Problem Set 5: Incomplete markets

Sciences Po - Macroeconomics 3 - Fall 2025
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The deadline submission is 30th, November 23:59. Upload a unique pdf file on Moodle and a zip file including the Julia file.

Literature: Distributional impact of aggregate risk

Present and comment on an **empirical** result or a stylized fact about "Distributional impact of aggregate risk" of an article of your choice, typically a table or a graph. Please confirm the article with me by email (paloma.peligry@sciencespo.fr). Once it has been approved, register the paper on the Google Sheet.

Exercise: Infinite horizon & small-heterogeneity model

Exercise based on Scheikman and Weiss (1986) and Lagos and Wright (2005).

Agents live an infinite amount of periods. Small-heterogeneity models are classes of equilibria where agents do save but where the equilibrium distribution of wealth endogenously features a finite state space. Three classes of equilibria can be found in the literature. In this exercise, we will study only one type, based on two assumptions:

- Agents choose their labor supply when employed, and the disutility of labor supply is linear. With c consumption and l the labor supply, the period utility function is $U(c,l) = u(c) - l$. The trick here is that the first-order condition for labor supply pins down the marginal utility of consumption of employed agents. Unemployed agents have home production $\delta < 1$.
- The second assumption is that the credit constraint is tighter than the natural borrowing limit (the loosest credit constraint, which ensures that consumption is always positive). Then, $\bar{a} > -\frac{\delta}{r}$ with r the steady-state interest rate. This ensures that unemployed agents will hit the credit constraint after a finite number of periods of unemployment; it is a trick to reduce the state space.

Notation: There is a continuum of agents of mass one. We study here simple partial equilibrium (the wage rate w and the real interest rate r are exogenous). e_t^i is the employment status, $e_t^i = 1$ when agents are employed and $e_t^i = 0$ when agents are unemployed. e_t^i is the entire history of shocks $e_t^i = \{e_k^i\}_{k=0}^t$. a_t^i is the savings of agent i.

Agents can be either employed or unemployed with the transition matrix:

$$T = \begin{pmatrix} \alpha & 1-\alpha \\ 1-\rho & \rho \end{pmatrix}$$

1. Write the program of an agent $i = \{e, u\}$ and the Bellman equations of each class of agents.
2. Find the Euler condition for both agents.
3. We know that the savings of unemployed agents decrease to reach the borrowing limit in a finite number of periods (not proved here; if interested in the proof, see Huggett 1993). Assume here that the credit constraint binds after one period of unemployment. Assume to simplify that $\bar{a} = 0$. How many different consumption levels are there in this economy? What are the consumption levels of each agent?

4. What is the condition for the employed agents to be borrowing constrained?
5. What is the condition for unemployed agents to be borrowing constrained?
6. Assume that employed agents are not borrowing constrained. Show that the employed agents consume the same amount. How is that possible? Express the saving level of employed workers as a function of α.
7. You now almost have everything to characterize the description of the agents' decision. But you need to follow the number of employed and unemployed agents of each type. What is the number of agents in each class? I.e., what are the employment and unemployment rates?
8. What is the total amount of savings in this economy (the sum of the savings of all unemployed plus all employed agents)?
9. How does α affect the total savings? Show that there are two effects: the effect on the individual saving rate and the effect on the number of workers.

Exercise (Julia): Own modeling project

You have to create your own modeling project, which has to include the following elements:

- A borrowing constraint
- Idiosyncratic risk
- At least two heterogeneity dimensions

Then at least one of the following "difficulty" should be included:

- Two choice variables
- Additional state variable
- Calibration exercise
- One price / market clearing condition different then the interest rate

You can also have some suggestions. All projects should be defined (at least the main Bellman equation) by Sunday 20th.

By the 30th of November, you should write a small presentation of the model, choice and state variables, the programming difficulties and a comment on the results. You should typically export 2 or 3 figures.

You can ask for an extension for the code and results of one week.