a) algoritmo de Numerov

consideramos el paso $h = x_n - x_{n-1} = x_{n+1} - x_n$
y expandimos en serie de taylor a $y(x)$

$$y(x_{n+1}) = y(x_n) + y'(x_n)h + \frac{y''(x_n)}{2!}h^2 + \frac{y'''(x_n)}{3!}h^3 + \frac{y''''}{4!}(x_n)h^4 + \frac{y'''''}{5!}(x_n)h^5 + \frac{y''''''(x_n)}{6!}h^6$$

$y(x_{n-1}) = y(x_n) - y'(x_n)h \ldots$ " " " " " con negativo en potencias impares

por tanto $y(n+1) + y(n-1) = 2y(x_n) + y''(x_n)h^2 + \frac{2y''''(x_n)}{4!}h^4 + O(h^6)$

y sabemos que $y''(x_n) = R_{x_n} y(x_n) + S(x_n)$

$$h^2 y''''(x_n) = R_{x_{n+1}} y_{x_{n+1}} + S_{x_{n+1}} - 2R_{x_n} y_{x_n} - 2S_{x_n} + R_{x_{n-1}} y_{x_{n-1}} + S_{x_{n-1}} + O(h^4)$$

reemplazamos en la ecuación

$$Y(n+1) + Y(n-1) = 2Y(x_n) + (R_{x_{n+1}} Y_{x_{n+1}} + S_{x_{n+1}} - 2R_{x_n} Y_{x_n} - 2S_{x_n} + R_{x_{n-1}} Y_{x_{n-1}} + S_{x_{n-1}})\frac{h^2}{12} + \delta(h^6)$$

$$Y_{x_{n+1}}\left[1 - \frac{h^2}{12} R_{x_{n+1}}\right] - 2Y_{x_n}\left[1 + \frac{5h^2}{12}\right] + Y_{x_{n-1}}\left[1 - \frac{h^2}{12} R_{x_{n-1}}\right] =$$

$$= \frac{h^2}{12}(S_{x_{n+1}} + 10S_{x_n} + S_{x_{n-1}}) + \delta(h^6)$$

b) Tenemos

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

Si usamos unidades naturales $m = 1$ $\omega = 1$ $\hbar = 1$

$$\frac{d^2\psi}{dx^2} - \frac{2m}{\hbar^2}V(x)\psi = -\frac{2m}{\hbar^2}E\psi$$

$$\frac{d^2\psi}{dx^2} - \frac{2m}{\hbar^2}V(x)\psi + \frac{2m}{\hbar^2}E\psi = 0$$

$$R_n = -\frac{2m}{\hbar^2}(V(x) - E) = -\frac{2m}{\hbar^2}\left(\frac{1}{2}m\omega^2x^2 - E\right)$$

$$R_n = -x^2 + E \qquad S_n = 0$$