MACROECONOMICS 3
PSET5

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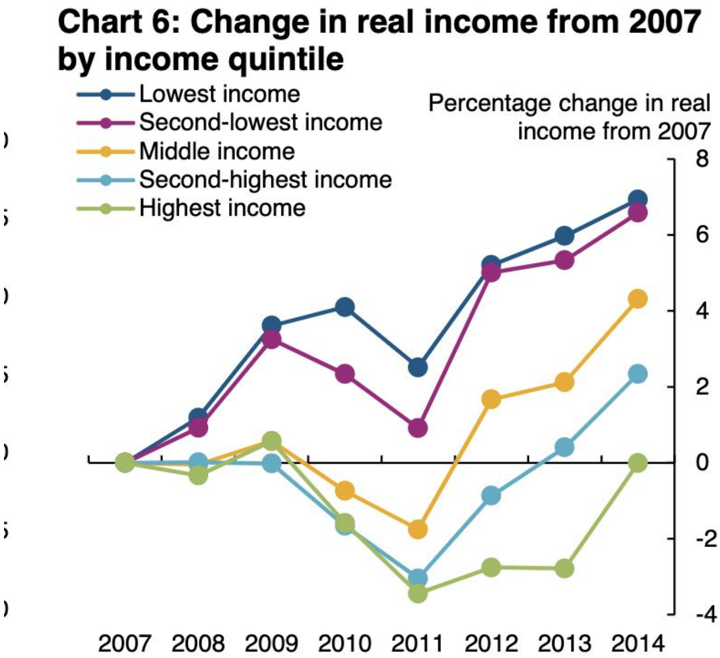
Literature Analysis



Source: FRS, ONS and authors' calculations.
(a) Mean income per household divided by the National Accounts consumption deflator. Incomes are measured before housing costs, net of direct taxes and inclusive of state benefits and tax credits. Data are for financial years.

Pugh, A., Bunn, P., & Yeates, C. (2018). The distributional impact of monetary policy easing in the UK between 2008 and 2014

Analysis

The chart displays the percentage change in real income from the 2007 baseline for different income quintiles in the UK, covering the period from 2007 to 2014, thus the aftermath of the 2008 financial crisis and the subsequent era of monetary policy easing. It aims to show how households towards the bottom of the income distribution experienced the fastest growth in incomes after 2007, although real income growth was still low for all groups relative to pre-crisis trends. In sharp contrast, the middle, second-highest, and highest income quintiles saw their real incomes decline after 2008. After 2011, there

was a clear divergence, with the lower quintiles seeing their real income growth accelerate dramatically, while the middle and higher quintiles began to recover from their earlier slump.
This pattern, especially the relative protection of lower incomes, could be partly attributed to the distributional impact of monetary policy easing. Low-income households often rely more on state benefits/tax credits and were significantly boosted by the improvement in employment rates that resulted from monetary easing. Higher-income households rely more on labor, financial, and rental income, so even though monetary easing boosted asset prices, the real labor income component for these groups grew slowly.
Finally, this resilience of low-income households could be attributed to the non-cyclical nature of state benefits and tax credits.

Exercise

Income $y = wl$.

$$V_e(a) = \max_{c,l,a'} \{u(c) - l + \beta\left[\alpha V_e(a') + (1-\alpha)V_u(a')\right]\}$$

s.t.

$$c + a' = (1+r)a + wl \quad and \quad a' \geq \bar{a}.$$

Bellman Equation for Unemployed Agent

Income $y = \delta$.

$$V_u(a) = \max_{c,a'} \{u(c) + \beta\left[(1-\rho)V_e(a') + \rho V_u(a')\right]\}$$

s.t.

$$c + a' = (1+r)a + \delta \quad and \quad a' \geq \bar{a}$$

2. Euler Condition for Both Agents

Let λ be the Lagrange multiplier on the credit constraint $a' \geq \bar{a}$.

Employed Agent

Substitute l from the budget constraint. The FOC w.r.t. c is:

$$\frac{\partial V_e}{\partial c} : u'(c_e) - \frac{1}{w} = 0 \implies u'(c_e) = \frac{1}{w}$$

The FOC w.r.t. a' is:

$$\frac{\partial V_e}{\partial a'} : -\frac{1}{w} + \beta\mathbb{E}_e[V'(a')] + \lambda_e = 0$$

Using $u'(c_e) = 1/w$ and the Envelope Condition $V'(a) = (1+r)u'(c)$, the Euler condition is:

$$u'(c_e) = \mathbb{E}_e[u'(c')] + \lambda_e$$

where $\mathbb{E}_e[u'(c')] = \alpha u'(c'_e) + (1-\alpha)u'(c'_u)$.

Unemployed Agent

The FOC w.r.t. a' is:

$$\frac{\partial V_u}{\partial a'} : -u'(c_u) + \beta\mathbb{E}_u[V'(a')] + \lambda_u = 0$$

Using $\beta(1+r) = 1$:

$$u'(c_u) = \mathbb{E}_u[u'(c')] + \lambda_u$$

where $\mathbb{E}_u[u'(c')] = (1-\rho)u'(c'_e) + \rho u'(c'_u)$.

3. Consumption Levels

Therefore, there are *three* distinct consumption levels

- All employed agents choose the same consumption c^e
- When a worker becomes unemployed, she starts her first unemployed period with assets a^e (the asset level chosen in employment) and chooses $a^u_{t+1} = \bar{a} = 0$ so that the constraint binds in the next period. Her consumption in that first unemployment period is

$$c^{u,1} = (1+r)a^e + \delta.$$

- From the next period onward, an unemployed agent has $a = 0$ and remains at the borrowing limit, with

$$c^{u,0} = \delta.$$

4. Condition for Employed Agents to be Borrowing Constrained

The constraint binds ($\lambda_u > 0$) if the Euler inequality holds at $a' = 0$: $u'((1+r)a_u + \delta) > \beta(1-\rho)u'(c_e) + \beta\rho u'(\delta)$ where a_u is the assets the agent carries into the period.

6. Employed Agents Consume and Save

Consumption: The FOC for labor supply l: $-1 + wu'(c_e) = 0$ implies $c_e = u'^{-1}(1/w)$, a constant. This is possible because the agent adjusts l_e, not c_e, to satisfy the budget constraint and saving goal.

Saving Level a (assuming $r = 0$ and $\beta = 1$): In the small-heterogeneity model, unconstrained employed agents save a constant amount A. The level A is determined by the agent who was employed (asset A) and just became unemployed (U_{NC}). This U_{NC} agent must be exactly indifferent between saving $a' > 0$ and hitting $a' = 0$. If $a' = 0$, $c_{u,NC} = A + \delta$. The unconstrained Euler equation holds ($\lambda_u = 0$):

$$u'(c_{u,NC}) = \mathbb{E}_u[u'(c')] = (1-\rho)u'(c_e) + \rho u'(c_{u,C})$$

Since $c_{u,C} = \delta$:

$$u'(A+\delta) = (1-\rho)u'(c_e) + \rho u'(\delta)$$

The saving level of employed workers ($a'_e = A$) is:

$$\mathbf{A} = \mathbf{u'^{-1}}\left((\mathbf{1}-\rho)\mathbf{u'}(\mathbf{c_e}) + \rho\mathbf{u'}(\delta)\right) - \delta$$

Note that $\mathbf{A}$ is independent of α.

Employed Not Constrained: Solve for $a_e(\alpha)$

(i) Employed Consumption: $c_e = (u')^{-1}\left(\frac{1}{w}\right)$

(ii) Employed Euler Equation (Using $u'(c_e) = 1/w$ and $\mathbb{E}_e[u'(c')]$):

$$\frac{1}{w} = \beta(1+r)\left[\alpha\frac{1}{w} + (1-\alpha)u'(c_u^{(1)})\right]$$

Solving for the marginal utility of the newly unemployed agent ($c_u^{(1)}$):

$$u'(c_u^{(1)}) = \frac{1}{w} \cdot \frac{1 - \beta(1+r)\alpha}{\beta(1+r)(1-\alpha)}$$

(iii) Recover a_e: Assuming $c_u^{(1)}$ is what the agent consumes when saving $a' = 0$, $c_u^{(1)} = (1+r)a_e + \delta$.

$$a_e = \frac{1}{1+r} \left\{ (u')^{-1} \left[\frac{1}{w} \cdot \frac{1 - \beta(1+r)\alpha}{\beta(1+r)(1-\alpha)} \right] - \delta \right\}$$

(iv) Effect of α on a_e: Assuming $\beta(1+r) < 1$: $\frac{da_e}{d\alpha} < 0$. *Intuition:* Higher job retention (α) reduces the risk of long-term unemployment, decreasing the need for precautionary savings (a_e).

7. Number of agents in each class: employment and unemployment rates

Let n_t^e and n_t^u be the fractions of employed and unemployed agents at time t; $n_t^e + n_t^u = 1$.

From the transition matrix,

$$\begin{aligned} n_{t+1}^e &= \alpha n_t^e + (1-\rho) n_t^u, \\ n_{t+1}^u &= (1-\alpha) n_t^e + \rho n_t^u. \end{aligned}$$

In a stationary equilibrium, $n_{t+1}^e = n_t^e = n^e$ and $n_{t+1}^u = n_t^u = n^u$, with $n^u = 1 - n^e$. Thus

$$n^e = \alpha n^e + (1-\rho)(1-n^e),$$

which gives

$$(1-\alpha)n^e = (1-\rho)(1-n^e) \quad \Rightarrow \quad \big[(1-\alpha) + (1-\rho)\big] n^e = 1 - \rho.$$

Hence

$$n^e = \frac{1-\rho}{(1-\alpha) + (1-\rho)}$$

$$n^u = 1 - n^e = \frac{1-\alpha}{(1-\alpha) + (1-\rho).}$$

8. Total amount of savings in the economy

Interpret "savings" as next-period asset choices a_{t+1}.

By assumption (credit constraint binds after one period of unemployment and $\bar{a} = 0$), unemployed agents choose

$$a_{t+1}^u = \bar{a} = 0,$$

so they do not accumulate savings. Employed agents all choose the same savings level $a^e(\alpha)$. With mass n^e of employed agents, total savings is therefore

$$\boxed{S(\alpha) = n^e(\alpha)\, a^e(\alpha).}$$

9. Effect of α on total savings: two channels

From the previous question,

$$S(\alpha) = n^e(\alpha)\, a^e(\alpha),$$

where

$$n^e(\alpha) = \frac{1-\rho}{(1-\alpha)+(1-\rho)} \quad \text{and} \quad a^e(\alpha) \text{ is implicitly defined by the employed Euler equation.}$$

Differentiate $S(\alpha)$:

$$\frac{dS}{d\alpha} = \frac{dn^e}{d\alpha}\, a^e(\alpha) + n^e(\alpha)\, \frac{da^e}{d\alpha}.$$

This shows two distinct effects:

- *Extensive-margin effect (number of workers):*

  $$\frac{dn^e}{d\alpha} > 0.$$

  A higher α (higher job-retention probability) increases the steady-state employment rate, so more agents are employed and able to save.

- *Intensive-margin effect (individual saving rate):*

  The saving choice $a^e(\alpha)$ solves the Euler equation

  $$u'(c^e) = \beta(1+r)\left[\alpha u'(c^e) + (1-\alpha)u'\big((1+r)a^e(\alpha)+\delta\big)\right].$$

  An increase in α means employed workers face lower unemployment risk, so the precautionary motive to save is weaker. This typically implies

  $$\frac{da^e}{d\alpha} < 0 :$$

  each employed worker saves less when jobs are more secure.

Therefore, the effect of α on total savings $S(\alpha)$ is the sum of:

$$\underbrace{\frac{dn^e}{d\alpha}\, a^e(\alpha)}_{\text{more employed savers}} \qquad + \qquad \underbrace{n^e(\alpha)\, \frac{da^e}{d\alpha}}_{\text{change in saving per employed worker}} \qquad .$$

The first term is positive; the second term is typically negative. The overall effect is ambiguous and depends on which force dominates.

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Results

Table 1 summarizes the key calibration moments and their model-implied values. The equilibrium interest rate of 2.97% is slightly below the 4% target, reflecting relatively high average patience in the model economy. The implied wealth Gini of 0.799 and top-decile wealth share of 65.6% indicate substantial concentration driven by permanent preference heterogeneity. Although the top-10% share falls short of the 75% benchmark, the model successfully reproduces the core mechanism whereby differences in discount factors generate persistent and pronounced wealth inequality.

Table 1: Calibration targets and model-implied moments

Moment	Target	Model
Real interest rate r^*	4%	2.97%
Wealth Gini	0.75	0.799
Top 10% wealth share	75%	65.6%

Saving Behavior and Wealth Accumulation

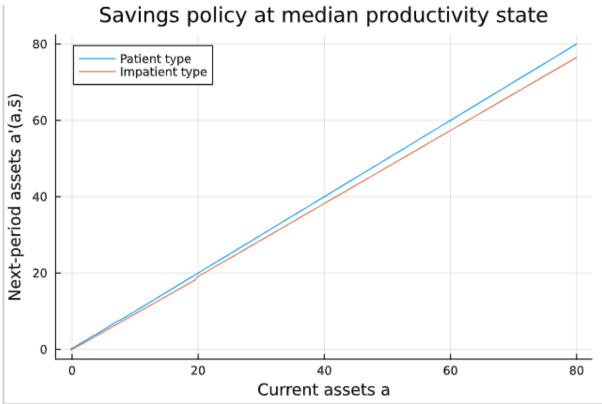

Figure 1: Savings policy functions for patient and impatient households at the median productivity state. Patient households save more at all wealth levels, generating long-run divergence in asset holdings.

Figure 1 illustrates the mechanism: patient households select higher next-period assets at each wealth level, while impatient households remain close to the borrowing constraint. This persistent behavior gap amplifies over time and produces a skewed cross-sectional wealth distribution.

Wealth Distribution

Figures 2 and 3 show strong inequality in the stationary distribution. The Lorenz curve is highly concave, reflecting the large mass of impatient agents at low wealth and the concentration of assets among patient households.

General Equilibrium

Figure 4 plots the capital demand and supply schedules. The curves intersect at an equilibrium interest rate slightly below the target, consistent with the relatively high savings of patient households.

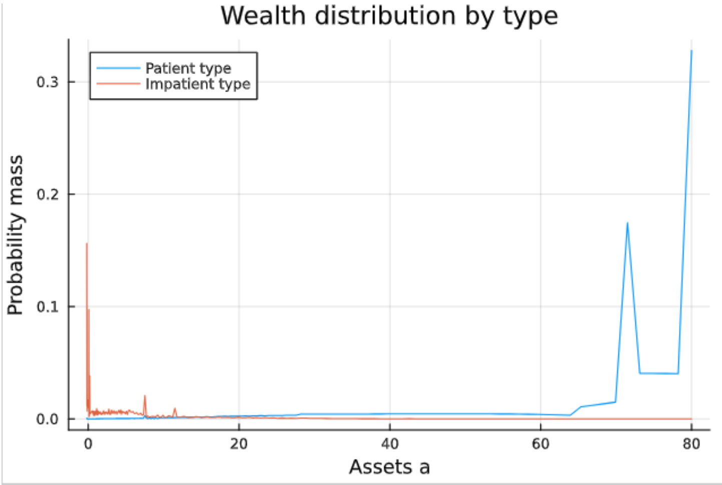

Figure 2: Wealth distribution across patient and impatient types. A small patient group holds most of the wealth, while impatient households cluster near low asset levels.

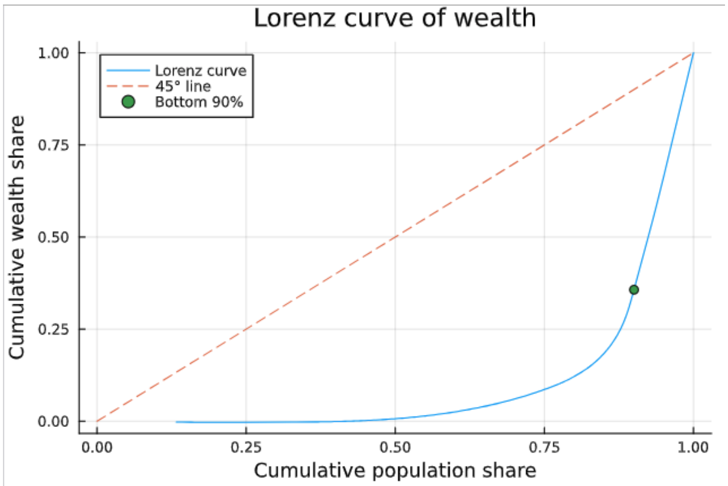

Figure 3: Lorenz curve of aggregate wealth. The bottom 90% of households hold a small fraction of wealth, consistent with the high Gini coefficient.

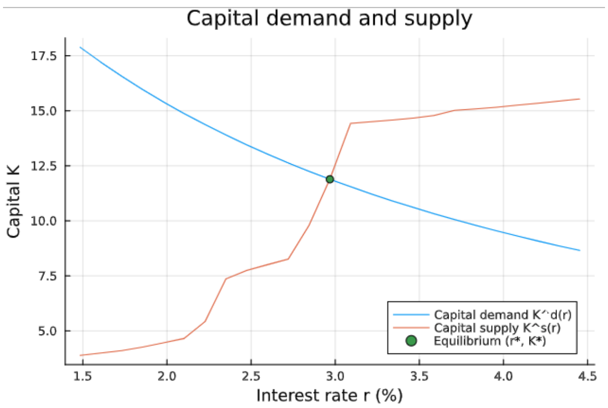

Figure 4: Capital demand and supply curves with the equilibrium interest rate r^*. The intersection determines the unique general-equilibrium price consistent with households' savings behavior and firms' factor demand.