# Ex 4.2    Q 3-14 , 37/50

Q 3)(a) $f(x) = 3x^2 - 6x + 1$ at $x = 1$

Sol:-

first derivative

$f'(x) = 6x - 6$

Let $f(x)' = 0$

$6x - 6 = 0$

$6x = 6 \Rightarrow x = \frac{\cancel{6}}{\cancel{6}} = 1$

$\boxed{x = 1}$

critical point

Test intervals at $x = 1$

| when $x < 1$ take $x = 0$ | when $x > 1$ take $x = 2$ |
|---|---|
| $f'(0) = 6(0) - 6 = -6$ | $f'(2) = 6(2) - 6 = 6$ |
| -ve | +ve |

since $f'(x)$ change from -ve to +ve it indicates relative minimum.

Second derivative.

$f'(x) = 6x - 6$

$f''(x) = 6$

$f''(1) = 6$

Since $f''(x) > 0$ so concave up

It has relative minimum at $x = 1$

3(b) $f(x) = x^3 - 3x + 3$ at $x = 1$ and $-1$

Sol:-

First derivative

$f'(x) = 3x^2 - 3$

let $f'(x) = 0$

$3x^2 - 3 = 0$

$3(x^2 - 1) = 0$

$x^2 = 1$

$x = \pm 1$

Critical points are $x = 1$ and $x = -1$

Test intervals

when $x = -1$

| | |
|---|---|
| L.H.S $x = -2$ | R.H.S $x = 0$ |
| $f'(-2) = 3(-2)^2 - 3 = 9$ | $f'(0) = 3(0)^2 - 3 = -3$ |
| +ve | -ve |

Since it changes from +ve -ve so it has relative maximum at $x = -1$

when $x = 1$

| | |
|---|---|
| L.H.S $x = 0$ | R.H.S $x = 2$ |
| $f'(0) = 3(0)^2 - 3 = -3$ | $f'(2) = 3(2)^2 - 3 = 9$ |
| -ve | +ve |

changes from -ve to +ve so it has relative minimum at $\boxed{x = 1}$

Since $f'(x)$ changes from $-ve$ to positive so it has relative minimum at $x=0$

Second derivative

$$f'(x) = \sin 2x$$
$$f''(x)\ 2\cos 2x$$

at $x=0$

$$f''(0) = 2\cos 2(0)$$
$$f''(0) = 2(1) = 2 \quad +ve$$

since $f''(x) > 0$ so concave up indicating relative minimum at $\boxed{x=0}$

4(b) $f(x) = \tan^2 x$

Sol:- First derivative

$$f'(x) = 2\tan x \sec^2 x$$

Let $f'(x) = 0$

$$2\tan x \sec^2 x = 0$$

$\because \sec^2 x \neq 0$ , $\cos x \neq 0$

So,

$$2\tan x = 0$$
$$\tan x = 0 \qquad x = 0, \pi, 2\pi$$

Critical point at $x=0$

Test interval around 0

| L.H.S $x = -\pi/4$ | R.H.S $x = \pi/4$ |
|---|---|
| $f(-\pi/4) = 2\tan(-\pi/4)\ \sec^2(-\pi/4)$ | $f(\pi/4) = 2\tan(\pi/4)\ \sec^2(\pi/4)$ |

Second derivative

$f'(x) = 3x^2 - 3$

$f''(x) = 6x$

Now

for $x = -1$

$f''(-1) = 6(-1) = -6$

$-ve$

$f''(x) < 0$, concave down so relative maximum

for $x = 1$

$f''(1) = 6(1) = 6$

$+ve$

concave up so relative minimum

$f(x) > 0$

Q 4(a) $f(x) = \sin^2 x$ at $x = 0$

First derivative

$f'(x) = 2\sin x \cos x$ $\because 2\sin x \cos x = \sin 2x$

$f'(x) = \sin 2x$

let $f'(x) = 0$

$\sin 2x = 0$ $\therefore 2x = n\pi$

$x = n\pi/2$

critical points at $x = 0, \pi, \pi/2$

Test interval around $x = 0$

L.H.S $x = -\frac{\pi}{4}$

$f'\left(\frac{-\pi}{4}\right) = \sin 2\left(\frac{-\pi}{4}\right) = \sin -\frac{\pi}{2}$

$f'(-\pi/4) = -1$ $-ve$

R.H.S $x = \pi/4$

$f'(\pi/4) = \sin 2\left(\frac{\pi}{4}\right) = 1$

$+ve$

$f(-\pi/4) = 2(-1)(2)$ | $f(\pi/4) = 2(1)(2)$

$f(-\pi/4) = -4$ | $f(\pi/4) = 4$

$-ve$ | $+ve$

Since $f'(x)$ changes from $-ve$ $+ve$ it indicates relative minimum at $x=0$.

2nd derivative

$$f'(x) = 2\tan x \sec^2 x$$

$$f''(x) = 2\tan x \, 2\sec x(\sec x \tan x) + \sec^2 x \, 2(\sec^2 x)$$

$$f''(x) = 4\tan^2 x \sec^2 x + 2\sec^4(x)$$

for $x = 0$

$$f''(0) = 4\tan^2(0)\sec^2(0) + 2\sec^4(0)$$

$$f''(0) = 4(0) \times 1 + 2(1)$$

$$f''(0) = 2$$

since $f''(x) > 0$ so concave up

therefore relative min at $x=0$

Q 4(c).

Both function are squares of non zeroes value $x$ is close to 0 but $x \neq 0$ and so are positive for values of $x$ near zero; both function are zero at $x=0$, so that must be a relative minimum.

Q 5(a) $f(x) = (x-1)^4$ , $g(x) = x^3 - 3x^2 + 3x - 2$

$x = 1$

Sol:-

$f(x) = (x-1)^4$

$f'(x) = 4(x-1)^3$

at $x = 1$

$f'(1) = 4(1-1)^3$

$f'(1) = 4(0)^3$

$f'(1) = 0$

$g(x) = x^3 - 3x^2 - 3(x) - 2$

$g'(x) = 3x^2 - 6x + 3$

$g'(1) = 3(1)^2 - 6(1) + 3$

$g'(1) = 6 - 6$

$g'(1) = 0$

Since $f'(1) = 0$ so the given point $x = 1$ is stationary.

Q5(b)

Second derivative

$f'(x) = 4(x-1)^3$

$f''(x) = 12(x-1)^2$

$f''(1) = 12(1-1)^2$

$f''(1) = 0$

no information

$g'(x) = 3x^2 - 6x + 3$

$g''(x) = 6x - 6$

$g''(1) = 6(1) - 6$

$g''(1) = 0$

no information.

Q5(c)

first derivative

$f'(x) = 4(x-1)^3$

Let $f'(x) = 0$

$4(x-1)^3 = 0$

$g'(x) = 3x^2 - 6x + 3$

Let $g'(x) = 0$

$3x^2 - 6x + 3 = 0$

Q6(a) $f(x) = 1 - x^5$ , $g(x) = 3x^4 + 8x^3$

$x = 0$

Sol:-

$f'(x) = -5x^4$

at $x = 0$

$f'(0) = -5(0)^4$

$f'(0) = 0$

Its stationary point

$g'(x) = 12x^3 + 24x^2$

$g'(0) = 12(0)^3 + 24(0)^2$

$g'(0) = 0$

Its a stationary point

Q6(b) Sol:-

Second derivative

$f'(x) = -5x^4$

$f''(x) = -20x^3$

at $x = 0$

$f''(0) = -20(0)^3$

$f''(0) = 0$

no info

$g'(x) = 12x^3 + 24x^2$

$g''(x) = 36x^2 + 48x$

$g''(0) = 36(0) + 48(0)$

$g''(0) = 0$

no info

Qno 6(c)

Test intervals in first derivative

first calculate critical points

$f'(x) = -5x^4$

let $f'(x) = 0$

$-5x^4 = 0$

$x^4 = 0$

$\boxed{x = 0}$

$g'(x) = 12x^3 + 24x^2$

let $g'(x) = 0$

$12x^3 + 24x^2 = 0$

$12x^2(x-2) = 0$

$\boxed{x = 2}$

critical points

$(x-1)^3 = 0$

Take cube root on b.s

$x-1 = 0$

$x = 1$

critical point

~~Put $x=1$ in $f'(x)$~~

~~$f'(1) = 4(1-1)^3 = 0$~~

~~1st derivative is zero at $x=1$, so stationary point at $x=1$~~

$3(x^2-2x+1)=0$

$(x^2-2x+1)=0$

$\because a^2-2ab+b^2=(a-b)^2$

$(x-1)^2 = 0$

sq on b.s

$x-1=0$

$x=1$

critical point

~~$g'(1) =$~~

for $f'(x) = 4(x-1)^3$

Test intervals

$f'(x)$ around $x=1$

for L.H.S $x=0$

$f'(0) = 4(0-1)^3$

$f'(0) = -4$ -ve

R.H.S $x=2$

$f'(2) = 4(2-1)^3$

$f'(2) = 4$ +ve

-ve to positive so relative min

Now $g'(x)$ $3x^2-6x+3$

L.H.S $x=0$

$f'(0) = 3(0)^2-6(0)+3$

$f'(0) = 3$ +ve

R.H.S $x=2$

$f'(0) = 3(2)^2-6(2)+3$

$f'(0) = 3$ +ve

No relative extremum

Test intervals

$f'(x) = -5x^4$

$x = 0$

L.H.S $x = -1$

$f'(-1) = -5(-2)^4 = -5$ -ve

R.H.S $x = 1$

$f'(1) = -5(1)^4 = -5$ -ve

No relative extremum

$g'(x) = 12x^3 + 24x^2$

$x = 0$

L.H.S $x = -1$

$g'(-1) = 12(-1)^3 + 24(-1)^4$

$g'(-1) = -12 + 24$

$g'(-1) = 24$ +ve

R.H.S $x = 1$

$g'(1) = 12(1)^3 + 24(1)^2$

$g'(1) = 12 + 24$

$g'(1) = 36$ +ve

No relative extremum.

Q7) $f(x) = 4x^4 - 16x^2 + 17$

Sol:-

$f'(x) = 16x^3 - 32x$

$f'(x) = 16x(x^2 - 2)$

Let $f'(x) = 0$

$16x(x^2 - 2) = 0$

$16x = 0$ , $x^2 - 2 = 0$

$\boxed{x = 0}$ $x^2 = 2$

$x = \pm\sqrt{2}$

putting critical point in $f'(x)$

$f'(0) = 16(0)^3 - 32(0) = 0$ stationary

$f'(\sqrt{2}) = 16(\sqrt{2})^3 - 32(\sqrt{2}) = 0$ stationary

$f'(-\sqrt{2}) = 16(-\sqrt{2})^3 - 32(-\sqrt{2}) = 0$ stationary

11) $= \sqrt[3]{x^2-25}$

Sol:- $f(x) = (x^2-25)^{1/3}$

$f'(x) = \frac{1}{3}(x^2-25)^{-2/3}(2x)$

$$f'(x) = \frac{2x}{(x^2-25)^{2/3}}$$

Let $f'(x) = 0$

$$\frac{2x}{3(x^2-25)^{2/3}} = 0$$

$2x = 0 \Rightarrow \boxed{x=0}$

$$f'(0) = \frac{2(0)}{(0^2-25)^{2/3}} = 0$$

Stationary

$x^2-25$

$x^2 = 25$

$x = \pm 5$

$$f'(5) = \frac{2(5)}{(5^2-25)^{2/3}}$$ not stationary

$$f'(-5) = \frac{2(-5)}{((-5)^2-25)^{2/3}}$$ Not stationary

Answer

$\boxed{x = -3}$ , $\boxed{x = 1}$

$$f'(-3) = \frac{3-2(-3)-(-3)^2}{((-3)^2+3)^2} = 0$$

Stationary

$$f'(1) = \frac{3-2(1)-(1)^2}{(1^2+3)^2} = 0$$

stationary

10) $\dfrac{x^2}{x^3+8}$ Sol:-

$$f'(x) = \frac{2x(x^3+8)-x^2(3x^2)}{(x^3+8)^2}$$

$$f'(x) = \frac{2x^4+16x-3x^4}{(x^3+8)^2}$$

$$f'(x) = \frac{16x-x^4}{(x^3+8)^2}$$

Let $f'(x)=0$

$$\frac{16x-x^4}{(x^3+8)^2} = 0$$

$$16x-x^4 = 0$$

$$x(16-x^3) = 0$$

$$x^3 = 16 \Rightarrow \boxed{x = \sqrt[3]{16}}$$

$x^3 - 16 = 0$

$x^3 = 16$

$x = \sqrt[3]{16}$

$\boxed{x = 2^{4/3}}$

$$f'(\sqrt[3]{16}) = \frac{16(\sqrt[3]{16})-(\sqrt[3]{16})^4}{((\sqrt[3]{16})^3+8)^2} = 0$$

stationary point.

$$f'(2^{4/3}) = \frac{16(2^{4/3})-(2^{4/3})^4}{((2^{4/3})^3+8)^2} = 0$$

stationary

8) $f(x) = 3x^4 + 12x$

Sol:-

$f'(x) = 12x^3 + 12$

Let $f'(x) = 0$

$12x^3 + 12 = 0$

$12x^3 = -12$

$x^3 = -\frac{\cancel{12}}{\cancel{12}}$ $\boxed{x = -1}$

$f'(1) = 12(-1)^3 + 12 = -12 + 12 = 0$

stationary

9) $f(x) = \frac{x+1}{x^2+3}$

Sol:-

$f'(x) = \frac{(x^2+3)(1) - (x+1)(2x)}{(x^2+3)^2}$

$f'(x) = \frac{x^2+3 - 2x^2 - 2x}{(x^2+3)^2}$

$f'(x) = \frac{3 - x^2 - 2x}{(x^2+3)^2}$

Let $f'(x) = 0$

$\frac{3 - x^2 - 2x}{(x^2+3)^2} = 0$

$-x^2 - 2x + 3 = 0$ $\times$ by $-1$ on b.s

$x^2 + 2x - 3 = 0$

$x^2 + 3x - x - 3 = 0$

$x(x+3) - 1(x+3) = 0$

$(x+3)(x-1)$

12) $f(x) = x^2(x-1)^{2/3}$

Sol:-

$$\cancel{2x(x-1)^{2/3} + x^2 \frac{1}{(x-1)^{-1/3}} \cdot \frac{2}{3}}$$

$$\cancel{2x(x-1)^{2/3} + x^2(x-1)^{-1/3} \cdot \frac{2}{3}}$$

$x$

$$f(x) = x^2(x-1)^{2/3}$$

$$f'(x) = x^2 \cdot \frac{2}{3}(x-1)^{-1/3} + 2x(x-1)^{2/3}$$

$$f'(x) = x(x-1)^{-1/3}\left(\frac{2}{3}x + 2(x-1)\right)$$

$$f'(x) = x(x-1)^{-1/3}\left(\frac{2}{3}x + 2x - 2\right)$$

$$f'(x) = x(x-1)^{-1/3}\left(\frac{2}{3}x + \frac{6x}{3} - \frac{6}{3}\right)$$

$$f'(x) = x(x-1)^{-1/3}\left(\frac{8x-6}{3}\right)$$

Let $f'(x) = 0$

$$f'(x) = x(x-1)^{-1/3}\left(\frac{8x-6}{3}\right) = 0$$

$\boxed{x = 0}$

$(x-1)^{-1/3}$ = undefined

$$\frac{8x-6}{3} = 0$$

$$\frac{2(4x-3)}{3} = 0$$

$4x - 3 = 0 \Rightarrow \boxed{x = \frac{3}{4}}$

critical points are $x = 0$ and $x = \frac{3}{4}$

$f'(0) = 0(0-1)^{-1/3}\left(\frac{8(0) - 6}{3}\right) = 0$

stationary

$f'\left(\frac{3}{4}\right) = \frac{3}{4}\left(\frac{3}{4} - 1\right)^{-1/3}\left(\frac{8(3/4) - 6}{3}\right) = 0$

stationary point.

13) $f(x) = |\sin x|$

Sol:-

$f(x) = \begin{cases} x \geq 0 & \sin x \\ x \leq 0 & -\sin x \end{cases}$

for $\sin x$

$f'(x) = \cos x$ let $f'(x) = 0$

$\cos x = 0$

critical points are $x = \frac{\pi}{2} + n\pi$ where $n \in \mathbb{Z}$ so $\frac{\pi}{2} + 0\pi$, $\frac{\pi}{2} + 1\pi$, $\frac{5\pi}{2}$ .....

for $-\sin x$

$f'(x) = -\cos x$ × by -1 on b.s let $f'(x) = 0$

~~critical~~ $\cos x = 0$

critical points are $\frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}$....

stationary points are $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ .....

14) $f(x) = \sin|x|$

Sol:-

$$f(x) = \begin{cases} x \geq 0, & x = \sin x \\ x < 0, & -x = -\sin x \end{cases}$$

Case $f(x) = \sin x$

$f'(x) = \cos x$ let $f'(x) = 0$

$\cos x = 0$

critical points are $x = \frac{\pi}{2} + n\pi$ where $n \in \mathbb{Z}$

$\boxed{x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}}$

case $f(x) = -\sin x$

$f(x) = -\cos x = \cos x = 0$

critical point are $x = \frac{\pi}{2} + n\pi$ $n \in \mathbb{Z}$

Stationary points $x = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}$

$\boxed{x = 0, \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \frac{5\pi}{2}, \ldots}$

Day: 

~~$\cos(x) = 0$~~

$\boxed{\cos(0) = 1}$

$x = \frac{\pi}{2} + n\pi, \; n \in \mathbb{Z}$

$\boxed{x = 0}$

37-50) Use any method to find relative extrema of the functions

37) $f(x) = x^4 - 4x^3 + 4x^2$

Sol:-

⇒ First derivative

$f'(x) = 4x^3 - 12x^2 + 8x$

⇒ Solve critical points

Set $f'(x) = 0$

$4x^3 - 12x^2 + 8x = 0$

$4x(x^2 - 3x + 2) = 0$

$4x(x-1)(x-2) = 0$

$\boxed{x = 0, \; x = 1, \; x = 2}$

⇒ Second derivative

$f''(x) = 12x^2 - 24x + 8$

At $x = 0$

$f''(0) = 12(0)^2 - 24(0) + 8 = 8 > 0$

$x = 0$ is minimum

At $x = 1$

$f''(1) = 12(1)^2 - 24(1) + 8 = -4 < 0$

$x = 1$ is maximum

$f''(2) = 12(2)^2 - 24(2) + 8 = 8 > 0$

$x = 2$ is minimum

⇒ Function values at critical points

Substitute critical points into $f(x)$

- $f(0) = (0)^4 - 4(0)^3 + 4(0)^2 = 0$
- $f(1) = (1)^4 - 4(1)^3 + 4(1)^2 = 1$
- $f(2) = (2)^4 - 4(2)^3 + 4(2)^2 = 0$

Final answer

- relative min 0 at $x = 0$
- relative max 1 at $x = 1$
- relative min 0 at $x = 2$

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At $x = \frac{-3}{5}$

$\cancel{f'''(-3/5) = 2(-3/5)(-3/5+1)(5(-3/5)+3)+}$

$f''(-3/5) = (-3/5)^2(10(-3/5)+8) + 2(-3/5)(-3/5+1)(5(-3/5)+3) = \frac{18}{25} > 0$

At $x = -3/5$ is minimum.

Substitute critical points into $f(x)$

$f(0) = (0)^3(0+1)^2 = 0$

$f(-1) = (-1)^3(-1+1)^2 = 0$

$f(-3/5) = (-3/5)^3(-3/5+1)^2 = -108/3125$

Final Answer

relative max 0 at $x = -1$

relative min $18/25$ at $x = -3/5$

inflection at $x = 0$

40) $f(x) = x^2(x+1)^3$

Sol:-

$f(x) = x^2(x+1)^3$

First derivative

$f'(x) = x^2 \cdot 3(x+1)^2 + (x+1)^3(2x)$

$f'(x) = 3x^2(x+1)^2 + 2x(x+1)^3$

$\cancel{f'(x) = x(x+1)\{3x(x+1)+2(x+1)\}}$

$f'(x) = x(x+1)^2\{3x + 2(x+1)\}$

$f'(x) = x(x+1)^2\{3x+2x+2\}$

$f'(x) = x(x+1)^2\{5x+2\}$

Critical points (Set $f'(x) = 0$)

$x(x+1)^2(5x+2) = 0$

$x = 0,\ x = -1,\ x = -2/5$

Second derivative

$f''(x) = x\frac{d\{(x+1)^2(5x+2)\}}{dx} + (x+1)^2(5x+2)\frac{dx}{dx}$

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$2 + \frac{3}{x^{1/3}} = 0$

$\frac{\cancel{2}}{x^{1/3}} = -\cancel{2}$

$\frac{1}{x^{1/3}} = -1$

$x^{1/3} = -1$

Cube on b.s

$\boxed{x = -1}$

Second derivative

$f''(x) = 2\left(-\frac{1}{3}\right)(x^{-4/3})$

$f''(x) = -\frac{2}{3}x^{-4/3}$

At $x = -1$

$f''(-1) = -\frac{2}{3}(-1)^{-4/3} = -\frac{2}{3} < 0$

$x = -1$ is maximum

Substitute critical points in $f(x)$

$f(-1) = 2(-1) + 3(-1)^{2/5} = 1$

Final answer

Relative maximum 1 at $x = -1$

42) $f(x) = 2x + 3x^{1/3}$

Sol:-

$f(x) = 2x + 3x^{1/3}$

First derivative

$f'(x) = 2 + 1x^{-2/3}$

$f'(x) = 2 + \frac{1}{x^{2/3}}$

Set $f'(x) = 0$

$2 + \frac{1}{x^{2/3}} = 0$

$\frac{1}{x^{2/3}} = -2$

(No solution as $x \neq 0$)





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43) $f(x) = \frac{x+3}{x-2}$

Sol:

$f(x) = \frac{x+3}{x-2}$

First derivative

$f'(x) = (x-2)\frac{d(x+3)}{dx} - (x+3)\frac{d(x-2)}{dx} \Big/ (x-2)^2$

$f'(x) = \frac{(x-2)-(x+3)}{(x-2)^2}$

$f'(x) = \frac{\cancel{x}-2-\cancel{x}-3}{(x-2)^2} = \frac{-5}{(x-2)^2}$

Critic Set $f'(x) = 0$

$\frac{-5}{(x-2)^2} = 0$ (No critical points)

$-5 \neq 0$ (No extrema)

44) $f(x) = \frac{x^2}{x^4+16}$

Sol:-

$f(x) = \frac{x^2}{x^4+16}$

First derivative

$f'(x) = \frac{(x^4+16)(2x) - x^2(4x^3)}{(x^4+16)^2}$

$f'(x) = \frac{2x^5+32x-4x^5}{(x^4+16)^2}$

$f'(x) = \frac{-2x^5+32x}{(x^4+16)^2} = \frac{2x(-x^4+16)}{(x^4+16)^2}$

Set $f'(x) = 0$

$\frac{2x(-x^4+16)}{(x^4+16)^2} = 0$

$2x(-x^4+16) = 0$

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Q no 44) $f(x) = \frac{x^2}{x^4+16}$

Sol:-

First derivative

$$f'(x) = \frac{(2x)(x^4+16) + (x^2)(4x^3)}{(x^4+16)^2}$$

$$f'(x) = \frac{2x^5 + 32x - 4x^5}{(x^4+16)^2}$$

$$f'(x) = \frac{-2x^5 + 32x}{(x^4+16)^2}$$

$$f'(x) = \frac{2x(-x^4+16)}{(x^4+16)^2}$$

Set $f'(x) = 0$

$$\frac{2x(-x^4+16)}{(x^4+16)^2} = 0$$

$$2x(-x^4+16) = 0$$

$$2x = 0 \quad , \quad -x^4 + 16 = 0$$

$$x = 0 \quad , \quad x^4 = 16$$

$$x = \pm 2$$

$x = 0, 2, -2$

(critical points)

Second derivative

$$f'(x) = \frac{-2x^5 + 32x}{(x^4+16)^2}$$

$$f''(x) = \frac{[(-10x^4+32)(x^4+16)^2] - [(-2x^5+32x)(8x^3(x^4+16))]}{(x^4+16)^4}$$

At $x=0$

$$f''(0) = \frac{[(-10(0)^4+32)((0)^4+16)^2] - [(-2(0)^5+32(0))(8(0)^3(0)^4+16)]}{(0^4+16)^4}$$

$$f''(0) = \frac{1}{8}$$

Relative minimum at $x=0$

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At $x=2$

$$f''(2)=\frac{[(-10(2)^4+32)((2)^4+16)^2]-[(-2(2)^5+32(2))(8(2)^3(2)^4+16)]}{(2^4+16)^4}$$

$$f''(2)=-\frac{1}{8}$$

Relative maximum at $x=2$

At $x=-2$

$$f''(-2)=\frac{[(-10(-2)^4+32)((-2)^4+16)^2]-[(-2(-2)^5+32(-2))(8(-2)^3(2)^4+16)]}{((-2)^4+16)^4}$$

$$f''(-2)=-\frac{1}{8}$$

Relative maximum at $x=-2$

Substituting critical points in $f(x)$

$$f(0)=\frac{0^2}{0^4+16}=0$$

$$f(2)=\frac{2^2}{2^4+16}=\frac{1}{8}$$

$$f(-2)=\frac{(-2)^2}{(-2)^4+16}=\frac{1}{8}$$

Result:-

Relative maximum of $\frac{1}{8}$ at $x=2$

Relative maximum of $\frac{1}{8}$ at $x=-2$

Relative minimum of $0$ at $x=0$

45) $f(x) = \ln(2+2x^2)$

Sol:-

$f(x) = \ln(2+x^2)$

First derivative

$f'(x) = \dfrac{1}{2+x^2} \cdot 2x$

$f'(x) = \dfrac{2x}{2+x^2}$

Set $f'(x) = 0$

$\dfrac{2x}{2+x^2} = 0$

$\boxed{x = 0}$

Second derivative

$\cancel{f''(x) = \dfrac{(2+x^2)(x) - 2x(2x)}{(2+x^2)^2}}$

$\cancel{f''(x) = \dfrac{2x+x^3 - 2(x^2)}{(2+x^2)^2}}$

$f''(x) = \dfrac{(2+x^2)(2) - 2x(2x)}{(2+x^2)^2}$

$f''(x) = \dfrac{4+2x^2-4x^2}{(2+x^2)^2}$

$f''(x) = \dfrac{4-2x^2}{(2+x^2)^2}$

At $x=0$

$f''(0) = \dfrac{4-2(0)^2}{(2+(0)^2)^2} = \boxed{1} > 0$

$x=1$ is minimum at $x=0$

Substitute critical points into $f(x)$

$f(0) = \ln(2+2(0)^2)$


Careb

$f(0) = \ln 2$

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Relative minimum $\ln 2$ at $x = 0$

$f(x) = \ln|2+x^3|$

Sol:-

$f(x)\ \ln|2+x^3|$

First derivative

$$f'(x) = \frac{1}{2+x^3}\ 3x^2$$

$$f'(x) = \frac{3x^2}{2+x^3}$$

Set $f'(x) = 0$

$$\frac{3x^2}{2+x^3} = 0$$

$3x^2 = 0$

$\boxed{x = 0}$

Second derivative

$$f''(x) = \frac{(2+x^3)(6x) - (3x^2)(3x^2)}{(2+x^3)^2}$$

$$f''(x) = \frac{12x + 6x^4 - 9x^4}{(2+x^3)^2}$$

$$f''(x) = \frac{12x - 3x^4}{(2+x^3)^2}$$

At $x = 0$

$$f''(0) = \frac{12(0) - 3(0)^4}{(2+(0)^3)^2} = 0$$

$x = 0$ is inflection point (inconclusive)

No relative extrema

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Q47/ $f(x) = e^{2x} - e^{x}$

Solve

$f'(x) = e^{2x}\cdot(2) - e^{x}$

$= 2e^{2x} - e^{x}$

$\Rightarrow 2e^{2x} - e^{x} = 0$

$e^{x}(2e^{x} - 1) = 0$ } → Critical points

$\Rightarrow e^{x} = 0$ (exponential function never zero)

$\Rightarrow 2e^{x} - 1 = 0$

$\Rightarrow e^{x} = 1/2$

$\Rightarrow x = \ln 1/2 \Rightarrow x = \ln 1 - \ln 2$

$\Rightarrow x = -\ln 2$

Relative min at $x = -\ln 2$

$f'(x)$ changes from $-$ to $+$

Q48/ $f(x) = (xe^{x})^{2}$

Solve.

$\Rightarrow f'(x) = 2(xe^{x})(e^{x} + xe^{x})$

$\Rightarrow f'(x) = 2xe^{x}\cdot(e^{x} + xe^{x})$

$\Rightarrow 2xe^{x}(e^{x} + xe^{x}) = 0$

$\Rightarrow (1)\ 2x = 0 \Rightarrow x = 0$

$\Rightarrow (2)\ e^{x} = 0$ (exponential form never zero).

$\Rightarrow (3)\ [e^{x} + xe^{x}] = 0$

$\Rightarrow e^{x}(1 + x) = 0$

$\Rightarrow 1 + x = 0 \Rightarrow x = -1$

Critical Points: $x = 0, -1$

Relative max at $x = -1$, $f'(x)$ changes $+$ to $-$

Relative min at $x = 0$, $f'(x)$ changes $-$ to $+$

Q49/ $f(x) = |3x - x^{2}|$

Solve

$$f(x) = \begin{cases} 3x - x^{2} & ; \text{ if } 0 \le x \le 3 \\ -(3x - x^{2}) = x^{2} - 3x & ; \text{ if } x < 0 \text{ or } x > 3 \end{cases}$$

$\Rightarrow f'(x) = 3x - x^{2} \Rightarrow 3 - 2x$ (case 1)

$\Rightarrow 3 - 2x = 0 \Rightarrow x = 3/2$

$\Rightarrow f'(x) = x^{2} - 3x \Rightarrow 2x - 3$ (case 2)

$\Rightarrow 2x - 3 = 0 \Rightarrow x = 3/2$

However, this critical point $x = 3/2$ does not lie in this region ($x < 0$ or $x > 3$), so it is not valid for this case.

Check endpoints & transitions at $x = 0, x = 3$ because $f(x)$ involves modulus

At $x = 0$

$\Rightarrow f(0) = |3(0) - (0)^{2}| = |0| = 0$

At $x = 3/2$

$\Rightarrow f(3/2) = 3(3/2) - (3/2)^{2} = 9/2 - 9/4 = 9/4$

At $x = 3$

$\Rightarrow f(3) = |3(3) - (3)^{2}| = |9 - 9| = 0$

Relative max at $x = 3/2$ ; $f(x) = 9/4$

Relative minim at $x = 0$ & $x = 3$ ; $f(x) = 0$

Q50/ $f(x) = |1 + \sqrt[3]{x}|$

Sol.

$$f(x) = \begin{cases} 1+\sqrt[3]{x} & ; \text{ if } x \geq -1 \\ -(1+\sqrt[3]{x}) = -1-\sqrt[3]{x} & ; \text{ if } x < -1 \end{cases}$$

$f'(x) = 1 + \sqrt[3]{x} = \frac{1}{3} x^{-2/3}$

$f'(x) = -1 - \sqrt[3]{x} = -\frac{1}{3} x^{-2/3}$

$f'(x) = \pm \frac{1}{3} x^{-2/3}$ are undefined at $x = 0$, we have critical points $x = -1$, where $f'(x)$ is undefined.

At $x = -1$

$f(-1) = |1 + \sqrt[3]{-1}| = |1-1| = 0$

At $x = 0$

$f(0) = |1 + \sqrt[3]{0}| = |1+0| = 1$

Relative Mini at $x = -1$ ; $f'(x)$ changes from $-$ to $+$ , $f(x) = 0$.

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38) $f(x) = x(x-4)^3$

Sol:-

⇒ First derivative

$f'(x) = x \cdot 3(x-4)^2 + (x-4)^3 (1)$

$f'(x) = 3x(x-4)^2 + (x-4)^3$

$f'(x) = (x-4)^2[3x + (x-4)]$

$f'(x) = (x-4)^2[3x + x - 4]$

$f'(x) = (x-4)^2[4x-4]$

$f'(x) = (x-4)^2 \cdot 4(x-1)$

$f'(x) = 4(x-4)^2(x-1)$

Solve for critical points

Set $f'(x) = 0$

$4(x-4)^2(x-1) = 0$

$x = 4, \quad x = 1$

Second derivative

$f''(x) = 4[(x-4)^2(1) + (x-1)2(x-4)]$

$f''(x) = 4[(x-4)^2 + 2(x-4)(x-1)]$

$f''(x) = 4(x-4)[(x-4) + 2(x-1)]$

$f''(x) = 4(x-4)[x-4+2x-2]$

$f''(x) = 4(x-4)(3x-3)$

$f''(x) = 4(x-4)3(x-1)$

At $x = 4$

$f''(4) = 4(4-4) \cdot 3(4-1) = 0$

~~At~~ $x = 4$ is inflection point

At $x = 1$

$f''(1) = 4(1-4) \cdot 3(1-1) \Rightarrow 36$

~~Relati~~ $x = 1$ is minimum

Function values at critical points

Substitute critical points into $f(x)$

$f(4) = (4)(4-4)^3 = 0$

$f(1) = (1)(1-4)^3 = -27$

Final answer:-

relative min $-27$ at $x = 1$

inflection at $x = 4$





Teacher's Signature

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39) $f(x) = x^3(x+1)^2$

Sol:-

$f(x) = x^3(x+1)^2$

First derivative

$f'(x) = x^3\{2(x+1)\} + (x+1)^2(3x^2)$

$f'(x) = x^3(2x+2) + 3(x+1)^2(3x^2$

$f'(x) = 2x^3(x+1) + 3x^2(x+1)^2$

$f'(x) = x^2(x+1)(2x+3(x+1))$

$f'(x) = x^2(x+1)(2x+3x+3)$

$f'(x) = x^2(x+1)(5x+3)$

=> Critical points

Set $f'(x) = 0$

$x^2(x+1)(5x+3) = 0$

$$\boxed{x = 0,\ x = -1,\ x = -\frac{3}{5}}$$

Second derivative

$f''(x) = \frac{d(x^2(x+1)(5x+3))}{dx}$

~~$f''(x) = 2x\{(x+1)(5)+(5x+3)(1)\}$~~

$f''(x) = x^2\frac{d(x+1)(5x+3)}{dx} + (x+1)(5x+3)\frac{dx^2}{dx}$

$f''(x) = x^2\{(x+1)(5)+(5x+3)(1)\} + (x+1)(5x+3)(2x)$

$f''(x) = x^2(5x+5+5x+3) + 2x(x+1)(5x+3)$

$f''(x) = x^2(10x+8) + 2x(x+1)(5x+3)$

At $x = 0$

$f''(0) = (0)^2(10(0)+8) + 2(0)(0+1)(5(0)+3) = 0$

$x = 4$ is inflection point

At $x = -1$

$f''(-1) = (-1)^2(10(-1)+8) + 2(-1)(-1+1)(5(-1)+3) \leq -2 < 0$

$x = -1$ is maximum.