Numerical Methods for Conservation Laws

Assignment 2 (System of Linear Equations, September 2021)

Solve the acoustic equations

$$p_t + K_o(x)u_x = 0 \tag{1}$$

$$\rho_o(x)u_t + p_x = 0 \tag{2}$$

$c_o(x) = \sqrt{(K_o(x)/\rho_o(x))}$ the speed of sound, using *both* first and second order forms of the flux difference splitting algorithm for a system of linear hyperbolic conservation laws with following initial data:

$u(x, 0) = 0.$ and

$$p(x,0) = \begin{cases} \overline{p}\sqrt{1-((x-x_o)/\overline{x})^2} & \text{if} \quad |x - x_o| < \overline{x}, \\ 0 & \text{otherwise} \end{cases}$$

$x_o = 0.4$, $\overline{x} = 0.075$, $\overline{p} = 0.2$, $\Delta x = 0.005$, $\Delta t = 0.004$, domain $[0, 1]$, both boundaries open. Solve for the following two cases and show results as stated:

1. $K_o(x) \equiv 1.$
   $\rho_o(x) = 1.$
   plot $p(x)$ at t= $0, 0.052, 0.26, 0.364, 0.6$

2. Plot $p(x)$ at t= $0.104, 0.26, 0.364$ after solving for additional discontinuous impedance
   $K_o(x) \equiv 1.$
   $$\rho_o(x) = \begin{cases} 1 & \text{if} \quad x < 0.6 \\ 4 & \text{if} \quad x > 0.6, \end{cases}$$