

Punto 6.

$$r(t + \Delta t) = r(t) + v(t)\Delta t + \frac{1}{2}a(t)(\Delta t)^2$$

$$v(t + \Delta t) = v(t) + \frac{\Delta t}{2}(a(t + \Delta t) + a(t))$$

$$r(t - \Delta t) = r(t) + v(t)\Delta t + \frac{1}{2}a(t)(\Delta t)^2$$

$$r(t + \Delta t) = r(t) + v(t)\Delta t + \frac{1}{2}a(t)(\Delta t)^2$$

$$r(t - \Delta t) = r(t) - v(t)\Delta t + \frac{1}{2}a(t)(\Delta t)^2$$

Restando las ecuaciones

$$r(t + \Delta t) - r(t - \Delta t) = 2v(t)\Delta t$$

$$v(t) = \frac{r(t + \Delta t) - r(t - \Delta t)}{2(\Delta t)}$$

$$v(t + \Delta t) = \frac{r(t + 2\Delta t) - r(t)}{2(\Delta t)}$$

$$v(t + \Delta t) = \frac{1}{2\Delta t} (r(t + 2\Delta t) - r(t) + r(t + \Delta t) - r(t))$$

$$r(t + \Delta t) + r(t - \Delta t) - r(t - \Delta t)$$

$$- r(t + \Delta t) = -2r(t + \Delta t) + r(t + \Delta t)$$

$$- r(t) = -2r(t) + r(t)$$

$$v(t + \Delta t) = \frac{1}{2\Delta t} [r(t + 2\Delta t) - 2r(t + \Delta t) + r(t + \Delta t)$$

$$+ r(t) + r(t + \Delta t) - 2r(t) - r(t - \Delta t) + r(t - \Delta t)]$$

$$v(t + \Delta t) = \frac{1}{2\Delta t} [r(t + 2\Delta t) - 2r(t + \Delta t) + r(t) +$$

$$r(t + \Delta t) - 2r(t) + r(t - \Delta t)] + \frac{1}{2\Delta t} [r(t + \Delta t) - r(t - \Delta t)]$$

$$v(t + \Delta t) = \frac{1}{2\Delta t} [r(t + 2\Delta t) - 2r(t + \Delta t) + r(t) +$$

$$- 2r(t) + r(t - \Delta t)] + v(t)$$

$$v(t+\Delta t) = \frac{\Delta t}{2} \left[\frac{r(t+2\Delta t) - 2r(t+\Delta t) + r(t)}{(\Delta t)^2} \right]$$

$$+ \left[\frac{r(t+\Delta t) - 2r(t) + r(t-\Delta t))}{(\Delta t)^2} \right] + v(t)$$

$$a(t) = \frac{2(r(t+\Delta t) - r(t) - v(t)\Delta t)}{(\Delta t)^2}$$

$$a(t+\Delta t) = \frac{2(r(t+2\Delta t) - r(t+\Delta t) - v(t+\Delta t)\Delta t)}{(\Delta t)^2}$$

$$a(t) = \frac{2r(t+\Delta t) - 2r(t) - r(t+\Delta t) + r(t-\Delta t))}{(\Delta t)^2}$$

$$a(t+\Delta t) = \frac{2r(t+2\Delta t) - 2r(t+\Delta t) - r(t+2\Delta t) + r(t)}{(\Delta t)^2}$$

$$a(t) + a(t+\Delta t) = \frac{2r(t+2\Delta t) + 2r(t+\Delta t) - 2r(t+\Delta t) - 2r(t) - r(t+\Delta t) + r(t-\Delta t) - r(t+2\Delta t) + r(t)}{(\Delta t)^2}$$

$$a(t) + a(t+\Delta t) = \frac{r(t+2\Delta t) - 2r(t+\Delta t) + r(t) + r(t+\Delta t) - 2r(t) + r(t-\Delta t))}{(\Delta t)^2}$$

$$v(t+\Delta t) = \frac{\Delta t}{2} \left[\frac{r(t+2\Delta t) - 2r(t+\Delta t) + r(t)}{(\Delta t)^2} \right] +$$

$$\left[\frac{r(t+\Delta t) - 2r(t) + r(t-\Delta t))}{(\Delta t)^2} \right] + v(t)$$

$$v(t+\Delta t) = v(t) + \frac{\Delta t}{2} [a(t+\Delta t) + a(t)]$$

$$\bullet \vec{r}(0) = [a(1+e), 0] \quad e = \frac{c}{a} \Rightarrow c = ea$$

$$R = c + a = ae + a \Rightarrow R = a(1+e)$$

$$\bullet \vec{v}(0) = \left[0, \sqrt{\frac{6M}{a(1+e)}} \right] \quad E = \frac{1}{2}mv^2 - \frac{6Mm}{r}$$

$$\frac{1}{2}mv_a^2 - \frac{6Mm}{r_a} = \frac{1}{2}mv_p^2 - \frac{6Mm}{r_p}$$

$$L = mrv$$

$$mr_p v_p = mr_a v_a \quad v_p = \frac{r_a}{r_p} v_a$$

$$\frac{1}{2}v_a^2 - \frac{1}{2}\frac{v_a^2}{\frac{r_p}{r_a}} = -\frac{6M}{r_p} + \frac{6M}{r_a}$$

$$v_a^2 = 26M \left(\frac{r_p(r_p - r_a)}{r_a(r_a^2 + r_p^2)} \right) \Rightarrow v_a = \sqrt{26M \frac{r_p}{r_a(r_a - r_p)}}$$

$$r_p = -ae + a \Rightarrow r_p = a(1-e)$$

$$v_a = \sqrt{26M \frac{a(1-e)}{a(1+e)(a(1+e) + a(1-e))}}$$

$$v_a = \sqrt{26M \frac{a(1-e)}{a(1+e)(2a)}}$$

$$v_a = \sqrt{\frac{6M(1-e)}{a(1+e)}} = \sqrt{\frac{6(1-e)}{a(1+e)}}$$

$$M=1$$