Homework 2.1: Computer assignment

Deep Learning Discrete Calculus for Engineering Applications, Special Topics in Mechanical Engineering (MECH-ENG 395-0-1), Prof. W.K. Liu, Spring 2023, Northwestern University.

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View code: https://github.com/Bakeey/ME395/tree/main/HW_02-1

Chichi Earthquake

We get the displacement signal for the chichi earthquake:

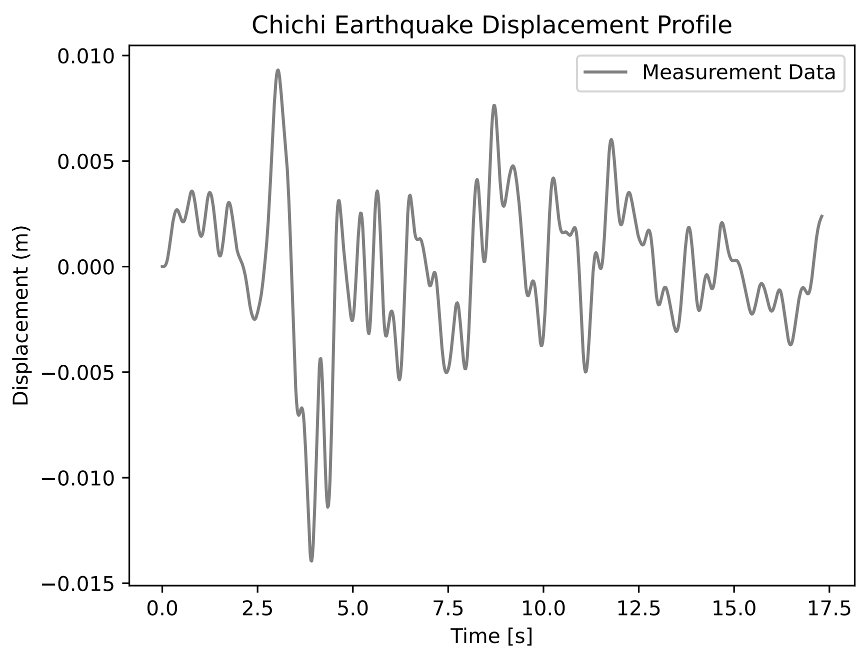

Figure 1: Chichi Earthquake Displacement Profile

Next, we get the velocity profile using a differentiation filter. We use the forward difference (Euler forward) and central difference filter and can compare these two methods to the given velocity measurement signal in figure 2.

We see that the computed velocity signal seems to be generally in agreement with the measured velocity signal, and plot the difference of the computed signal (differentiated from displacement using filter) to the measured signal in figure 3. Unsurprisingly, central difference is more accurate than forward difference.

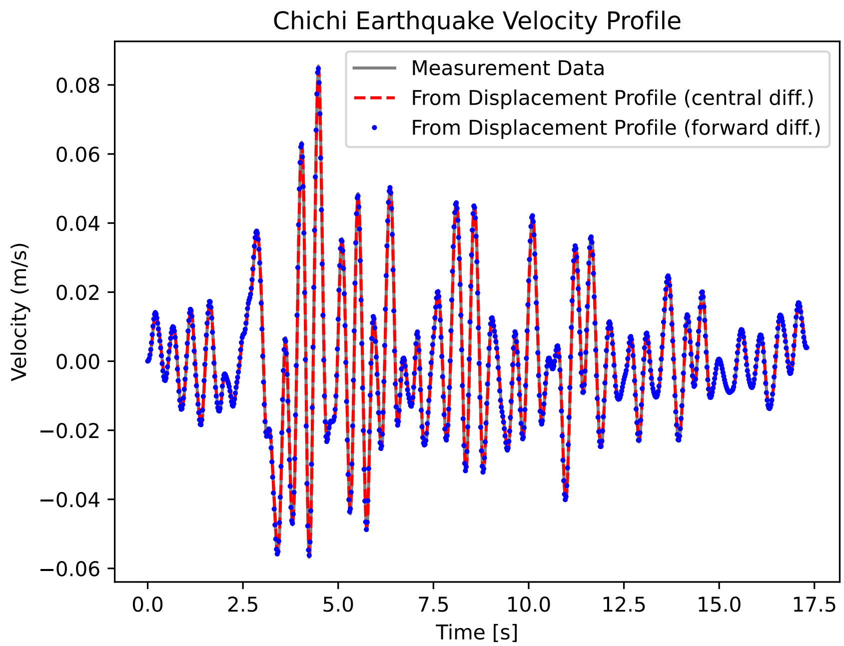

Figure 2: Chichi earthquake velocity, measurement and computed via filter.

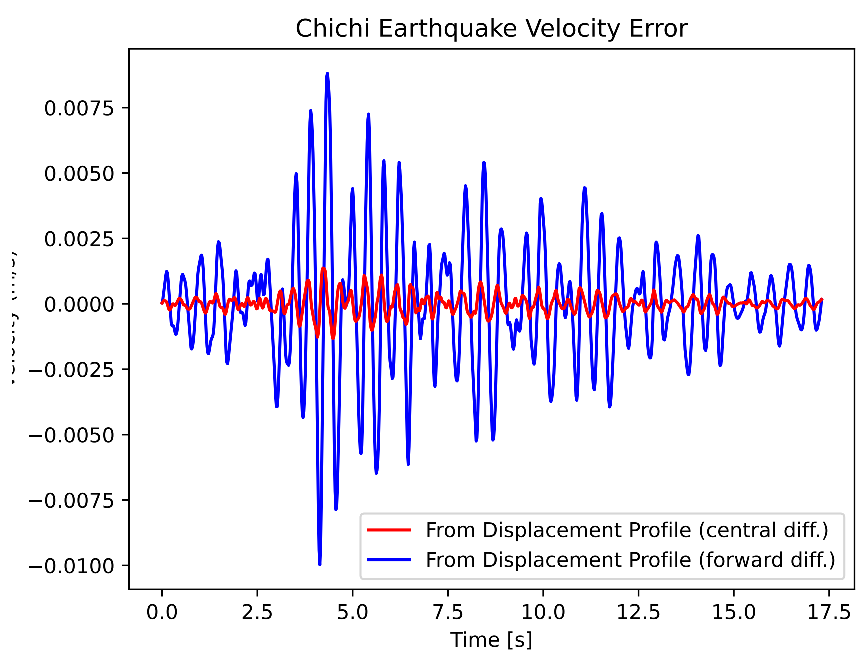

Figure 3: Chichi earthquake velocity error, comparing computed velocity differentiated from displacement to measurmeent velocity.

Lastly, we can derive the acceleration signal by differentiating again.

First, we perform differentiation using the central difference method on the velocity data computed from the displacement profile using central difference.

Secondly, we perform backward difference on the forward-difference velocity profile.

Third, we obtain the acceleration profile directly from the displacement measurement using second-order finite difference:

$$\frac{d^2u}{dt^2}(t_j) = \frac{u(t_{j+1}) - 2u(t_j) + u(t_{j-1})}{(t_j - t_{j-1})^2} \tag{1}$$

We plot all of thse derived acceleration signals in figure 4. Similarly to figure 3, compare the computed acceleration signal to the given acceleration measurement in figure 5.

We notice that the acceleration error for the signal using backwards differentiation is the same as for the 2nd order central difference method. This is because as seen in the handwritten homework, performing differentiation twice using forward and backward difference yields the same result as the 2nd order central difference method seen in equation 1.

Lastly, we see that performing first order central difference twice to receive a second order derivative results in a larger error. This is because since approximating the 1st order derivative via central differences can be written as

$$u'(x) \approx \frac{u(t_{j+1}) - u(t_{j-1})}{(t_{j+1} - t_{j-1})}$$

What is the main issue with applying again a central difference to compute $u''(t_j)$ is:

$$\frac{du'}{dt}(x) = \frac{u'(t_{j+1}) - u'(t_{j-1})}{(t_{j+1} - t_{j-1})} \approx \frac{u(t_{j+2}) - 2u(t_j) + u(t_{j-2})}{(t_{j+1} - t_{j-1})^2}$$

Which is less accurate than applying forward and backward difference!

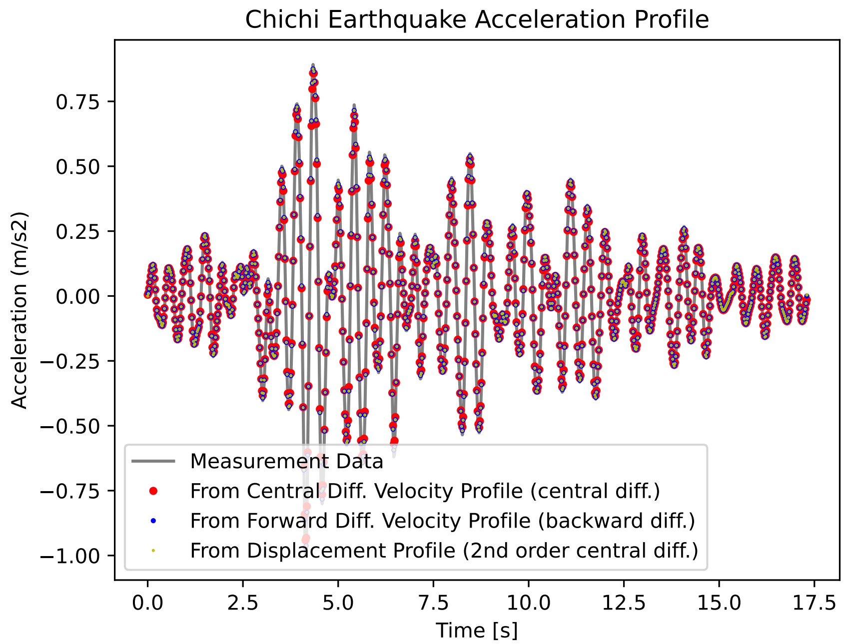

Figure 4: Chichi earthquake acceleration, measurement and computed via filter.

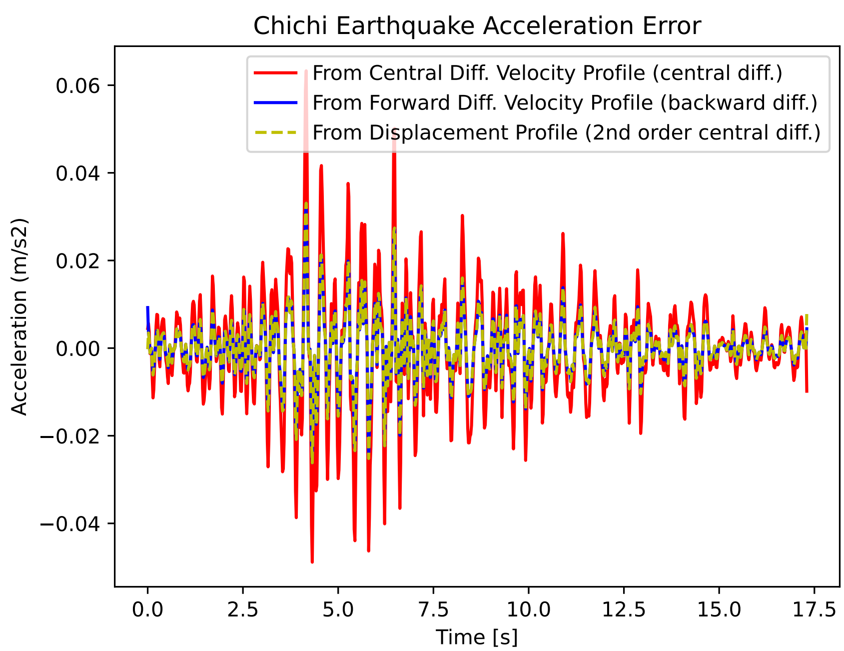

Figure 5: Chichi earthquake acceleration error, comparing computed acceleration to measurmeent acceleration.

Imperial Earthquake

We again get the displacement signal, this time for the imperial earthquake.

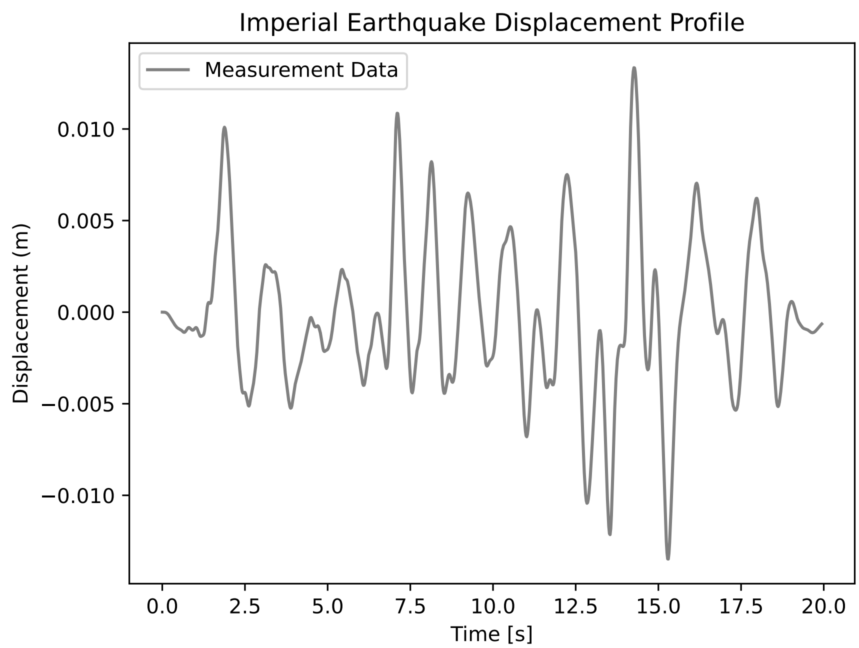

Figure 6: Imperial Earthquake Displacement Profile

Next, we get the velocity profile using a differentiation filter.

In general, the procedure is the same as for the Chichi earthquake.

We use the forward difference (Euler forward) and central difference filter in figure 7.

We see that the computed velocity signals seem to be generall in agreement. Now, we can derive the acceleration signal by differentiating again.

First, we perform differentiation using the central difference method on the velocity data computed from the displacement profile using central difference.

Secondly, we perform backward difference on the forward difference velocity profile.

Third, we obtain the acceleration profile directly from the displacement measurement using second-order finite difference. The results can be seen in figure 8

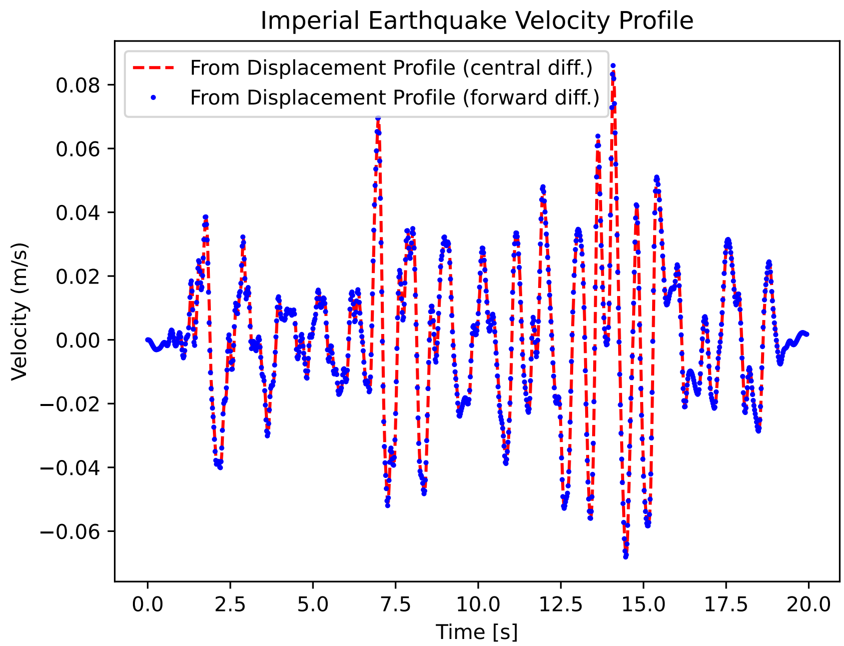

Figure 7: Imperial earthquake velocity, computed via filter.

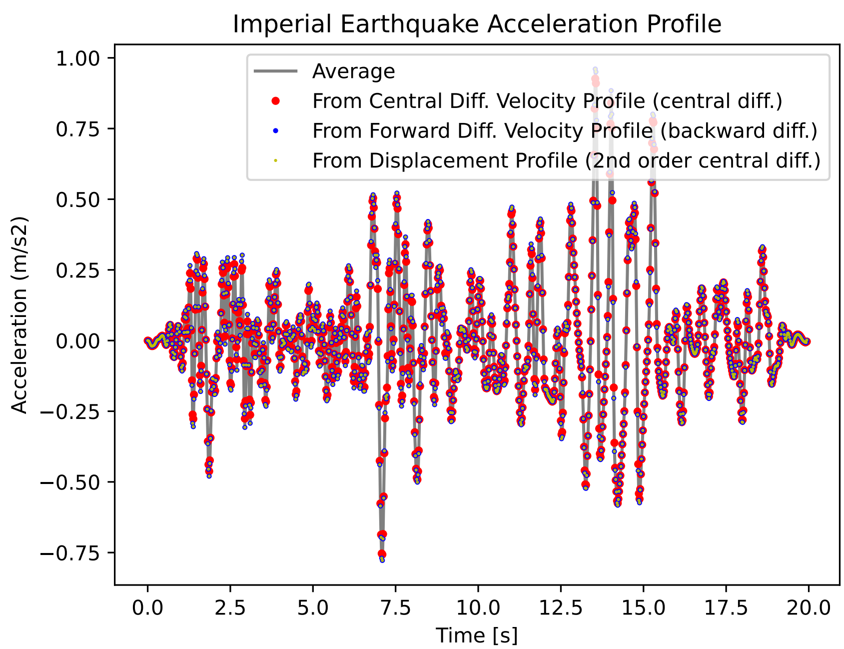

Figure 8: Imperial earthquake acceleration, computed via filter.

Lastly, we can plot the difference between the acceleration derived by differentiation from the velocity profiles to the one directly differentiated from the displacement profile. We can again see that there is no difference between taking forward and backward difference to taking second order central difference directly (apart from the edges, due to padding).

We can also again see that applying first-order central difference is less accurate.

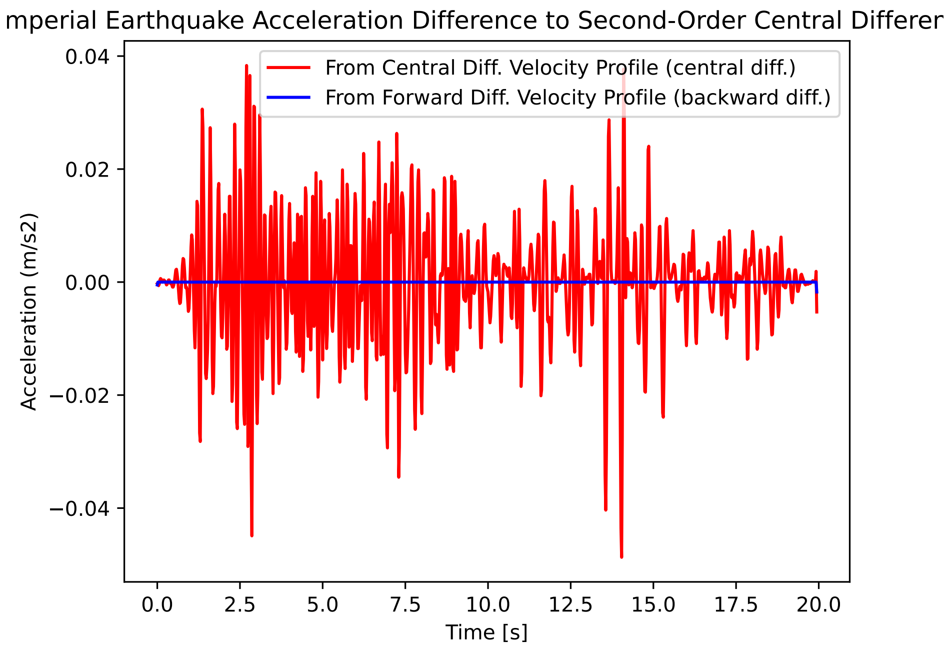

Figure 9: Chichi earthquake acceleration error, comparing computed acceleration to measurmeent acceleration.