Homework 3: Estimation
Active Learning for Robotics (ME 455), Spring 2023, Northwestern University.

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1 Particle Filter

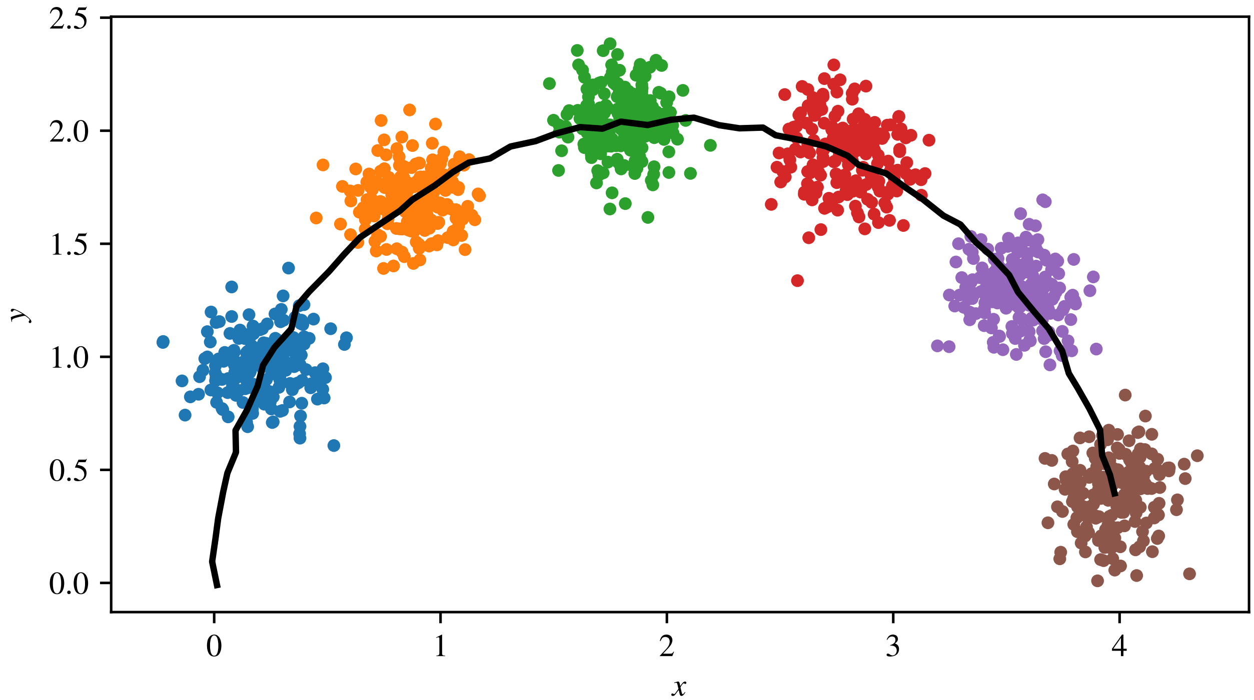

Figure 1.1: Each color represents a snapshot of the particles at a particular time. The black line is the state estimate of x and y every $dt = 0.1$ seconds.

2 Kalman Filter

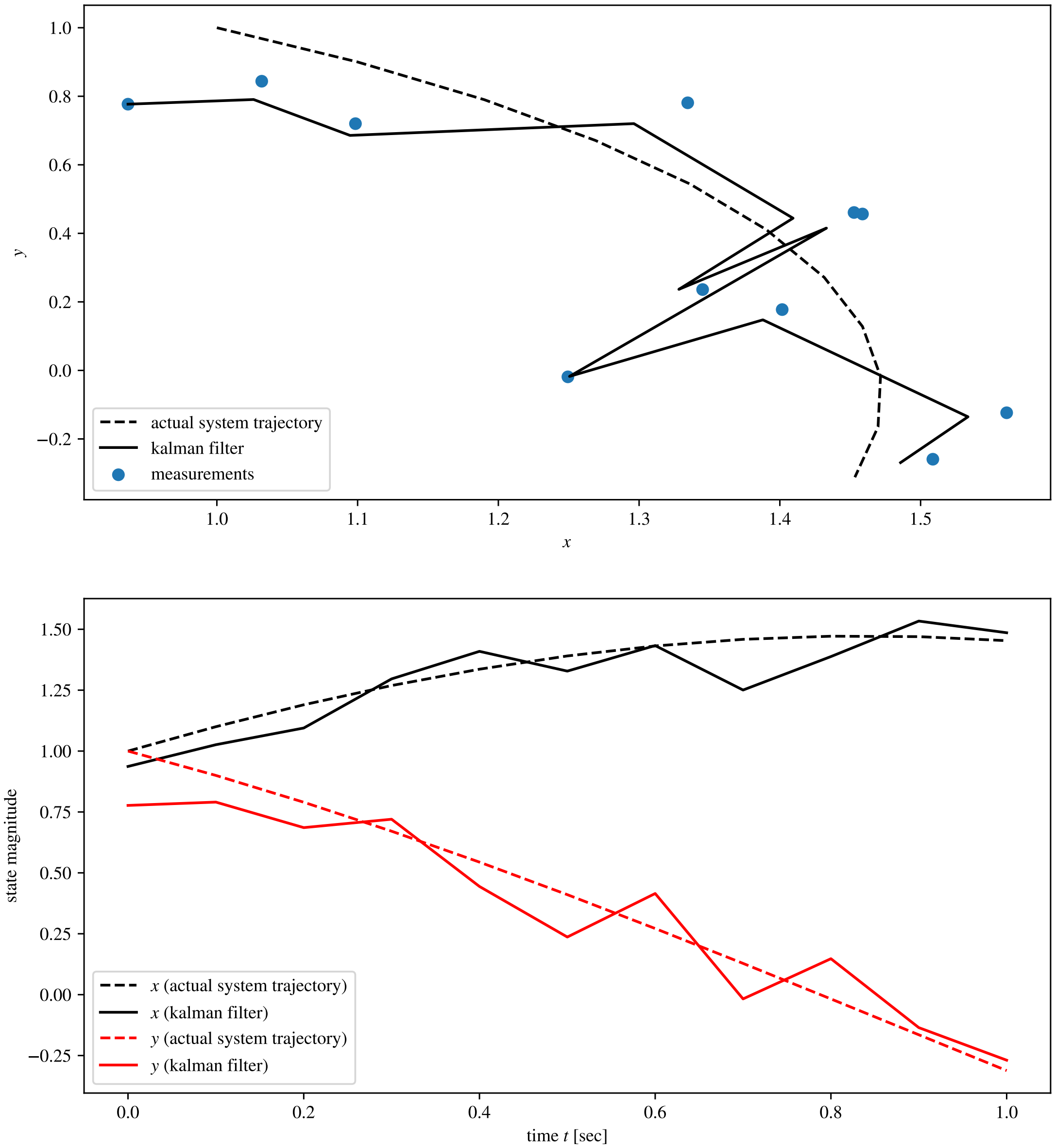

Figure 2.1: State evolution given measurements subject to normally distributed process and measurement noise with variance of 0.1. The nominal state evolution starting from initial condition $[1, 1], dt = 0.1$ is shown dashed. The filter state evolution is initialized at the first measurement with uncertainty (covariance of 0.1).

In figure 2.1, we can see in the upper plot that the filtered trajectory apparently "jumps" around the map. This is due to the large measurement errors (variance of 0.1 $\approx$ standard deviation of 0.3 – while our entire map only has size 0.5 by 1!). However, in the lower plot, we can see that the filter does in fact keep our filtered state roughly in the same region as the nominal system trajectory.

The filtered state has the following covariances:

Updated (a posteriori) estimate covariance $P_{k\|k} = \begin{bmatrix} \mathrm{Cov}(x,x) & \mathrm{Cov}(x,y) \\ \mathrm{Cov}(y,x) & \mathrm{Cov}(y,y) \end{bmatrix}$ with $\mathrm{Cov}(x,y) = \mathrm{Cov}(y,x) = 0$			
Measurement	t (sec)	$\mathrm{Cov}(x,x)$	$\mathrm{Cov}(y,y)$
1	0.0	0.1	0.1
2	0.1	0.06677740	0.06677740
3	0.2	0.07518671	0.07518671
4	0.3	0.08023872	0.08023872
5	0.4	0.08360925	0.08360925
6	0.5	0.08601793	0.08601792
7	0.6	0.08782497	0.08782497
8	0.7	0.08923071	0.08923071
9	0.8	0.09035542	0.09035542
10	0.9	0.09127568	0.09127568
11	1.0	0.09204254	0.09204254

We can see that the covariance is a diagonal matrix (since measurements of x and y are independent random variables) initialized at our state/sensor noise variance of 0.1.

This initial covariance gets reduced (no filtering for the first step since we initialize our filter at the first uncertain measurement), but thanks to the Kalman filter we can reduce the covariance below 0.1. The covariance grows in magnitude (additive noise over time) and presumably approaches 0.1 gradually.

3 Error and Optimality

In figure 3.1, we plot the mean error and standard deviation over 100 trajectories for a (scaled) Kalman gain. The x-axis denotes the scalar factor that the Kalman gain was multiplied by before computing the updated (a posteriori) state and covariance estimate.

We can see in the table below that the results are relatively noisy, possibly attibuted to the large variance in state and measurement noise. However, the mean error is lowest for the Kalman filter, indicating that it is indeed optimal.

Mean error and standard deviation for scaled Kalman gain $a \cdot K$			
No.	a	Mean Error	Standard Deviation
1	0.80	0.15626832	0.0834248
2	0.84	0.15434268	0.0813475
3	0.88	0.153419	0.0824559
4	0.92	0.1520322	0.0827037
5	0.96	0.15366641	0.0838657
6	1.00	**0.15154664**	0.0802521
7	1.04	0.15273871	0.0852308
8	1.08	0.15204482	0.0835434
9	1.12	0.15206308	0.0814685
10	1.16	0.1536205	0.0826514
11	1.20	0.1588911	0.0836619

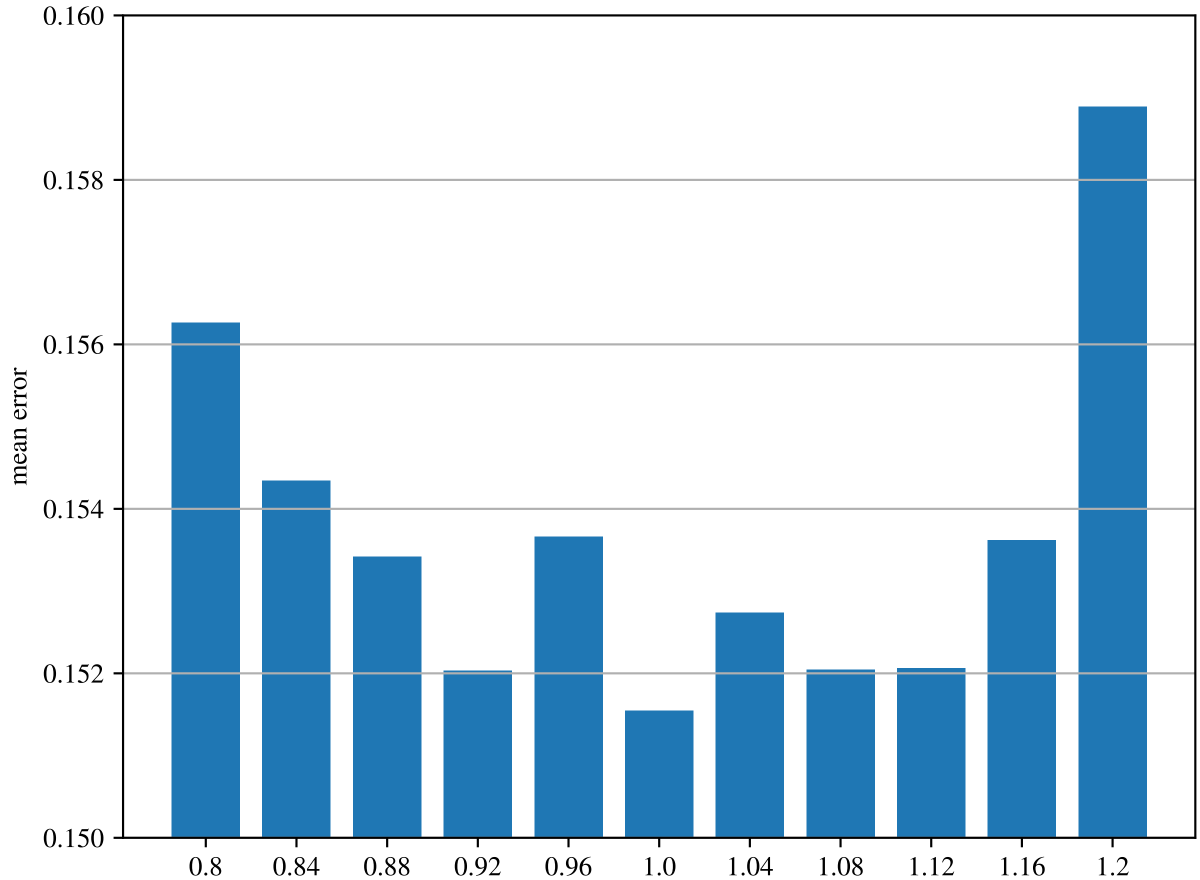

Figure 3.1: Mean error and standard deviation over 100 trajectories for a (scaled) Kalman gain. The mean error is lowest for a scaling factor of 1 (i.e. regular Kalman filter, indicating that it is indeed optimal).