



c)
$$\begin{aligned} x_L &= d \cos(\omega t) \\ y_L &= d \sin(\omega t) \\ x_N &= r \cos(\phi) \\ y_N &= r \sin(\phi) \end{aligned}$$

(C)

$$\begin{aligned} x_{rL} &= d \cos(\omega t) - r \cos(\phi) \\ y_{rL} &= r \sin(\phi) - d \sin(\omega t) \end{aligned}$$

$$r_L = (d^2 \cos^2(\omega t) - 2dr \cos(\omega t) \cos(\phi) + r^2 \cos^2(\phi) + r^2 \sin^2(\phi) - 2dr \sin(\omega t) \sin(\phi) + d^2 \sin^2(\omega t))^{1/2}$$

$$r_L = (d^2 + r^2 - 2dr(\cos(\omega t) \cos(\phi) + \sin(\omega t) \sin(\phi)))^{1/2}$$

$$r_L = \sqrt{d^2 + r^2 - 2r(t)d \cos(\phi + \omega t)} \quad * \cos(x-y) = \cos x \cos y + \sin x \sin y$$

d)
$$P_r = m\dot{r} \quad P_\phi = m\dot{\phi} r^2$$

$$L = T - U$$

$$T = \frac{m\dot{v}^2}{2}$$

$$U = -mgh$$

$$\vec{r} = r \cos(\phi) \hat{i} + r \sin(\phi) \hat{j}$$

$$\dot{\vec{r}} = (\dot{r} \cos(\phi) + r(-\sin(\phi))\dot{\phi}) \hat{i} + (\dot{r} \sin(\phi) + r \cos(\phi)\dot{\phi}) \hat{j}$$

$$\dot{\vec{r}}^2 = \dot{r}^2 \cos^2(\phi) + 2\dot{r}r(-\sin(\phi))\cos(\phi)\dot{\phi} + r^2 \sin^2(\phi)\dot{\phi}^2 + \dot{r}^2 \sin^2(\phi) + 2\dot{r}r \sin(\phi) \cos(\phi)\dot{\phi} + r^2 \cos^2(\phi)\dot{\phi}^2$$

$$\dot{\vec{r}}^2 = \dot{r}^2 + r^2 \dot{\phi}^2$$

$$T = \frac{m(\dot{r}^2 + r^2 \dot{\phi}^2)}{2}$$

$$U = -mgh = -\frac{Emm_T}{r} - \frac{Emm_L}{r_L}$$

$$L = \frac{m(\dot{r}^2 + r^2 \dot{\phi}^2)}{2} + \left(\frac{Emm_T}{r} + \frac{Emm_L}{r_L} \right)$$

$$H = P_r \dot{r} + P_\phi \dot{\phi} - L$$

$$H = \frac{P_r^2 r^2}{mr^2} + \frac{P_\phi^2}{mr^2} - \left(\frac{m\dot{r}^2 + mr^2 \dot{\phi}^2}{2} \right) = \left(\frac{Emm_T}{r} + \frac{Emm_L}{r_L} \right)$$

$$H = \frac{2P_r^2 r^2 + 2P_\phi^2}{2mr^2} - \frac{m^2 r^2 \dot{r}^2 + m^2 r^2 \dot{\phi}^2}{2} - \left(\frac{Emm_T}{r} + \frac{Emm_L}{r_L} \right)$$

$$H = \frac{P_r^2 r^2}{2mr^2} + \frac{P_\phi^2}{2mr^2} - \left(\frac{Emm_T}{r} + \frac{Emm_L}{r_L} \right)$$

$$H = \frac{P_r^2}{2m} + \frac{P_\phi^2}{2mr^2} - \frac{Emm_T}{r} - \frac{Emm_L}{r_L}$$

$$\textcircled{c} \quad \dot{r} = \frac{\partial H}{\partial P_r} = \frac{2P_r}{2m} = \frac{P_r}{m}$$

$$\dot{\phi} = \frac{\partial H}{\partial P_\phi} = \frac{2P_\phi}{2mr^2} = \frac{P_\phi}{mr^2}$$

$$\dot{P}_r = -\frac{\partial H}{\partial r} = - \left[-\frac{2P_\phi^2}{2mr^3} - (-1)\frac{Emm_T}{r^2} - \frac{Emm_L}{r_L^3} \left(\frac{-1}{2} \right) (2r - 2d \cos(\phi - \omega t)) \right] = \frac{P_\phi^2}{mr^3} - \frac{Emm_T}{r^2} - \frac{Emm_L}{r_L^3} [r - d \cos(\phi - \omega t)]$$

$$\dot{P}_\phi = -\frac{\partial H}{\partial \phi} = - \left[-\frac{Emm_L}{r_L^3} \left(\frac{-1}{2} \right) (2r d \sin(\phi - \omega t)) \right] = -\frac{Emm_L}{r_L^3} r d \sin(\phi - \omega t)$$

$$\textcircled{f} \quad \tilde{r} = \frac{r}{d} \quad \tilde{P}_r = \frac{P_r}{md} \quad \tilde{P}_\phi = \frac{P_\phi}{md^2}$$

$$\dot{r} = \frac{P_r}{m} = \tilde{P}_r d = \tilde{r} \dot{d} \quad \dot{P} = \tilde{P} \quad * \quad \tilde{r} = \frac{r}{d}$$

$$\dot{\phi} = \frac{P_\phi}{mr^2} = \frac{\tilde{P}_\phi d^2}{r^2} = \frac{\tilde{P}_\phi d^2}{\tilde{r}^2 d^2} = \frac{\tilde{P}_\phi}{\tilde{r}^2}$$

$$\Delta = \frac{Em_T}{d^3} \quad \mu = \frac{m_L}{m_T} \quad \tilde{r}' = \sqrt{1 + \tilde{r}^2 - 2\tilde{r} \cos(\phi - \omega t)}$$

$$\dot{P}_r = \frac{P_\phi^2}{mr^3} - \frac{Emm_T}{r^2} - \frac{Emm_L}{r_L^3} [r - d \cos(\phi - \omega t)]$$

$$\dot{\tilde{P}}_r = \frac{1}{md} \left[\left(\frac{\tilde{P}_\phi^2 m^2 d^4}{m \tilde{r}^3 d^3} \right) - \frac{Emm_T}{\tilde{r}^2 d^2} - \frac{Emm_L}{r_L^3} [\tilde{r} d - d \cos(\phi - \omega t)] \right]$$

$$\tilde{r}_L = \sqrt{\tilde{r}^2 d^2 + d^2 - 2\tilde{r} d^2 \cos(\phi - \omega t)} = d \tilde{r}'$$

$$\dot{\tilde{P}}_r = \frac{1}{md} \left[\frac{\tilde{P}_\phi^2 md}{\tilde{r}^3} - \frac{Emm_T}{\tilde{r}^2 d^2} - \frac{Emm_L d}{d^3 \tilde{r}'^3} [\tilde{r} - \cos(\phi - \omega t)] \right]$$

$$\ddot{\vec{r}} = \frac{\ddot{P}_0^2}{r^3} - \frac{Em_T}{P^2 d^3} - \frac{Em_L}{r^4 d^3} \frac{m_T}{m_T} [\vec{r} - \cos(\phi - \omega t)]$$

$$\ddot{\vec{r}} = \frac{\ddot{P}_0^2}{r^3} - \Delta \left[\frac{1}{r^2} + \frac{\mu}{r^3} [\vec{r} - \cos(\phi - \omega t)] \right]$$

$$\dot{P}_0 = - \frac{Em_L}{r^3} r d \sin(\phi - \omega t)$$

$$\ddot{P}_0 = \frac{1}{m d^2} \left[- \frac{Em_L}{d^3 r^3} \frac{m_T}{m_T} \vec{r} d^2 \sin(\phi - \omega t) \right]$$

$$\ddot{P}_0 = - \frac{\Delta \mu \vec{r}}{r^3} \sin(\phi - \omega t)$$

$$\textcircled{9} \quad \vec{P}_r^0 = x\ddot{x} + y\ddot{y} = \frac{r\cos(\phi)V_0\cos(\theta)}{rd} + \frac{r\sin(\phi)V_0\sin(\theta)}{rd}$$

$$= \frac{V_0}{d} (\cos(\phi)\cos(\theta) + \sin(\phi)\sin(\theta)) = \frac{V_0 \cos(\theta - \phi)}{d}$$

$$\vec{r} = \frac{r}{d} \Rightarrow V_0 = \vec{V}_0 d$$

$$\vec{P}_r^0 = \vec{V}_0 \cos(\theta - \phi)$$

$$\ddot{P}_0^0 = \frac{\vec{r}^2}{r^2} (\ddot{y}x - y\ddot{x}) = \frac{\vec{r}^2}{r^2} (V_0 \sin(\theta) r \cos(\phi) - V_0 \cos(\theta) r \sin(\phi))$$

$$\ddot{P}_0^0 = \frac{r}{d^2} (\sin(\theta)\cos(\phi) - \cos(\theta)\sin(\phi)) V_0$$

$$\ddot{P}_0^0 = \frac{\vec{r} d \vec{V}_0 d}{d^2} \sin(\theta - \phi) = \vec{V}_0 \vec{V}_0 \sin(\theta - \phi)$$