

Punto 1

$$a) \quad q' = f(q, p) = \frac{dq}{dt}$$

$$p' = g(q, p) = \frac{dp}{dt}$$

$$\frac{dE}{dt} = \frac{\partial E}{\partial q} \frac{dq}{dt} + \frac{\partial E}{\partial p} \frac{dp}{dt}$$

$$\frac{dE}{dt} = \begin{pmatrix} \frac{\partial E}{\partial q} & \frac{\partial E}{\partial p} \end{pmatrix} \begin{pmatrix} \frac{dq}{dt} \\ \frac{dp}{dt} \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial q} \frac{dq}{dt} & \frac{\partial}{\partial p} \frac{dq}{dt} \\ \frac{\partial}{\partial q} \frac{dp}{dt} & \frac{\partial}{\partial p} \frac{dp}{dt} \end{pmatrix} E = \begin{pmatrix} \frac{\partial f}{\partial q} & \frac{\partial f}{\partial p} \\ \frac{\partial g}{\partial q} & \frac{\partial g}{\partial p} \end{pmatrix} E$$

$$b) \quad x' = 2x - y$$

$$y' = x + 2y$$

$$M = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

$$\frac{dE}{dt} = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} E$$

$$E = E_0 \cdot e^{\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} t}$$

$$c) \det (M - \lambda I) = 0$$

$$(\lambda - 2)^2 + 1 = 0$$

$$\lambda^2 - 4\lambda + 5 = 0$$

$$\lambda_1 = 2 + i$$

$$\lambda_2 = 2 - i$$

$$(2 - 2 + i) V_1 - V_2 = 0$$

$$i V_1 + (2 - 2 + i) V_2 = 0$$

$$i V_1 = V_2$$

$$V_1 + i V_2 = 0$$

$$V_1 = \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$(2 - 2 - i) V_1 - V_2 = 0$$

$$V_1 + (2 - 2 - i) V_2 = 0$$

$$V_2 = \begin{pmatrix} 1 \\ i \end{pmatrix}$$