Numerical Methods for Conservation Laws

Assignment 1 (Scalar Laws, August 2021)

Use the first-order forms of **both** the flux difference splitting and finite volume method to numerically solve for the following scalar hyperbolic conservation laws:

1. $u_t + (u^2/2)_x = 0$, in the domain [-2,2], with initial conditions:

$$u(x,0) = 1, \qquad |x| < 1/3 \tag{1}$$

$$u(x,0) = 0, \qquad |x| > 1/3 \tag{2}$$

   Plot u versus x obtained numerically at $t = 2/3$ and $t = 4/3$ and compare with analytical solution.

2. Consider the traffic equation $\rho_t + (\rho u_{max}[1-\rho])_x = 0$, $0 \leq \rho \leq 1$, $u_{max} = 1.0$

   (a) Solve with initial conditions $\rho(x,0) = 0.25{+}0.75exp^{(-0.25x^2)}$, domain $[-30, 30]$. Plot $\rho(x, 25)$.

   (b) Solve with initial discontinuous data $\rho_l = 0.25, \rho_r = 1.0$, jump at $x = 0.0$, domain [-40,10]. Plot $\rho(x, 36)$.

   (c) Solve with initial discontinuous data $\rho_l = 1.0, \rho_r = 0.0$, jump at $x = 0.0$, domain [-30,20], Plot $\rho(x, 18)$.

Use 100 points in the domain and time step based on a CFL number of 0.8 for solving the above problems. Compare the solution when number of points are doubled to 200.