Punto teórico taller

$$D^4 f(x_j) \approx \frac{f(x_{j+2}) - 4f(x_{j+1}) + 6f(x_j) - 4f(x_{j-1}) + f(x_{j-2})}{h^4}$$

Demostración

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!} f^2(x) + \frac{h^3}{3!} f^3(x) + \frac{h^4}{4!} f^4(x) + \frac{h^5}{5!} f^5(x)$$

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2!} f^2(x) - \frac{h^3}{3!} f^3(x) + \frac{h^4}{4!} f^4(x) - \frac{h^5}{5!} f^5(x)$$

$$f(x+2h) = f(x) + 2hf(x) + 2h^2 f^2(x) + \frac{4}{3} h^3 f^3(x) + \frac{2}{3} h^4 f^4(x) + \frac{4}{15} f^5(x) h^5$$

$$f(x-2h) = f(x) - 2hf(x) + 2h^2 f^2(x) - \frac{4}{3} h^3 f^3(x) + \frac{2}{3} h^4 f^4(x) - \frac{4}{15} h^5 f^5(x)$$

$$4\left(f(x+h) + f(x-h)\right) = 4\left(f(x) + h^2 f^2(x)\right) + h^4 f^4(x) \cdot \frac{1}{3}$$

$$f(x+2h) + f(x-2h) = 2f(x) + 4h^2 f^2(x) + \frac{4}{3} h^4 f^4(x)$$

$$4\left(f(x+h) + f(x-h)\right) - \left(f(x+2h) - f(x-2h)\right) = 6f(x) - h^4 f^4(x)$$

$$f^4(x) = \frac{4f(x+h) + 4f(x-h) - f(x+2h) - f(x-2h) - 6f(x)}{-h^4}$$

$$f^4(x) = \frac{f(x+2) - 4f(x+1) + 6f(x) - 4f(x-1) + f(x-2)}{h^4}$$

The order is 2, $+ O(h^2)$
como en el punto de la sección 3.6
de las notas de clase