$\int 16 \cos^3$

1) $\int \cos^3(2x) \cdot \sin^3(2x)\, dx$

$\frac{1}{2} \int \cos^3(2x) \cdot \sin^3(2x) \cdot 2dx = \frac{1}{2} \int \cos(2x) \cdot \cos^2(2x) \cdot \sin^3(2x) \cdot 2dx =$

$= \frac{1}{2} \int \cos(2x) \cdot \sin^3(2x) \cdot (1 - \sin^2 2x) \cdot 2dx = \frac{1}{2} \int \cos 2x \cdot \sin^3 2x \cdot 2dx - \frac{1}{2} \int \cos 2x \cdot$

$\cdot \sin^3 x\, 2x \cdot 2dx$; $\left| \begin{matrix} \sin 2x = t \\ dx = \frac{1}{t'} dt \end{matrix} \right|$ $\frac{1}{2} \int \sin^3 2x \cdot \cancel{\cos 2x} \cdot \frac{1}{\cancel{\cos 2x} \cdot \cancel{2}} \cdot \cancel{2}\, dt -$

$= -\frac{1}{2} \int \sin^5(2x) \cdot \cancel{\cos 2x} \cdot \cancel{2} \cdot \frac{1}{\cancel{\cos 2x} \cdot \cancel{2}} dt = \frac{1}{2} \int t^3 \cdot dt - \frac{1}{2} \int t^5\, dt =$

$x) = \frac{1}{2} \cdot \frac{t^4}{4} - \frac{1}{2} \cdot \frac{t^6}{6} = \frac{\sin^4 2x}{8} - \frac{\sin^6 2x}{12} + C.$

$$\sin^2 x + \cos^2 x = 1$$

$\frac{1}{2} \int \sin^3 x \cdot dx \, (1 - \sin^2 x)$

$\frac{1}{(\sin 2x)'} = \frac{1}{\cos 2x \cdot 2}$

$$2)\ \int \cos^3(3x)\cdot \sin^5(3x)\cdot dx \quad \left[\begin{array}{l} u = \sin 3x \\ du = \cos 3x \cdot 3\,dx \\ dx = \dfrac{1}{\cos 3x \cdot 3}\,du \end{array}\right]$$

$$\frac{1}{3}\int \cos^3(3x)\cdot \sin^5(3x)\cdot 3\,dx = \frac{1}{3}\int \sin^5(3x)\cdot \cos^2 3x \cdot$$

$$= \frac{1}{3}\int \sin^5(3x)\cdot \cos^2(3x)\cdot \cos(3x)\,3dx =$$

$$\cdot\ \cancel{\cos(3x)}\cdot \cancel{3}\cdot \frac{1}{\cancel{\cos 3x}\ \cancel{3}} = \frac{1}{3}\int (1-\sin^2(3x))\cdot \sin^5(3x)\cdot du =$$

$$= \frac{1}{3}\int (1-u^2)\cdot u^5\cdot du = \frac{1}{3}\int u^5\,du - \frac{1}{3}\int u^7\,du = \frac{1}{3}\cdot\frac{u^6}{6} -$$

$$- \frac{1}{3}\cdot \frac{u^8}{8} + C = \frac{\sin^6 3x}{18} - \frac{\sin^8 3x}{24} + C.$$

$\sin|x| \cdot (1-$

4) $\int \cos^4(5x)\cdot \sin^5(5x)\,dx \quad \left|\begin{array}{l} u=\cos 5x \\ du=-\sin 5x\cdot 5 \end{array} \quad dx=-\frac{1}{5\sin 5x}\right|$

$\int \sin^{\frac{4}{5}}(5x)\cdot\cos^4(5x)\cdot\frac{1}{-\sin 5x\cdot 5}\,du = \int -\frac{\sin^4 5x\cdot\cos^4 5x}{5}\,du =$

$= \int -\frac{\sin^4(5x)\cdot u^4}{5}\,du = \int -\frac{(\sin^2 5x)^2\cdot u^4}{5}\,du = \int -\frac{(1-\cos^2 5x)^2\cdot u^4}{5}\,du = \int -\frac{(1-u^2)^2\cdot u^4}{5}\,du$

$= \int -\frac{u^4-2u^6+u^8}{5}\,du = -\frac{1}{5}\int (u^4-2u^6+u^8)\,du = -\frac{1}{5}\left(\int u^4\,du - \int 2u^6\,du + \int u^8\,du\right) =$

$= -\frac{u^5}{25} + \frac{2u^7}{35} - \frac{u^9}{45} = -\frac{\cos^5 5x}{25} + \frac{2\cos^7 5x}{35} - \frac{\cos^9 5x}{45} + C$

$-\frac{(\sin^2 5x)^2}{5}$ $\rightarrow$ $\sin^2 x + \cos^2 x = 1$

$\ldots u^2 + C$

5) $\int \cos^2(ax)\sin^2(ax)\,dx$.

$\int \frac{1}{4}\cdot 4\cos^2(ax)\cdot\sin^2(ax)\,dx = \frac{1}{4}\cdot\int \sin^2 2(ax)$.

$\boxed{2\sin(t)\cdot\cos(t) = \sin 2t}$

$\boxed{\sin^2 t = \frac{1}{2}\cdot(1-\cos 2t)}$

$\left| t = 2ax \quad dx = \frac{1}{2a}dt \right|$

$\ln|x| + \frac{1}{\ln} \frac{1}{4}\int \sin^2(2ax)\cdot\frac{1}{2a}\,dt = \frac{1}{4}\int\frac{\sin^2 t}{2a}\,dt = \frac{1}{8a}\int \sin^2 t\,dt =$

$| + C$. $= \frac{1}{8a}\int\frac{1}{2}\cdot(1-\cos(2t))\cdot dt = \frac{1}{16a}\int 1-\cos 2t\cdot dt = \frac{1}{16a}\cdot\int 1\,dt - \frac{1}{16a}\int\cos 2t\,dt =$

$= \frac{1}{16a}\cdot t - \frac{1}{16a}\cdot\frac{\sin 2t}{2} = \frac{4ax-\sin 4ax}{32a} + C$

$\boxed{2\sin(t)\cdot\cos(t) = \sin 2t}$

$\boxed{\sin^2 t = \frac{1}{2}\cdot(1-\cos 2t)}$

6) $\int \cos^2(ax) \cdot \sin^4(ax) \cdot dx$ $\left| \begin{array}{l} u = ax \\ du = a \\ dx = \frac{1}{a} du \end{array} \right|$

$$\int \cos^2 u \cdot \sin^4 u \cdot \frac{du}{a} =$$

$$= \frac{1}{a}\int \cos^2(u) \cdot \sin^4(u) \cdot du = \frac{1}{a}\int \sin^4 u \cdot (1 - \sin^2 u)\, du =$$

$$= \frac{1}{a}\int \sin^4 u \cdot du - \frac{1}{a}\int \sin^6 u \cdot du = \frac{\cos u \cdot \sin^5 u}{6a} \; \frac{\cos u \cdot \sin^3 u}{24a} -$$

$$- \frac{\cos u \cdot \sin u}{16a} + \frac{u}{16a} + C = \frac{\cos(ax) \cdot \sin^5(ax)}{6a} - \frac{\cos(ax) \cdot \sin^3(ax)}{24a} -$$

$$- \frac{\cos(ax) \cdot \sin(ax)}{16a} + \frac{ax}{16a} = \frac{\cos(ax) \cdot \sin^5(ax)}{6a} - \frac{\cos(ax) \cdot \sin^3(ax)}{24a} -$$

$$- \frac{\cos(ax) \cdot \sin(ax)}{16a} + \frac{x}{16} + C.$$

$$= \int \sin^4(u)\,du - \int \sin^6(u)\,du$$

Now solving:

$$\int \sin^4(u)\,du$$

Apply reduction formula:

$$\int \sin^{n}(u)\,du = \frac{n-1}{n}\int \sin^{n-2}(u)\,du - \frac{\cos(u)\sin^{n-1}(u)}{n}$$

with $n = 4$:

$$= -\frac{\cos(u)\sin^3(u)}{4} + \frac{3}{4}\int \sin^2(u)\,du$$

... or choose an alternative:

Apply product-to-sum formulas

Now solving:

$(y-1)^2 = 2(x-1)$

7) $\int 16 \cos^3(ax) \cdot \sin(ax) \cdot dx$ $\left| \begin{array}{l} u = \cos ax \\ du = -a \sin ax \\ dx = -\frac{1}{a \cdot \sin ax} du \end{array} \right|$

$16 \int -\cos^3(ax) \cdot \sin(ax) \cdot \frac{1}{\sin ax \cdot a} du = 16 \int -\cos^3(ax) \cdot \sin(ax) \cdot \frac{1}{a} dt =$

$= -\frac{16a}{a} \int \cos^3(ax) du = -\frac{16}{a} \int u^3 du = -\frac{16}{a} \cdot \frac{u^4}{4} = -\frac{4u^4}{a} = -\frac{\cos^4 ax \cdot 4}{a} + C$

8) $\int \cos(ax) \cdot \sin^3(ax)\, dx$ $\left| \begin{array}{l} u = \sin(ax) \\ du = \cos(ax) \cdot a \\ dx = \frac{1}{\cos(ax) \cdot a} du \end{array} \right|$

$\int \sin^3(ax) \cdot \cos(ax) \cdot \frac{1}{\cos(ax) \cdot a} \cdot du$

$\int \sin^3(ax) \cdot \frac{1}{a} du = \frac{1}{a} \int \sin^3(ax) \cdot du = \frac{1}{a} \int t^3 du = \frac{1}{a} \cdot \frac{t^4}{4} = \frac{\sin^4 ax}{4a} + C$

9) $\int 2\cos^2(ax)\cdot\sin(ax)\,dx$ $\left| \begin{array}{l} u=\cos ax \\ du=-a\cdot\sin(ax) \\ dx=-\frac{1}{\sin(ax)\cdot a}du \end{array} \right|$

$2\int\cos^2(ax)\cdot\sin(ax)\cdot dx$

$2\int-\cos^2(ax)\cdot\cancel{\sin(ax)}\cdot\frac{1}{\cancel{\sin(ax)}\cdot a}du=\frac{2}{a}\int-\cos^2 ax\cdot du=-\frac{2}{a}\int t^2\,du=$

$=-\frac{2}{a}\cdot\frac{t^3}{3}=-\frac{2\cdot\cos^3 ax}{3a}+C.$

10) $\int 5\cos^4(ax)\cdot\sin(ax)\,dx$ $\left| \begin{array}{ll} u=\cos(ax) & dx=-\frac{1}{\sin ax\cdot a} \\ du=-\sin(ax)\cdot a & \end{array} \right|$

$5\int-\cos^4(ax)\cdot\cancel{\sin(ax)}\cdot\frac{1}{\cancel{\sin(ax)}\cdot a}du=$

$=-\frac{5}{a}\int\cos^4(ax)\,du=-\frac{5}{a}\int t^4\,du=-\frac{\cancel{5}}{a}\cdot\frac{t^5}{\cancel{5}}=-\frac{\cos^5(ax)}{a}+C$