Punto 3.

Para 3 Puntos

$$\Omega = \{(t_{n-1}, f_{n-1}), (t_n, f_n), (t_{n+1}, f_{n+1})\}$$

$$P(t) = \frac{(t-t_n)(t-t_{n+1})}{(t_{n-1}-t_n)(t_{n-1}-t_{n+1})} f_{n-1} + \frac{(t-t_{n-1})(t-t_{n+1})}{(t_n-t_{n-1})(t_n-t_{n+1})} f_n$$

$$+ \frac{(t-t_{n-1})(t-t_n)}{(t_{n+1}-t_{n-1})(t_{n+1}-t_n)} f_{n+1}$$

$h = t_{n+1} - t_n = t_n - t_{n-1}$ $\quad t' = t - t_n$

$dt' = dt$

$t_{n+1} = h + t_n \ ; \ t_{n-1} = t_n - h$

$$P(t) = \frac{t'(t'-h)}{(-h)(-2h)} f_{n-1} + \frac{(t'+h)(t'-h)}{(h)(-h)} f_n + \frac{(t'+h)t'}{(2h)(h)} f_{n+1}$$

$$* \int_0^h \frac{t'(t'-h)}{2h^2} dt' f_{n-1} = \frac{-h}{12} f_{n-1}$$

$$* \int_0^h \frac{(t'+h)(t'-h)}{-h^2} dt' f_n = \frac{2h}{3} f_n$$

$$* \int_0^h \frac{(t'+h)t'}{2h^2} dt' f_{n+1} = \frac{5h}{12} f_{n+1}$$

$$y_{n+1} = y_n + \frac{h}{12}\left[5f_{n+1} + 8f_n - f_{n-1}\right]$$

Para 4 puntos

$$\Omega = \{(t_{n-2}, f_{n-2}), (t_{n-1}, f_{n-1}), (t_n, f_n), (t_{n+1}, f_{n+1})\}$$

$$P(t) = \frac{(t-t_{n-1})(t-t_n)(t-t_{n+1})}{(t_{n-2}-t_{n-1})(t_{n-2}-t_n)(t_{n-2}-t_{n+1})} f_{n-2} + \frac{(t-t_{n-2})(t-t_n)(t-t_{n+1})}{(t_{n-1}-t_{n-2})(t_{n-1}-t_n)(t_{n-1}-t_{n+1})} f_{n-1} + \frac{(t-t_{n-1})(t-t_{n-2})(t-t_{n+1})}{(t_n-t_{n+1})(t_n-t_{n-2})(t_n-t_{n-1})} f_n + \frac{(t-t_{n-2})(t-t_{n-1})(t-t_n)}{(t_{n+1}-t_{n-2})(t_{n+1}-t_{n-1})(t_{n+1}-t_n)} f_{n+1}$$

$$P(t) = \frac{(t'+h)(t')(t'-h)}{(-h)(-2h)(-3h)} f_{n-2} + \frac{(t'+2h)(t')(t'-h)}{(h)(-h)(-2h)} f_{n-1} + \frac{(t'+h)(t'+2h)(t'-h)}{(2h)(h)(-h)} f_n + \frac{(t'+2h)(t'+h)(t')}{(3h)(2h)(h)} f_{n+1}$$

$$* \int_0^h \frac{(t'+h)(t')(t'-h)}{-6h^3} dt' f_{n-2} = \frac{h}{24} f_{n-2}$$

$$* \int_0^h \frac{(t'+2h)(t')(t'-h)}{2h^3} dt' f_{n-1} = \frac{-5h}{24} f_{n-1}$$

$$* \int_0^h \frac{(t'+h)(t'+2h)(t'-h)}{-2h^3} dt' f_n = \frac{19h}{24} f_n$$

$$* \int_0^h \frac{(t'+2h)(t'+h)(t')}{6h^3}\, dt' \, f_{n+1} = \frac{3h}{8} f_{n+1}$$

$$y_{n+1} = y_n + \frac{h}{24}\left[9f_{n+1} + 19f_n - 5f_{n-1} + f_{n-2}\right]$$