Consider the following problem:

s.t.

and . Then on the set

The optimal transportation plan is

Now we convert the OPT problem into OT problem, the corresponding OT problem is:

such that

where:

, and is an isolated point and

, and is an isolated point.

The optimal plan is the following:

Chart, line chart

Description automatically generated

The problem is, there is cross in the optimal plan, i.e. we cannot solve the new OT problem by sorting and .

Why this problem happens?

My understanding is: the reason in can be solved by sorting is the space has an order "" and "" **is constant with** .

**In particular, if , then . (see the graph below).**

**(It is a sufficient condition, but may not be necessary)**

Then

.

But, when is the isolate point, then violates this condition, thus we have:

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