Computer Assignment 2

- Find steady-state temperature distribution on a 2D rectangular plate as shown in Figure 1 below by solving Laplace equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{1}$$

with the specified temperatures at the boundaries as indicated in the figure.

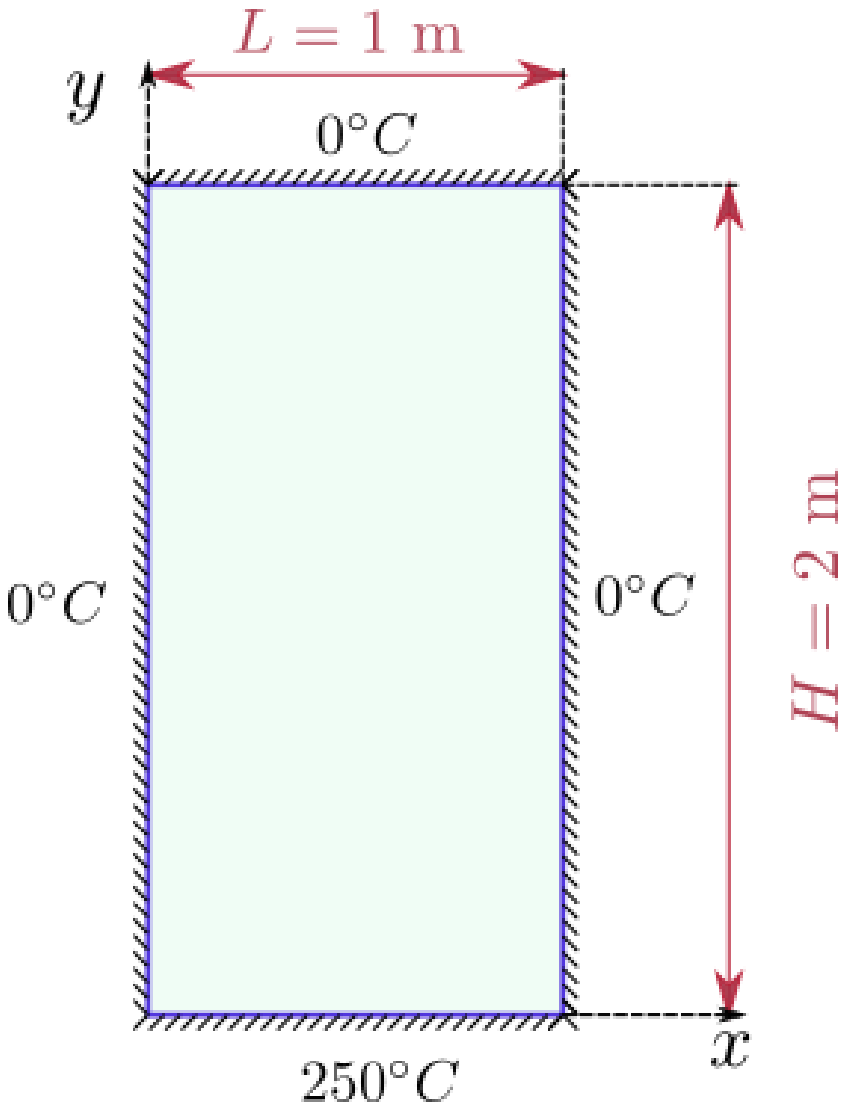

Figure 1

- In order to solve the Laplace equation (1) and present the numerical results, **strictly** follow the instructions given below:

  1. Deiscretize the domain using $IM \times JM$ (i.e. 41×81) grid points.
  2. Use (i) Point Jacobi (PJ), (ii) Point Gauss-Seidel (PGS), (iii) Point Succesive Over Relaxation (PSOR) with optimum relaxation parameter ω_{opt} as given below for soving the system of equations obtained from the discretized Laplace equation.

     $$\omega_{opt} = \frac{2 - 2\sqrt{1-a}}{a}$$

     where

     $$a = \left[\frac{\cos\left(\frac{\pi}{IM-1}\right) + \beta^2 \cos\left(\frac{\pi}{JM-1}\right)}{1+\beta^2} \right]^2$$

     and

     $$\beta = \frac{\Delta x}{\Delta y}$$

  3. Impose convergence criterion

     $$\text{ERROR} < 10^{-3} \tag{2}$$

     where

     $$\text{ERROR} = \sum_{i,j} |T_{i,j}^{n+1} - T_{i,j}^{n}|$$

  4. Show convergence history as a plot '$log_{10}(ERROR)$' Versus 'Number of iterations' for PJ, PGS, PSOR schemes.
  5. Note down the number of iterations required by PJ, PGS, PSOR schemes to achieve convergence criterion given in equation (2). Write your comments on the observation.
  6. Plot variations of temperature T along the midlines; i.e. (i) along $x = 0.5$ line and (ii) along $y = 1.0$ line. Compare these numerical results with the exact solutions as given by

     $$T(x,y) = (250) \left[2 \sum_{n=1}^{\infty} \frac{1-(-1)^n}{n\pi} \frac{Sinh\left(\frac{n\pi(H-y)}{L}\right) Sin\left(\frac{n\pi x}{L}\right)}{Sinh\left(\frac{n\pi H}{L}\right)} \right]$$

  7. Contour plot the final result using ParaView software.
  8. As far as possible, use variable names in the code matching with the class notations.

General Instructions

- **Checklist for submission:**
  1. Flowchart of the C code written by you.
  2. The code (written in C only)[1] with proper inline documentation for each function. (Use meaningful variable names).
  3. A "README.txt" file which contains the proper description on how to run your code and get the plots.
  4. Brief report, in pdf format, where all the above mentioned results in terms of plots and comments are included. The plots submitted by you must be reproducible independently by the TA's from your code(s).
- **Instruction for submission:**
  - Rename your program file as your roll number (example: 204010006.c).
  - Rename the report as your roll number (example: 204010006.pdf)
  - Submit all the **4 documents** as stated in the Checklist above.
- **Notes:**
  - Marks will be given only if the program is working and showing correct result. No step marks will be given.
  - Assignment will not be evaluated if "instruction for submission" are not followed properly.
- **Copying program from each other or from any other source will lead to severe penalty.**

– E N D –

[1] You will be given zero mark for the entire assignment if any other computer language is used.