IMO Stream Test 2

June Camp 2017

Time: $4\frac{1}{2}$ hours

1. Let ABC be a triangle with $AB = AC \neq BC$ and let I be its incentre. The line BI meets AC at D, and the line through D perpendicular to AC meets AI at E. Prove that the reflection of I in AC lies on the circumcircle of triangle BDE.

2. Let n, m, k and l be positive integers with $n \neq 1$ such that $n^k + mn^l + 1$ divides $n^{k+l} - 1$. Prove that either

   - $m = 1$ and $l = 2k$; or
   - $l \mid k$ and $m = \frac{n^{k-l}-1}{n^l-1}$.

3. Let $n \geq 3$ be a positive integer. Find the maximum number of diagonals of a regular n-gon one can select, so that any two of them do not intersect in the interior or are perpendicular to each other.