Punto 2.

(a) Tenemos el algoritmo de Verlet de la forma:

$$x_{n+1} = 2x_n - x_{n-1} + h^2 x''_n$$

Por otro lado, podemos expresar x_n, x_{n-1}, x_{n+1} de la forma

$$x_n = \tilde{x}_n + \varepsilon_n$$

y la aceleración

$$a_n = a(\tilde{x}_n + \varepsilon_n) \overset{S.T.}{\approx} a(\tilde{x}_n) + \varepsilon_n a'_n$$

Reemplazando

$$\tilde{x}_{n+1} + \varepsilon_{n+1} = 2(\tilde{x}_n + \varepsilon_n) - \tilde{x}_{n-1} - \varepsilon_{n-1} + h^2(a(\tilde{x}_n) + \varepsilon a'_n)$$

$$\varepsilon_{n+1} - (2 + h^2 a'_n)\varepsilon_n + \varepsilon_{n-1} = \underbrace{-\tilde{x}_{n+1} + 2\tilde{x}_n - \tilde{x}_{n-1} + h^2(a(\tilde{x}_n))}_{0}$$

$$\varepsilon_{n+1} - (2 + h^2 a'_n)\varepsilon_n + \varepsilon_{n-1} = 0$$

(b) Partiendo de $2R = h^2\omega^2$ y la función anterior:

Relacionamos a con $\omega \Rightarrow a = -x\omega^2$ por lo tanto, $a' = -\omega^2$

Reemplazando,

$$\varepsilon_{n+1} - (2 - h^2\omega^2)\varepsilon_n + \varepsilon_{n-1} = 0$$
$$\varepsilon_{n+1} - (2 - 2R)\varepsilon_n + \varepsilon_{n-1} = 0$$
$$\varepsilon_{n+1} - 2(1 - R)\varepsilon_n + \varepsilon_{n-1} = 0$$

(c) $\varepsilon_n = \varepsilon_0 \lambda^n$

$$\varepsilon_0\lambda^{n+1} - 2(1-R)\varepsilon_0\lambda^n + \varepsilon_0\lambda^{n-1} = 0$$
$$\varepsilon_0\lambda^n(\lambda - 2(1-R) + \lambda^{-1}) = 0$$
$$\lambda(\lambda - 2(1-R) + \lambda^{-1}) = 0$$
$$\lambda^2 - 2(1-R)\lambda + 1 = 0$$

$$\lambda_\pm = \frac{2(1-R) \pm \sqrt{4(1-R)^2 - 4}}{2}$$

$$\lambda_\pm = (1-R) \pm \sqrt{(1-R)^2 - 1}$$
$$\lambda_\pm = (1-R) \pm \sqrt{\cancel{1} - 2R + R^2 - \cancel{1}}$$

$$\lambda_{\pm} = (1-R) \pm \sqrt{R^2-2R}$$

(d) $|\lambda_{\pm}| \leq 1 \Rightarrow \lambda_{\pm}$ puede ser 1 ó -1

$$1 \geq (1-R) \pm \sqrt{R^2-2R}$$
$$1-1+R \geq \pm\sqrt{R^2-2R}$$
$$R^2 \geq R^2-2R$$
$$0 \geq -2R$$
$$R=0$$

$$-1 \geq (1-R) \pm \sqrt{R^2-2R}$$
$$-1-1+R \geq \pm\sqrt{R^2-2R}$$
$$-2+R \geq \pm\sqrt{R^2-2R}$$
$$4-4R+R^2 \geq R^2-2R$$
$$4 \geq 2R$$
$$2 \geq R$$
$$R \leq 2$$

$$2R = h^2\omega^2 \Rightarrow R = \frac{h^2\omega^2}{2} \leq 2 \quad h^2 \leq \frac{4}{\omega^2} \quad \boxed{h \leq \frac{2}{\omega}}$$