Computer Assignment 1: Blind Experiments with Finite Difference Schemes

- Consider a linear advection equation

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$$

with five different Initial Conditions (i.e. solution at time t=0) as described below. Take the value of wave speed $a = 1.0$ m/s.

1. Discontinuous initial solution as described in Figure 1

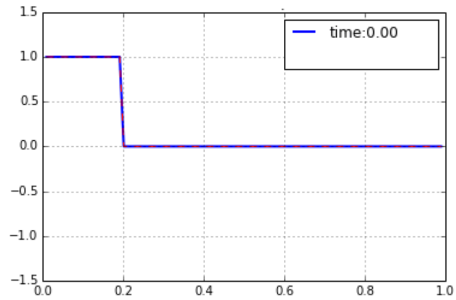

Figure 1: Initial condition 1

$$u(x,0) = \begin{cases} 1 & \text{if} \quad 0.2 \leq x \\ 0 & \text{Otherwise} \end{cases}$$

2. Initial solution having two periods as described in Figure 2

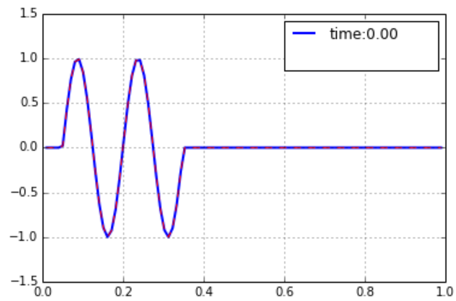

Figure 2: Initial condition 2

$$u(x,0) = \begin{cases} 0 & \text{for } 0 \leq x < 0.05 \\ sin(4\pi(\frac{x-0.05}{0.3})) & \text{for } 0.05 \leq x < 0.35 \\ 0 & \text{for } 0.35 \leq x \leq 1 \end{cases}$$

3. Initial solution having four periods as described in Figure 3

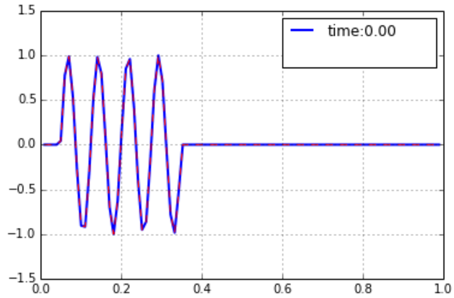

Figure 3: Initial condition 3

$$u(x,0) = \begin{cases} 0 & \text{for } 0 \leq x < 0.05 \\ sin(8\pi(\frac{x-0.05}{0.3})) & \text{for } 0.05 \leq x < 0.35 \\ 0 & \text{for } 0.35 \leq x \leq 1 \end{cases}$$

4. Initial solution having six periods as described in Figure 4

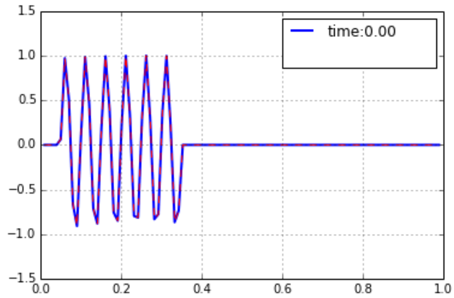

Figure 4: Initial condition 4

$$u(x,0) = \begin{cases} 0 & \text{for } 0 \leq x < 0.05 \\ sin(12\pi(\frac{x-0.05}{0.3})) & \text{for } 0.05 \leq x < 0.35 \\ 0 & \text{for } 0.35 \leq x \leq 1 \end{cases}$$

5. Initial solution having Gaussian curve as described in Figure 5

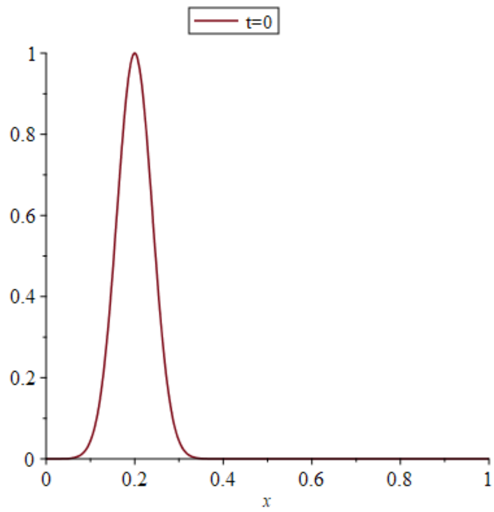

Figure 5: Initial condition 5

$$u(x,0) = e^{-50\dfrac{(x-0.2)^2}{\sigma^2}} \qquad \text{where} \quad \sigma = 0.4$$

- Consider 101 grid points (as shown in Figure 6) for discretizing the computational space $0 \leq x \leq 1$ for all the five initial conditions described earlier.

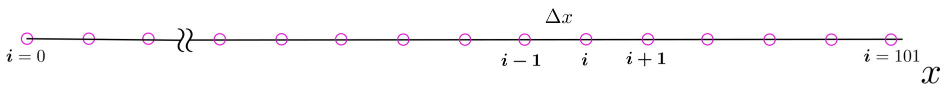

Figure 6: Grid for the linear advection equation.

- Apply the following Finite Difference Schemes (derivations of which will be explained in future) to obtain solutions of the linear advection equations at time t>0 as described below. At present, you need not know the background details of these schemes.

  1. Forward Time Forward Space (FTFS) scheme

  $$u_i^{n+1} = u_i^n - \nu(u_{i+1}^n - u_i^n)$$

  2. Forward Time Central Space (FTCS) scheme

  $$u_i^{n+1} = u_i^n - \frac{\nu}{2}(u_{i+1}^n - u_{i-1}^n)$$

3. Forward Time Backward Space (FTBS) scheme

$$u_i^{n+1} = u_i^n - \nu(u_i^n - u_{i-1}^n)$$

4. Lax-Wendroff (LW) scheme

$$u_i^{n+1} = u_i^n - \frac{\nu}{2}(u_{i+1}^n - u_{i-1}^n) + \frac{\nu^2}{2}(u_{i+1}^n - 2u_i^n + u_{i-1}^n)$$

5. Beam-Warming (BW) scheme

$$u_i^{n+1} = u_i^n - \frac{\nu}{2}(3u_i^n - 4u_{i-1}^n + u_{i-2}^n) + \frac{\nu^2}{2}(u_i^n - 2u_{i-1}^n + u_{i-2}^n)$$

6. Fromm (FR) scheme

$$\left(u_i^{n+1}\right)_{FR} = \frac{1}{2}\left[\left(u_i^{n+1}\right)_{LW} + \left(u_i^{n+1}\right)_{BW}\right]$$

where ν is Courant Friedrichs Lewy and it is given by $\nu = \dfrac{a\Delta t}{\Delta x}$, where Δt is the time step by which solution is advanced. Meaning of ν will be explained later.

- For implementing boundary conditions at the two end of the computational domain (that is, at x=0 and x=1), do the following:

$$u_0 = u_1 \qquad u_{101} = u_{100}$$

at all time. Since the number of time steps (typically below 100) to be considered by you should be such that the disturbances in the solution will not reach the boundaries within that time interval.

- By the above statement, it is clear that you need to update your solution (i.e. obtain u_i^{n+1}) only at the interior grid points $i = 2, \ldots, 100$.

- While computing solutions, consider three values of ν for each scheme; namely $\nu = 0.5$, $\nu = 1.0$, $\nu = 1.5$.

- Create a report (pdf file), which should include the following:

  1. The "critical observations" in a tabular form for each results in order to compare different schemes.
  2. All the plots of the solution obtained after 40 time steps for all the schemes considering all the initial conditions and all the prescribed values of ν.

- Use the plotting range for x-axis as $0 \leq x \leq 1$ and that for y-axis as $-1.5 \leq u \leq 1.5$ (as shown in the Figures 1-5 for initial conditions) uniformly.
- Incorporate all possible finite difference schemes, initial conditions and values of ν in a single code. You need to submit one code which will produce results for all the possible cases with user's option.
- Therefore, your submission folder should contain (1) one computer code as stated above, (2) a report as explained above and (3) "README.txt" file which contains the proper description on how to run the code and get the plots.

General Instructions

- **Checklist for submission:**
  - The code (written in C/C++ only) with proper inline documentation for each function. (Use meaningful variable names).
  - A "README.txt" file which contains the proper description on how to run the code and get the plots.
  - The plots shown in the figures inside your report must be reproducible independently by the TA's from your code based on what you have described in "README.txt".
- **Instruction for submission:**
  - Rename your program file as your roll number (example: 184010006.c).
  - Keep the C/C++ code, report and README.txt file (See the checklist) in a folder named it as your roll number (example 184010006).
- **Notes:**
  - There is no mark for this assignment.
  - However, your class test on this topic will not be evaluted unless you submit this assignment within due date and your submitted computer program is working and showing correct results.
  - Assignment will not be considered as complete if "instructions for submission" are not followed properly.
- **Copying program from each other will lead to severe penalty.**
- **Warning:** Your computer code will be checked with plagiarism checker.

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