《计算流体力学基础》第一次作业附加题目

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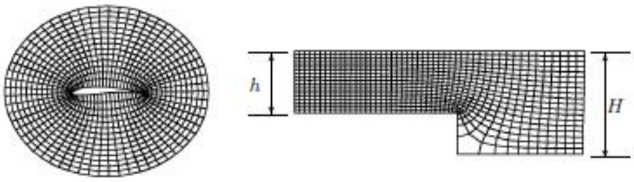
也可以借助于复变换，生成绕儒科夫斯基翼型的 O 型网格或者扩张通道内的 H 型网格。



(a)　　(b)

图 2: 儒科夫斯基翼型的 O 型网格 (a) 和扩张通道的 H 型网格 (b)

儒科夫斯基翼型的 O 型网格可以直接用流体力学教科书中给出的复变换计算，对于扩张通道的 H 型网格，可以通过如下变换，把一个上半平面变换为需要的区域

$$z = \frac{H}{\pi}\ln t_1 - \frac{h}{\pi}\ln t_2$$

其中

$$t_1 = \frac{1+t}{1-t}, \qquad t_2 = \frac{b+t}{b-t}, \qquad t = \left(\frac{w-b^2}{w-1}\right)^{1/2}$$

参数 $b = H/h$。此变换把 $w-$ 平面的上半平面变换为一个扩张管道，于是，我们可以首先在 $w-$ 上半平面 $(r_1, r_2)\times(0,\pi)$ 划分极坐标网格，然后再把每一点都映射到 $z-$ 平面上。要注意到复变函数中的开方和对数运算中的多值性，在对 $w-$ 平面坐标轴上的点做变换时，为了避免微小的数值误差引起最终结果较大的变化，可以把区间 $(0,\pi)$ 用一个小参数 ε 修改为 $(\varepsilon, \pi-\varepsilon)$，另外，径向区域应满足 $r_1 < 1, r_2 > 1$，最好沿径向不要均匀取点，所取的点应该越向外越稀疏。

O 网格：
对于茹科夫斯基翼型外流场的 O 网格划分：
考虑下面的复变换

$$z = kw + \frac{k_1}{w}$$

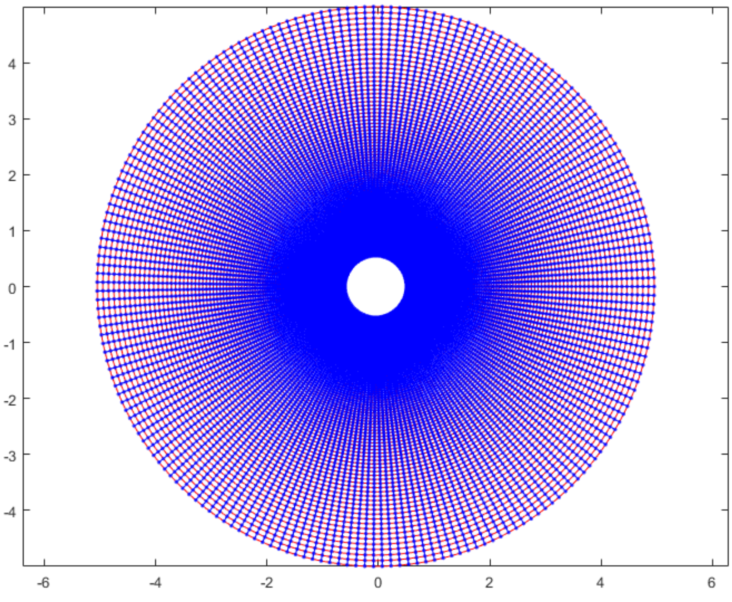
取 $k = \frac{1}{2}, k_1 = \frac{c^2}{2}$ 可以得到茹科夫斯基变换

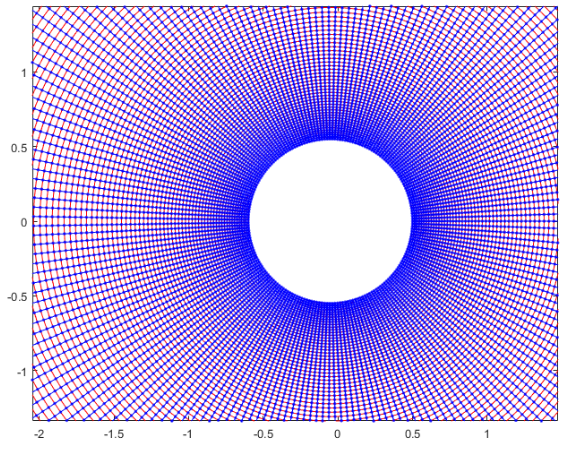
$$z = \frac{1}{2}\left(w + \frac{c^2}{w}\right)$$

这个复变换将 W 平面上的一个圆变为 Z 平面上的翼型，更一般的我们定义 W 平面上这样一个圆：

$$w = -\lambda c + (1+\lambda) c e^{i\theta}$$

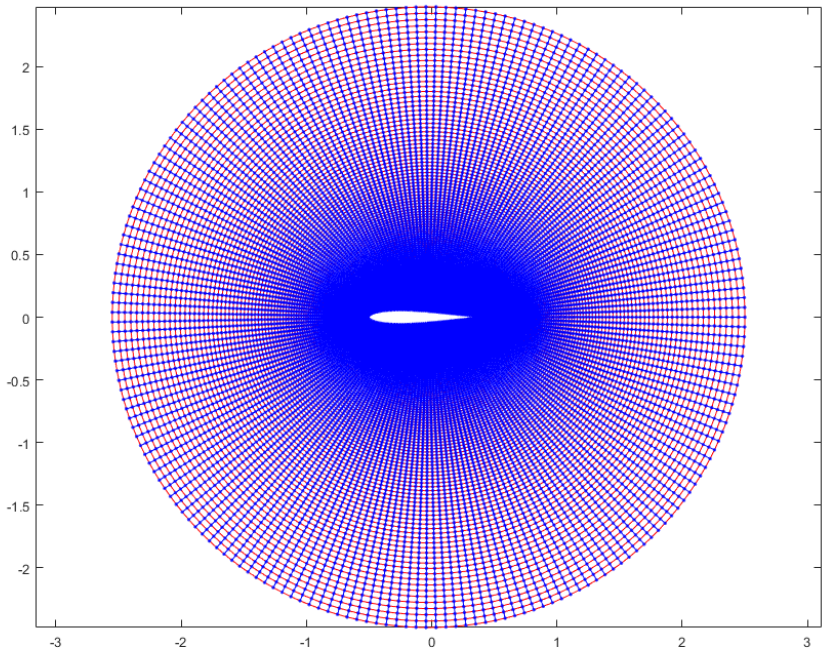
显然其中的参数 λ 决定了圆心的位置而 c 决定了圆的半径。而且这个圆横过点 $(c,0)$，在计算的过程中取 $\lambda = 0.1$，$c = 0.5$。然后再 W 平面上的圆环区域划分极坐标网格，为了让翼型表面的网格从密到稀疏，划分圆环区域极坐标网格的时候半径越大网格越稀疏，网格间距采用一个比例系数来控制。
W 平面网格划分效果图：
整体和局部：

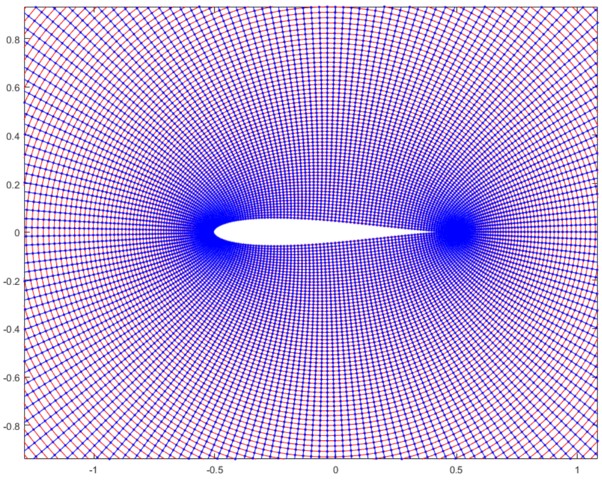


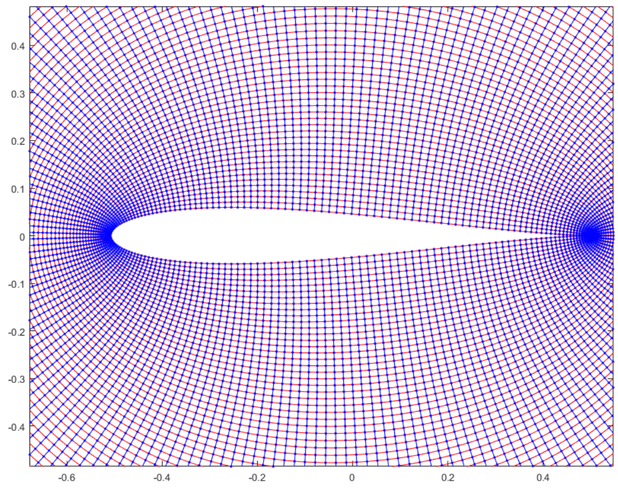


将网格点利用茹科夫映射变换会 Z 平面，并且保持原来网格点之间之间的顺序可以得到 Z 平面上的网格，也就是茹科夫斯基翼型外面的 O 网格：

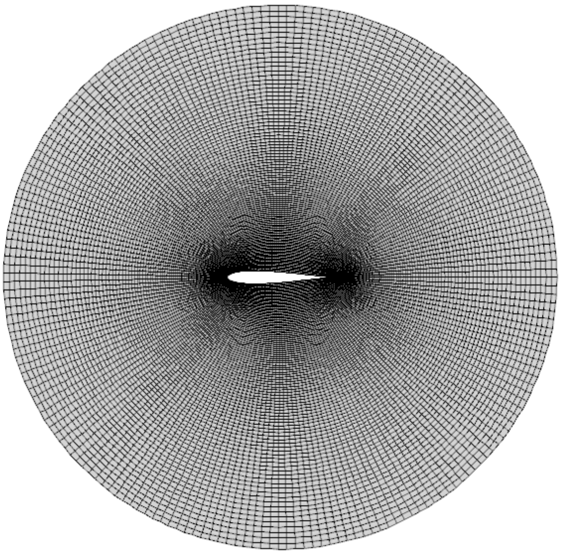
网格的整体和局部：

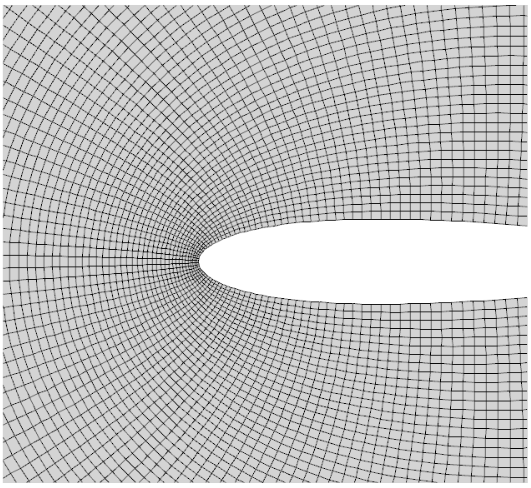






将生成的文件导入 tecplot 中绘图：

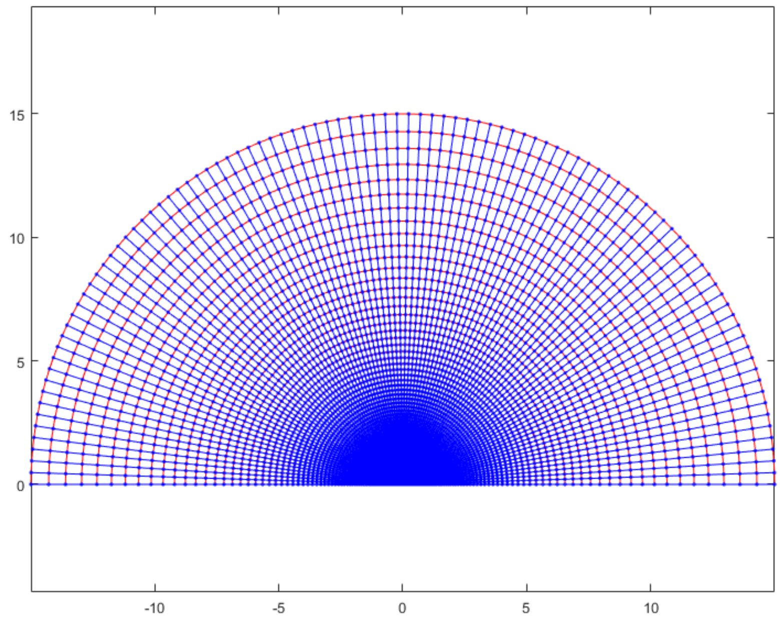




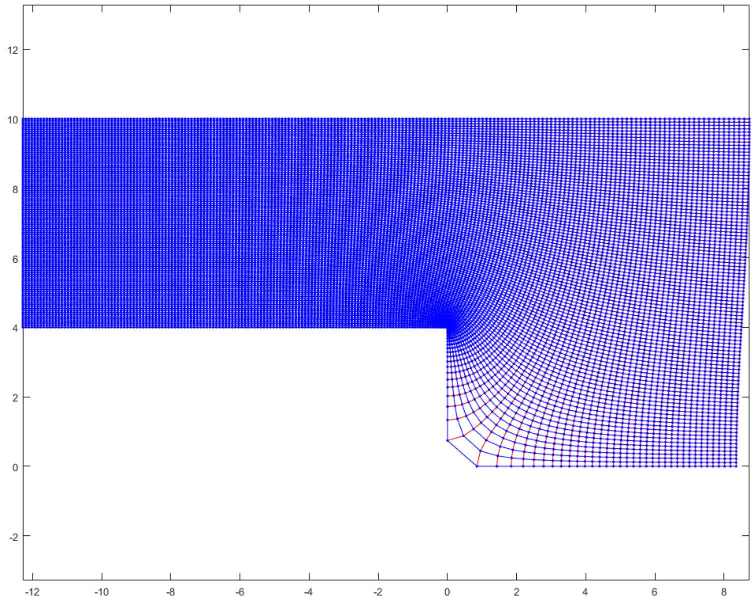
扩张通道内的 H 网格：
根据题目中的复变换和题目中提示的网格密度控制方法进行划分为了避免奇异的情况将扰动取为：$\varepsilon = eps = 2^{-52}$。

在 W 上半平面内按照极坐标的方式划分从内到外由密到疏的网格，划分的效果如下图：为了得到一个形状较为好的通道其中通道的几何参数取值如下

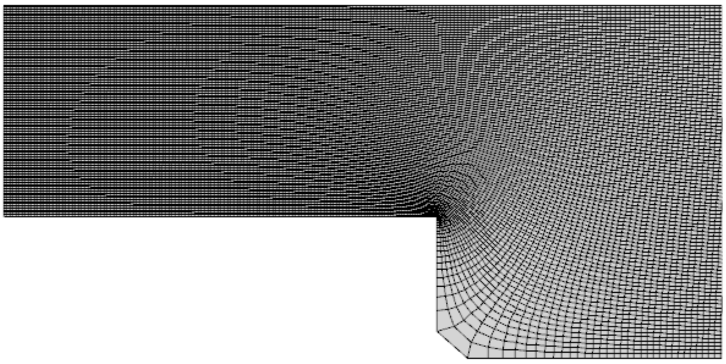
$$H = 10, h = 6$$

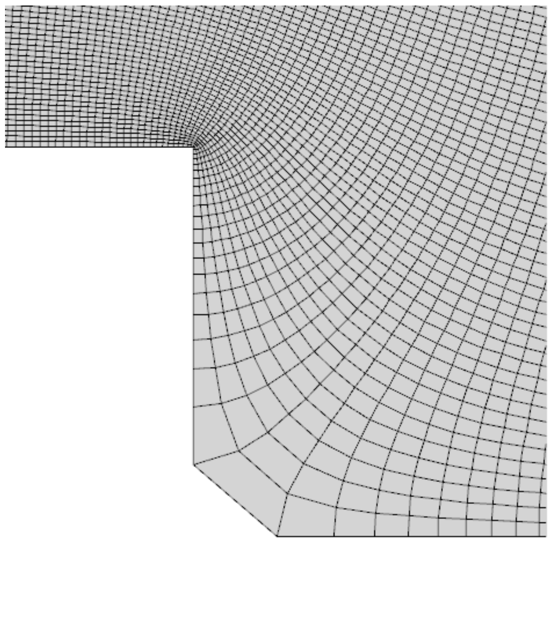


Z 平面上的网格：



用 Tecplot 作图：





MATLAB 程序附录

茹科夫斯基翼型 O 网格 MATLAB 文件

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%% Program name: JoukowskiOMesh.m
%%%% Program Prupose: Generate Joukowski
Airfoil O mesh
%%%% Aurthor : Yang Yang
%%%% Date : 2015.09.21
%%%% Version: 1.0
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Jukowski Airfoil parameters
c = 0.5;
lambda = 0.1;
%% Mesh parameters
N_r = 100;
N_theta = 200;
r_span = [(1+lambda)*c,5];
theta_span = [0,2*pi];
%% Generate mesh for Jukowski Airfoil
r = meshfun(r_span,N_r);
```

```
theta =
linspace(theta_span(1),theta_span(2),N_theta);
[R,THETA] = meshgrid(r,theta);
x = R.*cos(THETA) -lambda*c;
y = R.*sin(THETA) ;
W = x + sqrt(-1)*y;

%% confroaml mapping
Z = 0.5*(W + c^2*(W.^(-1)) );
X = real(Z);
Y = imag(Z);

%% plot Mesh in Z plane
[imax jmax] = size(X);
figure('Color',[1 1 1]);
% plot phi
for j = 1:jmax
   for i = 1:imax-1

plot([X(i,j),X(i+1,j)],[Y(i,j),Y(i+1,j)],'r.-')
       hold on;
   end
end
```

```
% plot varphi
for i = 1:imax
   for j = 1:jmax - 1

plot([X(i,j),X(i,j+1)],[Y(i,j),Y(i,j+1)],'b.-')
   end
end

axis equal

%% Plot mesh in W plane
figure('Color',[1 1 1]);
% plot r
for j = 1:jmax
   for i = 1:imax-1

plot([x(i,j),x(i+1,j)],[y(i,j),y(i+1,j)],'r.-')
        hold on;
   end
end

% plot theta
for i = 1:imax
```

```
   for j = 1:jmax - 1

plot([x(i,j),x(i,j+1)],[y(i,j),y(i,j+1)],'b.-')
   end
end
axis equal

%% Output Mesh to tecplot file
fp = fopen('JoukowskiOMesh.dat','w');
fprintf(fp,'Title = JoukowskiOMesh\n');
fprintf(fp,'VARIABLES = "X", "Y"\n');
fprintf(fp,'ZONE I =%d, J =%d, F =
point\n',imax,jmax);

for j = 1:jmax
   for i = 1:imax
        fprintf(fp,'%e, %e\n',X(i,j),Y(i,j));
   end
end
fclose(fp);
```

```
function r = meshfun(r_span,N_r)
% This function control mesh nodes distribution
```

```
factor = 1.02;
r = zeros(1,N_r);          % array to store r;
r(1) = r_span(1);          % Initial value for r
delta = zeros(1,N_r-1);     % array to store
delta;
delta(1) = (r_span(2) - r_span(1))*(1 -
factor)/(1 - factor^(N_r-1));

for i = 2:N_r -1
    delta(i) = factor*delta(i-1);
    r(i) = r(i-1) + delta(i-1);
end
r(N_r) = r(N_r-1) + delta(N_r-1);

end
```

扩张通道 H 网格 MATLAB 文件

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%% Program name: ChannelHMesh.m
%%%% Program Prupose: Generate Channel H mesh
%%%% Aurthor : Yang Yang
```

```
%%%% Date : 2015.09.21
%%%% Version: 1.0
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% H-Zone parameters
H = 10;
h = 6 ;

%% Mesh parameters
N_r = 200;
N_theta = 100;
r_span = [0.001,15];
theta_span = [0+eps,pi-eps];
%% Generate mesh for Jukowski Airfoil
r = meshfun(r_span,N_r);
theta =
linspace(theta_span(1),theta_span(2),N_theta);
[R,THETA] = meshgrid(r,theta);
x = R.*cos(THETA) ;
y = R.*sin(THETA) ;
W = x + sqrt(-1)*y;

% Conformal mapping
```

```
b = H/h;
t = sqrt( (W-b^2) ./ (W-1) );
t1 = (1+t)./(1-t);
t2 = (b+t)./(b-t);

Z = H/pi*log(t1) - h/pi*log(t2);
X = real(Z);
Y = imag(Z);

%% plot mesh
[imax jmax] = size(X);
figure('Color',[1 1 1]);
% plot mesh in z plane
for j = 1:jmax
   for i = 1:imax-1

plot([X(i,j),X(i+1,j)],[Y(i,j),Y(i+1,j)],'r.-')
       hold on;
   end
end

% plot mesh in z plane
for i = 1:imax
```

```
   for j = 1:jmax - 1

plot([X(i,j),X(i,j+1)],[Y(i,j),Y(i,j+1)],'b.-')
   end
end

axis equal
hold off

figure('Color',[1 1 1]);
% plot phi
for j = 1:jmax
   for i = 1:imax-1

plot([x(i,j),x(i+1,j)],[y(i,j),y(i+1,j)],'r.-')
       hold on;
   end
end

% plot varphi
for i = 1:imax
   for j = 1:jmax - 1
```

```
plot([x(i,j),x(i,j+1)],[y(i,j),y(i,j+1)],'b.-')
   end
end
axis equal
hold off
%% Output Mesh in tecplot file form
fp = fopen('ChannelHMesh.dat','w');
fprintf(fp,'Title = ChannelHMesh\n');
fprintf(fp,'VARIABLES = "X", "Y"\n');
fprintf(fp,'ZONE I =%d, J =%d, F =
point\n',imax,jmax);

for j = 1:jmax
   for i = 1:imax
       fprintf(fp,'%e, %e\n',X(i,j),Y(i,j));
   end
end
fclose(fp);
```

```
function r = meshfun(r_span,N_r)
% This function control mesh nodes distribution
factor = 1.05;
r = zeros(1,N_r);          % array to store r;
```

```
r(1) = r_span(1);          % Initial value for r
delta = zeros(1,N_r-1);    % array to store
delta;
delta(1) = (r_span(2) - r_span(1))*(1 -
factor)/(1 - factor^(N_r-1));

for i = 2:N_r -1
    delta(i) = factor*delta(i-1);
    r(i) = r(i-1) + delta(i-1);
end
r(N_r) = r(N_r-1) + delta(N_r-1);

end
```