

General adjustment formulas

Denote by V_t the market value of the CDS (assuming that B and S are default-free), by $\tau = \min\{\tau_R, \tau_S, \tau_B\}$ the first default time and by $\xi \in \{R, S, B\}$ the identity of first defaulting firm. Recall that $x^+ = \max(x, 0)$ and $x^- = -\min(x, 0)$ and denote by $D(0, t)$ the discount factor over the period $[0, t]$ (with constant interest rate, $D(0, t) = e^{-rt}$).

It can be shown that

$$\begin{aligned}\text{CVA} &= \mathbb{E}^{\mathbb{Q}}(I_{\{\tau < T\}} I_{\{\xi=S\}} D(0, \tau) \delta^S V_{\tau}^+) \\ \text{DVA} &= \mathbb{E}^{\mathbb{Q}}(I_{\{\tau < T\}} I_{\{\xi=B\}} D(0, \tau) \delta^B V_{\tau}^-)\end{aligned}$$

Comments.

- CVA gives loss of B due to premature default of S; DVA gives loss of S due to premature default of B.
- The value adjustments involve an option on the market value $V = (V_t)_{t \leq T}$ of the swap with strike $K = 0$ (a call for the CVA and a put for the DVA).
- Similar formula holds if V is the market value of another derivative such as an interest swap or even a reinsurance contract.
- DVA is a bit problematic: a worsening credit quality of B leads to an accounting profit for B.

A simplified formula

In order to evaluate the CVA and DVA formulas one needs a model with stochastic credit spreads that takes **dependence** between the default of S, B and the **market value** V of the CDS into account (a **dynamic portfolio credit risk model**). Markets often work with a simpler formula that assumes that the default of S and B and V are **independent**:

$$\begin{aligned}\text{CVA}^{\text{indep}} &= \delta^S \int_0^T \bar{F}_B(t) D(0, t) \mathbb{E}^{\mathbb{Q}}(V_t^+) f_S(t) dt, \\ \text{DVA}^{\text{indep}} &= \delta^B \int_0^T \bar{F}_S(t) D(0, t) \mathbb{E}^{\mathbb{Q}}(V_t^-) f_B(t) dt.\end{aligned}$$

Here f_S is the density of τ_S and \bar{F}_B resp \bar{F}_S is the survival function of τ_B resp τ_S .

Comments.

- In order to evaluate the simplified formula one only needs to determine the marginal distribution of τ_S and τ_B and the so-called **expected exposures** $\mathbb{E}^{\mathbb{Q}}(V_t^+)$ and $\mathbb{E}^{\mathbb{Q}}(V_t^-)$.
- The independence assumption underlying the simplified value adjustment formula between the price of the CDS on R , that is V_t , and the default event of S and B is often unrealistic; in practice this is known as **wrong way risk**.

Examples:

- ▶ CDS on a financial institution: given that S defaults it is quite likely that credit quality of R is low.
- ▶ Reinsurance.

For further reading on counterparty risk see Gregory (2012).

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