

Punto 2.

Para 3 puntos

$$\Omega = \{(t_{n-2}, f_{n-2}), (t_{n-1}, f_{n-1}), (t_n, f_n)\}$$

$$P(x) = \prod_{i \neq j, i=0}^n \frac{x - x_i}{x_j - x_i}$$

$$P(t) = \frac{(t - t_n)(t - t_{n-1})f_{n-2}}{(t_{n-2} - t_n)(t_{n-2} - t_{n-1})} + \frac{(t - t_n)(t - t_{n-2})f_{n-1}}{(t_{n-1} - t_n)(t_{n-1} - t_{n-2})} \\ + \frac{(t - t_{n-2})(t - t_{n-1})f_n}{(t_n - t_{n-2})(t_n - t_{n-1})}$$

\* Tomando  $h = t_p - t_{p-1}$

$$P(t) = \frac{(t - t_n)(t - t_{n-1})f_{n-2}}{2h^2} + \frac{(t - t_n)(t - t_{n-2})f_{n-1}}{-h^2} \\ + \frac{(t - t_{n-2})(t - t_{n-1})f_n}{2h^2}$$

$$* t_{p-1} = t_p - h$$

$$* t' = t - t_n$$

$$* dt' = dt$$

$$P(t) = \frac{(t - t_n)(t - t_n + h)f_{n-2}}{2h^2} + \frac{(t - t_n)(t - t_n + 2h)f_{n-1}}{-h^2} \\ + \frac{(t - t_n + 2h)(t - t_n + h)f_n}{2h^2}$$

$$P(t) = \frac{t'(t' + h)f_{n-2}}{2h^2} + \frac{t'(t' + 2h)f_{n-1}}{-h^2} + \frac{(t' + 2h)(t' + h)f_n}{2h^2}$$



Haciendo la integral:

$$Y_{n+1} - Y_n = \int_0^h \left[ \frac{t'(t'+h)}{2h^2} f_{n-2} + \frac{t'(t'+2h)}{-h^2} f_{n-1} + \frac{(t'+2h)(t'+h)}{2h^2} f_n \right] dt'$$

$$\star \int_0^h \frac{t'(t'+h)}{2h^2} f_{n-2} dt' = \left( \frac{h}{6} + \frac{h}{4} \right) f_{n-2} = \frac{5h}{12} f_{n-2}$$

$$\star \int_0^h \frac{t'(t'+2h)}{-h^2} f_{n-1} dt' = \left( -\frac{h}{3} - h \right) f_{n-1} = -\frac{4h}{3} f_{n-1}$$

$$\star \int_0^h \frac{(t'+2h)(t'+h)}{2h^2} f_n dt' = \frac{23h}{12} f_n$$

$$Y_{n+1} - Y_n = \frac{5h}{12} f_{n-2} - \frac{4h}{3} f_{n-1} + \frac{23h}{12} f_n$$

$$Y_{n+1} = Y_n + \frac{h}{12} (23f_n - 16f_{n-1} + 5f_{n-2})$$

Para 4 puntos

$$\Omega = \{(t_{n-3}, f_{n-3}), (t_{n-2}, f_{n-2}), (t_{n-1}, f_{n-1}), (t_n, f_n)\}$$

$$P(t) = \frac{(t-t_{n-2})(t-t_{n-1})(t-t_n)}{(t_{n-3}-t_{n-2})(t_{n-3}-t_{n-1})(t_{n-3}-t_n)} f_{n-3} + \frac{(t-t_{n-3})(t-t_{n-1})(t-t_n)}{(t_{n-2}-t_{n-3})(t_{n-2}-t_{n-1})(t_{n-2}-t_n)} f_{n-2}$$

$$+ \frac{(t-t_{n-3})(t-t_{n-2})(t-t_n)}{(t_{n-1}-t_{n-3})(t_{n-1}-t_{n-2})(t_{n-1}-t_n)} f_{n-1} + \frac{(t-t_{n-3})(t-t_{n-2})(t-t_{n-1})}{(t_n-t_{n-3})(t_n-t_{n-2})(t_n-t_{n-1})} f_n$$



$$P(t) = \frac{(t-t_n+2h)(t-t_n+h)(t-t_n)}{(-h)(-2h)(-3h)} f_{n-3}$$

$$+ \frac{(t-t_n+3h)(t-t_n+h)(t-t_n)}{(h)(-h)(-2h)} f_{n-2}$$

$$+ \frac{(t-t_n+3h)(t-t_n+2h)(t-t_n)}{(2h)(h)(-h)} f_{n-1}$$

$$+ \frac{(t-t_n+3h)(t-t_n+2h)(t-t_n+h)}{(3h)(2h)(h)} f_n$$

$$P(t) = \frac{(t'+2h)(t'+h)t'}{-6h^3} f_{n-3} + \frac{(t'+3h)(t'+h)t'}{2h^3} f_{n-2}$$

$$+ \frac{(t'+3h)(t'+2h)t'}{-2h^3} f_{n-1} + \frac{(t'+3h)(t'+2h)(t'+h)}{6h^3} f_n$$

$$\star \int_0^h \frac{t'(t'+2h)(t'+h)}{-6h^3} dt' f_{n-3} = -\frac{3h}{8} f_{n-3}$$

$$\star \int_0^h \frac{t'(t'+3h)(t'+h)}{2h^3} dt' f_{n-2} = \frac{37h}{24} f_{n-2}$$

$$\star \int_0^h \frac{t'(t'+3h)(t'+2h)}{-2h^3} dt' f_{n-1} = -\frac{59h}{24} f_{n-1}$$

$$\star \int_0^h \frac{t'(t'+3h)(t'+2h)(t'+h)}{6h^3} dt' f_n = \frac{55h}{24} f_n$$

$$y_{n+1} = y_n + \frac{h}{24} (55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3})$$