Equation for Global Warming

Derivation and Application

Contents

1. Amazing carbon dioxide

How can a small change in carbon dioxide (CO_2) content make a critical difference to the actual global surface temperature of the Earth?

2. Derivation of IPCC equation $\Delta F = 5.35\, ln\, (C/C_0)$

Equation gives the increase in heat flux density, ΔF (in Watts/m^2) when CO_2 concentration increases from C_0 to C ppm.

Two distinctly different derivations are given:

2.1 Derivation One - uses an equation derived from the Heat Transfer Equation.

2.2 Derivation Two – mostly uses first principles and includes:

- Derivation of the basic equation: $F = 5.35\, ln\, C$
- Setting of initial conditions giving: $\Delta F = 5.35\, ln\, (C/C_0)$
- Calculation of CO_2 flux density and comparison with the other non-condensing greenhouse gases that maintain a temperature structure for the atmosphere.

3. Derivation of temperature increase equation: $\Delta T = 1.66\, ln\, (C/C_0)$

Equation gives the temperature increase (ΔT) when CO_2 concentration increases from C_0 to C ppm.

4. Calculation of temperature increase for doubling CO_2 content

Calculation of the temperature increase for instant doubling the atmospheric CO_2 content when there is:

- no feedback; and
- feedback from a change in water vapour opacity due to a change in temperature.

Evaluation of the temperature increase for instant doubling the atmospheric CO_2 content when all feedbacks are included

5.   References

1.   Amazing carbon dioxide

Why is a trace gas, such as carbon dioxide, (only 0.04% of air) referred to as the control knob of the Earth's thermostat? How can a small change in carbon dioxide (CO_2) content make a critical difference to the actual global surface temperature of the Earth? Nitrogen and oxygen comprise the bulk of the atmosphere but do not absorb the earth's heat radiation. Although water vapour and clouds together absorb 75% of the Earth's heat radiation[1] they cannot determine the temperature of the atmosphere. Water vapour and clouds depend on temperature and air circulation in ways that CO_2 does not. They condense and cannot maintain a *temperature structure* for the atmosphere. CO_2 accounts for 80% of the non-condensing gases that maintain the temperature structure of the Earth and acts as the control knob of the Earth's thermostat. It controls the amount of water vapour and clouds.

CO_2 absorption is strong as it absorbs in the frequency range where the Earth's heat emission (Planck field) is strongest. The instant *doubling* of CO_2 content (e.g. from pre-industrial 280 ppm to 560 ppm) would reduce the Earth's emission of heat radiation to space by about 4 Watts for every square metre of the Earth's surface. CO_2 absorption is that strong. The atmospheric temperature must be raised to radiate an extra 4 Watts per square metre to restore the Earth's energy balance. The increased surface temperature of 1.2^oC from the instant doubling of CO_2 content allows an increased water vapour content by maintaining a constant relative humidity. The extra water vapour increases the overall absorption by water vapour itself raising the surface temperature further by about 1.2^oC. The total increase is about **3^oC** when all feedbacks are included.

Although the temperature of Mars, Earth and Venus are affected by their distance from the Sun and by the sunlight they reflect to space, their surface temperature is strongly determined by their atmospheric density of carbon dioxide and water vapour as shown in the table:

	Mars	Earth	Venus
CO_2 density	very low	Significant	extremely high
Water vapour	little (0.03%)	global average 0.4%	little left (0.002%)
Average surface temp.	**minus 50^oC**	**15^oC**	**460^oC**
Greenhouse effect	**minus ($5C^o$)**	**significant (+$33C^o$)**	**"runaway" (+$400C^o$)**

Table 1 - Surface temperature is strongly determined by the atmospheric density of carbon dioxide and water vapour

Figure 1 gives the climate impact for increasing concentrations of CO2:
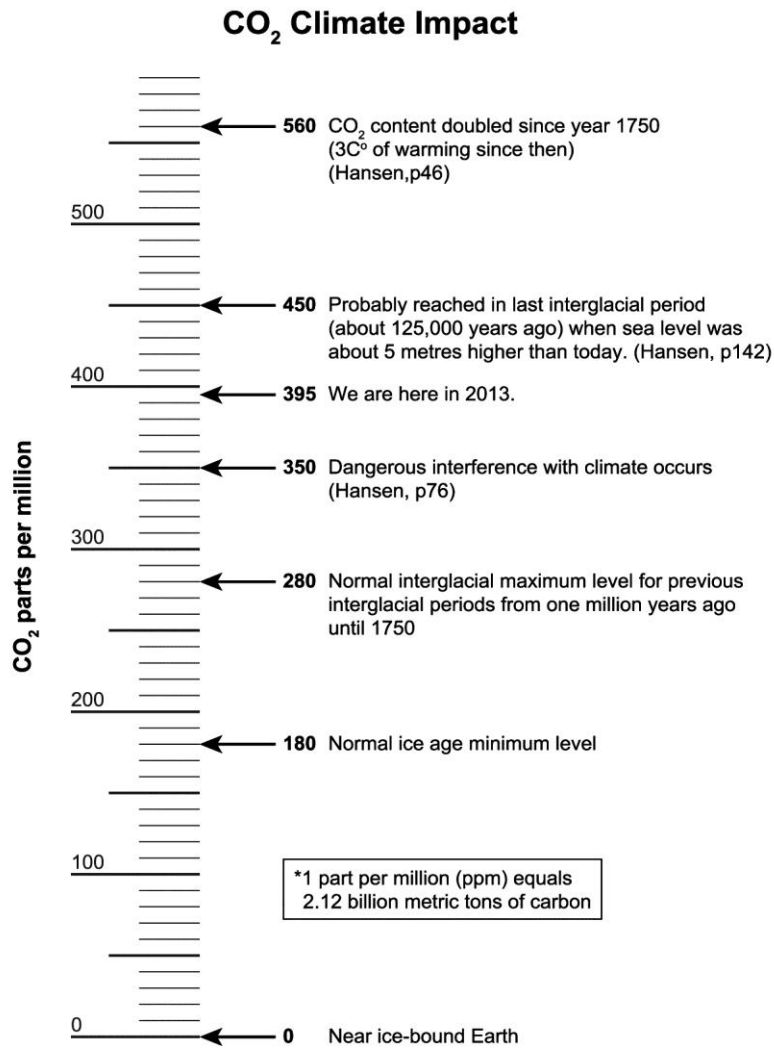
CO₂ Climate Impact

Figure 1 – Climate impact for increasing concentrations of CO$_2$

2.   Derivation of IPCC expression ΔF = 5.35 ln (C/C$_0$)

2.1   Derivation One

The assumptions we will make allow us to represent the real atmosphere. This remarkably reasonable representation of the real atmosphere is due in part to the small mean optical thickness of the Earth's atmosphere. We assume that the atmosphere is transparent to visible radiation and heating only occurs at the Earth's Finally, we assume local thermodynamic equilibrium. This means that in a localised atmospheric volume below 40kms we consider it to be isotropic (emission is non-directional) with a uniform temperature. Here emissivity equals absorptivity. Two temperatures (T$_e$ and T$_s$) are important. The *effective emission temperature* (T$_e$) is the temperature the Earth would have without an atmosphere just taking into account its reflectivity and its distance from the sun. The Earth radiates as a black body in the Infrared spectrum. We calculate the effective emission temperature by assuming the rate of the Earth's energy absorption equals the rate of emission.

$$S\pi r^2\left(1-\ \alpha_p\right)=4\pi r^2\sigma T_e^4$$

Where the solar constant, S = 1366 W/m^2 and the planetary albedo, α$_p$ = 0.3244

Stefan-Boltzmann law for the Earth as a black body (or perfect radiator) gives:

F = σT^4
where   F is the flux density emitted in W/m^2
             σ is the Stefan-Boltzmann constant, and
             T is the absolute temperature.

$$F=\sigma T_e^4\ =\ F_e$$

Therefore, the flux (F) absorbed by the climate system is:

$$F_e\ =S\frac{\left(1-\alpha_p\right)}{4} \tag{4}$$

T$_s$ is the surface air temperature and **F**$_{g▶a}$ (**g**round to **a**tmosphere) is the upward flux density (heat) radiated from the surface (σT_s^4).

First we calculate the vertical opacity of the atmosphere (τ_g) from the Chamberlain[4] expression that he derived from the general heat transfer equation:

$$T_s^4=\ T_e^4\left(1+\frac{3}{4}\tau_g\right) \tag{5}$$

$$T_s^4=\ \frac{F_e}{\sigma}\left(1+\frac{3}{4}\tau_g\right)=\frac{\left(1-\alpha_p\right)S}{4\sigma}\left(1+\frac{3}{4}\tau_g\right)\ from\ Equations\ 4\ and\ 5$$

$$F = \sigma T_S^4 = \frac{(1-\alpha_p)S}{4}\left(1 + \frac{3}{4}\tau_g\right) = F_{g\blacktriangleright a} \tag{6}$$

We differentiate F with respect to τ

$$\frac{dF}{d\tau} = \frac{(1-\alpha_p)S}{4} \; X \; \frac{3}{4} \quad or \quad \Delta F = \frac{3S(1-\alpha_p)}{16}\Delta\tau \tag{7}$$

The following formula[2] is used to calculate $\Delta\tau$:

$$\tau_{CO_2} = 1.73\,(CO_2)^{0.263} \quad where\ CO_2\ is\ in\ ppmv\ X\ 10^{-6} \tag{8}$$

$$or\ \tau_{CO_2} = 0.457\,(CO_2)^{0.263} \quad where\ CO_2\ is\ in\ ppmv$$

Here $\tau = aC^b$ \hfill (9)

where a and b are constants

The initial conditions are $\tau_o = aC_0^b$ \hfill (10)

On dividing Equation (9) by Equation (10) and taking the natural logarithm of both sides we get:

$$\Delta\tau = \tau_0\left\{e^{b\ln\left(\frac{C}{C_0}\right)} - 1\right\}\ \text{where b =0.263} \tag{11}$$

$$\Delta F = \frac{3S(1-\alpha_p)}{16}\,\tau_0\left\{e^{0.263\ln\left(\frac{C}{C_0}\right)} - 1\right\} \tag{12}$$

Equation 29 below can be expressed:

$$\frac{dF_{a\blacktriangleright g}}{dT} = \frac{4\sigma T^3}{A} = \frac{1}{A}X\frac{dF_{g\blacktriangleright a}}{dT} \quad where\ A = 1 + \frac{3}{4}\tau_g$$

$$\int_{T_1}^{T_2}\frac{dF_{a\blacktriangleright g}}{dT}dT = \frac{1}{A}X\int_{T_1}^{T_2}\frac{dF_{g\blacktriangleright a}}{dT}dT$$

$$\boxed{\Delta F_{a\blacktriangleright g} = f_a\ X\ \Delta F_{g\blacktriangleright a}\quad where\ \ f_a = \frac{1}{A}\ and\ A = 1 + \frac{3}{4}\tau_g}$$

$f_a = 0.6$ is the fraction of flux returned downward to the Earth, absorbed or re-emitted by CO2 . This is consistent with the IPCC result. The flux density directed downward to warm the surface further is the CO_2 greenhouse flux density ($\Delta F_{a\blacktriangleright g}$) and the equation for the CO2 greenhouse flux density (W/m^2) is:

$\Delta F = \Delta F_{a \blacktriangleright g} = f_a \ X \ \Delta F_{g \blacktriangleright a}$

Therefore, $\Delta F = f_a \dfrac{3S(1 - \alpha_p)}{16} \tau_0 \left\{ e^{0.263 \ln\left(\frac{C}{C_0}\right)} - 1 \right\}$ (13)

We can use the following
identity to clarify the exponential term: $e^x \doteq 1 + x \ for \ x < 1$

$We \ set \ x = 0.263 \ \ln\left(\dfrac{C}{C_0}\right) \ then \ e^{0.263 \ln\left(\frac{C}{C_0}\right)} - 1 \doteq 0.263 \ \ln\left(\dfrac{C}{C_0}\right)$

$as \ \ 0.263 \ \ln\left(\dfrac{C}{C_0}\right) < 1 \ for \ C_0 = 280 \ ppm \ and \ 280 \le C \le 1000 \ ppm$

ΔF = 0.6 X 173.0 X 0.201 X 0.263 ln (C/C$_0$)

ΔF = 5.487 ln (C/C$_0$)   (14)

But the IPCC equation includes a small amount of CO_2 absorption of high frequency solar radiation[6] that reduces ΔF by 0.06 W/m^2. Adjusting Equation 14 for CO_2 solar absorption gives:

ΔF = 5.40 ln (C/C$_0$)   (15)

The coefficient, 5.40 is within the one percentage point margin of error of the IPCC result of 5.35 for their coefficient. We will use the IPCC result later.

2.2 Derivation Two

T_s is the surface air temperature and $F_{g \blacktriangleright a} = \sigma T_s^4$ is upward flux density (heat) radiated from the surface. f is the fraction of Earth's heat radiation in the spectral interval over which CO_2 absorption is significant. f X $F_{g \blacktriangleright a}$ is the amount of Earth's heat radiation in the spectral interval over which CO_2 absorption is significant. Some of the heat radiation emanating from the surface will be absorbed by CO_2 before passing through the remainder of the atmosphere. f X $F_{g \blacktriangleright a}$X $(1 - T_d)$ is the amount of flux density from the surface absorbed by CO_2 in the upper atmosphere and re-emitted equally in all directions both upward and downward. $(1\text{-}T_d)$ is the fractional absorption. f_a is the fraction of flux density returned downward to the Earth, absorbed or re-emitted by CO_2. The flux density directed downward to warm the surface further is the CO_2 greenhouse flux density ($F_{a \blacktriangleright g}$) and the equation for the CO2 greenhouse flux density is:

F = f_a X f X $F_{g \blacktriangleright a}$X $(1 - T_d)$
where:

       F is the CO_2 greenhouse flux density in W/m^2 ($F_{a \blacktriangleright g}$)

f_a is the fraction of flux returned downward to the Earth, absorbed or re-emitted by CO_2

f is the Planck Blackbody Fraction (The fraction of Earth's heat radiation in the spectral interval over which CO_2 absorption is significant)

$F_{g \blacktriangleright a}$ is the total flux density emitted by the Earth's surface (σT_s^4)

T_d is the diffuse transmittance

$(1-T_d)$ is the fractional absorption

Opacity measures the degree of opaqueness. Infrared opacity (or optical depth) of carbon dioxide (τ_{CO_2}) measures the degree to which infrared radiation sees carbon dioxide as opaque. It describes the extent of absorption and scattering of infrared radiation and is measured downwards from the top of the atmosphere. The following formula[2] is used:

$$\tau_{CO_2} = 1.73 \, (CO_2)^{0.263} \quad where \; CO_2 \; is \; in \; units \; of \; ppmv \; X \; 10^{-6}$$

The spectral interval from 550 cm^{-1} to 1015 cm^{-1} is chosen to calculate the Planck Fraction.

The formula for $1 - T_d$ expressed in terms of τ is:

$$1 - T_d = 1 - 2 \int_0^1 \mu e^{-\tau/\mu} d\mu \; = 1 - 2E_3(\tau)$$

$E_3(\tau) \; is \; the \; exponential \; integral \; and \; E_3(0) = \dfrac{1}{2}$

This gives the formula: $1 - T_d = 0.05371 \, \textbf{\textit{ln}} \, C$ where $1 \le C \le 1000$ ppmv
Using this formula we can derive the IPCC result[3] as follows:

F = 0.6 X 0.4256 X390 X 0.05371 *ln* C

(16)

$$\boxed{F = 5.35 \; \textit{ln} \; C}$$

The CO2 GHG flux density (F_0) at initial concentration C_0 is given by:

$F_0 = 5.35 \; ln \; C_0$ (17)

Equation 16 – Equation 17 gives:

$\Delta F = F - F_0 = 5.35 \; ln \; C - 5.35 \; ln \; C_0 = 5.35 \; ln \; (C/C_0)$

$$\boxed{\Delta F = 5.35 \; \textbf{\textit{ln}} \; (C/C_0)}$$

(18)

Equation 18 is the IPCC result. The coefficient (5.35) in the IPCC result has an uncertainty of 1%. This gives the derivation from first principles of the IPCC simplified equation. We now calculate ΔF for instant doubling of the atmospheric CO_2 content by setting $C = 2C_0$ in Equation 18 and we get $\Delta F = 3.71$ W/m^2.

The logarithmic relationship between CO_2 concentration and radiative forcing (ΔF) means that each further doubling of CO_2 content gives an extra 3.71 W/m^2 and a further temperature increase. Even on Venus which has 10,000 times more CO_2 than Earth and an average surface temperature of 460°C, the CO_2 absorption is not saturated. This is because on Earth, Mars and Venus it is always possible to find a higher layer of the atmosphere (with lower partial pressure, lower τ_{CO_2} and more narrow absorption lines) to absorb the heat and then radiate it up to space and down to the ground.

The equation F = 5.*35 ln* C, Equation 16 above, is important for calculating CO_2 flux density (F) at concentration C in the atmosphere. CO_2 concentration reached 400 ppm on 11th May 2013 and F is 32 W/m2 or about 20% of total greenhouse gas flux density of the Earth's Greenhouse effect. However, it is in the comparison with the *non-condensing* greenhouse gases where CO_2 gets its controlling influence. CO_2 accounts for 80% of the greenhouse gas flux density of the *non-condensing* greenhouse gases that maintain the temperature structure of the Earth and acts as the control knob of the Earth's thermostat.

3.   Derivation of the temperature increase equation:

$\Delta T = 1.66\, ln\ (C/C_0)$

The assumptions we will make allow us to represent the real atmosphere. This remarkably reasonable representation of the real atmosphere is due in part to the small mean optical thickness of the Earth's atmosphere. "Instant" doubling means there is no feedback from a change in water vapour opacity due to a change in temperature. We assume that the atmosphere is transparent to visible radiation and heating only occurs at the Earth's surface (Grey atmosphere). There is no convection and scattering can be neglected. Finally, we assume local thermodynamic equilibrium. This means that in a localised atmospheric volume below 40kms we consider it to be isotropic (emission is non- directional) with a uniform temperature. Here Kirchhoff's Law is applicable so that emissivity equals absorptivity. Two temperatures (T_e and T_s) are important. The *effective emission temperature* (T_e) is the temperature the Earth would have without an atmosphere just taking into account its reflectivity and its distance from the sun. The flux (F) absorbed by the climate system as:

$$F_e = S\frac{(1-\alpha_p)}{4} \tag{19}$$

$where\ \ solar\ constant, S = 1366 \ {}^{W}/_{m^2} \ and\ the\ planetary\ albedo, \alpha_p = 0.32$

Stefan-Boltzmann law for the Earth as a black body (or perfect radiator) gives:

F = σT**4**
where   F is the flux density emitted in W/m^2
                    σ is the Stefan-Boltzmann constant, and
                    T is the absolute temperature.

$$F = \sigma T_e^4 \ = \ F_e \tag{20}$$

T_s is the surface air temperature and $F = \sigma T_s^4$ is intermediate upward flux density (heat) radiated from the surface. ϵ is the fraction of the upward flux (σT_s^4) that is absorbed by the atmosphere and equals that subsequently emitted hence $F = \ \epsilon \sigma T_s^4$

The radiation absorbed in the upper atmosphere at temperature T_a is re-emitted equally in all directions, half upward and half downward. Hence,

$$Hence, \ \epsilon \sigma T_s^4 = 2\epsilon \sigma T_a^4 \ and \ T_a^4 = \frac{T_s^4}{2} \tag{21}$$

$If \ T_s = 288.15^o K \ then \ T_a = 242.3^o K \ in\ the\ upper\ atmosphere.$

The flux density out of the top of the atmosphere is given by:

$$F_{out} = \ \epsilon \sigma T_a^4 + (1 - \ \epsilon)\sigma T_s^4$$

Parameterisation gives:

$$\Delta F_{out} = \ \Delta \epsilon (\sigma T_a^4 - \ \sigma T_s^4) = \ \Delta \epsilon \sigma \frac{T_s^4}{2} \ \ from \ Equation \ 21 \tag{22}$$

First we calculate the vertical opacity of the atmosphere (τ_g) from the Chamberlain[4] expression that he derived from the general heat transfer equation:

$$T_s^4 = \ T_e^4 \left(1 + \frac{3}{4}\tau_g\right) = \ T_e^4 \ A \ \ where \ A = 1 + \frac{3}{4}\hat{o}_g \tag{23}$$

$$T_s^4 = \ A \frac{F_e}{\sigma} = A \frac{(1 - \alpha_p)S}{4\sigma} \ from \ Equations \ 19 \ and \ 20$$

$$\sigma T_s^4 = \ A \frac{(1 - \alpha_p)S}{4} \tag{24}$$

We now determine the relation of T_s to ΔF through τ using:

$$\frac{dT_s}{dF} = \frac{dT_s}{d\tau} X \frac{d\tau}{d\epsilon} X \frac{d\epsilon}{dF} \tag{25}$$

$$\epsilon = 2\left(1 - \frac{1}{1+\frac{3}{4}\tau_g}\right)$$

$$\frac{d\epsilon}{d\tau} = \frac{3}{2} X \frac{1}{\left(1+\frac{3}{4}\tau_g\right)^2} \ or \ \frac{d\tau}{d\epsilon} = \frac{2}{3} A^2 \tag{26}$$

Taking the derivative of Equation (23) and substituting for T_e we have:

$$\frac{dT_s}{d\tau} = \frac{3T_s}{16\left(1+\frac{3}{4}\tau_g\right)} = \frac{3T_s}{16A} \tag{27}$$

$$\frac{d\epsilon}{dF} = \frac{2}{\sigma T_s^4} \tag{28}$$

Substituting Equations 24,26,27 and 28 in Equation 25 we have:

$$\frac{dT_s}{dF} = \frac{3T_s}{16A} X \frac{2}{3} A^2 X \frac{2}{\sigma T_s^4} = \frac{A}{4\sigma T_s^3} = \frac{T_s}{(1-\acute{a}_p)S} \tag{29}$$

$$\boldsymbol{\Delta T_s} = \frac{T_s}{(1-\alpha_p)S} \boldsymbol{\Delta F} = \frac{T_s}{928.88} \boldsymbol{\Delta F} = \boldsymbol{0.31\ \Delta F} \tag{30}$$

Substituting for ΔF from Equation 18, ΔF = 5.35 ln (C/C$_0$)

$$\boldsymbol{\Delta T_s} = \frac{T_s}{(1-\alpha_p)S} X\ 5.35\ \ln(C/C_0) = \frac{T_s}{173.62} \ln(\boldsymbol{C/C_0}) \tag{31}$$

Substituting for T_s, in Equation 31 gives:

$$\boxed{\Delta T = 1.66\ ln\ (C/C_0)} \tag{32}$$

Greenhouse gases, including carbon dioxide and water vapour, keep the Earth's surface about 33°C warmer than it would otherwise be. How much warming does carbon dioxide itself contribute to the current surface temperature of the Earth? We can calculate the CO_2 flux density (F) at concentration C in the current atmosphere using Equation 16, F = 5.35 ln C, from Section 2 above. CO_2 concentration reached 400 ppm on 11th May 2013 and therefore F is 32.05 W/m^2. From Equation 30 we have:

$$\boxed{\Delta T = 0.31\ \Delta F} \tag{33}$$

Therefore, ΔT = 0.31 X 32.05 = **10°C**

Water vapour adds a further 75 W/m^2 giving total ΔF = 107.05 W/m^2. Surface temperature increase ΔT = 0.31 X 107.05 = **33°C**. That is, CO_2 and water vapour increase the surface temperature of the Earth by 33°C.

4.   Calculation of temperature increase for doubling CO_2 content

We have shown that an *instant doubling* of the CO_2 content increases the opacity (opaqueness) of the atmosphere to heat radiation reducing its emission to space by about 4 Watts/m^2. We need to calculate the rise in temperature needed to warm up the Earth's atmosphere to radiate an extra 4 Watts/m^2 to restore the Earth's energy balance. This disturbance to the Earth's energy balance is referred to as a climate forcing of 4 Watts/m^2. First let's calculate the temperature increase for instant doubling the atmospheric CO_2 content when there is no feedback.

We calculate ΔF for instant doubling of the atmospheric CO_2 content by setting C = $2C_0$ in Equation 32:

ΔT = 1.66 *ln* (C/C_0) = 1.66 X 0.693 = 1.2^{o}C

$$Instant\ doubling\ of\ CO_2\ gives\ the\ classic\ result\ \Delta T_s = 1.2^o\ C$$

Let's now calculate the temperature increase for instant doubling the atmospheric CO_2 content when there is feedback from a change in water vapour opacity due to a change in temperature. The increased surface temperature from the instant doubling of CO_2 content allows an increased water vapour content by maintaining a constant relative humidity. According to the Clausius-Clapeyron Equation (36 below) the 1.2 ^{o}C increase in the Earth's surface temperature due to CO2 itself gives a full 8% increase in the amount of water vapour held at saturation in the warmer atmosphere or a 6.2% increase at 0.77 global average relative humidity. The extra water vapour opacity increases the overall absorption by water vapour itself raising the surface temperature further.

The opacity of water vapour is a function of the water vapour partial pressure (P) and is given[2] by:

$$\tau = 0.0126\ P^{0.503} \tag{34}$$

On differentiation we get:

$$\Delta\tau = 0.00634\ \frac{\Delta P}{P_0^{0.497}} \tag{35}$$

The water vapour partial pressure is a function of temperature and relative humidity[2]

$$P = H(P_0 e^{-(L/RT)})$$ (36)

where $R = 8.3145$ Jmol^{-1}K^{-1}, molar gas constant

$L = 43655$ Jmol^{-1}, latent heat per mole of water

$P_0 = 1.4 \times 1011$ Pa, water vapour saturation constant

$H = 0.77$, global average relative humidity

Using Equation 36 and T = 288.15°K we obtain P_0 = 1315.86 Pa. The extra water vapour contained by the warmer atmosphere raises the partial pressure. If P_1 and P_2 are the partial pressures at two temperatures T_1 and T_2 respectively, Equation 36 takes the form:

$$P_2 = P_1 \, e^{\left\{-\frac{L}{R}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)\right\}}$$ (37)

Using Equation 37 let's calculate the increase in partial pressure when the temperature increases from 288.15 °K to 289.35 °K.

$$P_2 = 1315.86 \, e^{\left\{-\frac{43655}{8.3145}\left(\frac{1}{289.35}-\frac{1}{288.15}\right)\right\}}$$
$$= 1419.15 \, Pa$$

ΔP = P_2 – P_1 =103.30 Pa

From Equation 35 we have:

$$\Delta\tau = 0.00634 \, \frac{103.3}{(1315.86)^{0.497}} = 0.01844$$

Now the extra water vapour does its own absorption raising the surface temperature further. From Equations 24 and 27 we have:

ΔT = 32.09 Δτ = 0.5917°C

This further increase in surface temperature will cause another cycle of water vapour feedback and so on. The temperature converges to 290.54°K after 12 cycles of the water vapour feedback loop as shown in Table 2 below:

Cycle	T_0	T_1	P_0	P_1	ΔP	$\Delta \tau$	ΔT
1	288.15	289.35	1315.86	1419.15	103.3	0.01844	0.5917
2	289.35	289.94	1419.15	1472.69	53.54	0.00921	0.2954
3	289.94	290.24	1472.69	1500.09	27.4	0.00462	0.1484
4	290.24	290.39	1500.09	1514.02	13.93	0.00233	0.0748
5	290.39	290.46	1514.02	1521.09	7.065	0.00118	0.0377
6	290.46	290.5	1521.09	1524.67	3.577	0.00059	0.0191
7	290.5	290.52	1524.67	1526.48	1.81	0.0003	0.0096
8	290.52	290.53	1526.48	1527.39	0.915	0.00015	0.0049
9	290.53	290.53	1527.39	1527.85	0.463	7.7E-05	0.0025
10	290.53	290.53	1527.85	1528.09	0.234	3.9E-05	0.0012
11	290.53	290.54	1528.09	1528.21	0.118	2E-05	0.0006
12	290.54	290.54	1528.21	1528.27	0.06	9.9E-06	0.0003

Table 2 – Each row gives the calculations for one cycle of the positive feedback loop for water vapour. The temperature converges after 12 cycles giving 2.4°C as the total surface temperature increase.

In general, the water vapour feedback induced by the initial CO_2 content increase will <u>double</u> the sensitivity of the global surface temperature.

Water vapour provides a fast feedback after a temperature increase. Ice sheet melting is a slow feedback but still positive. Fortunately, we can evaluate the climate sensitivity for all fast feedbacks from 20,000 years ago to the present from the ice core data as shown by Hansen[5].

$$Climate\ sensitivity = \frac{Temperature\ change}{Climate\ forcing}$$

Ice sheets have trapped air bubbles of the atmosphere over that period. The ice core data show a total climate forcing of about 6.5 Watts/m^2 maintained an equilibrium temperature of about 5 Celsius degrees, giving a climate sensitivity of about 0.75. That is, we get about 3 degrees temperature increase for a 4 Watts/m^2 disturbance of the Earth's energy balance for instant doubling of CO_2 content. The resulting uncertainty is 0.5C°. This is a precise evaluation of climate sensitivity without using climate models[5].

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