Punto 1

a) $\quad q' = f(q,p) = \dfrac{dq}{dt}$ $\qquad\qquad p' = g(q,p) = \dfrac{\partial p}{\partial t}$

$$\dfrac{dE}{dt} = \dfrac{\partial E}{\partial q}\dfrac{dq}{dt} + \dfrac{\partial E}{\partial p}\dfrac{dp}{dt}$$

$$\dfrac{dE}{dt} = \begin{pmatrix} \dfrac{\partial E}{\partial q} & \dfrac{\partial E}{\partial p} \end{pmatrix}\begin{pmatrix} \dfrac{dq}{dt} \\ \dfrac{dp}{dt} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial}{\partial q}\dfrac{dq}{dt} & \dfrac{\partial}{\partial p}\dfrac{dq}{dt} \\ \dfrac{\partial}{\partial q}\dfrac{dp}{dt} & \dfrac{\partial}{\partial p}\dfrac{dp}{dt} \end{pmatrix} E = \begin{pmatrix} \dfrac{\partial f}{\partial q} & \dfrac{\partial f}{\partial p} \\ \dfrac{\partial g}{\partial q} & \dfrac{\partial g}{\partial p} \end{pmatrix} E$$

b) $\quad x' = 2x - y \qquad\qquad y' = x + 2y$

$$M = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

$$\dfrac{dE}{dt} = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} E \qquad\qquad E = E_0 \cdot e^{\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} t}$$

c) $\det(M - \lambda I) = 0$

$$\det\begin{pmatrix} 2-\lambda & -1 \\ 1 & 2-\lambda \end{pmatrix} = 0$$

$(2-\lambda)^2 + 1 = 0 \qquad \lambda^2 - 2\lambda + 5 = 0$

$\lambda_1 = 1 \qquad\qquad \lambda_2 = 3$

$\begin{pmatrix} 2-1 & -1 \\ 1 & 2-1 \end{pmatrix} V_1 = 0 \qquad\qquad \begin{pmatrix} 2-3 & -1 \\ 1 & 2-3 \end{pmatrix} V_2 = 0$

$$V_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad\qquad V_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$