# 17 Introduction to counterparty risk

## 17.1 Introduction

- A substantial part of all derivative transactions is carried out over the counter and there is no central clearing counterparty to guarantee fulfilment of the contractual obligations.

- These trades are subject to the risk that a contracting party defaults during the transaction, thus affecting the cash flows that are actually received by the other party. This is known as counterparty credit risk.

- Counterparty risk received a lot of attention during the financial crisis of 2007-2009 as some of the institutions heavily involved in derivative transactions experienced worsening credit quality or—in the case of Lehman Brothers—even a default event.

- Counterparty risk management is now a key issue for all financial institutions and the focus of many new regulatory developments.

**Example of Interest-Rate Swap**

- Two parties $A$ and $B$ agree to exchange a series of interest payments on a given nominal amount of money for a given period.

- $A$ receives payments at a fixed interest rate and makes floating payments at a rate equal to the three-month LIBOR rate.

- Suppose that $A$ defaults at time $\tau_A$ before the maturity of the contract.

- If interest rates have risen relative to their value at inception of contract:

  ▸ The fixed interest payments have decreased in value and the value of the contract has increased for $B$.

  ▸ The default of $A$ constitutes a loss for $B$; the loss size depends on the term structure of interest rates at $\tau_A$.

- If interest rates have fallen relative to their value at $t = 0$:

  ▸ The fixed payments have increased in value so that the swap has a negative value for $B$.

- $B$ will still has to pay the value of the contract into the bankruptcy pool, and there is no upside for $B$ in $A$'s default.

- If $B$ defaults first the situation is reversed: falling rates lead to a counterparty-risk-related loss for $A$.

**Management of counterparty risk**

- Counterparty risk has to be taken into account in pricing and valuation. This has led to the notion of credit value adjustments (CVA).

- Counterparty risk needs to be controlled using risk-mitigation techniques such as netting and collateralization.

- Under a netting agreement the value of all derivatives transactions between $A$ and $B$ is computed and only the aggregated value is subject to counterparty risk; since offsetting transactions cancel each other out, this has the potential to reduce counterparty risk substantially.

- Under a collateralization agreement the parties exchange collateral (cash and securities) that serves as a pledge for the receiver. The value of the collateral is adjusted dynamically to reflect changes in the value of the underlying transactions.

## 17.2 Credit value adjustments

General definition. The price (for the protection buyer ) satisfies

$$\text{True price} = \text{(counterparty) risk-free price}$$
$$- \text{ adjustment for default of seller (CVA)}$$
$$+ \text{ adjustment for default of buyer (DVA)},$$

where CVA and DVA stand for Credit Value Adjustment and Debt Value Adjustment respectively.

### General adjustment formulas

Denote by $V_t$ the market value of the CDS (assuming that $B$ and $S$ are default-free), by $\tau = \min\{\tau_R, \tau_S, \tau_B\}$ the first default time and by $\xi \in \{R, S, B\}$ the identity of first defaulting firm. Recall that $x^+ = \max(x, 0)$ and $x^- = -\min(x, 0)$ and denote by $D(0, t)$ the discount factor over the period $[0, t]$ (with constant interest rate, $D(0, t) = e^{-rt}$).

It can be shown that

$$\mathsf{CVA} = \mathbb{E}^{\mathbb{Q}}(I_{\{\tau < T\}} I_{\{\xi = S\}} D(0, \tau) \delta^S V_\tau^+)$$
$$\mathsf{DVA} = \mathbb{E}^{\mathbb{Q}}(I_{\{\tau < T\}} I_{\{\xi = B\}} D(0, \tau) \delta^B V_\tau^-)$$

**Comments.**

- CVA gives loss of B due to premature default of S; DVA gives loss of S due to premature default of B.

- The value adjustments involve an option on the market value $V = (V_t)_{t \leq T}$ of the swap with strike $K = 0$ (a call for the CVA and a put for the DVA).

- Similar formula holds if $V$ is the market value of another derivative such as an interest swap or even a reinsurance contract.

- DVA is a bit problematic: a worsening credit quality of B leads to an accounting profit for B.

## A simplified formula

In order to evaluate the CVA and DVA formulas one needs a model with stochastic credit spreads that takes dependence between the default of S, B and the market value $V$ of the CDS into account (a dynamic portfolio credit risk model). Markets often work with a simpler formula that assumes that the default of S and B and $V$ are independent:

$$\mathrm{CVA}^{\mathsf{indep}} = \delta^S \int_0^T \bar{F}_B(t) D(0,t) \mathbb{E}^{\mathbb{Q}}(V_t^+) f_S(t) \, \mathrm{d}t,$$

$$\mathrm{DVA}^{\mathsf{indep}} = \delta^B \int_0^T \bar{F}_S(t) D(0,t) \mathbb{E}^{\mathbb{Q}}(V_t^-) f_B(t) \, \mathrm{d}t.$$

Here $f_S$ is the density of $\tau_S$ and $\bar{F}_B$ resp $\bar{F}_S$ is the survival function of $\tau_B$ resp $\tau_S$.

**Comments.**

- In order to evaluate the simplified formula one only needs to determine the marginal distribution of $\tau_S$ and $\tau_B$ and the so-called expected exposures $\mathbb{E}^{\mathbb{Q}}(V_t^+)$ and $\mathbb{E}^{\mathbb{Q}}(V_t^-)$.

- The independence assumption underlying the simplified value adjustment formula between the price of the CDS on $R$, that is $V_t$, and the default event of $S$ and $B$ is often unrealistic; in practice this is known as wrong way risk.

  Examples:

  ▶ CDS on a financial institution: given that S defaults it is quite likely that credit quality of R is low.

  ▶ Reinsurance.

For further reading on counterparty risk see Gregory (2012).