

PAMO Stream Test 3

June Camp 2017

Time: $4\frac{1}{2}$ hours

1. In a triangle, let r be the radius of the incircle, r_A , r_B and r_C be the radii of the excircles opposite A , B and C respectively, and s be the semiperimeter (i.e. $s = (AB + BC + CA)/2$). Prove that

$$r_A r_B r_C = r s^2.$$

2. Let x , y and z be positive real numbers such that $x + y + z = 3$. Show that

$$\sqrt{x} + \sqrt{y} + \sqrt{z} \geq xy + yz + zx.$$

3. Two players, Lauren and Dylan play the following game with a big slab of chocolate divided into a grid of squares: with Lauren playing first, they take turns choosing a square of chocolate that hasn't been eaten yet, and eating that square together with all the squares below it and to its right. For instance, if in a 5 by 6 slab of chocolate Lauren chooses square (3,4), she also eats squares (3,5), (3,6), (4,4), (4,5), (4,6), (5,4), (5,5) and (5,6). However, the top left square (at position (1,1)) has been poisoned, so whoever eats that square dies, and even worse, loses.
 - (a) If the slab of chocolate has dimensions 2017 by 2017, who has the winning strategy and what is it?
 - (b) If the slab of chocolate has dimensions m by n , where $m \neq n \in \mathbb{N}$, which player has the winning strategy?