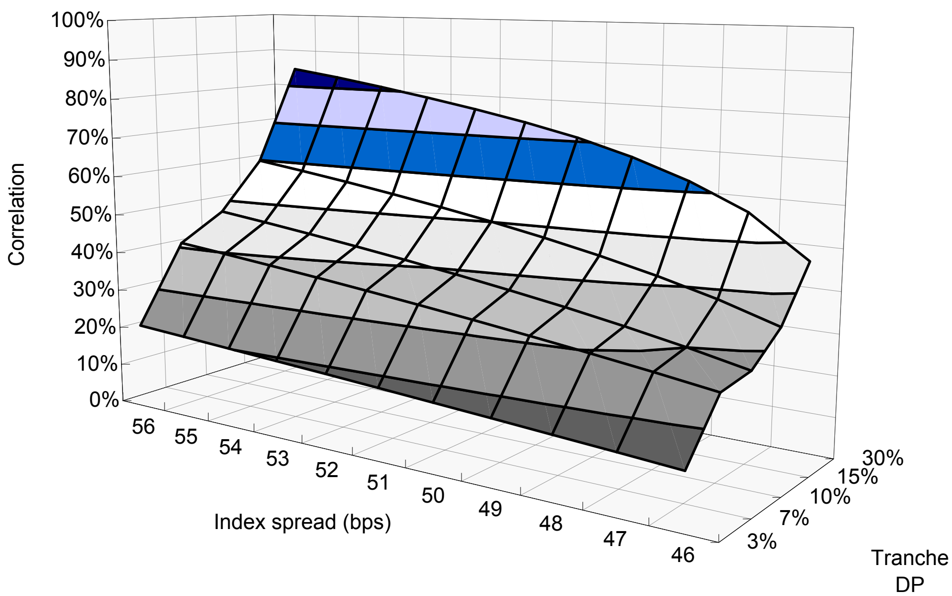
To examine shifts in relative pricing, we can start from the low detachment point (DP) towards higher DPs by examining the base correlation at each point. If base correlation for a low DP moves up or down, it is likely that the base correlation for higher points are also affected. On the other hand, if the base correlation curve moves only at high DPs only, it suggests that relative pricing of one or more higher tranches, but not low tranches, are changing.

B. Multiple Tranches Moving Around

Unfortunately, we rarely see just one index tranche move in price. For example, changes in the overall index level may cause individual tranche spreads to move more or less than as predicted by tranche deltas.[5] In fact, a change in the index spread often causes fluctuations in base correlation of individual tranches. In particular, it can be very difficult to isolate such effects for a more senior tranche.

Graph 9 plots how the base correlation curve shifts as the overall index spread changes, while all the tranche spreads remain unchanged. If tranche spreads do not change, base correlation rises as the overall index spread increases (*i.e.*, the tranches are outperforming the overall index).

Graph 9 : Shifting Base Correlation Curve



Source: Nomura

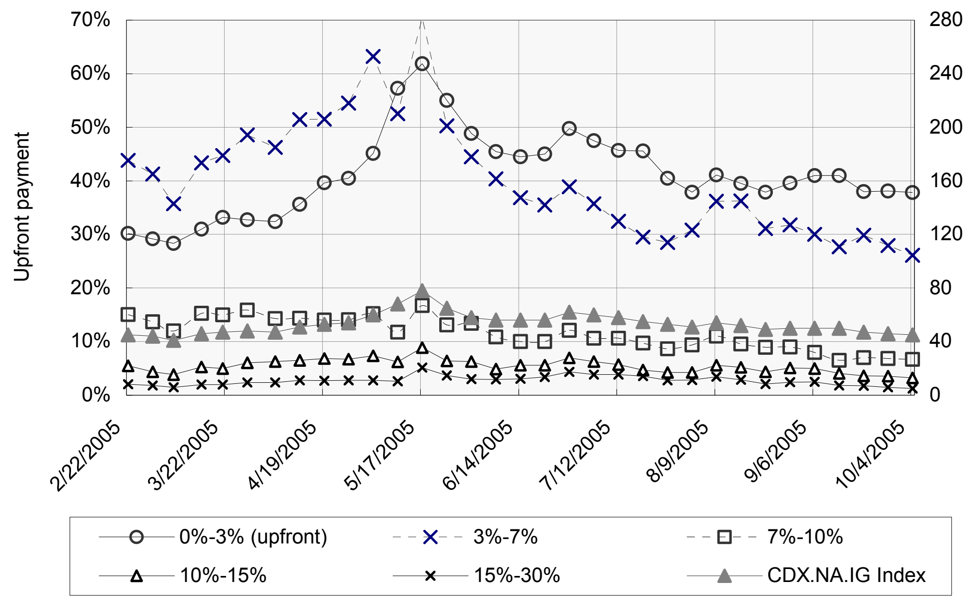
Moreover, the curve becomes steeper as the index spread increases. As we have discussed, tranche outperformance "adds up" as the detachment point increases. Here, all tranches (up to 30%) are outperforming the index. Base correlation at each detachment point reflects relative pricing of all tranches up to that point. In the graph, as the index spread increases from 46 bps to 56 bps, base correlation at the 3% point rises from 13% to 22%, while that at 15% point increases from 34% to 60%.

[5] For a detailed discussion of delta, please see: "*The Bespoke [bispóuk]" – A Guide to Single-Tranche Synthetic CDOs*, Nomura Fixed Income Research (17 November 2004).

Delta & Correlation: The May 2005 Correlation Shakeout

As we saw above, it is often difficult to extract relative value information among index tranches from the base correlation curve. Alternatively, it may be helpful to examine how tranche spreads move in relation to the overall index. The major correlation re-pricing that occurred in May 2005 may provide a useful insight. Graph 10 plots spread movements of the North American investment-grade CDS index and its tranches from early 2005 to the recent weeks.

Graph 10 : Index vs.Tranche Spreads



Source: Creditflux

The index tranche market experienced a great deal of volatility in the second week of May 2005, after S&P downgraded GM and Ford to junk status. Shortly after that, hedge funds began to reduce leverage in the index tranche market and reversed a popular strategy of long equity tranche and short mezzanine tranche.[6] The position was so-called "delta neutral," where spread movements at the index level would have had an offsetting impact between long position in the equity tranche and short position in the mezzanine position.

However, unwinding of the strategy caused a huge sell-off in the equity tranche while the mezzanine tranche was short squeezed. As a result, the implied correlation dropped for the both tranches, but the base correlation curve steepened. The spread movement in each leg of the long-short strategy in fact was NOT offset, resulting large mark-to-market losses to hedge funds that had pursued the strategy.

In the first week of May, the overall index spread widened from 60 bps to 68 bps. However, the tranche spreads moved in complete disarray, leaving many delta-hedged positions deeply in the red. For example, the 0%-3% tranche's upfront payment soared by 12% to 57%. The impact was significantly more than predicted by the tranche's delta. More surprisingly, *the mezzanine tranches actually tightened* in the wake of the index spread widening. In other words, the *realized* deltas for these tranches were negative!

In theory, tranche delta should be positive for all tranches, as long as we hold correlation *constant*. In reality, however, a tranche's spread can move more or less than its delta indicates. Such a move may be signaling a change in the implied correlation and a shift in the relative valuation of tranches.

[6] For more details, see the 16 May 2005 edition of *CDO-CDS Update*, Nomura Fixed Income Research.

CDX.NA.IG Index Tranche Spread Changes						
	May 10		May 3		Change	
Overall index spread (bps)	68		60		+ 8	
	Spread	Correlation	Spread	Correlation	Spread	Correlation
0%-3% (upfront payment + running spread of 500 bps)	57.3%	16%	45.1%	22%	+ 12.2%	- 6%
3%-7%	210 bps	34%	253 bps	34%	- 43 bps	0%
7%-10%	47 bps	44%	61 bps	42%	- 14 bps	+ 2%
10%-15%	25 bps	57%	30 bps	54%	- 5 bps	+ 3%
15%-30%	10 bps	84%	11 bps	79%	- 1 bps	+5%

Source: Creditflux and Nomura

Following the correlation shakeout of May 2005, many market participants seem to have lost confidence in the effectiveness of delta hedging. However, in our view, delta can be useful for inferring shifting relative pricing across the index capital structure, as an additional tool to base correlation.

V. Other Factors to Consider

Although the index tranche market has embraced a simple model that assumes a portfolio of identical credit risk (*i.e.*, uniform spread), some people also focus on "dispersion" of the portfolio. The allocation among tranches is arguably different between a portfolio comprised of identical reference credits and one with significant differences among reference credits. Credits with different risk levels tend to default at different times. Other things being equal, spread dispersion in a portfolio tends to result in lower correlation for the portfolio. Assuming that all credits in the portfolio are identical (as in the LHP model), and using the average spread to characterize the portfolio's risk, tends to yield base correlation that is lower than that for a heterogeneous portfolio.

VI. Conclusion

In this note, we have revisited the two measures of implied correlation; compound correlation and base correlation. Both attempt to capture the level of correlation in a portfolio, as indicated by tranche spreads in the market. Implied correlation is simpler to understand, but there are many problems, including; (1) mezzanine tranche correlation may not be well defined, and (2) the curve exhibits a "smile" which is difficult to use for a non-standard tranche. Base correlation is an improvement in these two aspects but is more difficult to interpret, and other problems remain unsolved. While the market awaits development of models that better explain market pricing, understanding the dynamics of base correlation and tranche delta can still guide investors and provide helpful insight into relative value across the capital structure of a credit risk portfolio.

— E N D —

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