Klemens ITEN

ME 395 Handwritten Homework

Given forward taylor & backward taylor expansion:

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2}h^2 + \mathcal{O}(h^3) \qquad (I)$$

$$f(x_{i-1}) = f(x_i) - f'(x_i)h + \frac{f''(x_i)}{2}h^2 - \mathcal{O}(h^3) \qquad (II)$$

Adding (I) and (II) gives us:

$$f(x_{i+1}) + f(x_{i-1}) = 2f(x_i) + f''(x_i)h^2 + \mathcal{O}(h^4)$$

Rearranging terms gives us:

$$f''(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1}) + \mathcal{O}(h^4)}{h^2}$$

$$= \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2} + \mathcal{O}(h^2)$$

Thus, the 2^{nd} order central finite difference filter is given by:

$$f''(x_i) \approx \frac{1}{h^2}\begin{bmatrix} 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} f(x_{i-1}) \\ f(x_i) \\ f(x_{i+1}) \end{bmatrix}$$

which is accurate to $\mathcal{O}(h^2)$.