ECMA 31140. PERSPECTIVES ON COMPUTATIONAL MODELING FOR ECONOMICS
PROBLEM SET 9
DUE DATE: MARCH 11TH

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You are encouraged to work and discuss in groups, but you must submit your work individually. Answers must be legibly hand-written or typed. All assignments are due electronically on Canvas, attach code. Assignments are due at 12:30 PM. Late problem sets *will not be accepted*

1. Computing policies for the monetary model. (100 points)

In class we have consider the a monetary model where the Bellman equation is given by:

$$\mathcal{V}(m,\theta) = \max_{c,m'} \left[\theta u(c) + \beta \sum_{\theta' \in \Theta} \mathcal{V}(m',\theta') Q(\theta,\theta') \right],$$

subject to:

$$m' = \frac{m}{1+\gamma} - c + y + \tau$$

and subject to the CIA constraint:

$$c \leq \frac{m}{1+\gamma}$$

Assume that the utility function is given by $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, and that:

$$\sigma = 2, \beta = 0.98, y = 1, \gamma = 0.02, \tau = 0.0234$$

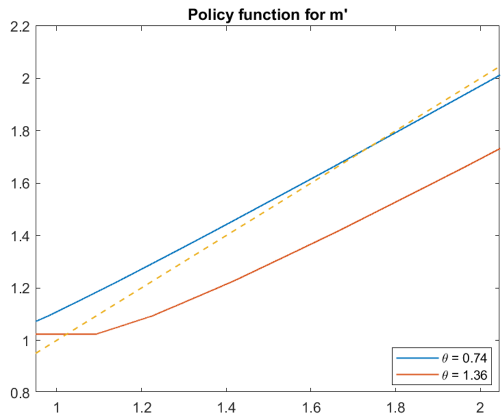
Assume also two possible values for the shock $\Theta = \{\theta_1, \theta_2\} = \{0.74, 1.36\}$, and the Markov transition matrix is given by:

$$Q(\theta,\theta') = \Pr(\theta'|\theta) \equiv \begin{bmatrix} Pr(\theta_1|\theta_1) & Pr(\theta_2|\theta_1) \\ Pr(\theta_1|\theta_2) & Pr(\theta_2|\theta_2) \end{bmatrix} = \begin{bmatrix} 0.73 & 0.27 \\ 0.27 & 0.73 \end{bmatrix}$$

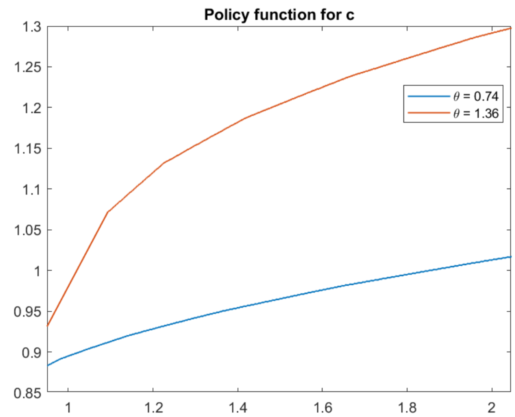
a) Use a grid of 1000 points to solve for the policy functions $g(m,\theta), c(m,\theta)$ using VFI. (50)

Answer.
With Value Function Iteration, we have the policy functions:
It takes 35 seconds to finish the iteration.

Date: March 11, 2025.



(a) $g(m, \theta)$

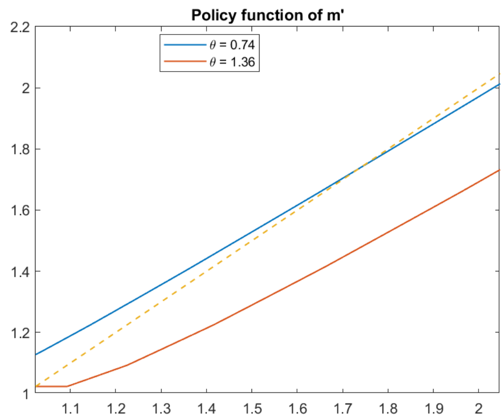


(b) $c(m, \theta)$

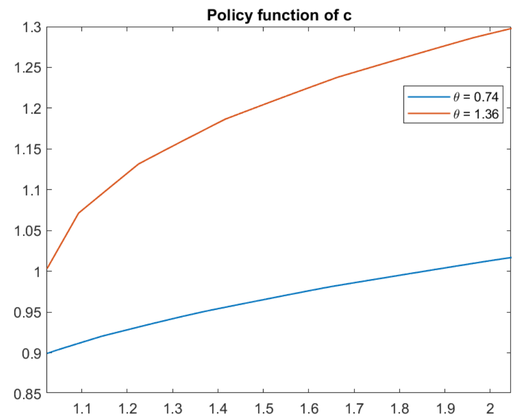
FIGURE 1. Policy Function: VFI Result

b) Use a grid of 1000 points to solve for the policy functions $g(m, \theta), c(m, \theta)$ using EGM. (50)

Answer.
With Endogenoue Grid Method, we have the policy functions:



(a) $g(m, \theta)$



(b) $c(m, \theta)$

FIGURE 2. Policy Function: EGM Result

It takes 0.5 second to finish the iteration.

2. Computing distributions. (100 points)

Once you obtained policies $g(m,\theta_1)$ and $g(m,\theta_2)$, in this exercise you will compute distributions. Take into account all information regarding parameters provided in Question 2. Consider however a grid for the distributions that is finer than the grid used in Question 2. Let π_1, π_2 be the ergodic distribution corresponding to the matrix $Q(\theta,\theta')$.

a) Consider the following recursions:

$$\Psi_{s+1}(m',\theta_j) = \Psi_s\left(g^{-1}(m',\theta_j),\theta_j\right)$$

with the initial distribution given by:

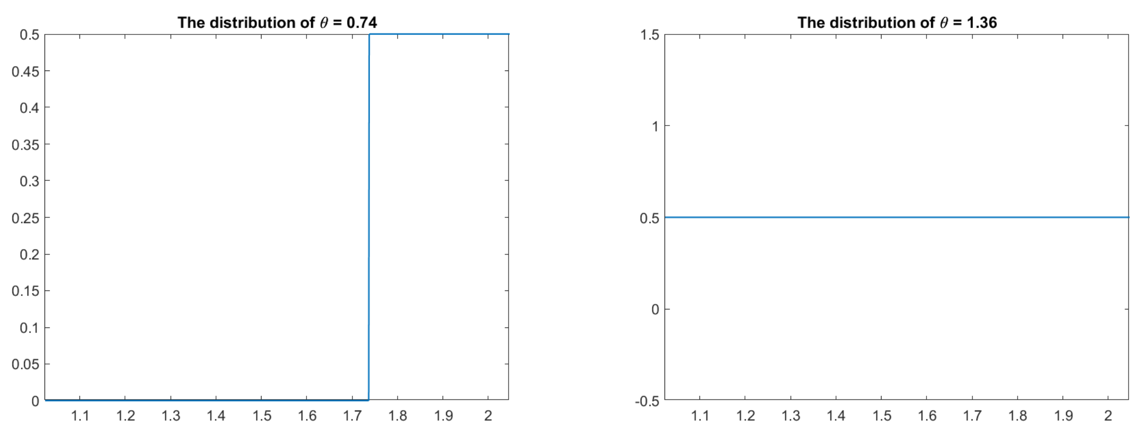
$$\Psi_0(m_i,\theta_j) = \left\{\Psi_0(m_1,\theta_j) + [\Psi_0(m_M,\theta_j) - \Psi_0(m_1,\theta_j)]\frac{m_i - m_1}{m_M - m_1}\right\} \times \pi_j$$

with: $\Psi_0(m_1,\theta_j) = 0, \Psi_0(m_M,\theta_j) = 1$.

Find the limiting distributions for each case ($j = 1, 2$), by iterating on the recursions. (50)

Answer.

With the rule of recursion (which is not the true solution), we have the limiting distribution:



(a) $\theta = 0.74 \quad (j = 1)$ (b) $\theta = 1.36 \quad (j = 2)$

FIGURE 3. Distribution Iteration Result

b) Consider the following recursions now:

$$\Psi_{s+1}(m',\theta_j) = \sum_{\theta\in\Theta} Q(\theta,\theta_j)\Psi_s\left(g^{-1}(m',\theta),\theta\right)$$

$j = 1, 2$, with initial distributions given by:

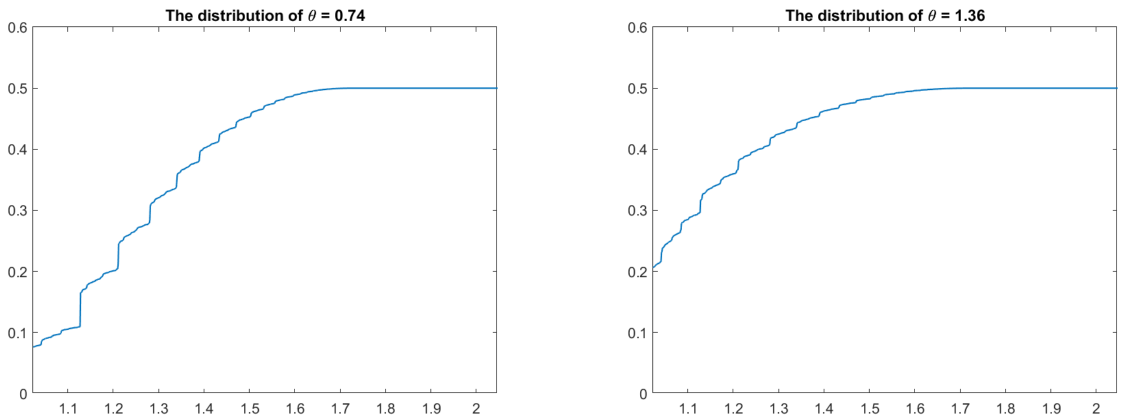
$$\Psi_0(m_i, \theta_j) = \left\{ \Psi_0(m_1, \theta) + [\Psi_0(m_M, \theta) - \Psi_0(m_1, \theta)] \frac{m_i - m_1}{m_M - m_1} \right\} \times \pi_j$$

with: $\Psi_0(m_1, \theta_j) = 0, \Psi_0(m_M, \theta_j) = 1$.

Find the limiting distributions for each case ($j = 1, 2$), by iterating on the recursions. (50)

Answer.

With the rule of recursion (which is the true recursion needed to solve the model), we have the limiting distribution:



(a) $\theta = 0.74 \quad (j = 1)$ (b) $\theta = 1.36 \quad (j = 2)$

Figure 4. Distribution Iteration Result

3. Computing equilibrium. (120 points)

Consider the same set up as in Questions 2 and 3, but omit the information that $\tau = 0.0234$.

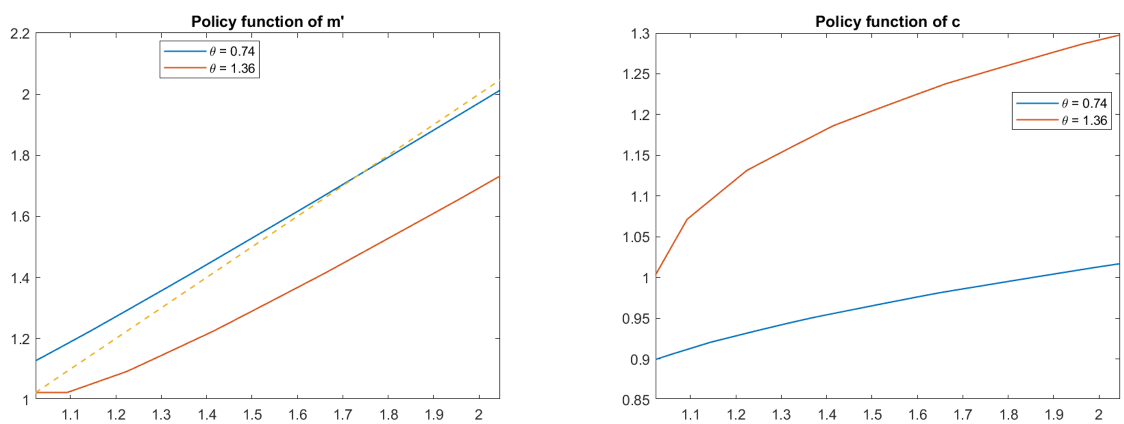
a) Find the equilibrium of the model. Use either the iterative scheme discussed in class or Bisection method. (100)

   Answer.
   We used the iterative method shown in class, (bisection methods delivers the same result). We find that when $\tau = 0.0234$, the market is cleared.

   This elapsed time is 71 seconds.

b) Plot policy functions and distributions in equilibrium. (20)

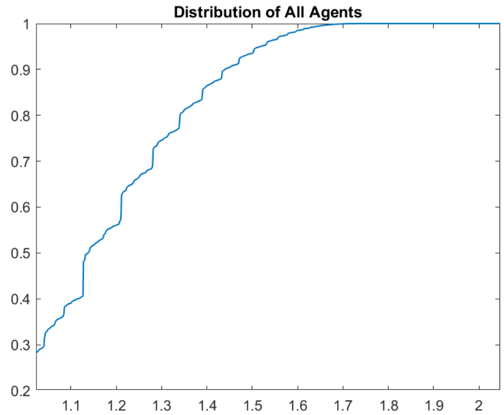
   Answer.
   In equilibrium, the policy functions should look like:



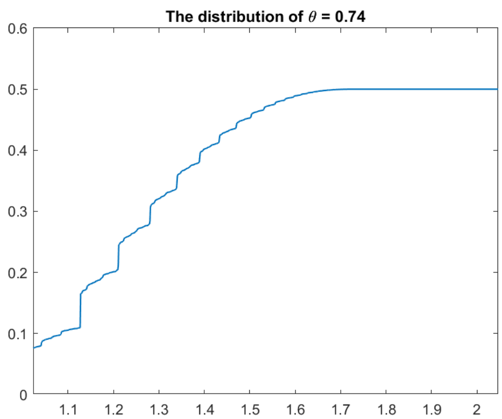
(a) $g(m, \theta)$ (b) $c(m, \theta)$

FIGURE 5. Policy Function at Equilibrium

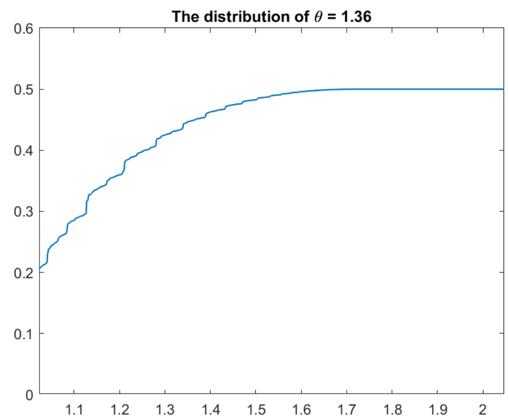
The distribution should look like:



(a) Overall Distribution



(b) Distribution of $\theta = 0.74$



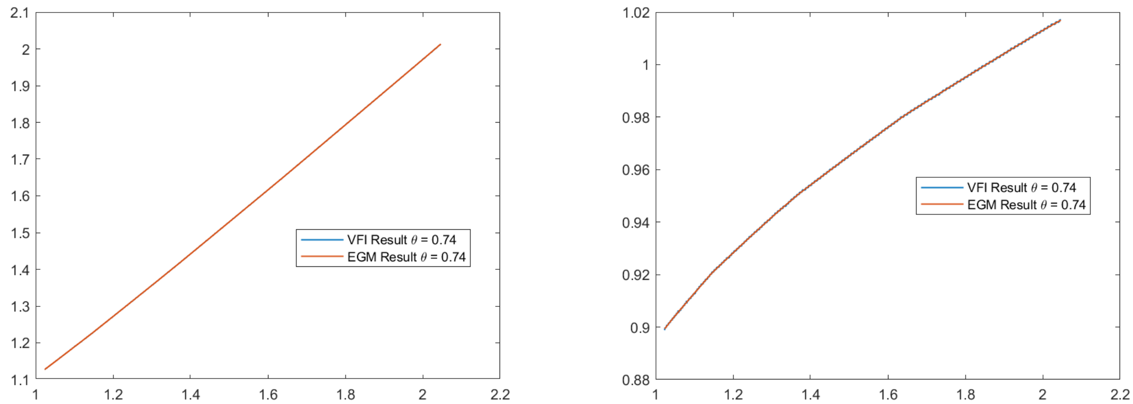
(c) Distribution of $\theta = 1.36$

FIGURE 6. Distribution at Equilibrium

Discussion

In this discussion section, we will compare the policy function obtained with VFI and endogenous grid method.

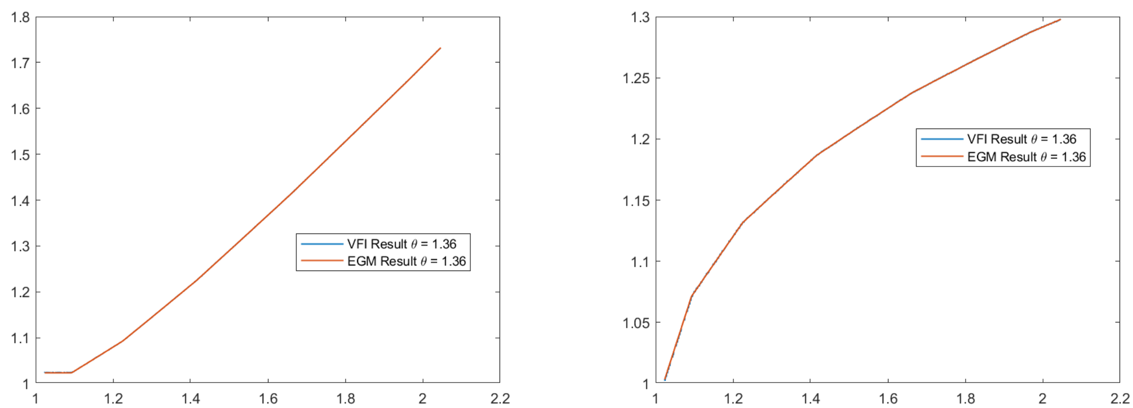
- $\theta = 0.74$



(a) Investment (b) Consumption

FIGURE 7. Policy Function Comparison: $\theta = 0.74$

- $\theta = 1.36$



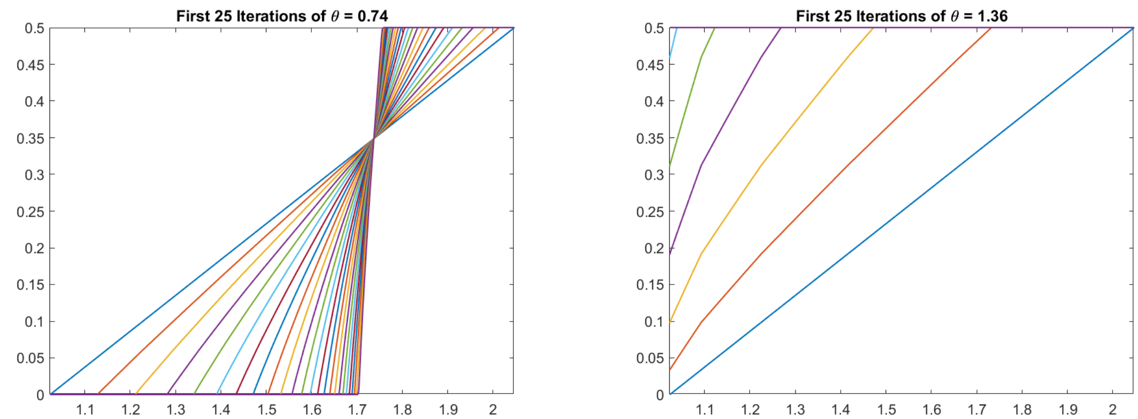
(a) Investment (b) Consumption

FIGURE 8. Policy Function Comparison: $\theta = 1.36$

This is the visual evidence that VFI and EGM should generate the same result. However, since VFI takes around 35 seconds to finish running and EGM take 0.5 second to finish, EGM is significantly more efficient.

For the iteration of distribution, we have the distribution process:

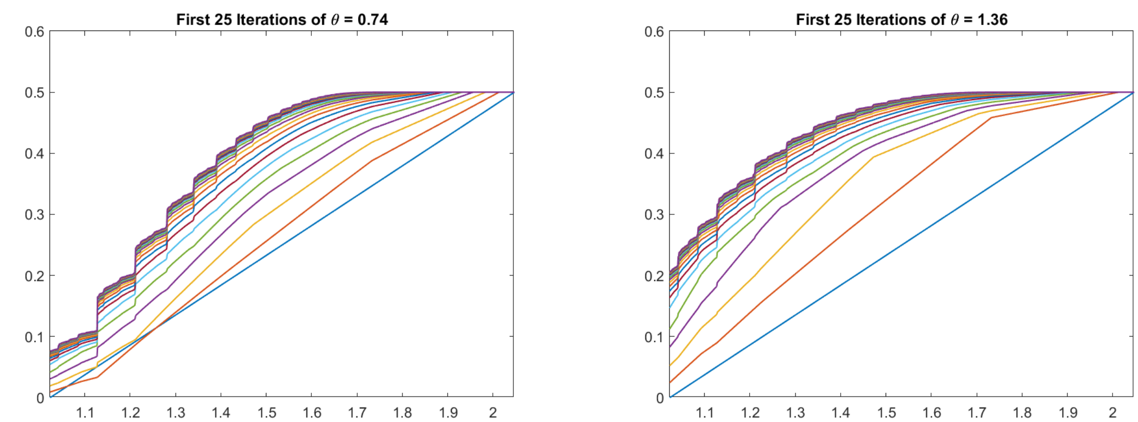
- Recursion rule 1 (incomplete rule):



(a) $\theta = 0.74 \quad (j = 1)$ (b) $\theta = 1.36 \quad (j = 2)$

FIGURE 9. Distribution Iteration Process

- Recursion rule 2 (complete rule):



(a) $\theta = 0.74 \quad (j = 1)$ (b) $\theta = 1.36 \quad (j = 2)$

FIGURE 10. Distribution Iteration Process