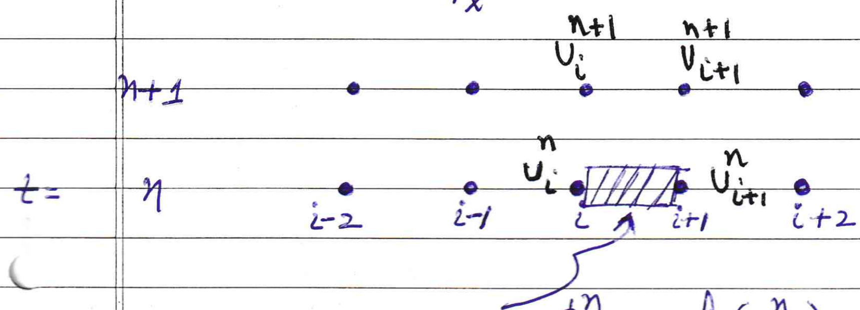
NUMERICAL ALGORITHMS — SCALAR LAW

- FLUX DIFFERENCE SPLITTING

$$u_t + f(u)_x = 0$$



EXPLICIT

$$\rightarrow \quad \phi^n_{i+1/2} = f(u^n_{i+1}) - f(u^n_i)$$

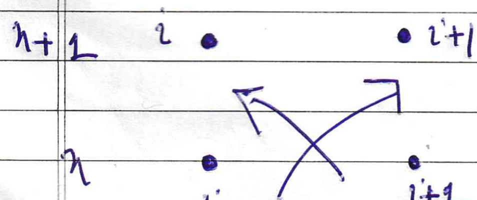
$$u_i^{n+1} = u_i^n - \frac{\Delta t}{\Delta x}\Delta F^n \qquad = f^n_{i+1} - f^n_i = \Delta f^n_{i+1/2}$$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + \frac{\Delta F^n}{\Delta x} = 0 \qquad = \text{fluctuation @ } i+1/2, n$$

$$\rightarrow \quad \text{signal} \quad \frac{\Delta t}{\Delta x}\phi^n_{i+1/2} = \frac{\Delta t}{\Delta x}\Delta f^n_{i+1/2}$$

$$\rightarrow \quad \text{target} \quad \nabla^n_{i+1/2} = \operatorname{sgn}\left(\frac{f^n_{i+1} - f^n_i}{u^n_{i+1} - u^n_i}\right)$$

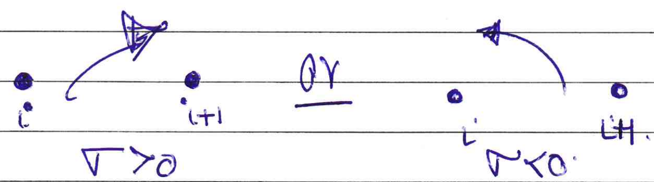
$$= \operatorname{sgn}\left(\frac{\Delta f^n_{i+1/2}}{\Delta u^n_{i+1/2}}\right)$$

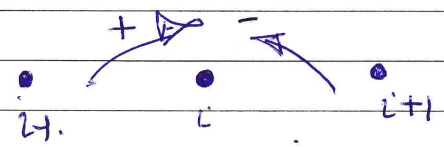


$$\hat{\nabla}_{i+1/2} = \operatorname{sgn}\left(\hat{a}^n_{i+1/2}\right) = \nabla^n$$

+1 or −

$$\text{target} = x_{\left(i+1/2+\frac{\nabla^n}{2}\right)} \quad \text{(first order)}$$





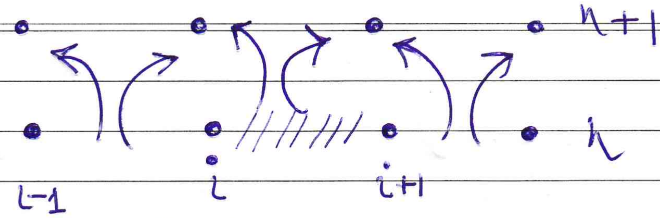
$\sigma = +1$ or -1

$$O(1) \qquad U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x}\left[\Delta f^n_{i-1/2}\Big|_{\sigma^n_{i-1/2}>0} + \Delta f^n_{i+1/2}\Big|_{\sigma^n_{i+1/2}<0} \right]$$

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x}\left[\frac{(1+\sigma^n_{i-1/2})}{2}\Delta f^n_{i-1/2} + \frac{(1-\sigma^n_{i+1/2})}{2}\Delta f^n_{i+1/2} \right]$$

Stability $\qquad D^n_{i+1/2} = \frac{\Delta f^n_{i+1/2}}{\Delta U^n_{i+1/2}}\frac{\Delta t}{\Delta x} = \hat{a}^n_{i+1/2}\frac{\Delta t}{\Delta x}$

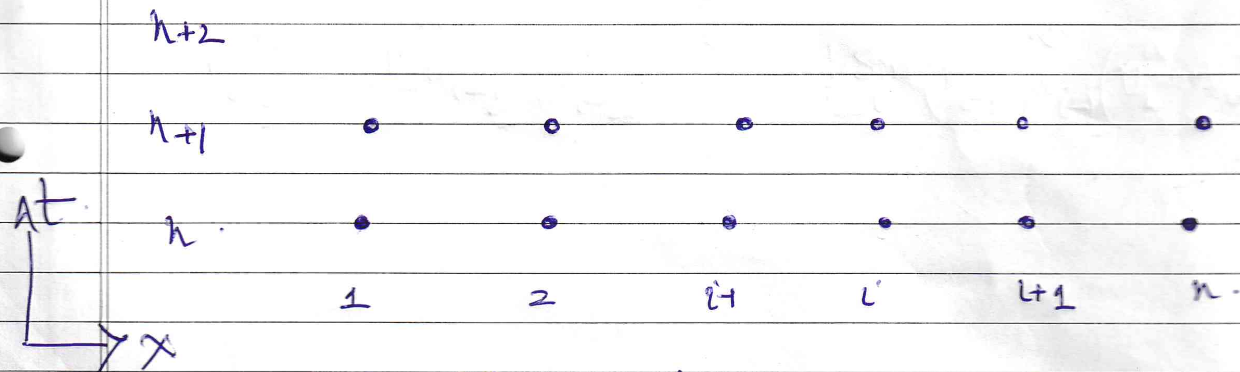
$O(2)$



$$\frac{(1-\nu)}{2} \quad \frac{(1+\nu)}{2}$$

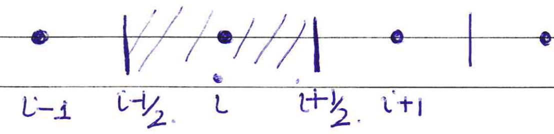
$$\nu^n_{i+1/2} = \frac{\Delta f^n_{i+1/2}}{\Delta U^n_{i+1/2}} \frac{\Delta t}{\Delta x}$$

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x}\left[\frac{(1+\nu^n_{i-1/2})}{2}\Delta f^n_{i-1/2} + \frac{(1-\nu^n_{i+1/2})}{2}\Delta f^n_{i+1/2}\right]$$



n+1 points, n intervals.

FINITE VOLUME FORM. $O(1)$.



$$u_t + f_x(u) = 0$$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = -\frac{\Delta f}{\Delta x}$$

$$u_i^{n+1} = u_i^n - \frac{\Delta t}{\Delta x}\left[F_{i+1/2}^n - F_{i-1/2}^n\right]$$

$$F_{i+1/2} \sim f(u_i^n) + \hat{a}^-_{i+1/2}(u_{i+1}^n - u_i^n) \qquad \hat{a}^- = \min(\hat{a}, 0)$$

$$\hat{a}^-_{i+1/2} = \frac{f_{i+1}^n - f_i^n}{u_{i+1}^n - u_i^n} \quad \text{if } < 0 \quad \text{else } = 0$$

$$F_{i-1/2} \sim f(u_{i-1}^n) + \hat{a}^-_{i-1/2}(u_i^n - u_{i-1}^n)$$

$\hat{a} > 0$ (uniformly) $\qquad u_i^{n+1} = u_i^n - \frac{\Delta t}{\Delta x}\left[f_i^n - f_{i-1}^n\right]$

$\hat{a} < 0$ $\qquad u_i^{n+1} = u_i^n - \frac{\Delta t}{\Delta x}\left[f_{i+1}^n - f_i^n\right]$

$$u_i^{n+1} = u_i^n - \frac{\Delta t}{\Delta x}\left[\left\{f_i^n + \hat{a}^-_{i+1/2}(u_{i+1}^n - u_i^n)\right\} - \left\{f_{i-1}^n + \hat{a}^-_{i-1/2}(u_i^n - u_{i-1}^n)\right\}\right]$$