Our model introducing *permanent preference heterogeneity*: households are either patient or impatient, with distinct discount factors $\beta_P > \beta_I$. This generates differences in savings behavior even when households face the same idiosyncratic shocks. Alongside this, agents continue to experience idiosyncratic productivity risk, so heterogeneity arises from both type and income. The model preserves a simple structure, one asset, one choice variable, and a borrowing constraint, but becomes richer in its ability to generate realistic wealth dispersion. We then calibrate (β_P, β_I) and the population shares so that the stationary wealth distribution of the model matches a simple empirical moment from European economies. According to the European Central Bank's Household Finance and Consumption Survey Finance and Network (2013), the top 10% of households typically hold between 55% and 65% of total net wealth. We will target a similar concentration in our model by choosing parameters that reproduce a top–10% wealth share within this range.

Model Setup

Households

Time is discrete, $t = 0, 1, 2, \ldots$. The economy is populated by a continuum of households of measure one. Each household is born with a permanent type

$$z \in \{P, I\},$$

where P denotes a *patient* type and I an *impatient* type. Type is fixed for life. Population shares satisfy $\lambda_P \in (0, 1)$ and $\lambda_I = 1 - \lambda_P$.

Preferences

A household of type z maximizes expected discounted utility

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta_z^t \, u(c_t) \right],$$

where $\beta_P > \beta_I$ and utility is CRRA:

$$u(c) = \begin{cases} \dfrac{c^{1-\mu}}{1-\mu}, & \mu \neq 1, \\ \log c, & \mu = 1. \end{cases}$$

Idiosyncratic Productivity Risk

Each household receives an idiosyncratic labor productivity shock

$$s_t \in \mathcal{S} = \{s_1, \dots, s_{N_s}\},$$

which evolves according to a finite-state Markov chain with transition matrix

$$\Pi(s' \mid s) = \Pr(s_{t+1} = s' \mid s_t = s), \qquad \sum_{s'} \Pi(s' \mid s) = 1.$$

At wage w, effective labor income equals ws_t.

Assets and Borrowing Constraint

Households save in a single risk-free asset paying interest rate r. Let a_t denote beginning-of-period assets and a_{t+1} next-period assets. The borrowing constraint is

$$a_{t+1} \geq \underline{a}.$$

Budget Constraint

Given prices (r, w), consumption satisfies

$$c_t + a_{t+1} = (1 + r)a_t + ws_t,$$

or equivalently,

$$c_t = (1 + r)a_t + ws_t - a_{t+1}, \qquad c_t \geq 0.$$

State and Choice Variables

For a household of type z:

- State variables: (a_t, s_t, z), with z fixed for life.
- Choice variable: a_{t+1}.

Bellman Equation

For each type $z \in \{P, I\}$, the value function satisfies

$$V_z(a,s) = \max_{a' \geq \underline{a}} \left\{ u((1+r)a + ws - a') + \beta_z \sum_{s' \in \mathcal{S}} \Pi(s' \mid s)\, V_z(a', s') \right\}.$$

The associated optimal saving policy is

$$g_z(a,s) \in \arg\max_{a' \geq \underline{a}} \left\{ u((1+r)a + ws - a') + \beta_z \sum_{s'} \Pi(s' \mid s)\, V_z(a', s') \right\}.$$

References

Finance, Eurosystem Household and Consumption Network (2013). *The Eurosystem Household Finance and Consumption Survey-results from the first wave*. Tech. rep. ECB statistics paper.