17 Introduction to counterparty risk

17.1 Introduction

- A substantial part of all derivative transactions is carried out over the counter and there is no central clearing counterparty to guarantee fulfilment of the contractual obligations.
- These trades are subject to the risk that a contracting party defaults during the transaction, thus affecting the cash flows that are actually received by the other party. This is known as counterparty credit risk.
- Counterparty risk received a lot of attention during the financial crisis of 2007-2009 as some of the institutions heavily involved in derivative transactions experienced worsening credit quality or—in the case of Lehman Brothers—even a default event.
- Counterparty risk management is now a key issue for all financial institutions and the focus of many new regulatory developments.

Example of Interest-Rate Swap

- Two parties A and B agree to exchange a series of interest payments on a given nominal amount of money for a given period.
- A receives payments at a fixed interest rate and makes floating payments at a rate equal to the three-month LIBOR rate.
- Suppose that A defaults at time τ_A before the maturity of the contract.
- If interest rates have risen relative to their value at inception of contract:
  - The fixed interest payments have decreased in value and the value of the contract has increased for B.
  - The default of A constitutes a loss for B; the loss size depends on the term structure of interest rates at τ_A.
- If interest rates have fallen relative to their value at $t = 0$:
  - The fixed payments have increased in value so that the swap has a negative value for B.

  - B will still has to pay the value of the contract into the bankruptcy pool, and there is no upside for B in A's default.
- If B defaults first the situation is reversed: falling rates lead to a counterparty-risk-related loss for A.

Management of counterparty risk

- Counterparty risk has to be taken into account in pricing and valuation. This has led to the notion of credit value adjustments (CVA).
- Counterparty risk needs to be controlled using risk-mitigation techniques such as netting and collateralization.
- Under a netting agreement the value of all derivatives transactions between A and B is computed and only the aggregated value is subject to counterparty risk; since offsetting transactions cancel each other out, this has the potential to reduce counterparty risk substantially.

- Under a collateralization agreement the parties exchange collateral (cash and securities) that serves as a pledge for the receiver. The value of the collateral is adjusted dynamically to reflect changes in the value of the underlying transactions.

17.2 Credit value adjustments

General definition. The price (for the protection buyer) satisfies

$$\begin{aligned}\text{True price} = \; & \text{(counterparty) risk-free price} \\ & - \text{ adjustment for default of seller (CVA)} \\ & + \text{ adjustment for default of buyer (DVA)} \,,\end{aligned}$$

where CVA and DVA stand for Credit Value Adjustment and Debt Value Adjustment respectively.

General adjustment formulas

Denote by V_t the market value of the CDS (assuming that B and S are default-free), by $\tau = \min\{\tau_R, \tau_S, \tau_B\}$ the first default time and by $\xi \in \{R, S, B\}$ the identity of first defaulting firm. Recall that $x^+ = \max(x, 0)$ and $x^- = -\min(x, 0)$ and denote by $D(0, t)$ the discount factor over the period $[0, t]$ (with constant interest rate, $D(0, t) = e^{-rt}$).

It can be shown that

$$\text{CVA} = \mathbb{E}^{\mathbb{Q}}\big(I_{\{\tau<T\}} I_{\{\xi=S\}} D(0,\tau)\delta^S V_\tau^+\big)$$
$$\text{DVA} = \mathbb{E}^{\mathbb{Q}}\big(I_{\{\tau<T\}} I_{\{\xi=B\}} D(0,\tau)\delta^B V_\tau^-\big)$$

Comments.

- CVA gives loss of B due to premature default of S; DVA gives loss of S due to premature default of B.
- The value adjustments involve an option on the market value $V = (V_t)_{t \le T}$ of the swap with strike $K = 0$ (a call for the CVA and a put for the DVA).
- Similar formula holds if V is the market value of another derivative such as an interest swap or even a reinsurance contract.
- DVA is a bit problematic: a worsening credit quality of B leads to an accounting profit for B.

A simplified formula

In order to evaluate the CVA and DVA formulas one needs a model with stochastic credit spreads that takes dependence between the default of S, B and the market value V of the CDS into account (a dynamic portfolio credit risk model). Markets often work with a simpler formula that assumes that the default of S and B and V are independent:

$$\mathrm{CVA}^{\mathsf{indep}} = \delta^S \int_0^T \bar{F}_B(t) D(0,t) \mathbb{E}^{\mathbb{Q}}(V_t^+) f_S(t)\,\mathrm{d}t,$$
$$\mathrm{DVA}^{\mathsf{indep}} = \delta^B \int_0^T \bar{F}_S(t) D(0,t) \mathbb{E}^{\mathbb{Q}}(V_t^-) f_B(t)\,\mathrm{d}t.$$

Here f_S is the density of τ_S and $\bar{F}_B$ resp $\bar{F}_S$ is the survival function of τ_B resp τ_S.

Comments.

- In order to evaluate the simplified formula one only needs to determine the marginal distribution of τ_S and τ_B and the so-called expected exposures $\mathbb{E}^{\mathbb{Q}}(V_t^+)$ and $\mathbb{E}^{\mathbb{Q}}(V_t^-)$.
- The independence assumption underlying the simplified value adjustment formula between the price of the CDS on R, that is V_t, and the default event of S and B is often unrealistic; in practice this is known as wrong way risk.

  Examples:

  - CDS on a financial institution: given that S defaults it is quite likely that credit quality of R is low.
  - Reinsurance.

For further reading on counterparty risk see Gregory (2012).