R-Project Proof

Sunday, November 8, 2020 4:19 PM

Show that the minimum order statistic converges in probability to zero

* Hint: CDF $Exp(\lambda) = f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & o.w. \end{cases}$

Start with probability you want to show from the Definition of convergence in probability to zero.

$P(|Y_{(1)} - 0| < \epsilon)$ and take lim as $n \to \infty$ and show that this probability converges to $\underline{1}$.

Let $Y_1, Y_2, Y_3, \ldots, Y_n \sim exp(\lambda)$

Y_1 = minimum value of $c[Y_1 : Y_n]$

$$\lim_{n\to\infty} P(|Y_n - 0| < \epsilon) = \lim_{n\to\infty} P(|Y_n| < \epsilon)$$

Absolute value inequal. property

$$= \lim_{n\to\infty} P(-\epsilon < Y_n < \epsilon)$$

$$= \lim_{n\to\infty} P(0 < Y_n < \epsilon)$$

$$= \lim_{n\to\infty} P(F_Y(y))$$

$\text{CDF} \sim \text{Exp}(\lambda) = 1 - e^{-\lambda x}$

$\lambda = 1$

$$= \lim_{n \to \infty} P(1 - e^{-\lambda n})$$

$$= \lim_{n \to \infty} P(1 - e^{-(1)n})$$

$$= \lim_{n \to \infty} P(1) - \lim_{n \to \infty} (e^{-n})$$

$$= 1 - 0$$

$$= 1$$

$$\lim_{n \to \infty} P(|Y_1 - 0| \geq \epsilon) = \lim_{n \to \infty} P(|Y_1 - 0| \geq \epsilon)$$

$$= \lim_{n \to \infty} P(|Y_1| \geq \epsilon)$$

$$= \lim_{n \to \infty} P(-\epsilon > Y_1 \geq \epsilon)$$

$$= \lim_{n \to \infty} P(0 > Y_1 \geq \epsilon)$$

$$= \lim_{n \to \infty} P(1 - (1 - e^{-\lambda x}))$$

$\text{CDF} \sim \text{Exp}(\lambda) = 1 - e^{-\lambda x}$

$\lambda = 1$

$$= \lim_{n \to \infty} P(e^{-(1)x})$$

$$= 0$$