PAMO Stream Test 2

June Camp 2017

Time: $4\frac{1}{2}$ hours

1. Let $\mathbb{Z}$ and $\mathbb{Q}$ be the sets of itnegers and rationals respectively. Does there exist a partition of $\mathbb{Z}$ into three non-empty subsets A, B, C such that the sets $A+B$, $B+C$ and $C+A$ are disjoint?

2. Let ABC be a triangle with orthocentre H. Prove that the triangle formed by the angle bisectors of AH, BH and CH is congruent to ABC.

3. Find all positive integers $n, k_1, k_2, \ldots, k_n$ such that $k_1 + k_2 + \cdots + k_n = 5n - 4$ and
$$\frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_n} = 1.$$