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# Introduction

The growing field of Artificial Intelligence over the past decades has profoundly influenced our daily lives, altering the way we communicate with each other, learn, educate, interact with technology and numerous other aspects of life. Being able to teach machines how to learn and make predictions from data with Machine Learning opened many doors to scientists and businesses, enabling unprecedented levels of innovation, efficiency, and personalized customer services experiences across various industries.

One of the fields that has been influenced by AI is robotics, allowing the machine to learn and interact with the outside world through its actuators controlled by input sensory data processed with ML learning algorithms. Robotic Learning is what lies in the intersection of Robotics and Machine learning, and it takes advantage of Reinforcement Learning, a subfield of ML that teaches machines through trial and error allowing robots or agents in videogames to make intelligent decisions in complex environments.

Humans have the ability to learn through a process of trial and error, enabling complex locomotive tasks such as walking down the stairs and talking at the same time. This learning paradigm has already been thoroughly mimicked in virtual environments with digital agents [1], [2]. However, there are still many ongoing challenges regarding real-life scenarios, particularly in the realm of robotics and Reinforcement Learning . When working with real-world scenarios, more things have to be taken into account, such as unpredictable changes [3], resilience and adaptability necessity [4] and safety operations [5], [6], among many others.

Although applying RL to a robot in real life introduces new challenges, it has already been researched [7]. In this context, Multi-Agent Reinforcement Learning [8] becomes an important area of focus. This technique naturally introduces the challenge of not only the need to adapt to the environment but considering the other agent actions. Despite this unique challenge, being able to make multiple robots cooperate with each other to accomplish a task would advance our capabilities in various fields, ranging from autonomous vehicles to collaborative robots in manufacturing, warehouses, and healthcare.

## Historical overview

[ TO BE FILLED]

## Identification of the problem

As previously mentioned, despite the extensive research conducted on RL and MARL in virtual environments and in some cases in real-life scenarios, there are still quite a few challenges opened and waiting to be solved, like a lack of sample efficiency, setting goals and specifying rewards in dynamic environments, generalization to new and different tasks and data collection without human supervision [4].

Among these ongoing challenges, we think that one of the main issues is the **reality gap** that exists when trying to implement a Reinforcement Learning policy (algorithm) learned in a simulation into the corresponding real-life agent. Addressing this particular problem can help solving other issues as well, like diminishing the number of samples needed, saving time, mechanical wear and consequently, money.

## Rationale

The amount of time to invest in this project is very limited, therefore a study [9] has been taken as a reference point. This is to have a benchmark for comparing simulation results, defining the task, and narrowing down the choice of algorithms to be implemented.

This project aims to reduce the reality gap, also called sim-to-real transfer problem, with a specific task involving multiple agents (MARL) and it is divided in three main parts:

1. Simulation of the environment and robots using Gazebo [10] and ROS 2 [11] for control and algorithm implementation.
2. Real implementation of the environment and robots, mimicking as much as possible Gazebo’s simulation, while trying to minimize the reality gap.
3. Discussion, comparison, and analysis of the results.

Improving the sim-to-real-transfer in a muti-agent scenario can help the community prosper and create or improve applications where robots must cooperate with each other to solve a problem.

## Objectives

The academic goals for this project englobe the implementation of RL algorithms in a simulated environment as well as in a real-case scenario and the construction of two robotic arms that cooperate with each other.

Besides the research goals, improving programming abilities with Python, C, and other languages, getting familiar with software and frameworks such as Linux systems and ROS 2 as well as applying electronics, robotics and AI knowledge learnt while pursuing my bachelor are the main goals of this project. Some questions that I want to answer and may help the reader are the following:

* Is it possible to reduce the reality gap using the existent techniques [12], [13]?
* Can I build two working robots that communicate and cooperate with each other using MARL algorithms?
* How much can the reality gap can be reduced?

## Scope and limitations

While it would be ideal to fabricate the robot via a manufacturing partner and use generative design techniques, the constrained timeframe of this project regrettably does not allow such approaches.

That is why the scope of this project is building two functional […] joints robotic arm (I STILL NEED TO DECIDE IF 3 or 4 JOINTS) that interact with each other to successfully perform a task, recreating the environment in a physics simulator and in addition, implement MARL algorithms in both real-case and virtual-based while trying to minimize the reality gap.

Having a limited amount of time and resources also makes certain aspects of the project inevitably limited. This is particularly evident while training the MARL models not only in the simulation but more notably in the real-world application. The challenge extends beyond the initial development of algorithms, encompassing optimization and improvement when receiving new data and feedback.

## Overview of methodology and resources

The study [9] has been chosen as a pipeline for this project, since it provides a specific task for a MARL case scenario application so therefore, the same physics simulator will be used. In this case they used Gazebo in Linux Ubuntu OS (more supported), this software will be thoroughly explained in further sections.

Python and VSCode [14] will be used to develop the models and algorithms due to the richness of libraries and resources they provide [15], [16]. To be able to communicate Python in an organized and efficient way, Robot Operating System 2 (ROS 2) will be used as a bridge. ROS 2 will also be deeply commented in its own section.

Regarding the implementation of the robot, the hardware control will be a Raspberry Pi 3/ ESP32/ Arduino UNO/STM32 (I STILL NEED TO DECIDE, PROBABLY Raspberry Pi 3), the sensors will be [ …] and the structure of the robots will be [ …].

The algorithms applied both in simulation and implementation will be Distributed Approximate RL, Game-Theoretic RL [9], and/or PPO [17].

## Organization of the Study

# ROS2 and Gazebo

As previously said, this project is divided in two main parts, simulation, and implementation. Focusing on the simulation, two main frameworks have been used, ROS and Gazebo, which its main characteristics and how have they been used are commented out below.

## ROS

ROS or Robot Operating System [11] is a useful framework to control robots both in simulation and real life. To avoid confusions, there is a need to clarify that ROS is a set of open-source software frameworks, not an operating system, as one may thing at first instance.

The use of ROS in this project comes from benefitting from several powerful features which makes communication between certain software and frameworks more convenient. Among these features, in hierarchical order, **workspaces**, **packages** and **nodes** will be used to organize different parts of the algorithm.

Before continue explaining ROS features, it needs to be said that in this project ROS2 [18] is being used, specifically ROS 2 Humble, due to its compatibility with other software of interests such as Gazebo.

All recent ROS2 versions (Foxy, Galactic, Humble and Iron) rely on workspaces, which is a ROS term for the location of the development space on the system. This is quite convenient as it allows different ROS2 distributions to run in the same computer, switch between them and keep track of all the ongoing changes in a project.

Furthermore, a workspace is organized in packages. Packages offer a controlled way to separate and execute files and usually similar files are stored in the same package. But how can the user control robots or agents? The answer is by using **nodes**, **topics,** and **services**.

A node is a core concept in ROS 2 and an important element of what is referred to as the “ROS 2 Graph”, which is a network of ROS 2 elements processing data in a collective way at the same time. Each node is and should be responsible for a single, modular purpose, e.g., controlling joint motors or publishing sensor data from an ultrasonic sensor. Although, how can the nodes communicate and share data between them and other frameworks?

This is where topics and services come into the system. Topics serve as a communication mechanism within the ROS 2 Graph. Nodes can publish data to a topic, and other nodes can subscribe to that topic to receive and process the information. This pub-sub model facilitates a decentralized and modular architecture, allowing nodes to communicate seamlessly without direct dependencies on each other. Topics are crucial for real-time data exchange in robotic systems, enabling coordination between diverse components, such as sensor data input and motor control commands as explained before.

In contrast, services offer a request-response pattern of communication. A node providing a service advertises its availability, and other nodes can send requests to it. The service-providing node then processes the request and sends a response back. This mechanism is particularly useful for scenarios where a specific task needs to be executed on-demand, such as querying a sensor for specific information or requesting a robot to perform a specific action.

A diagram of a service

Description automatically generated

**Figure 1.** Multiple nodes sharing information via topics and services.

A full robotic system, as in this project, contains multiple nodes working in concert. In ROS 2, a single package can contain multiple nodes written in different programming languages such as C++ or Python.

A very useful tool to visualize active nodes, topics and services is *rqt\_graph*. With this command it is possible to see in a graph, real time changes and connections between actives nodes in a project. Below, there is an example with several active nodes that share data between them and Gazebo, the simulation software used.

[ADD RQT GRAPH WHEN DDPG AGENT IS RUNNING]

## Gazebo

Since the aim of this project is to decrease the reality gap, an error that is introduced when transitioning from simulations to real-world applications, Gazebo Sim emerges as a core tool. Gazebo, an open-source robot simulation software, plays a crucial role in bridging this gap by offering a realistic and dynamic environment for testing and refining robotic algorithms and control systems before their implementation on physical hardware.

Not only can Gazebo generate accurate 3D simulations but incorporate physics engines that faithfully replicate the dynamics of various robotic platforms. This realistic and dynamic simulation allows researchers, among other things, to train algorithms and test its performance across a wide variety of scenarios, ensuring a more accurate representation of real-world challenges.

In line with the project's goal, Gazebo seamlessly integrates with ROS2. This integration becomes extremely useful when deploying ROS 2 nodes within Gazebo simulations, providing a virtual environment for debugging and refining algorithms before they are executed on physical robots. The symbiotic relationship between Gazebo and ROS 2 enhances the overall capabilities of both platforms.

Furthermore, it is possible to construct multi-robot environments and the interactions between numerous robotic agents, which works perfectly in the current case of a MARL agent. On top of this, the simulation software supports a wide array of sensors, including cameras, lidars, and sonars.

Given these considerations, Gazebo stands out as a highly suitable choice for the current project. Its consistent integration of ROS 2, dynamic simulation capabilities, support for multi-robot (MARL) environments and diverse sensor simulation make this software a valuable tool in narrowing down the reality gap [12].

## Bridging Gazebo and ROS2

## Bridging ESP32 and ROS2

# Model design

As previously said, one of the project’s goals is to reduce the reality gap from simulation to real-case scenario when applying MARL. Because [9] is being taken as a reference to design RL algorithms and environment, Distributed RL (DRL) and Game-Theoretic RL (GTRL) are explained in detail below.

To further understand how RL algorithms work, there is a need to first understand Markov Decision Process. A reinforcement learning agent is designed to make a series of sequential decisions through interactions with its surroundings [19]. This environment is usually structured as an infinite-horizon discounted Markov decision process, or in a simpler way, MDP. See *Definition 1.*

**Definition 1.** A Markov decision process is defined by a tuple (S, A, P , R, γ), where S and A denote the state and action spaces, respectively; P : S × A → ∆(S) denotes the transition probability from any state s ∈ S to any state s’ ∈ S for any given action a ∈ A; R : S × A × S → R is the reward function that determines the immediate reward received by the agent for a transition from (s, a) to s’ ; γ ∈ [0, 1) is the discount factor that trades off the instantaneous and future rewards.

Parallel implementations of single-agent RL scale well on large multi-agent systems although it suffers from issues such as learning stability due to a continuous-changing environment [20] that each agent faces, therefore, to approach this challenge all agents should be jointly trained in a distributed manner.

To address this issue, [9] introduces two distributed RL algorithms, Distributed Approximate Reinforcement Learning and Game Theoretic RL. Explain why only DARL is used!

## Distributed Reinforcement Learning

The use of distributed reinforcement learning can be explained as follows. Deep Reinforcement Learning has been successfully applied in [4], [21], [22], mostly in single-agent control problems. When it comes to multiple-agent control, managing the large number of degrees of freedom, heterogeneous physical constraints and partial or asymmetric observations for different robots, demands scalability.

That is why in DRL each agent applies individual RL algorithms and reward functions which are then coupled and aligned to the goal of the cooperative task. The defined system for a MARL algorithm is, as Markov Decision Process (MDP) dictates, the tuple (1).

(1)

Where:

* The set representing the number of agents.
* The set representing all possible states of the environment.
* The set representing available actions to the agent .
* The state transition function .
* The policy for the agent .
* The accumulative reward of an agent (also called Value function).

In this type of model, each agent aims to maximize the value function with the starting state at time as shown in (2).

(2)

Where:

* is the expected value taken over a random value which is the sum of discounted rewards.
* is the discount factor that weights the future rewards.
* is the reward received by the agent at the time *.*

### States

In [9] the state of a robot at time is given by (3), where denotes the joint angles and is the global coordinate of the robot’s end effector (gripper).

(3)

### Policy

DDPG agent (2 main networks Actor and Critic, 2 sub clone networks that stabilize training)

Joint torque values from Actor’s network

### Value function

### Reward function

Since the reinforcement learning algorithm settings work in a distributed manner, one of the most important things is to correctly define individual reward functions that captures both the specific robot goal and the common task goal.

The main constituents defined in [9] for the reward task are: i) those that capture the object displacement from target, (4) and (5) respectively for both robots, and ii) that which captures the object posture deviation (6).

(4)

(5)

(6)

Furthermore, two reward function structures are built and tested out, RS-1 and RS-2. On one hand each robot is concerned with both its end effector displacement to the target and the object posture deviation, as we can see in (7). On the other hand, one robot is concerned with the object displacement to the target, while the other is concerned with the object posture deviation, shown in (8).

RS-1: (7)

RS-2: (8)

Where:

[ ADD Here 𝑑 (𝑝, 𝑝′ ) = ∥𝑝 − 𝑝 ′ ∥1 characterizes the distance between 𝑝 and 𝑝 ′ , and 𝑎(𝑝 1 , 𝑝2 ) is the absolute angle between the vector (𝑝 1 − 𝑝 2 ) and the target (𝑝 1 𝑡𝑎𝑟𝑔𝑒𝑡 − 𝑝 2 𝑡𝑎𝑟𝑔𝑒𝑡)]

The paper results show that RS-2 leads to better performance compared to RS-1, therefore, equation (8) will be the structure used for the reward function in this project, and it is explained in depth below.

# Simulation

Gazebo [10] will be used as the physics simulator due to the use of this software in [9]. Specifically, Gazebo Fortress will be used, due to its compatibility with the software robot control, ROS 2 Humble. Newer versions of Gazebo and ROS 2 (such as Gazebo Garden and ROS 2 Iron, respectively) have been used, although they have presented so many versions’ incompatibilities and errors that their use was ultimately avoided.

Since [9] do not provide the resources for the simulation, this has been built from scratch, only taking as a reference the environment setup showed in figure \_.



**Figure 2**. Environment setup reference [9]

To be able to represent and run a simulation in Gazebo two things are needed. Firstly, an *.sdf* file that describes the world (physics, models, plugins) and secondly an external software dedicated to control the simulation.

As previously said, the control software used is ROS 2, which allows the user to communicate with Gazebo through Python programmable *Nodes*. Nodes are the place where control and receiving and publishing data is happening.

[ … ]

## Algorithm implementation

### System structure

To apply distributed reinforcement learning to the simulated environment multiple steps will be followed consequently. The first step is to acquire the real-time state of each robot as well as the state of the object to be manipulated. The observed current state () will be sent to the policy () where an algorithm will decide which action to take next. Once the action () is executed, the environment will respond with a new state (). Based on the action’s () effectiveness, the reward function () will provide a reward as a scalar value. Finally, the agent learns from this data (, , and ) and the cycle is repeated.

### States

As previously explained in the Distributed Reinforcement Learning section, in [9] they define the state set taking into account only the joints angles and the end-effector global position, as shown in (3).

Because of the limitations of real-world number of sensors and other drawbacks, using only two parameters is a good way to go and a good beginning. It is also true that the more parameters or states are defined, the more precise will be the simulation, with a higher computational cost as well. This may be suitable for other applications although not so much for the current approach since the aim of this project is to minimize the reality gap, being the number of sensors in real world very limited due to lack of space, dynamics, sample amount and money expenses.

These reasons point into using the fewer and most critical number of sensors that will become the states of the robots. Therefore, taking [9] as a reference the states set will be defined as (3).

(3)

Consequently, there is a need to find a way to extract each joint angles () and both end-effector global coordinates (). To obtain these values, several components are required. Firstly, the *PosePublisher* plugin for Gazebo is utilized, which provides the General Coordinates () and a Quaternion (, see *Definition 2*) describing the orientation of each link. Additionally, a ROS2 node is necessary, subscribing to the topic where Gazebo publishes the *PosePublisher* data. Lastly, a bridge between ROS2 and Gazebo is established using the ros\_gz\_bridge, facilitating data exchange between these systems. Then, assuming that *'segment4\_1'* and *'segment4\_2'* represent the grippers of the respective robots, their global coordinates and quaternions () can be directly obtained from the simulator.

**Definition 2.** A quaternion is a mathematical concept [23] that extend complex numbers first described by Sir William Rowan Hamilton in 1843. It is composed by four components, one real part and three imaginary parts, and can be written in the form .It is used in different fields to represent three-dimensional rotations and orientations since it provides certain advantages over other methods such as Euler angles.

In this case quaternions are used instead of Euler notation to avoid ***gimbal lock*** [24] and due to the fact that they are more compact and computationally efficient than rotation matrices.

### Termination state design

The terminal state is a crucial concept that defines the conditions under which an episode concludes. Once the system reaches a terminal state, the ongoing episode ends, and the environment will be reset to its initial state for the start of a new episode. The design of the terminal state is essential for shaping the learning process and achieving specific goals in the training of an agent and can be triggered by the fulfillment of one or many conditions, such as task completion, fatal states, safety concerns, learning process stagnation or run out of time.

Since all conditions expressed before are important enough to reset the simulation if accomplished, the design of the terminal state will be the following state. Let *done* be a Boolean variable triggered by:

### Expected cumulative future reward

# Implementation

## Material

## Frameworks

# Reality Gap

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# Appendix

## Quaternions