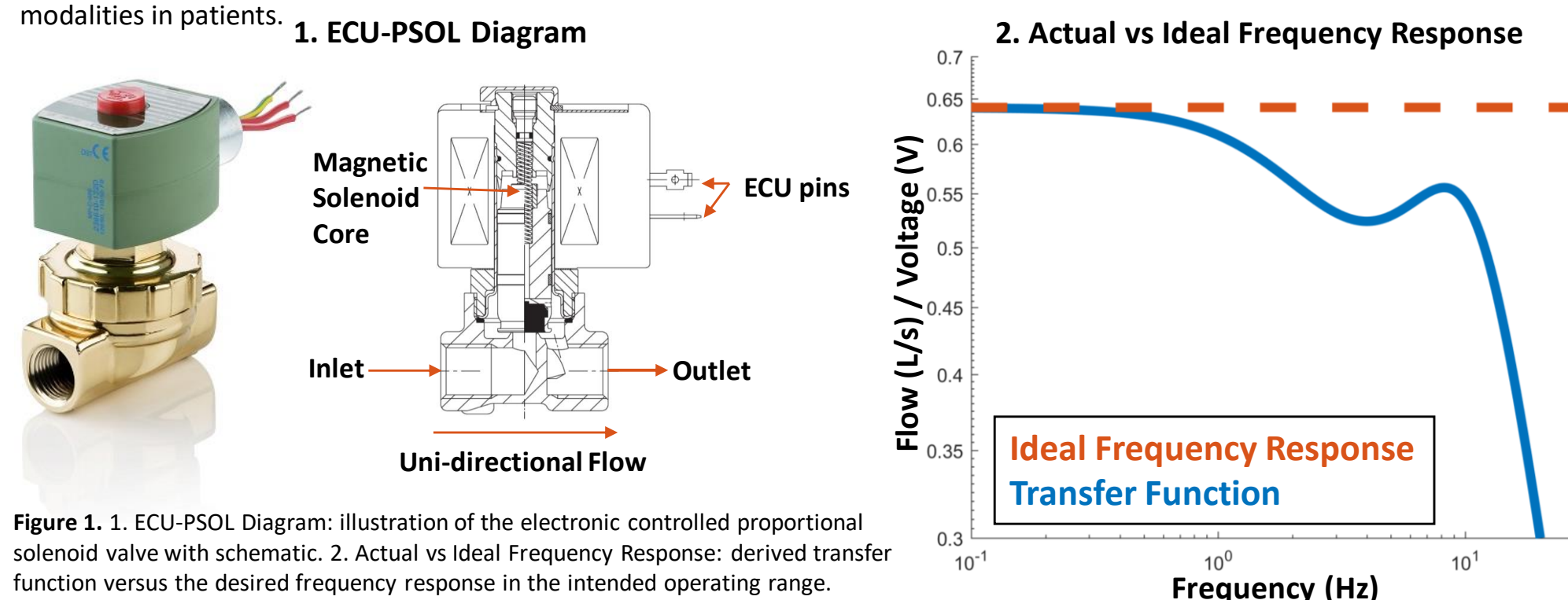


## Introduction

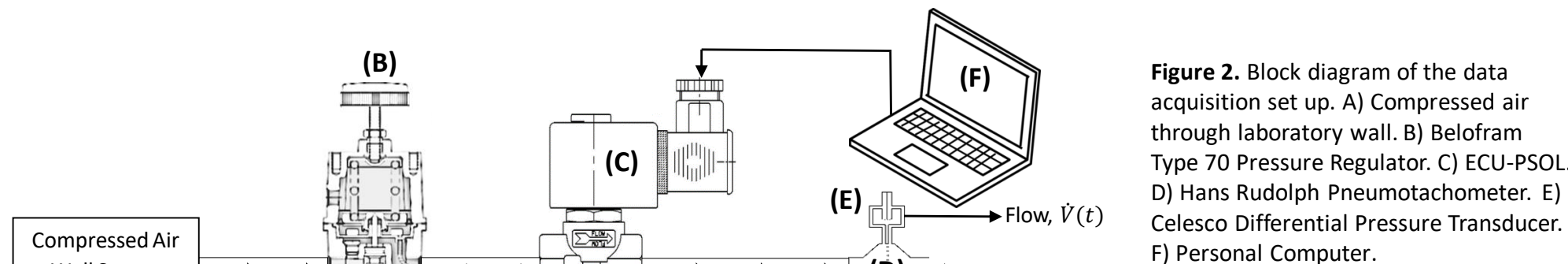
### Modeling Mechanical Ventilation with Proportional Solenoid Valve

Proportional solenoid (PSOL) valves are an important component of modern mechanical ventilators[3]. However the use of PSOL valves in high frequency oscillators is limited due to incomplete characterization of their dynamic responses, as well as the necessity to incorporate them into sophisticated closed-loop systems for precise airway pressure control and patient safety[4]. The goal of this study was to characterize the dynamic response of high-flow PSOL valve using a linear transfer function, which may allow for robust simulation and design of closed-loop controllers for various ventilator and oscillatory modalities in patients.



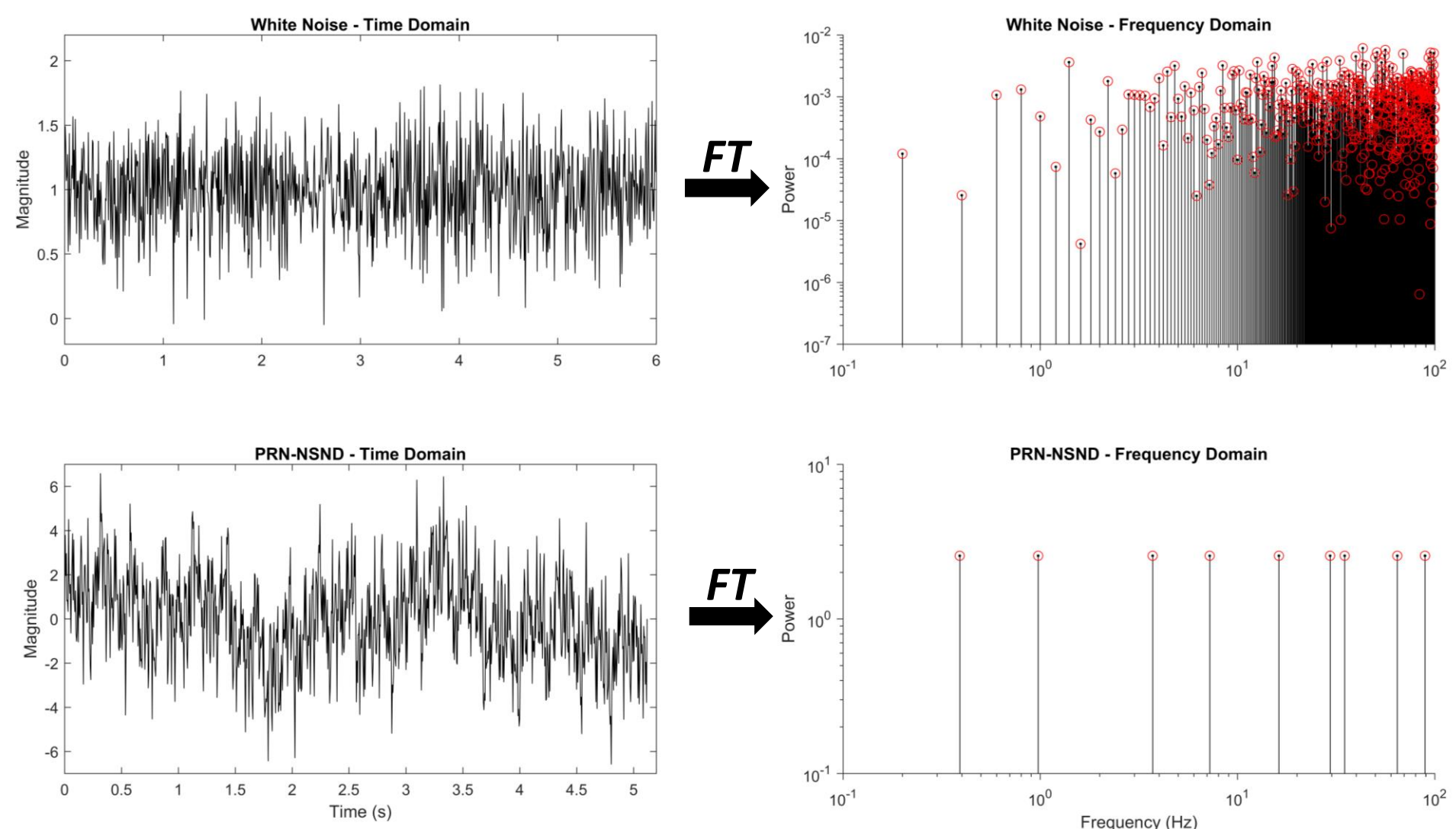
## Methods

### Data Acquisition of Dynamic Response of Proportional Solenoid Valve



### PRN-NSND and White Noise Waveforms

For exciting the ECU-PSOL system at multiple frequencies in order to describe the relationship between the input and output, pseudorandom noise (PRN) waveforms were used. PRN comprises a series of sinusoids at specific harmonics of a fundamental frequency. To mitigate the effects of cross-talk and harmonic distortion during the measurement of the load, non-sum-non-difference PRN (PRN-NSND) were used[5]. These waveforms restrict energy to harmonics which are not multiples of each other.



## Methods

### Voltage-Flow Transfer Function

The relationship between output and input of our ECU-PSOL valve can be defined by the following transfer function:

$$H_{\text{obs}}(s) = \frac{\dot{V}(s)}{v(s)} = \frac{\text{Measured Flow}}{\text{Controlled Voltage}}$$

where the output  $\dot{V}(s)$  is the flow rate of compressed air through the valve while the input, defined by  $v(s)$ , is the value of the control voltage (0-10 V) to the ECU common pin.

### Transfer Function Model

To quantify the relationship between flow and voltage, polynomials of varying order are used in both the numerator and denominator. The transfer function can now be better described by the following:

$$H_{\text{fit}}(s) = \Gamma \prod_{m=1}^M (s + \alpha_m) / \prod_{n=1}^N (s + \beta_n)$$

where  $\alpha_m$  and  $\beta_n$  denote the zeros and poles, respectively of  $H_{\text{fit}}(s)$ , and  $M \leq N$ . The parameters  $\Gamma$ ,  $\alpha_m$  and  $\beta_n$  were estimated using a nonlinear gradient search technique (MATLAB v9.0, The Mathworks Inc.). The optimal number of zeros and poles for the ECU-PSOL system ( $M, N$ ) were determined based on negative real values for the roots of the poles.

### Model Fitting

To measure a model's fit to data, the sum of squared residuals ( $SSR$ ) is commonly used which can be described by:

$$R_k = H_{\text{obs}}(s_k) - H_{\text{fit}}(s_k) \quad , \quad k = 1, 2, \dots, K$$

$$SSR = \sum_{k=1}^K [\Re(R_k)]^2 + \sum_{k=1}^K [\Im(R_k)]^2$$

where  $R_k$  is the residual between actual and model values while the  $SSR$  describes the sum of squared residuals for both real and imaginary values. Generally, more parameters will yield more accurate data.  $SSR$  assesses accuracy via the difference between actual results and model predicated results[2]. For our specific data set, to better distinguish the differences between the varying number of poles and zeros a mean squared residuals was used:

$$MSR = \frac{SSR}{2K}$$

where  $2K$  is the total number of samples. However, because any form of the  $SSR$  will almost always be smaller for a model with more parameters, a different statistic must be used to determine the significance of one model over another based on  $SSR$  values as well as the number of parameters.

### Model Comparison

Akaike Information Criterion (AIC) provides a measure of model quality obtained by simulating where the model is tested on a different data set[1]. After computing several different models (or transfer functions), AIC is used as the main criterion of fit. The AIC will reward a model for reducing the  $SSR$ , but punish it for employing more parameters. Given the finite sample size of our data set, where 22 NSND frequencies were used, the small sample-size corrected AIC ( $AIC_c$ ) was used. This version induces a greater penalty for extra parameters. This criterion is described the following formula:

$$AIC_c = 2K \ln \left( \frac{SSR}{2K} \right) + \underbrace{\frac{2(M+N+1)}{\text{penalty for } \uparrow \text{ number of parameters}}}_{\text{advantage for reducing } SSR} + \underbrace{2(M+N) \left( \frac{M+N+1}{2K - (M+N) - 1} \right)}_{\text{correction for small } K}$$

where  $N$  is the number of data points used and  $M$  is the number of estimated parameters. Theoretically, the model with the smallest AIC is the most accurate.

## Results

### Best Fit Transfer Function

For our ECU-PSOL system, a best fit transfer function of 5 poles and 3 zeros was determined as:

$$H_{\text{Fit}} = \frac{-5756 s^3 + 2.333 \cdot 10^6 s^2 + 1.863 \cdot 10^8 s + 2.919 \cdot 10^9}{s^5 + 562.9 s^4 + 1.25 \cdot 10^5 s^3 + 1.017 \cdot 10^7 s^2 + 4.518 \cdot 10^8 s + 4.563 \cdot 10^9}$$

This transfer function passed the required constraint that the roots of the poles had negative real parts. In addition, it had the lowest AICc score of -7.5153.

In order to excite the ECU-PSOL at varying frequencies, multiple PRN-NSND waveforms were used then their results were combined. From this data set, a best fit transfer function was determined. To validate the general frequency response of the ECU-PSOL system, the valve was excited with uniform Gaussian white noise.

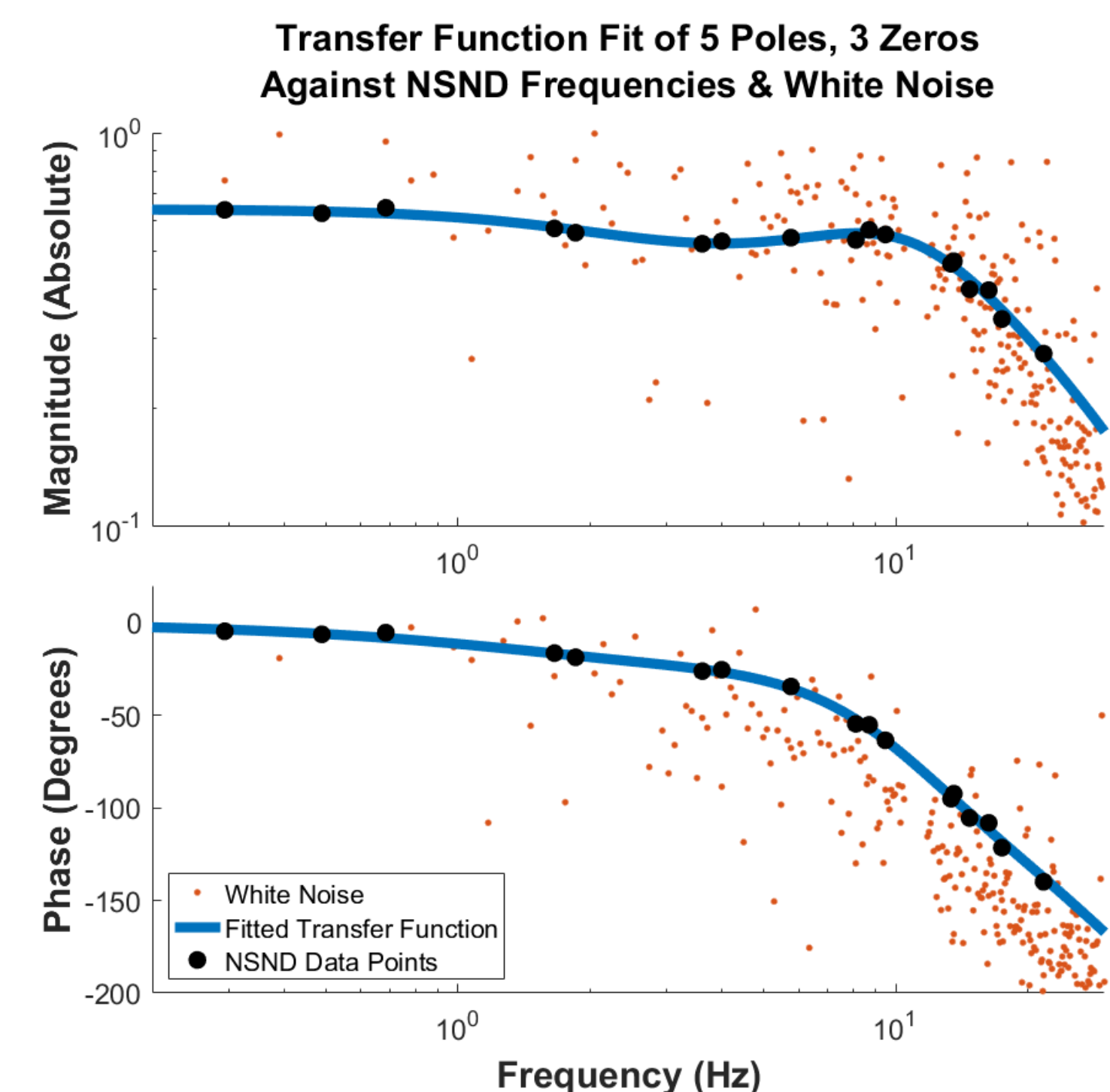


Figure 5. Bode plot of the ECU-PSOL excited by NSND waveforms and white noise with the transfer function fit for the NSND data points. Top plot is the magnitude of ECU-PSOL system in absolute while the bottom plot is the phase in degrees.

### Assessing Quality of Fit: AIC and MSR

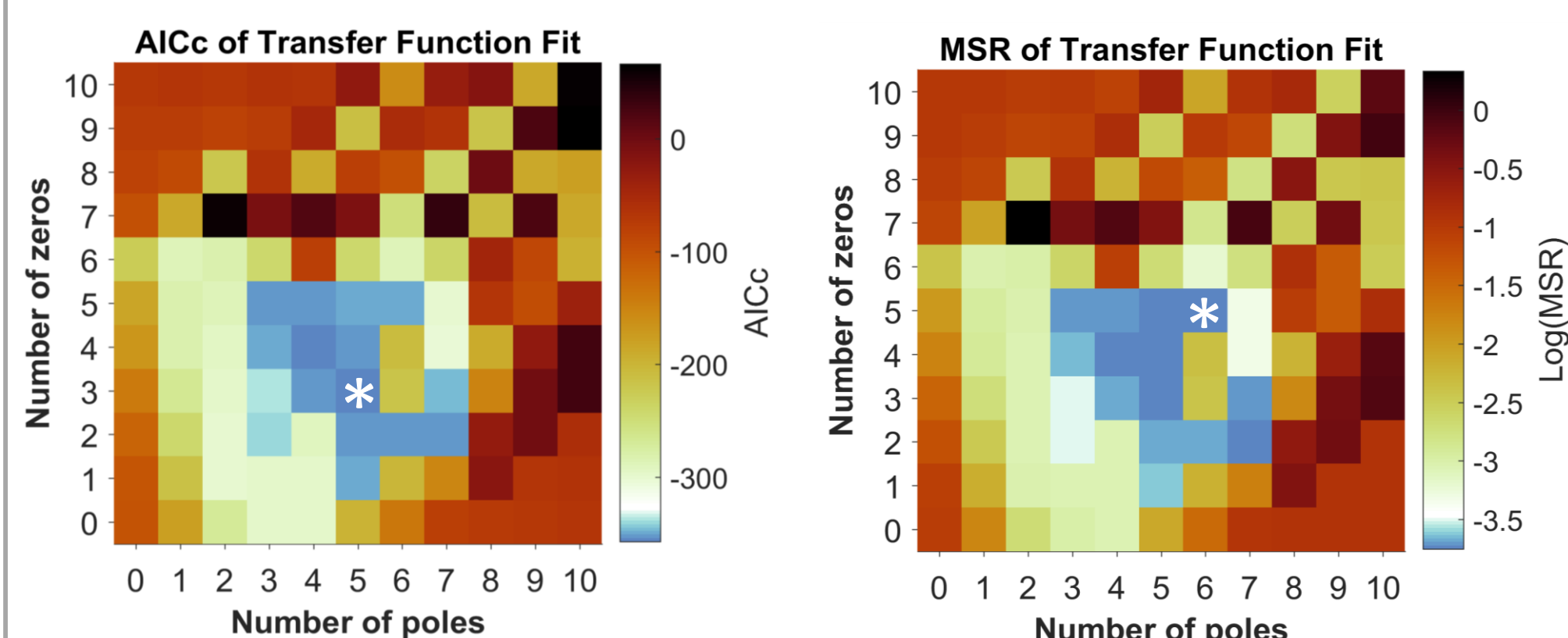


Figure 6. Heat maps of AICc and MSR of varying number of poles and zeros for transfer function fit. The asterisk symbols found in both plots indicate the best fit transfer function according to lowest value of AICc or MSR.

## Conclusions

- The dynamic frequency response of a high-flow PWM PSOL valve can be adequately described using a linear transfer function.
- A parametric transfer function will allow for robust simulation and design of a closed-loop controller in a device capable of generating various ventilatory and oscillatory modalities in patients with acute respiratory failure.
- Future work will involve optimizing the frequency response of the valve in a closed loop system, and validating the optimization in real time.

## Acknowledgments

This work was supported by the Office of the Assistant Secretary of Defense for Health Affairs, through the Peer Reviewed Medical Research Program under Award No. W81XWH-16-1-0434. Opinions, interpretations, conclusions and recommendations are those of the author and are not necessarily endorsed by the Department of Defense.

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