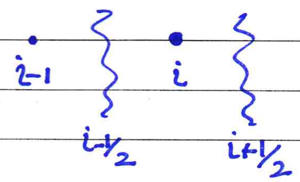
NONLINEAR SYSTEMS — ALGORITHM



$$\Delta \underline{U}^n_{i+1/2} = \underline{U}^n_{i+1} - \underline{U}^n_{i} = \sum_p \hat{\alpha}^n_{p,i+1/2}\, \hat{\underline{r}}^n_{p,i+1/2}$$

$$\Delta \underline{f}^n_{i+1/2} = \hat{A}^n \Delta\underline{U}_{i+1/2} = \sum_p \hat{\lambda}^n_{p,i+1/2}\, \hat{\alpha}^n_{p,i+1/2}\, \hat{\underline{r}}^n_{p,i+1/2}$$

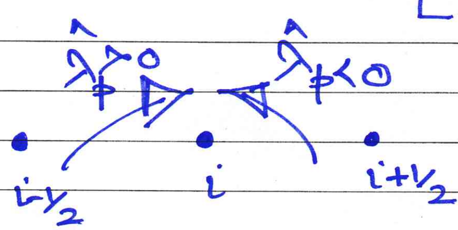
$$= \Delta\hat{\underline{f}}_{1,i+1/2} + \Delta\hat{\underline{f}}_{2,i+1/2} + \Delta\hat{\underline{f}}_{3,i+1/2}$$

$$\Delta t^n \le \frac{\Delta x\, CFL}{|\hat{\lambda}_{max}|}$$

$O(1)$

$$\underline{U}^{n+1}_i = \underline{U}^n_i - \frac{\Delta t}{\Delta x}\left[\sum_{p} \Delta \underline{f}^n_{p,i+1/2} + \sum_{p} \Delta \underline{f}^n_{p,i-1/2}\right]$$

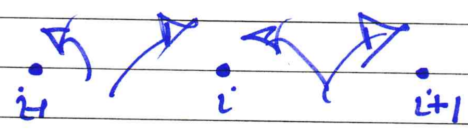
$\hat{\lambda}_{p,i+1/2} < 0$ $\qquad$ $\hat{\lambda}_{p,i-1/2} > 0$

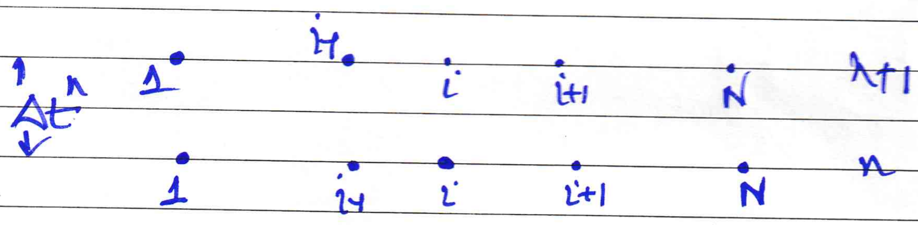


$O(2)$

$$\hat{\nu}^n_{p,i+1/2} = \hat{\lambda}^n_{p,i+1/2} \frac{\Delta t}{\Delta x}$$

$$\underline{U}_i^{n+1} = \underline{U}_i^{n} - \frac{\Delta t}{\Delta x} \sum_{p} \left\{ \left(\frac{1+\nu_{p,i+\frac{1}{2}}}{2} \right)^{n} \Delta \underline{f}_{p,i-\frac{1}{2}} + \left(\frac{1-\nu_{p,i+\frac{1}{2}}}{2} \right)^{n} \Delta \underline{f}_{p,i+\frac{1}{2}} \right.$$





$$\Delta t^{n} \leq \frac{\Delta x \, CFL}{|\hat{\lambda}_{max}|}.$$

Roe - numerical flux

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2}(\gamma-3)u^2 & (3-\gamma)u & \gamma-1 \\ \frac{1}{2}(\gamma-1)u^3 - uH & H-(\gamma-1)u^2 & \gamma u \end{bmatrix}$$

$$\underline{U}_i^{n+1} = \underline{U}_i^{n} - \frac{\Delta t}{\Delta x}\left[\underline{F}^{n}_{i+\frac{1}{2}}(\underline{U}) - \underline{F}^{n}_{i-\frac{1}{2}}(\underline{U})\right]$$

$$\underline{F}^{n}_{i+\frac{1}{2}} = \frac{1}{2}\left[\underline{f}^{n}_{i+1} + \underline{f}^{n}_{i}\right] - \frac{1}{2}\left|\hat{A}^{n}_{i+\frac{1}{2}}\right|\left(\underline{U}^{n}_{i+1} - \underline{U}^{n}_{i}\right)$$

$$= \frac{1}{2}\left[\underline{f}^{n}_{i+1} + \underline{f}^{n}_{i}\right] - \frac{1}{2}\sum_{p}\left|\lambda^{n}_{i+\frac{1}{2},p}\right| \alpha^{n}_{i+\frac{1}{2},p}\, r^{n}_{i+\frac{1}{2},p}$$

INVISCID EULER

$$\underline{U}_t + \underline{f}(\underline{U})_x = 0 \; ; \quad \underline{U} = \begin{Bmatrix} \rho \\ \rho u \\ e \end{Bmatrix} ; \; \underline{f} = \begin{Bmatrix} \rho u \\ \rho u^2 + p \\ u(p+e) \end{Bmatrix}$$

$$e = \text{total energy} = \frac{p}{\gamma - 1} + \frac{1}{2}\rho u^2$$

$$H = \frac{e+p}{\rho} = \text{total sp. enthalpy}$$

$$H = \frac{\gamma p}{(\gamma-1)\rho} + \frac{u^2}{2} \;, \qquad a^2 = (\gamma-1)\left(H - \frac{u^2}{2}\right)$$

$$\underline{r_1} = \begin{pmatrix} 1 \\ u-a \\ H-ua \end{pmatrix} ; \quad \underline{r_2} = \begin{pmatrix} 1 \\ u \\ u^2/2 \end{pmatrix} ; \quad \underline{r_3} = \begin{pmatrix} 1 \\ u+a \\ H+ua \end{pmatrix}$$

$$\lambda_1 = u-a \qquad \lambda_2 = u \qquad \lambda_3 = u+a$$

$$\hat{u} = \frac{\sqrt{\rho_L}\, u_L + \sqrt{\rho_R}\, u_R}{\sqrt{\rho_L} + \sqrt{\rho_R}} \;, \qquad \hat{H} = \frac{\sqrt{\rho_L}\, H_L + \sqrt{\rho_R}\, H_R}{\sqrt{\rho_L} + \sqrt{\rho_R}}$$

$$\hat{a}^2 = (\gamma-1)\left(\hat{H} - \frac{\hat{u}^2}{2}\right)$$

$$\hat{u}_{i+\frac{1}{2}} = \frac{\sqrt{\rho_i}\, u_i + \sqrt{\rho_{i+1}}\, u_{i+1}}{\sqrt{\rho_i} + \sqrt{\rho_{i+1}}} \qquad \underbrace{\overset{\bullet}{i} \qquad \overset{\bullet}{i+1}}_{i+\frac{1}{2}} \qquad \hat{H}_{i+\frac{1}{2}} = \frac{\sqrt{\rho_i}\, H_i + \sqrt{\rho_{i+1}}\, H_{i+1}}{\sqrt{\rho_{i+1}} + \sqrt{\rho_i}}$$

Shallow Water eq's

$$\begin{bmatrix} h \\ hu \end{bmatrix}_t + \begin{bmatrix} uh \\ hu^2 + \frac{1}{2}gh^2 \end{bmatrix}_x = 0 \qquad \underline{U} = \begin{bmatrix} h \\ hu \end{bmatrix} ; \underline{f}(\underline{U}) = \begin{bmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \end{bmatrix}$$

$$f'(\underline{U}) = \begin{bmatrix} 0 & 1 \\ -u^2 + gh & 2u \end{bmatrix}$$

$$\lambda_1 = u - \sqrt{gh} \qquad \lambda_2 = u + \sqrt{gh}$$

$$\underline{r}_1 = \begin{bmatrix} 1 \\ u - \sqrt{gh} \end{bmatrix} \qquad \underline{r}_2 = \begin{bmatrix} 1 \\ u + \sqrt{gh} \end{bmatrix}$$

$$\bar{h} = \hat{h}_{i+\frac{1}{2}} = \frac{1}{2}(h_i + h_{i+1}) ; \qquad \hat{u}_{i+\frac{1}{2}} = \frac{\sqrt{h_i}\, u_i + \sqrt{h_{i+1}}\, u_{i+1}}{\sqrt{h_i} + \sqrt{h_{i+1}}}$$

$$\hat{c} = \sqrt{g\hat{h}}$$

$$\hat{\lambda}_1 = \hat{u} - \hat{c} \qquad \hat{\lambda}_2 = \hat{u} + \hat{c}$$

$$\hat{r}_1 = \begin{bmatrix} 1 \\ \hat{u} - \hat{c} \end{bmatrix} \qquad \hat{r}_2 = \begin{bmatrix} 1 \\ \hat{u} + \hat{c} \end{bmatrix}$$