IMO Stream Test 1

June Camp 2017

Time: $4\frac{1}{2}$ hours

1. The leader of an IMO team chooses positive integers n and k with $n > k$, and announces them to the deputy leader and a contestant. The leader then secretly tells the deputy leader an n-digit binary string, and the deputy leader writes down all n-digit binary strings which differ from the leader's in exactly k positions. (For example, if $n = 3$ and $k = 1$, and if the leader chooses 101, the deupty leader would write down 001, 111 and 100.) The contestant is allowed to look at the strings written by the deputy leader and guess the leader's string. What is the minimum number of guesses (in terms of n and k) needed to guarantee the correct answer?

2. Let ABC be a triangle with circumcircle Γ and incentre I. Let M be the midpoint of the side BC. Denote by D the foot of the perpendicular from I to side BC. The line through I perpendicular to AI meets sides AB and AC at F and E respectively. Suppose the circumcircle of triangle AEF intersects Γ at a point X other than A. Prove that the lines XD and AM meet on Γ.

3. Denote by $\mathbb{R}^+$ the set of all positive real numbers. Find all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ such that

$$xf(x^2)f(f(y)) + f(yf(x)) = f(xy)\left(f(f(x^2)) + f(f(y^2))\right)$$

for all positive real numbers x and y.